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Excitation allocation for generic identifiability of linear dynamic networks with fixed modules

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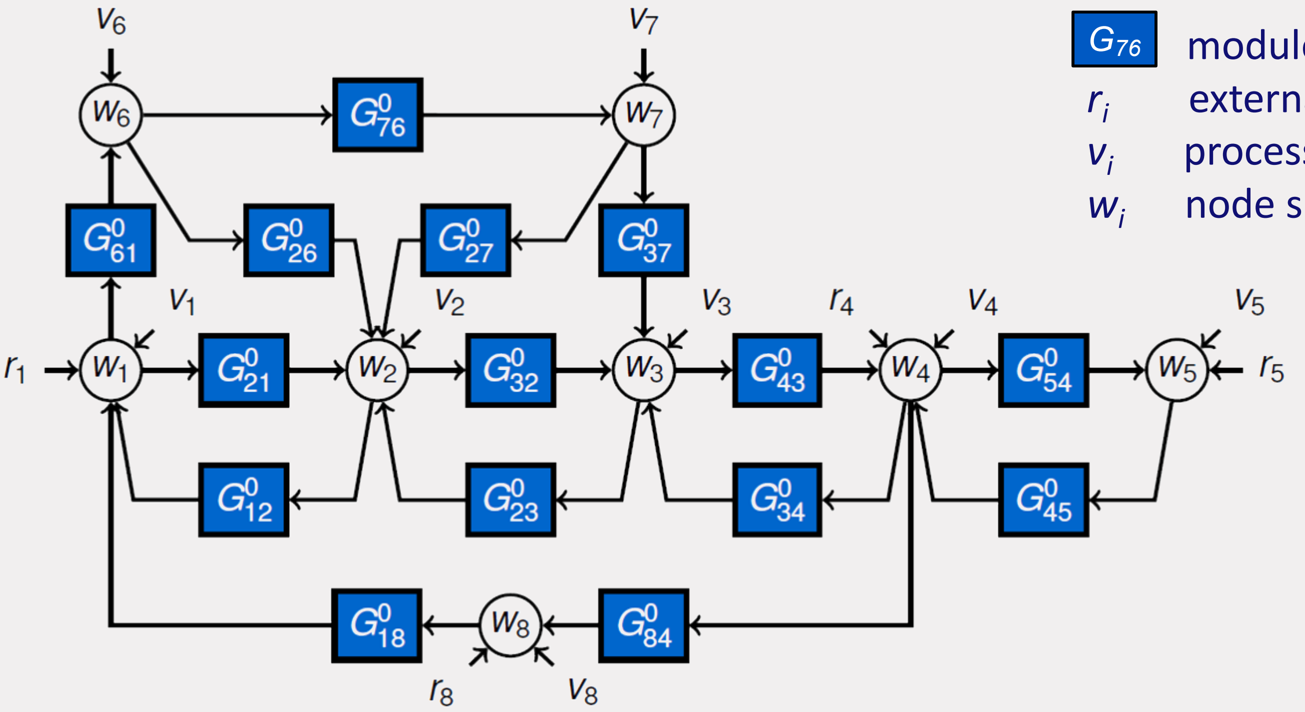
erc European Research Council

