A Sequential Least Squares algorithm for ARMAX dynamic network identification

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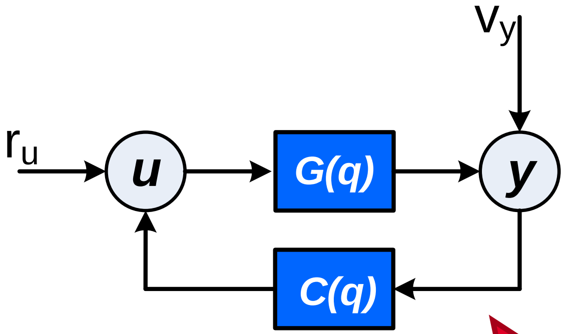
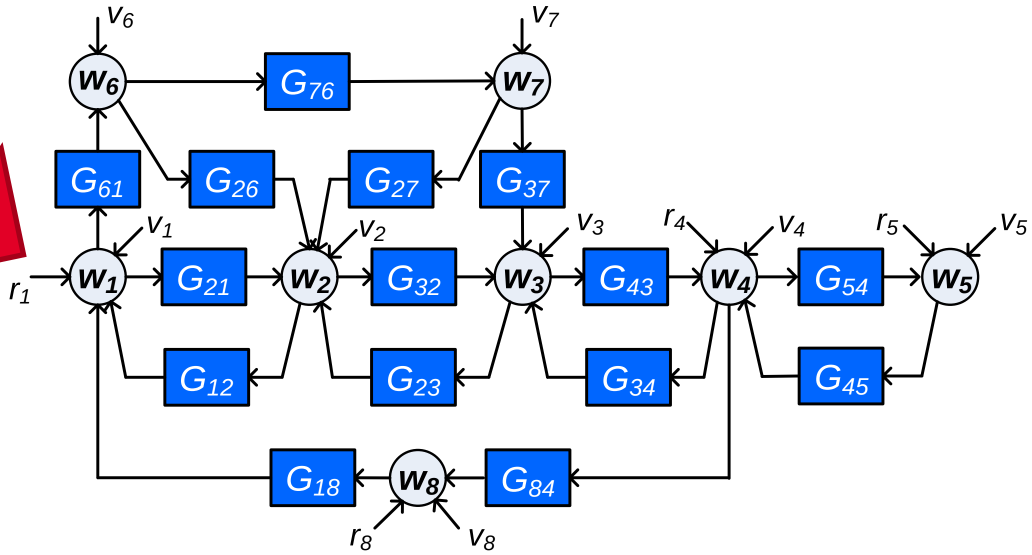
Distributed systems & control





Systems interconnected into a distributed control problem

Identify dynamic network models!

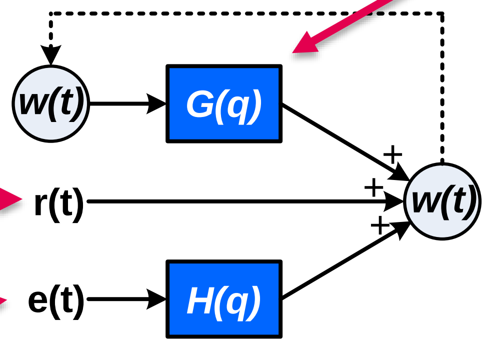
Dynamic network model

Modules

Internal variables / nodes

External variables / references

Process noise



Network setup from[1]

- Stable & well-posed
- Noise may be correlated
- Rational transfer functions

$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_L \end{bmatrix}}_{w = G^0 w + R^0 r + H^0 e}$$

[1] P.M.J. Van den Hof, A.G. Dankers, P.S.H. Heuberger and X. Bombois. (2013). *Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates*. Automatica.

TU/e

Dynamic network identification

Identification of all dynamic modules with joint-direct method[1]

- Encodes network topology in G^0
- Can deal with correlated noise
- Consistency & minimum variance

- Non-convex optimization
- May scale poorly

What algorithm estimates dynamic networks without running into local minima?

