EINDHOVEN UNIVERSITY OF TECHNOLOGY

Local module identification in dynamic networks with correlated noise – the full input case

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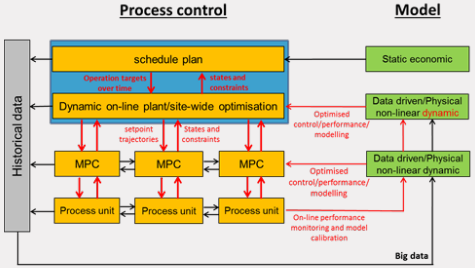
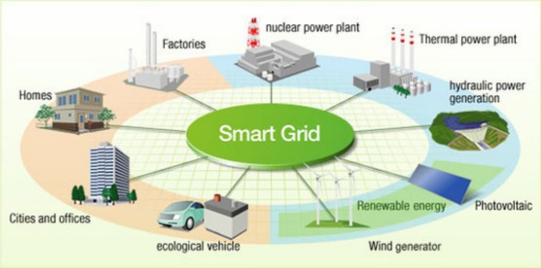
Introduction – dynamic networks

Decentralized process control



Smart power grid

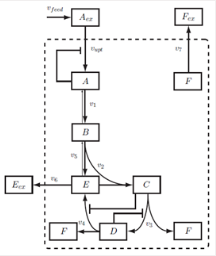


Pierre et al. (2012)

Autonomous driving

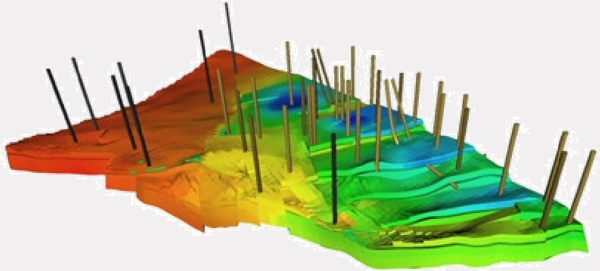
www.envidia.com

Metabolic network



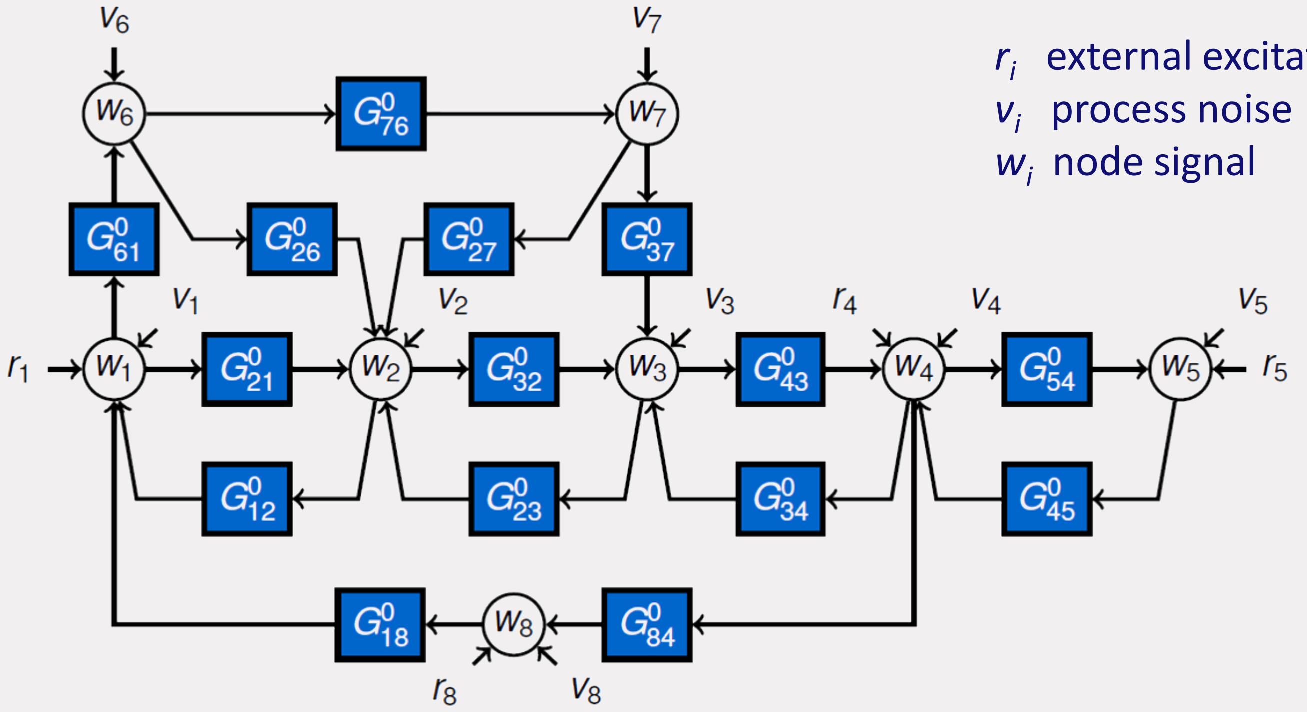
Hillen (2012)