Dynamic network



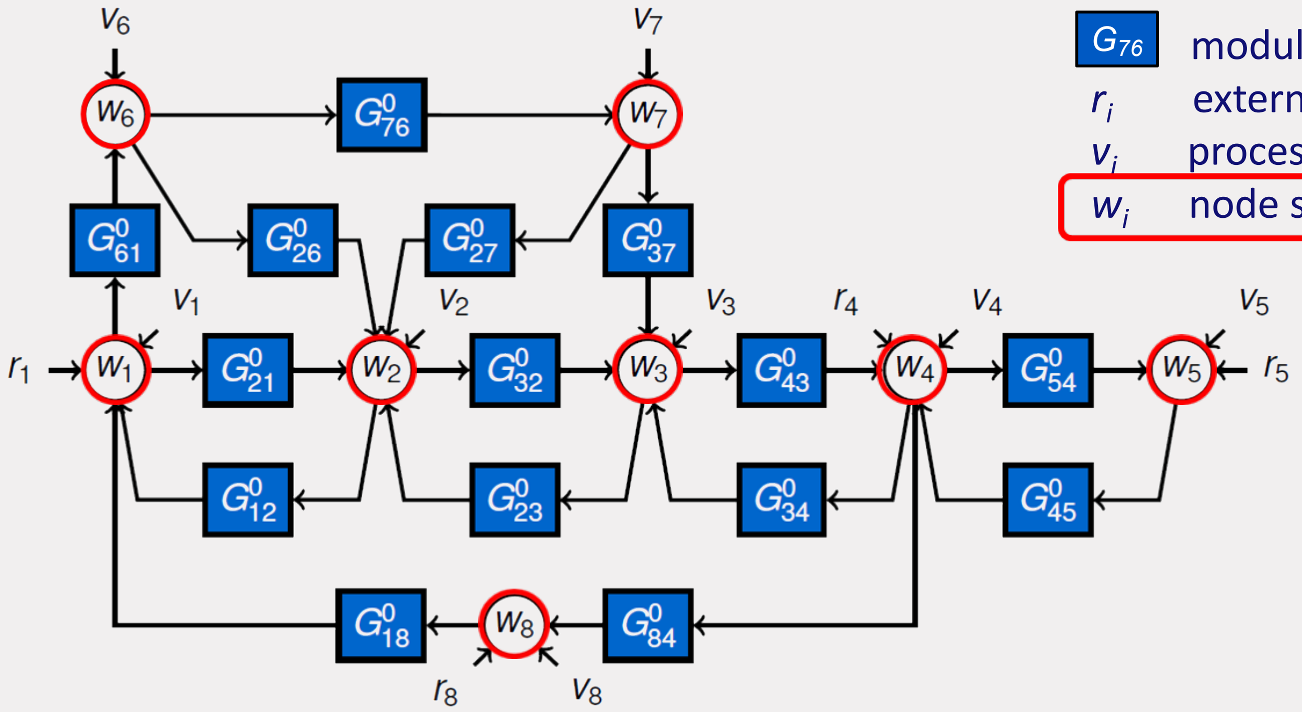
G_{76} module – transfer function

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



G_{76}	module
r_i	external excitation
v_i	process noise
w_i	node signal

Dynamic network setup

Collecting all equations:

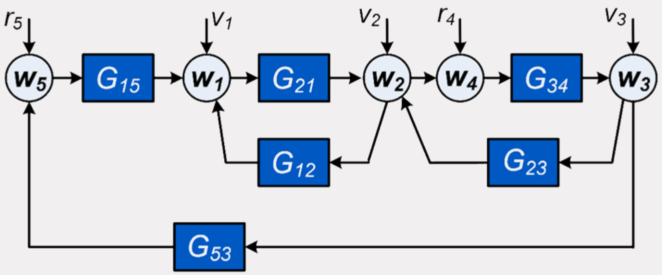
$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

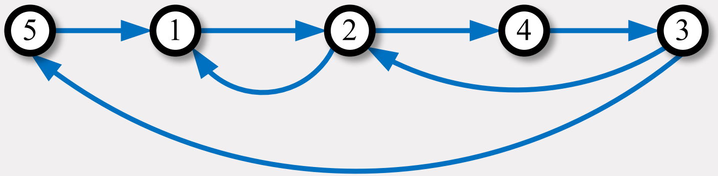
$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t); \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r and e are called external signals.