[1] H.H.M. Weerts, P.M.J. Van den Hof, A.G. Dankers: *Prediction error identification of linear dynamic networks with rank-reduced noise.* To appear in Automatica, arXiv preprint arXiv:1711.06369

Outline

- Durbin’s method
- Sequential Least Squares algorithm
- Analysis
- Simulations
- Conclusions

Durbin's method

Estimation of SISO ARMA model $\mathcal{A}^0(q)y = \mathcal{C}^0(q)e$ parameterized as $\varepsilon = \frac{\mathcal{A}(q,\theta)}{\mathcal{C}(q,\theta)}y$

$$V = \frac{1}{N}\sum_{t=1}^{N}\varepsilon^2$$

1. Estimate innovation with high order AR $\quad \varepsilon_{AR} = \boldsymbol{A}y \quad \Rightarrow \quad \hat{\varepsilon}_{AR}$

Linear in parameters

2. Use estimated innovation as input → consistency[1] $\quad \varepsilon_{L2} = \mathcal{A}y - \mathcal{C}\hat{\varepsilon}_{AR} \quad \Rightarrow \quad \hat{\mathcal{A}}, \hat{\mathcal{C}}$

Linear in parameters

3. Re-estimate → asymptotic efficiency[2] $\quad \varepsilon_{L3} = \hat{\mathcal{C}}^{-1}\left(\mathcal{A}y - \mathcal{C}\hat{\varepsilon}_{AR}\right) \quad \Rightarrow \quad \hat{\mathcal{A}}, \hat{\mathcal{C}}$

[1] J. Durbin, (1960). *The fitting of time-series models*. Revue de l'Institut International de Statistique

[2] D. Mayne and F. Firoozan, (1982). *Linear identification of ARMA processes*. Automatica

ARMAX parameterization

ARMAX model of every 'row' of $w = G^0 w + R^0 r + H^0 e$

→ Polynomial matrices $D(q,\theta),\ N_G(q,\theta),\ N_H(q,\theta),\ N_R(q,\theta)$

$G(q,\theta) = D^{-1}(q,\theta)N_G(q,\theta),$ → $N_G(q,\theta)$ has 0 on diagonal

$H(q,\theta) = D^{-1}(q,\theta)N_H(q,\theta),$ → $N_H(q,\theta)$ is monic

$R(q,\theta) = D^{-1}(q,\theta)N_R(q,\theta),$

For algorithm: $D(q,\theta)$ is diagonal and monic

$Dw = N_G w + N_R r + N_H e \quad \Rightarrow \quad \underbrace{\mathcal{A} = D - N_G, \quad \mathcal{B} = N_R, \quad \mathcal{C} = N_H}_{\text{MIMO ARMAX}}$

G and N_G have the same structure

Sequential Least Squares: step k=1

unstructured open-loop ARX model of the network

→ Polynomial matrices $A(q,\theta),\ B(q,\theta)$

$$\varepsilon_{ARX}(\eta) := A(\eta)w - B(\eta)r \qquad \Rightarrow \qquad \hat{\varepsilon}_{ARX}$$

$$V = \frac{1}{N}\sum_{t=1}^{N} \varepsilon_{ARX}^T \varepsilon_{ARX}$$

Innovation & dynamics consistently estimated if model order high enough

Sequential Least Squares: step k=2+

ARMAX estimate: Use innovation as input

$$\varepsilon_{L2}(t,\theta) = (D(\theta) - N_G(\theta))w - N_R(\theta)r - N_H(\theta)\hat{\varepsilon}_{ARX}$$

Linear in parameters

$$V = \frac{1}{N}\sum_{t=1}^{N} \varepsilon_{Lk}^T \varepsilon_{Lk}$$

Improve ARMAX estimate: re-estimate with noise model

$$\varepsilon_{Lk}(t,\theta) = N_H^{-1}(\hat{\theta}^{[k-1]})\Big((D(\theta) - N_G(\theta))w - N_R(\theta)r - N_H(\theta)\hat{\varepsilon}_{ARX}\Big)$$