Hydrocarbon reservoirs



Mansoori (2014)

Dynamic network setup



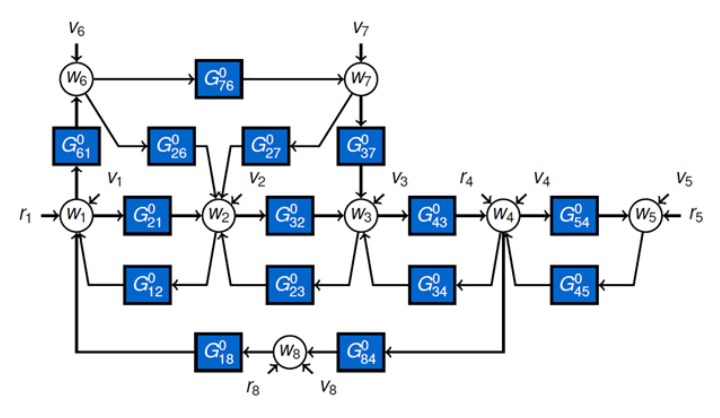
r_i external excitation

v_i process noise

w_i node signal

P.M.J. Van den Hof, A.G. Dankers, P.S.C. Heuberger and X. Bombois. Automatica, 2013.

Dynamic network setup



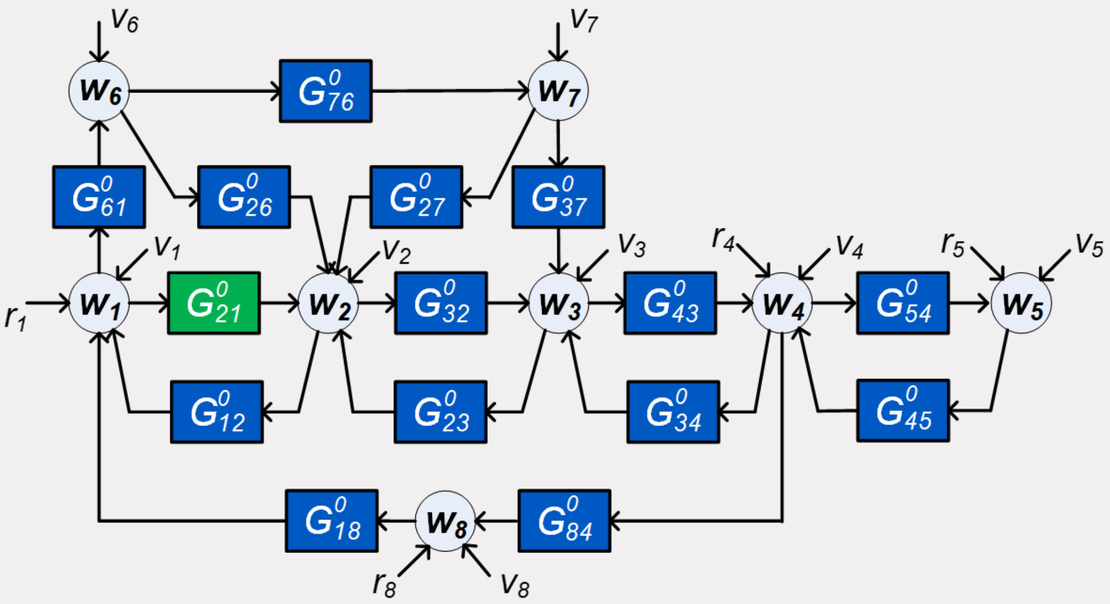
Assumptions:

- Total of L nodes
- Network is well-posed and stable
- Modules are dynamic, may be unstable
- Disturbances are stationary stochastic and can be correlated