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

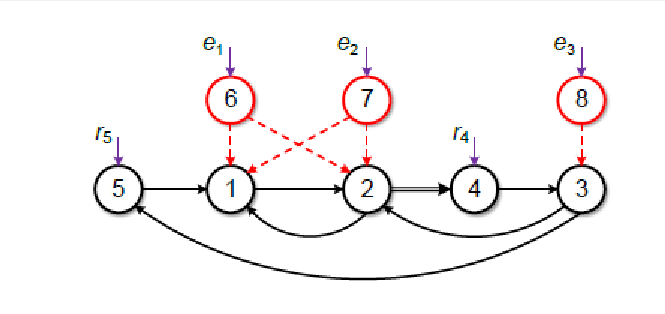
Dynamic network setup – directed graph



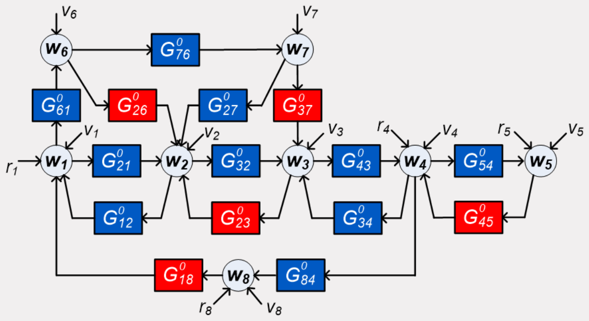


Nodes are vertices; modules/links are edges

Extended graph:
including the external signals
and disturbance correlations



Network identifiability



The identifiability problem:

The network **model**:

$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

can be transformed with any rational $P(q)$:

$$P(q)w(t) = P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\}$$

$$w(t) = (I - P(q))w(t) + P(q)\{G(q)w(t) + R(q)r(t) + H(q)e(t)\}$$

to an **equivalent model**:

$$w(t) = \tilde{G}(q)w(t) + \tilde{R}(q)r(t) + \tilde{H}(q)e(t)$$

→ **Nonuniqueness,** unless there are structural constraints on G, R, H.

[1] Weerts, Linder et al., *Automatica*, 2019.
[2] Bottegal et al., *SYSID 2018*

Network identifiability

Consider a **network model set**:

$$\mathcal{M} = \{(G(\theta), R(\theta), H(\theta))\}_{\theta \in \Theta}$$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

Generic identifiability of $\mathcal{M}$:

- There do not exist distinct equivalent models (generating the same data)
- for almost all models in the set.

Full measurement set-up: all w and r signals are assumed to be measured.

[1] Weerts et al., Automatica, March 2018;
[2] Bazanella, CDC2017; Hendrickx et al., IEEE-TAC, 2019.

Contents

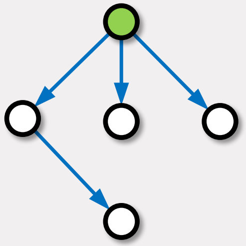
- Dynamic networks and identifiability
- Current solution with pseudotree covering
- Limitation of the current solution
- Extension with SIMUGs to reduce conservatism

Synthesis solution for network identifiability

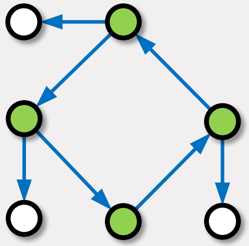
Allocating external signals for generic identifiability:

1. Define **Pseudo-tree:** a directed network where each vertex has indegree ≤ 1

Tree with root in green



Cycle with outgoing trees;
Any node in cycle is root



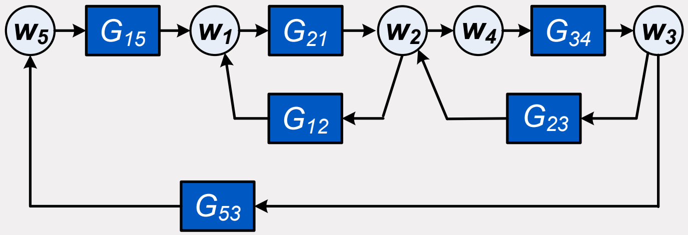
2. Cover the network (model set) graph with **edge-disjoint** pseudo-trees, where all out-neighbors of a vertex are in the same pseudo-tree

3. Assign an independent external signal (r or e) at a root of each pseudo-tree.

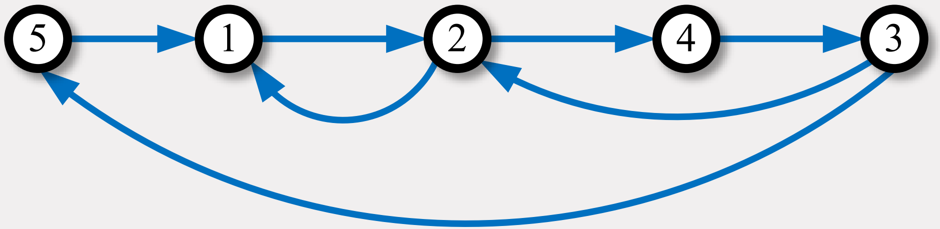
This guarantees generic identifiability of the model set.