Linear in parameters

Asymptotic results

Equivalence with WNSF[1] → (uses $A(\hat{\eta})$ and $B(\hat{\eta})$ instead of $\hat{\varepsilon}_{ARX}$)

Consistency at k=2[2]

Asymptotic efficiency in number of samples at k=3[2]

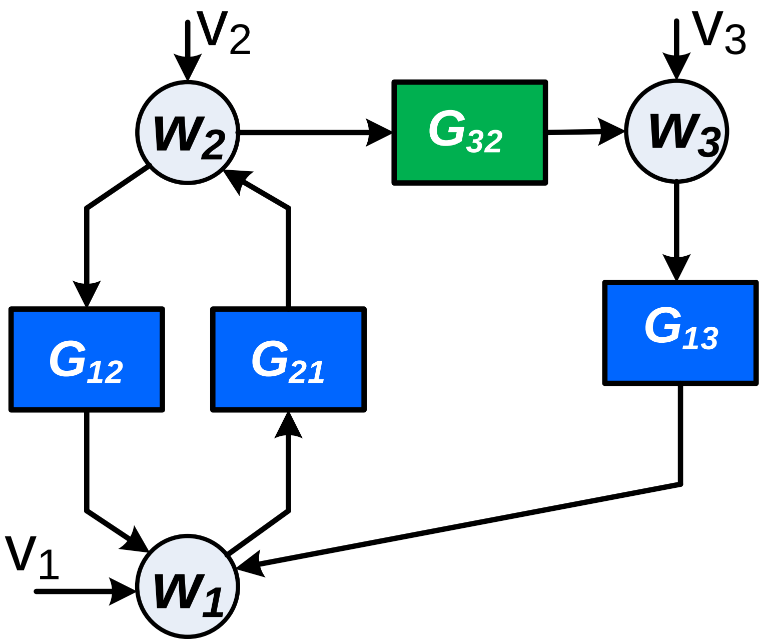
Iterations $k \geq 4$ may improve the model