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t)$$

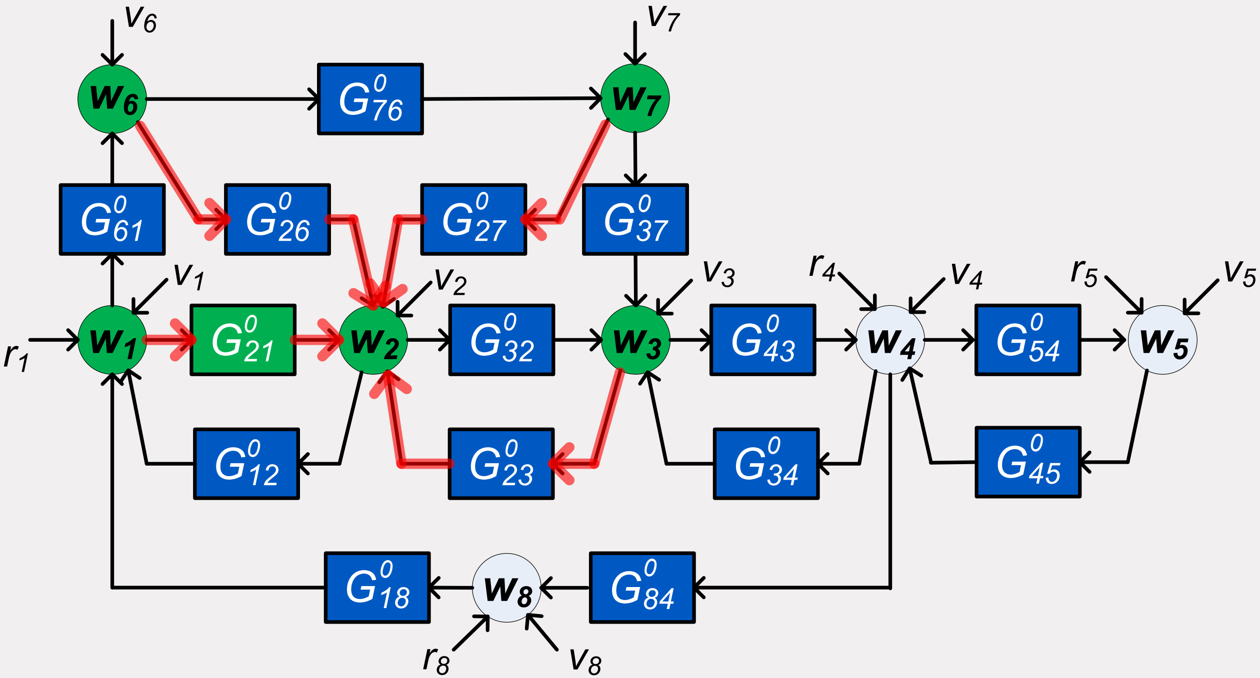
Single module identification



For a network with known topology:

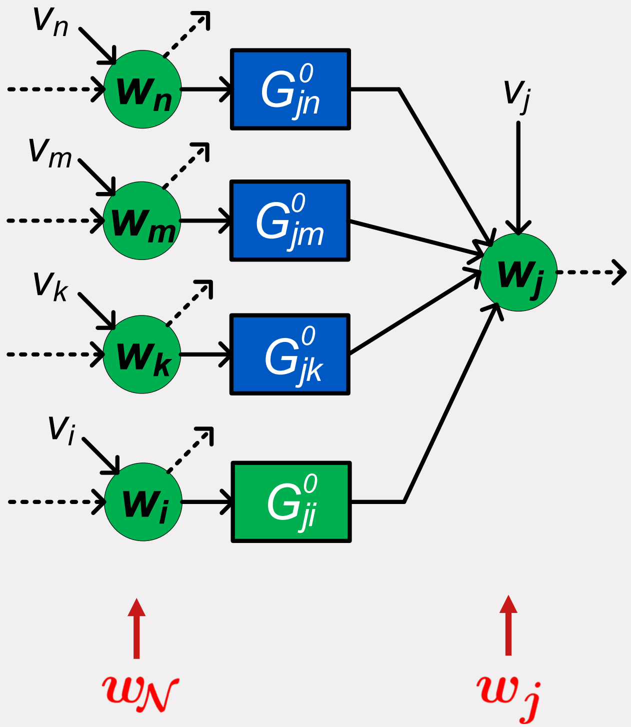
- Identify G_{21}^0 on the basis of measured signals
- Which signals to measure? Preference for local measurements