[1] X. Cheng, S. Shi and PVdH, IEEE-TAC, February 2022.

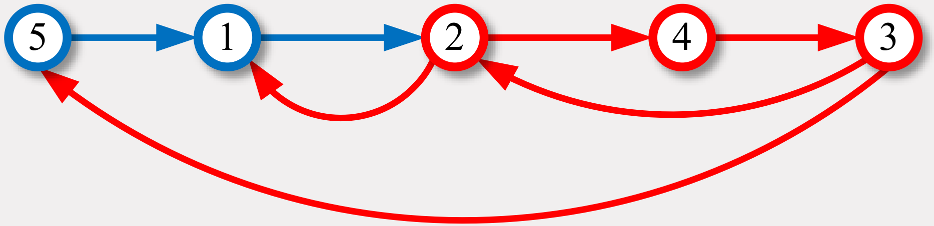
Where to allocate external excitations for network identifiability?



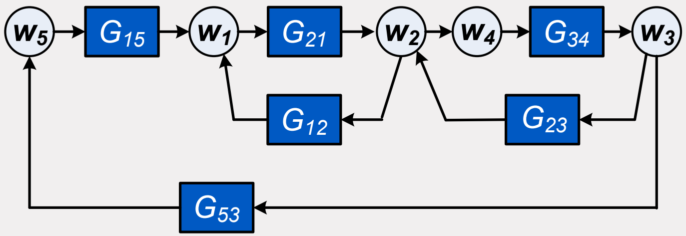
All indicated modules are parametrized

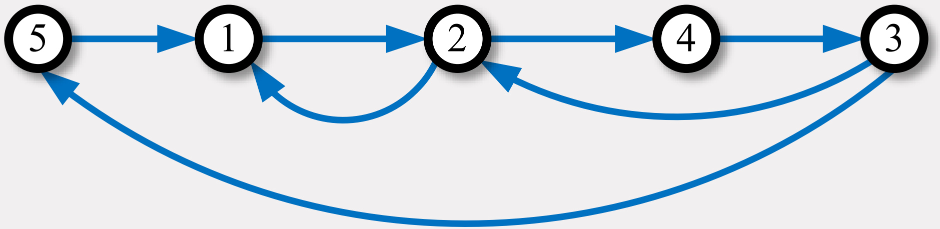


Two disjoint pseudo-trees

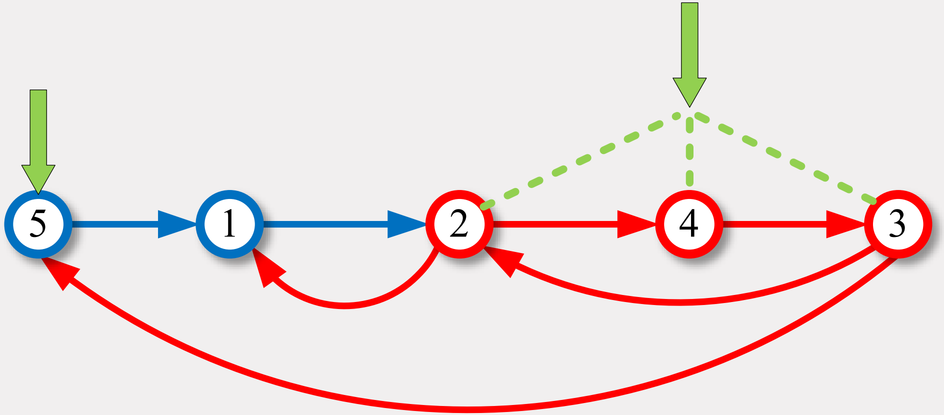


Where to allocate external excitations for network identifiability?