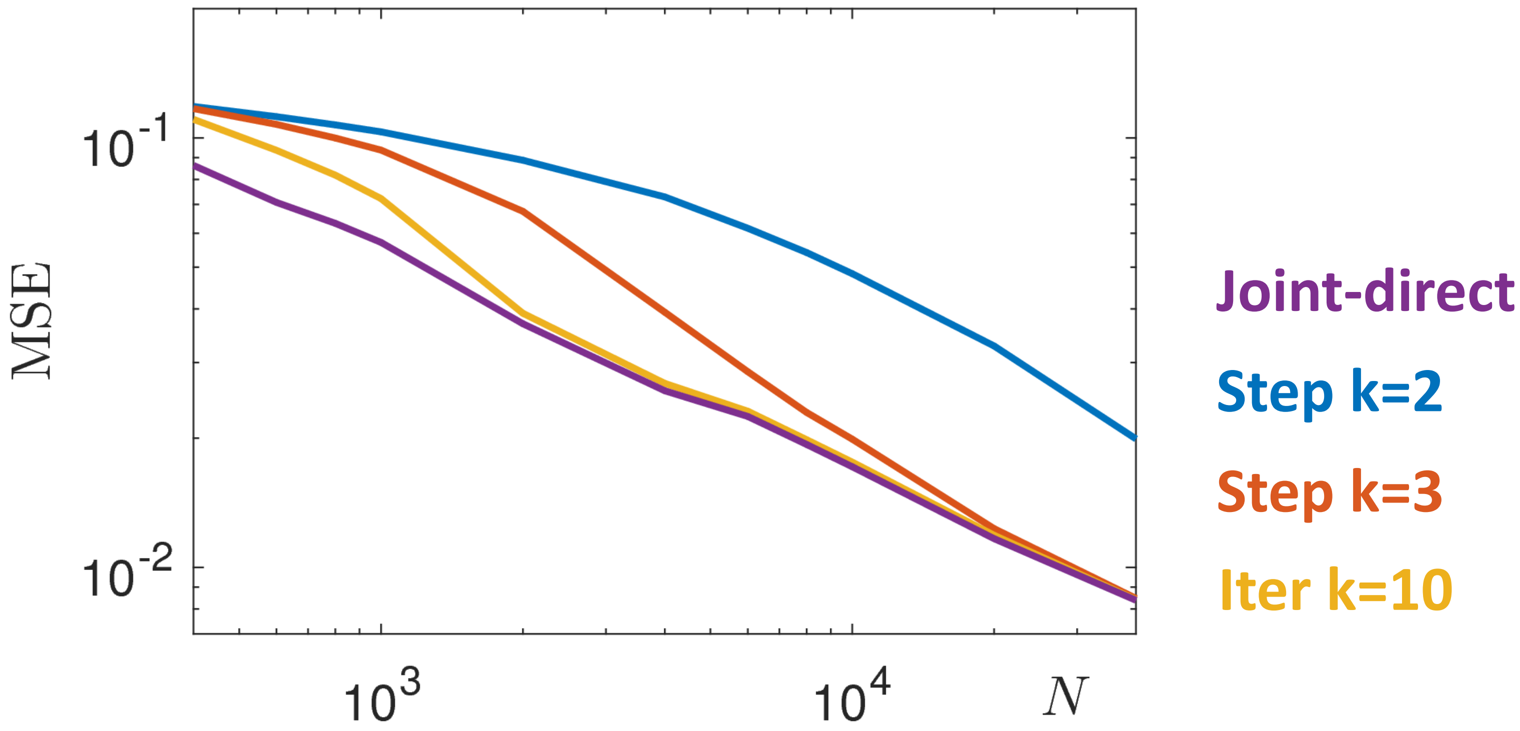
[1] M. Galrinho, C. Rojas, H. Hjalmarsson, (2014). *A weighted least-squares method for parameter estimation in structured models*. CDC
[2] M. Galrinho, C. Rojas, H. Hjalmarsson, (2017). *Parametric identification using weighted null-space fitting.* arXiv preprint arXiv:1708.03946

Simulation: Convergence

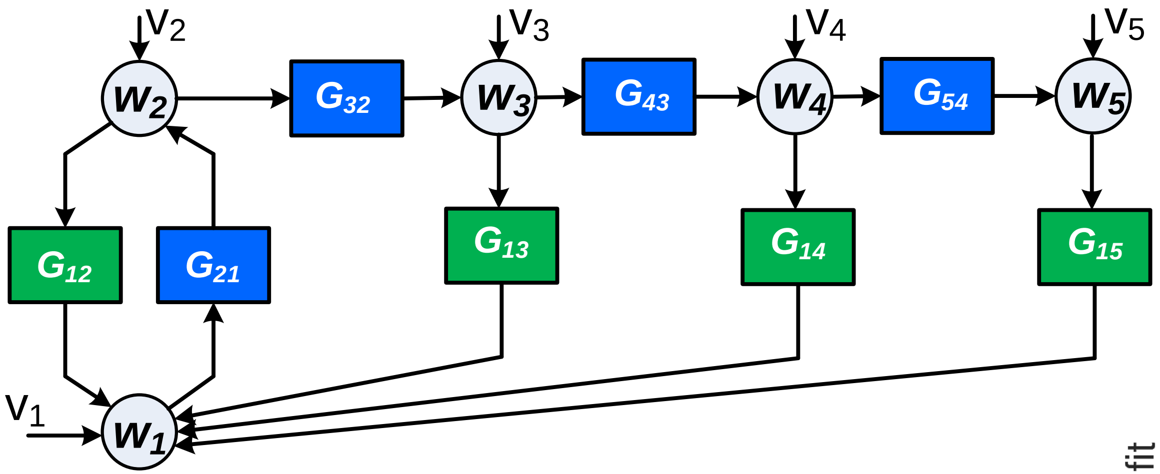
- 3rd order network ARMAX model
- 100 Monte-Carlo runs for each N
- MSE of impulse response of green module