Single module identification



Identifiying G^0_{21} is part of a multi-input, single-output problem

Single module identification



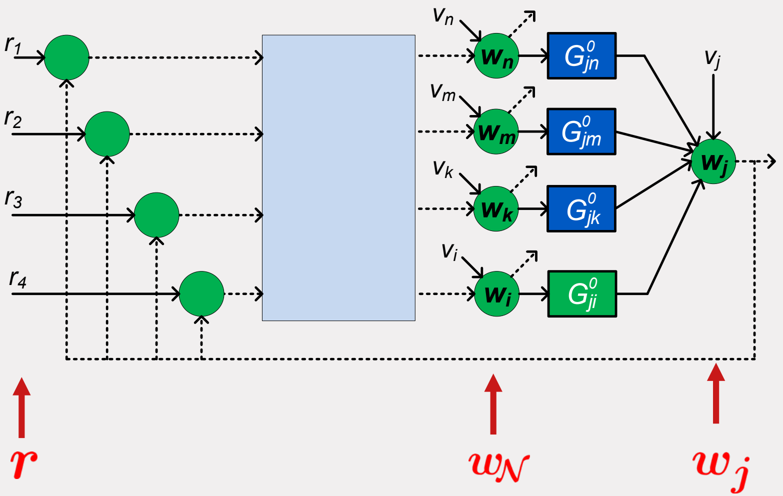
Multi-input single-output identification problem

to be addressed by a closed-loop identification method

Options:

1. **Indirect identification**
2. **Direct identification**

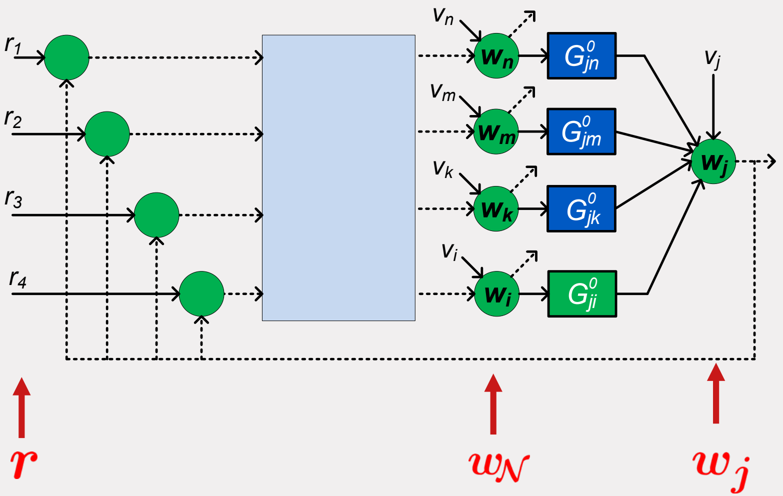
Single module identification



1. **Indirect identification**[1][2]
   - sufficient number of external excitations r
   - estimate $T_{\mathcal{N}r}$ and T_{jr} consistently, and determine
     $$\hat{G}_{j\mathcal{N}} = \hat{T}_{jr}\hat{T}_{\mathcal{N}r}^{-1}$$
   - consistent estimate, also if v correlated
   - noise signals not used for estimation (no minimum variance)
   - freedom in location of r-signals (e.g. directly on $w_\mathcal{N}$)
   - we do not necessarily need all inputs to w_j to be included in $w_\mathcal{N}$[3]

[1] VdHof et al., Automatica 2013
[2] Gevers et al., SYSID 2018; Bazanella et al., CDC 2019
[3] Dankers et al., IEEE-TAC, 2016

Single module identification



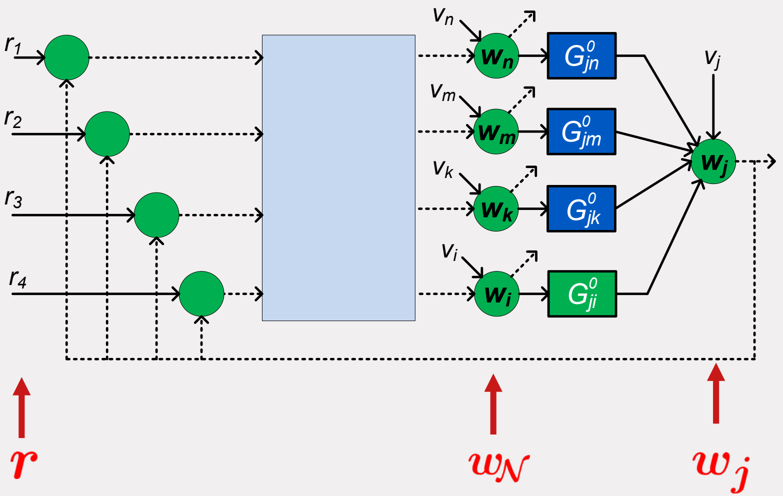
2. Direct identification[1]