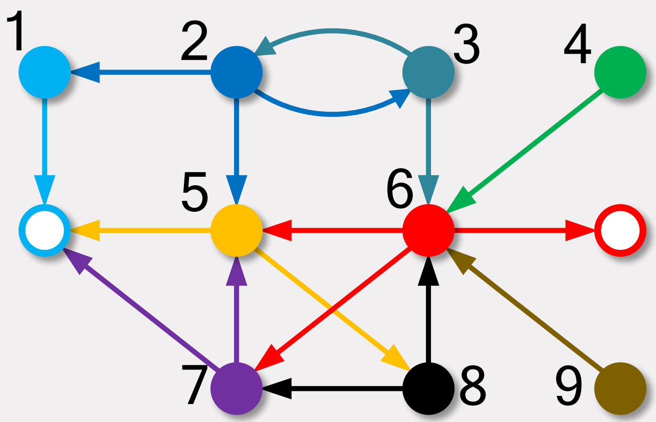




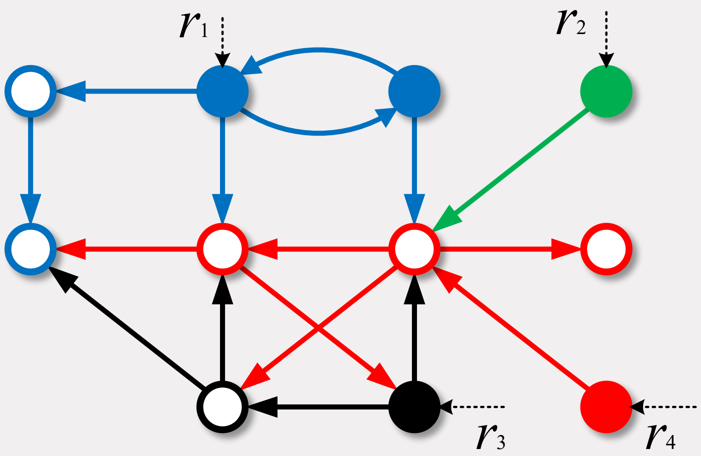
Two independent excitations
guarantee
generic network identifiability



Finding the pseudo-trees: merging algorithm



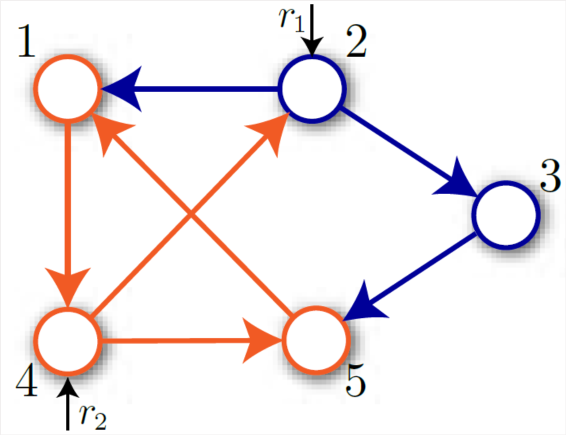
Pseudo-tree merging algorithm [1]



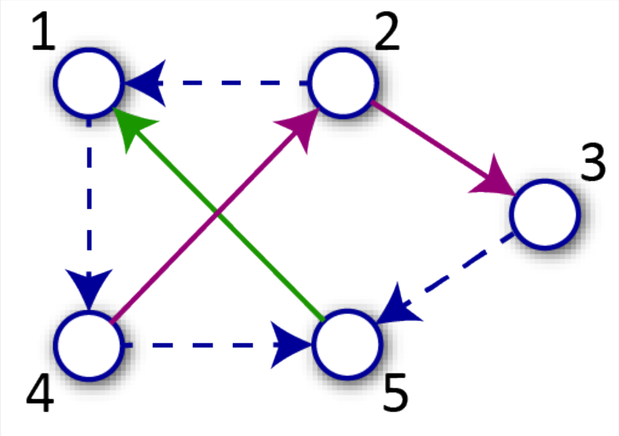
- Merging is implemented through an algebraic procedure
- Known (fixed) edges do not need to be covered

[1] X. Cheng, S. Shi and PVdH, IEEE-TAC, February 2022.