$$MSE(N) = \frac{1}{100}\sum_{i=1}^{100} \|g(\theta_0) - g(\theta_i(N))\|_2$$





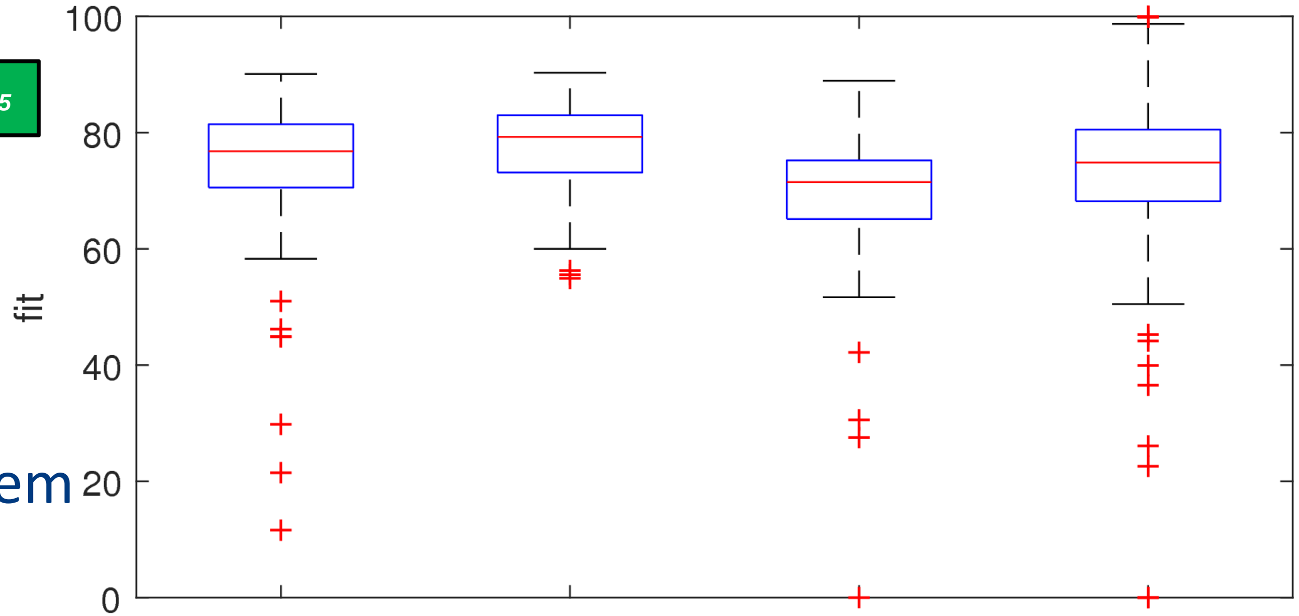
Simulation: random networks



In order to compare: 4-input-1-output problem

100 Monte-Carlo runs,
random modules & signals

$$\text{fit} = 100\left(1 - \frac{\|G^0_{12} - \hat{G}_{12}\|_2}{\|G^0_{12}\|_2}\right)$$



Presented SLS algorithm | SSARX[1] | Matlab `ARMAX()` | Matlab `ARMAX()` true initialization

Take home messages

Dynamic networks can efficiently be estimated with Sequential Least Squares

- No local minima
- Can handle large-scale
- Can encode network topology
- Can handle correlated noise
- Estimates all modules
- Can be extended, e.g.
  - topology detection
  - single module identification

European Research Council

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