- Estimate transfer $w_{\mathcal{N}} \rightarrow w_j$ and model the disturbance process on the output.
- consistent estimate and ML properties
- provided there is enough excitation,
- and v_j uncorrelated with other v signals
- input signal set $w_{\mathcal{N}}$ can be further reduced[2]

[1] VdHof et al., Automatica 2013

[2] Dankers et al., IEEE-TAC, 2016; Dankers et al., IFAC 2017

Single module identification



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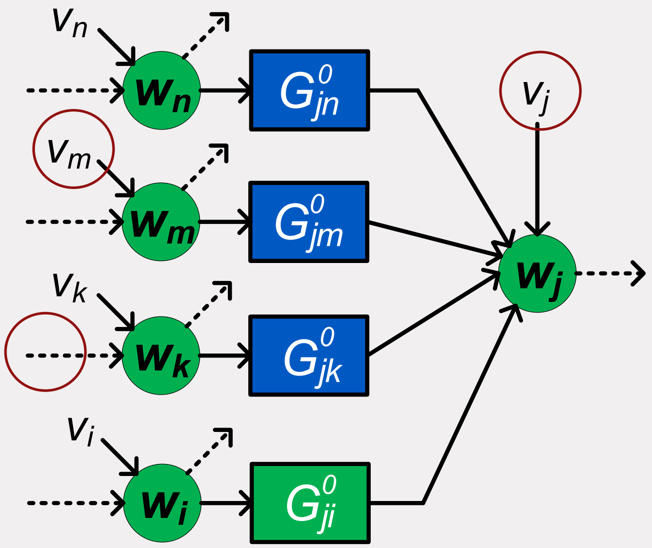
How to deal with correlations between v signals in the direct method?

[1] VdHof et al., Automatica 2013

[2] Dankers et al., IEEE-TAC, 2016; Dankers et al., IFAC 2017

Direct identification



Currently available results

For a consistent and minimum variance estimate (direct method) there is one additional condition:

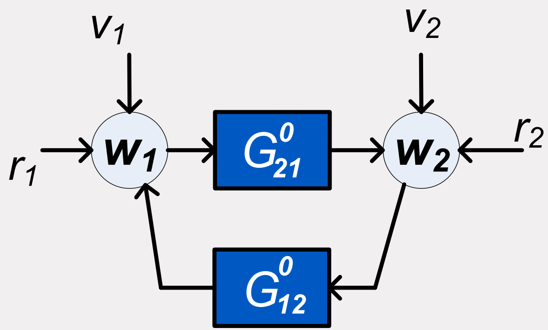
- absence of **confounding variables,** [1][2] i.e. correlated disturbances on inputs and outputs

Two different types of confounding variables:

- **Direct-type**: v_j is correlated to any term in $v_{\mathcal{N}}$
- **Indirect-type**: v_j is correlated to any other $v_\alpha, \alpha \notin \mathcal{N}$ that has a path to $w_{\mathcal{N}}$

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Direct confounding variables



Back to the (classical) closed-loop problem:

Direct identification of G_{21}^0 can be consistent provided that v_1 and v_2 are uncorrelated

In case of correlation between v_1 and v_2 (direct confounding variable): MIMO approach joint prediction of w_1 and w_2 leads to ML results,

$\Longrightarrow$ model w_1 and w_2 both as input and output,
and model the joint disturbance process

Joint estimation of G_{21}^0 and G_{12}^0 : Joint–direct method[1,2,3,4].

Direct confounding variables: **add a predicted output** and model the correlated disturbances

[1] P.M.J. Van den Hof et al. *Proc. 56th IEEE CDC*, 2017
[3] T.S. Ng, G.C. Goodwin, B.D.O. Anderson, *Automatica*, 1977
[2] H.H.M. Weerts et al., *Automatica*, Dec. 2018.
[4] B.D.O. Anderson and M. Gevers, *Automatica* 1982.