Conservatism of current solution

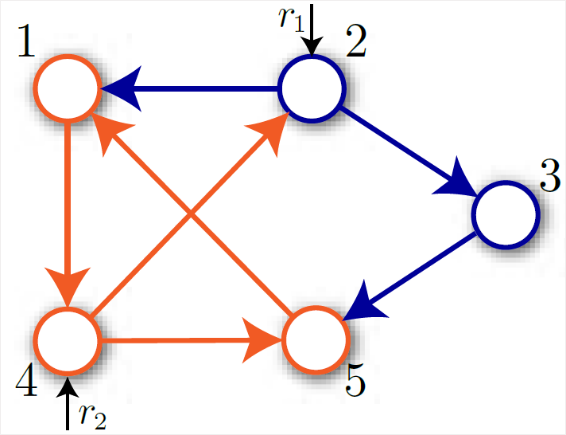


2 pseudo-trees for all edges parametrized

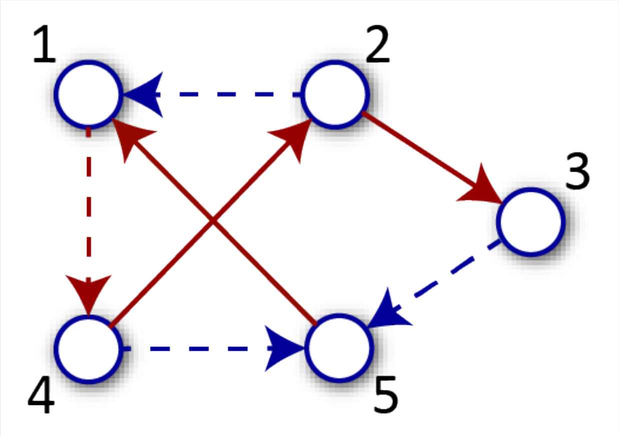


Dashed edges are fixed.
Parametrized edges covered by 2 pseudo-trees (4-2-3, 5-1)

Conservatism of current solution



2 pseudo-trees for all edges parametrized



Dashed edges are fixed.
Parametrized edges covered by 2 pseudo-trees (4-2-3, 5-1)

But: all parametrized edges can be covered by 1 pseudo-tree: (5-1-4-2-3)

New approach: explicit incorporation of fixed edges

Define **multi-rooted graph:** directed graph for which there is a nonempty set of roots from all of which there exist paths to every vertex in the graph.

Define **single source identifiable multi-rooted graph (SIMUG):**
multi-rooted graph where each vertex has an indegree *of parametrized edges* ≤ 1.

Define **edge-disjoint SIMUGs:**
no common edges and for each vertex all outgoing edges are in the same SIMUG.

Result:
Generic network identifiability holds if:

- All *parametrized edges* in the network graph are covered by edge-disjoint SIMUGs, and
- An external signal (r or e) is applied to one vertex in the root set of every SIMUG.