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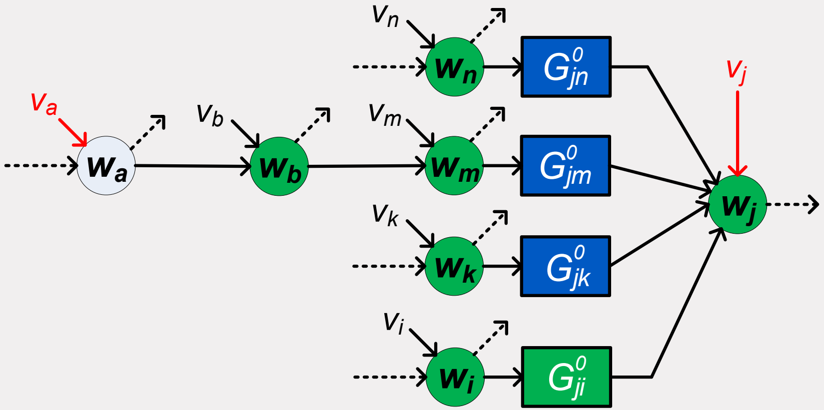
Indirect confounding variables



If v_j and v_a are correlated, then the effect of the confounding variable can be "blocked" by

- measuring a node on each path from w_a to $w_{\mathcal{N}}$, and
- including the "blocking nodes" as predictor inputs in the model

A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

General algorithm philosophy

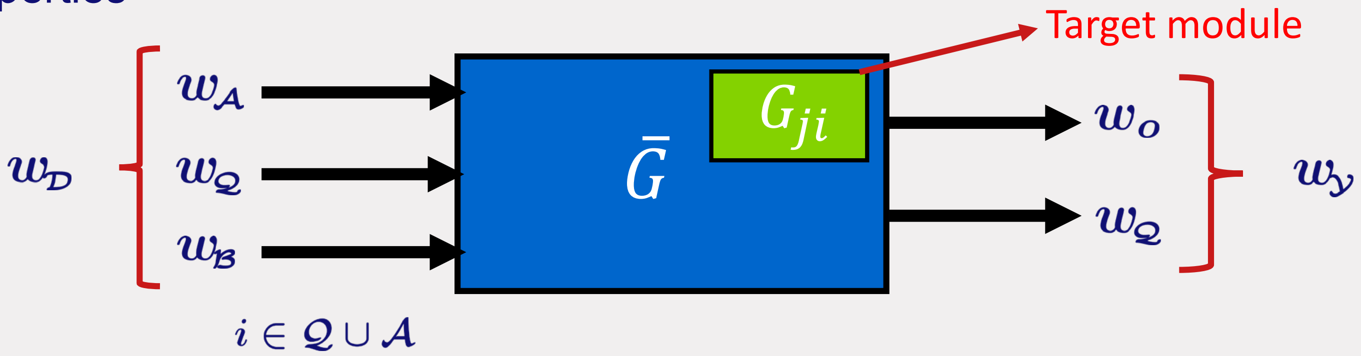
1) Start with output of target module and its predictor inputs
2) Handle direct confounding variables
   - Add inputs to predicted outputs $\longrightarrow w_{\mathcal{Q}}$
   - Add predicted inputs for the modified outputs $\longrightarrow w_{\mathcal{A}}$
   - Repeat step 2
3) Handle indirect confounding variables
   - Add predictor inputs $\longrightarrow w_{\mathcal{B}}$

MIMO identification setup

Sets of signals:

- Only predictor inputs → $w_{\mathcal{A}}, w_{\mathcal{B}}$
- Only predicted output → $w_{\mathcal{O}}$
- Both predictor inputs and predicted outputs → $w_{\mathcal{Q}}$

Direct identification $w_{\mathcal{D}} \longrightarrow w_{\mathcal{Y}}$: Identification of G_{ji} can be consistent with ML properties



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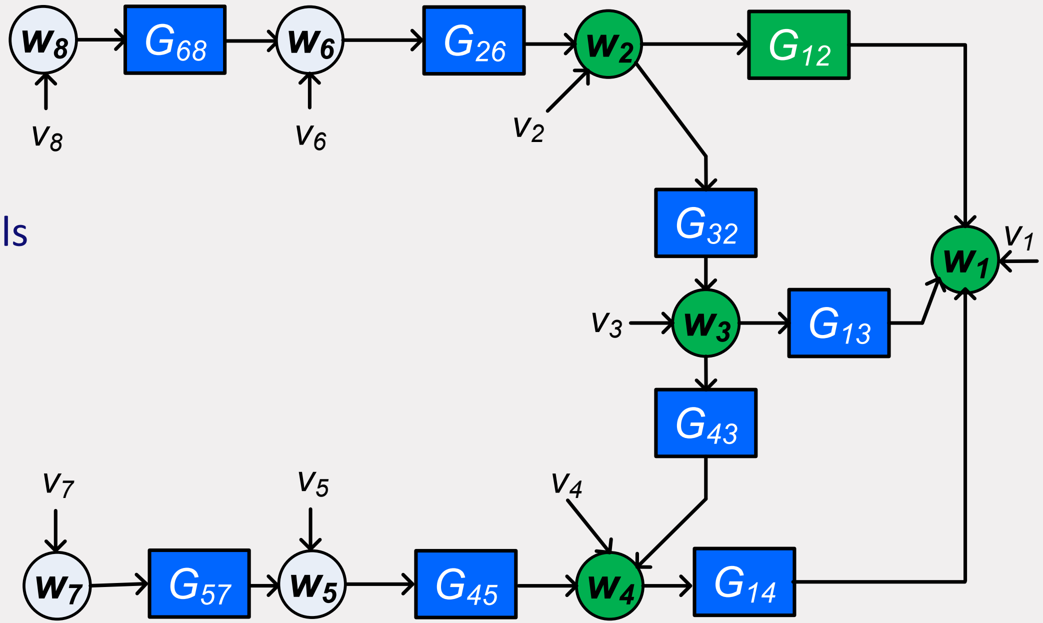
Full input case