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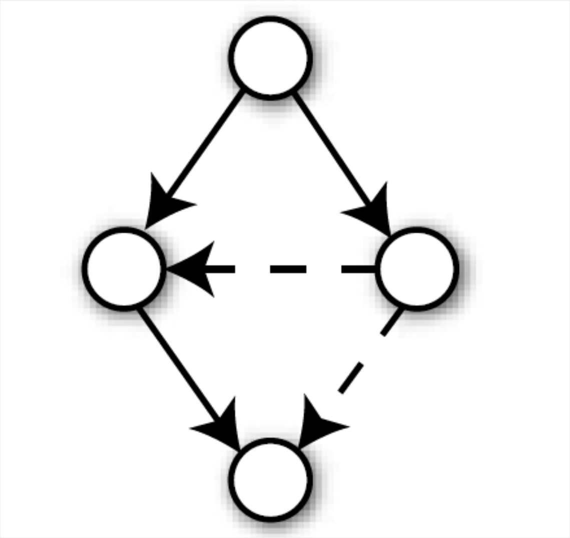
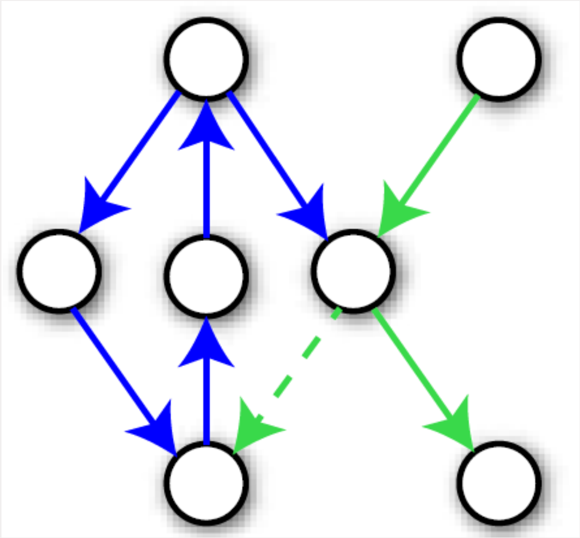
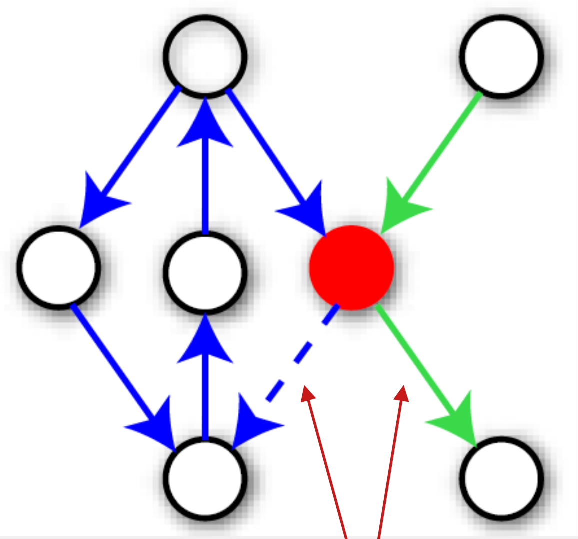
Examples of SIMUGs



One SIMUG



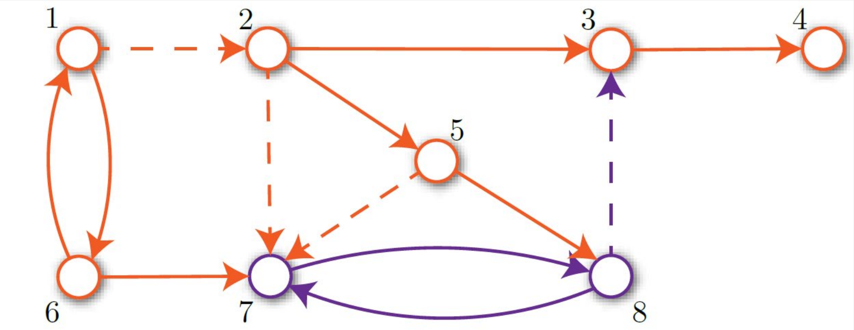
Two edge-disjoint SIMUGs

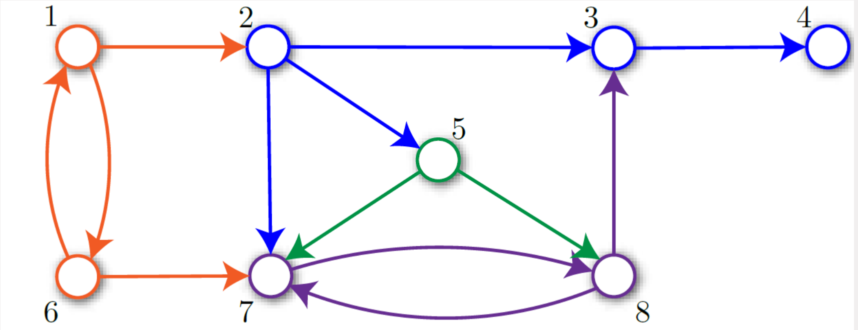


Two SIMUGs,
but not edge-disjoint

8-node example with fixed links:

Dashed links are fixed (known modules)



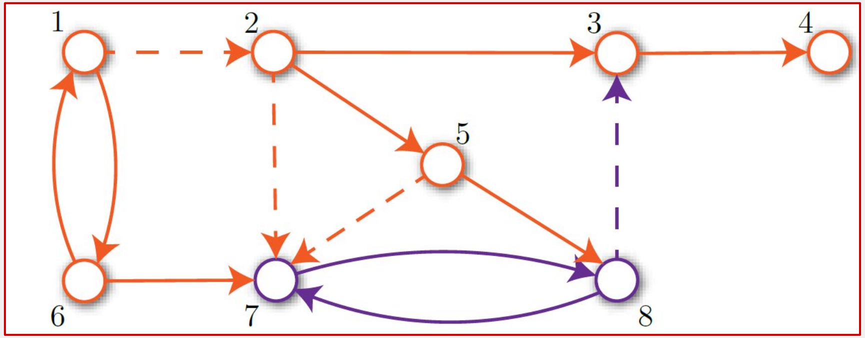


Network graph covered by 2 edge-disjoint SIMUGs
Two external signals required for identifiability: (1 or 6) and (7 or 8)

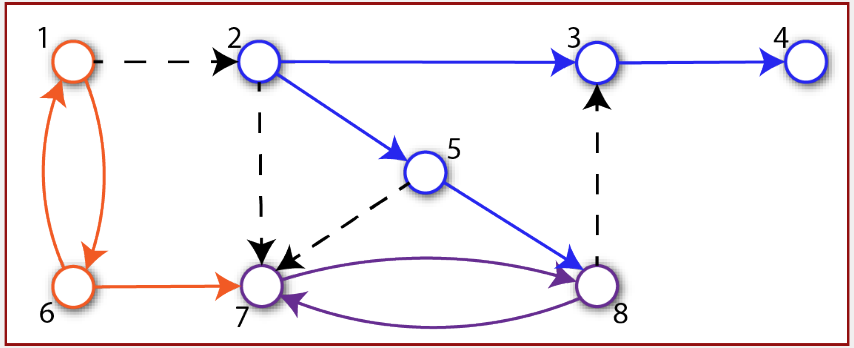
When discarding the fixed-property of links: Network graph covered by 4 edge-disjoint pseudo-trees
Four external signals required for identifiability: (1 or 6), (7 or 8), 2 and 5

8-node example with fixed links:

Dashed links are fixed (known modules)



Network graph covered by 2 edge-disjoint SIMUGs
Two external signals required for identifiability: (1 or 6) and (7 or 8)



When applying pseudo-tree covering of the parametrized edges:
graph covered by 3 pseudo-trees
Three external signals required for identifiability: (1 or 6), (7 or 8), and 2.

Merging algorithm

Consider the disjoint SIMUG covering: $\{\mathcal{T}_1, \cdots \mathcal{T}_n\}$

Construct the mergability matrix $\mathfrak{M}$ with

$$\mathfrak{M}_{ij} = \begin{cases} 1 & \text{if } \mathcal{T}_i \text{ is mergeable to } \mathcal{T}_j \\ \varnothing & \text{if there are no vertices in } \mathcal{T}_i \cup \mathcal{T}_j \text{ with multiple parametrized incoming links,} \\ 0 & \text{otherwise} \end{cases}$$

Merging is represented by an algebraic row- and column operation on $\mathfrak{M}$

Summary

Identifiability of network model sets is determined by

- Presence and location of external signals,
- Correlation of disturbances
- Topology of network: parametrized/fixed modules

- Graph-based method for synthesizing allocation of external signals
- that effectively exploits the presence of fixed (known) modules
- and can be executed through algebraic operations
- But reaching the minimum number of excitations is not guaranteed

The end