We include all in-neighbors of the predicted outputs as predictor inputs

Maximum use of information in the signals

$$w_{\mathcal{Y}} = \{1\} \quad w_{\mathcal{D}} = \{2, 3, 4\}$$



Network with v_1 correlated with v_3 and v_6.

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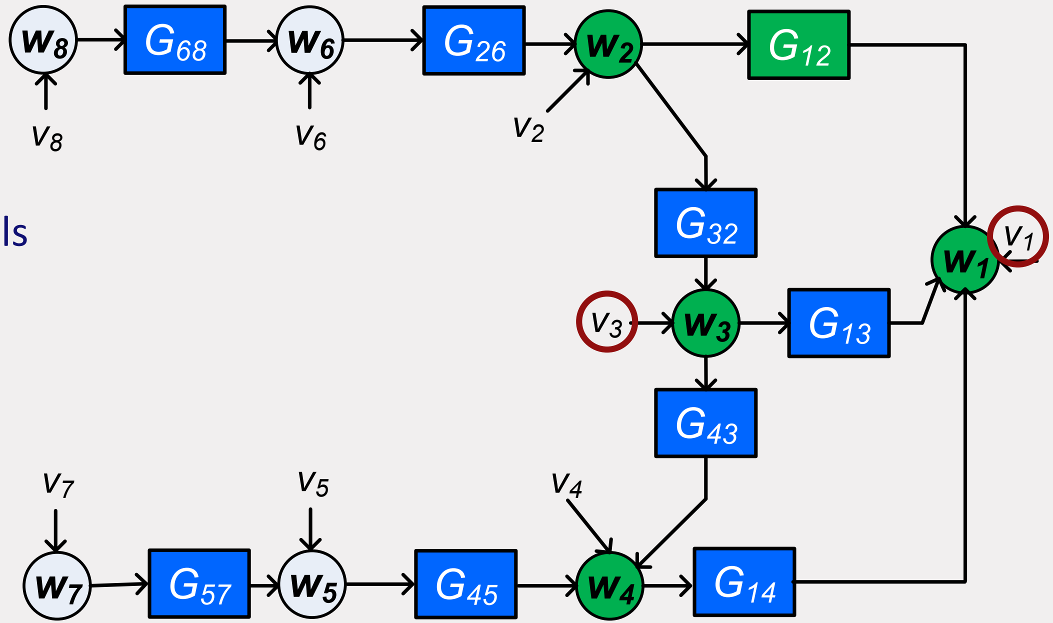
Full input case

We include all in-neighbors of the predicted outputs as predictor inputs

Maximum use of information in the signals

$$w_{\mathcal{Y}} = \{1, \textcolor{red}{3}\} \quad w_{\mathcal{D}} = \{2, 3, 4\}$$

Handling direct confounding variable



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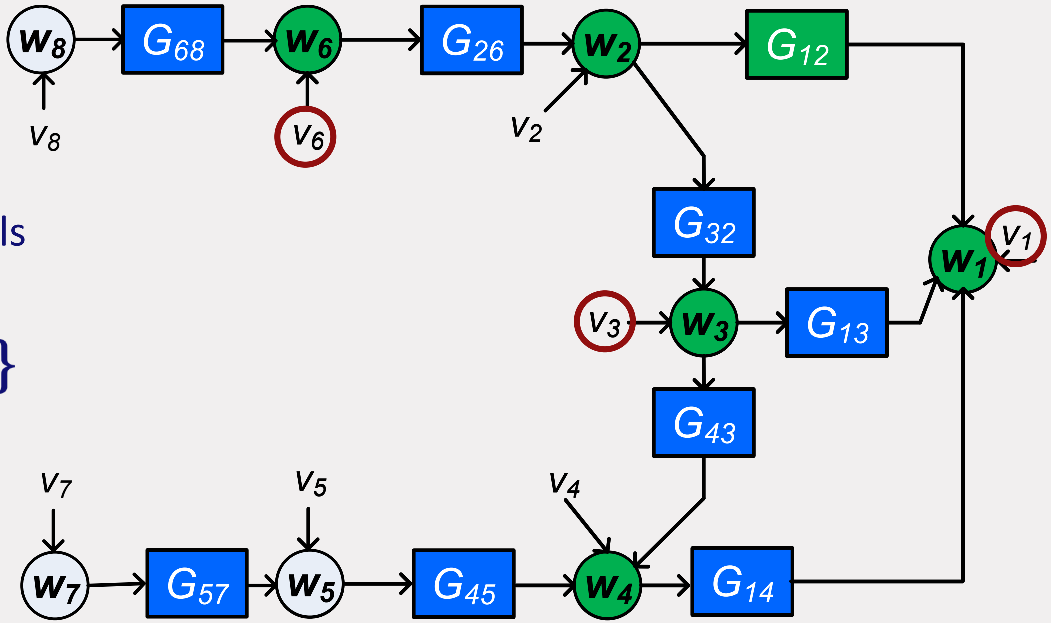
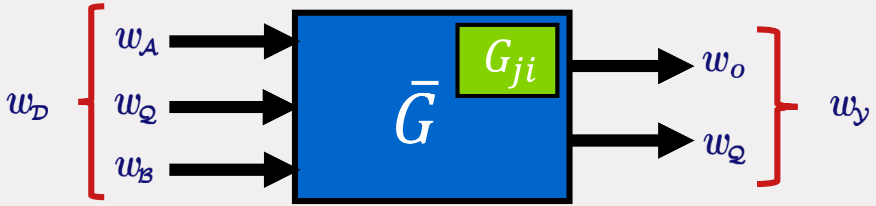


Handling indirect confounding variable

Network with v_1 correlated with v_3 and v_6.

Direct identification: $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$

Generalization of result



Conditions for consistent (and ML) estimation of G_{ji} :

- System in the model set
- Any indirect confounding variable for $w_\mathcal{A} \rightarrow (w_\mathcal{Q}, w_o)$ is blocked by a node in $w_\mathcal{B}$
- There are no confounding variables for $w_\mathcal{B} \rightarrow w_\mathcal{A}$
- There are no direct or unmeasured paths from $(w_i \cup w_j)$ to $w_\mathcal{B}$
- There is persistence of excitation, i.e. $\Phi_\kappa(\omega) > 0$ at a sufficient number of frequencies, with

$$\kappa = \begin{bmatrix} w_\mathcal{D} \\ \xi_\mathcal{Q} \\ w_0 \end{bmatrix}$$

and $\xi_\mathcal{Q}$ the innovation process of $w_\mathcal{Q}$

- All modules in G are strictly proper or satisfy some technical delay conditions

Summary

- Methods for **consistent** and **minimum variance** estimation of a single module
- For direct method: treatment of confounding variables / correlated disturbances
- Particular situation: full-input case.
  Can be generalized to other setups to create more flexibility in choice of sensors[1].
- A priori known modules can be accounted for
- Generalizing towards combining **direct** and **indirect** approach:
  Ramaswamy et al. (later in this session)

[1] K..R. Ramaswamy et al., ArXiv, 2018.

Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
- A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois (2016). Identification of dynamic models in complex networks with predictior error methods - predictor input selection. *IEEE Trans. Autom. Contr.*, *61 (4),* pp. 937-952, 2016.
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- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Prediction error identification of linear dynamic networks with rank-reduced noise. *Automatica, 98,* pp. 256-268, December 2018.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Single module identifiability in linear dynamic networks. Proc. 57th IEEE CDC 2018, ArXiv 1803.02586.
- K.R. Ramaswamy, G. Bottegal and P.M.J. Van den Hof (2018). Local module identification in dynamic networks using regularized kernel-based methods. Proc. 57th IEEE CDC 2018.
- K.R. Ramaswamy, P.M.J. Van den Hof and A.G. Dankers(2019). Generalized sensing and actuation schemes for local module identification in dynamic networks. Proc. 58th IEEE 2019 CDC.
- K.R. Ramaswamy and P.M.J. Van den Hof (2019). A local direct method for module identification in dynamic networks with correlated noise. Submitted for publication. ArXiv:1908.00976.

Papers available at www.pvandenhof.nl

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