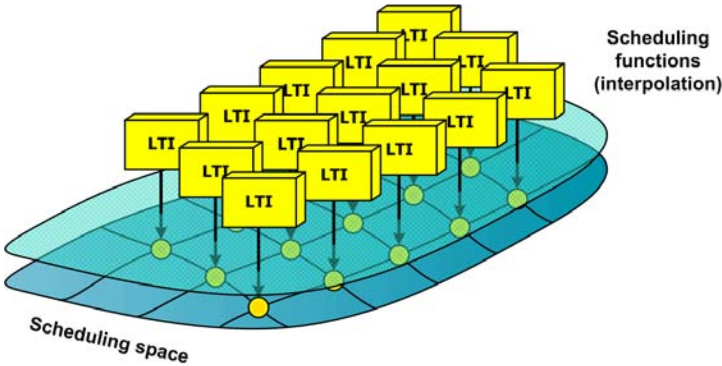


Model structures for identification of linear parameter-varying (LPV) models

Paul M. J. Van den Hof

October 19, 2009, Chinese Academy of Sciences, Beijing, China

Joint work with Roland Tóth and Peter S.C. Heuberger

Delft University of Technology

Oldest and largest of 3 Technical Universities in the Netherlands:
Delft – Eindhoven - Twente

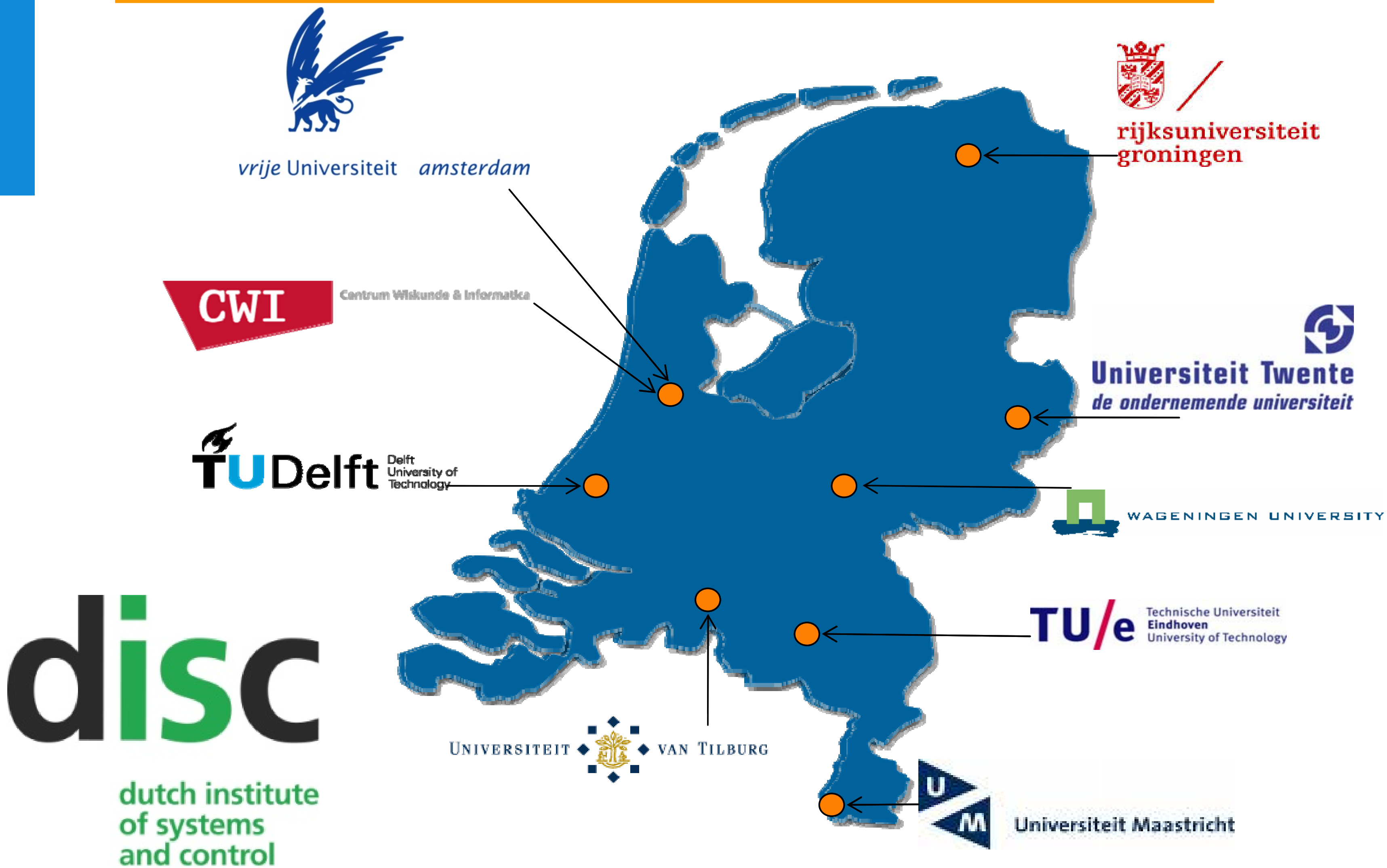
Founded in 1842 as an engineering school

Now: 5000 employees, of which 2700 scientists, 15,000 students, distributed over 8 engineering faculties:

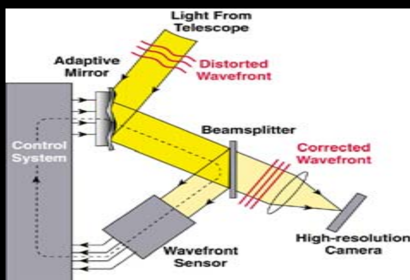
- Electrical, Math, Comp. Science
- Aerospace Engineering
- Applied Sciences
- Mechanical, Maritime, Mat. Eng.
- Civil Engin., Earth Sciences
- Industrial Design
- Techn. Policy Making and Manag
- Architecture

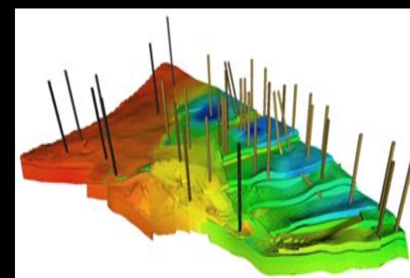
Delft

Dutch Universities in Control



Delft Center for Systems and Control





University wide center and department within Mechanical Engineering

- 16 scientific (tenured) staff
- 40 PhD students
- 15 Postdocs
- 40 MSc students / year

- BSc programs ME, EE, AP,
- MSC programs ME, EE, AP
- 3TU MSc Systems and Control
- PhD programme DISC

Fundamentals:

- Modelling, control and optimization of complex, non-linear and hybrid systems
- Signal analysis, signal processing and data-based modelling (identification for control)

Mechatronics and Microsystems:

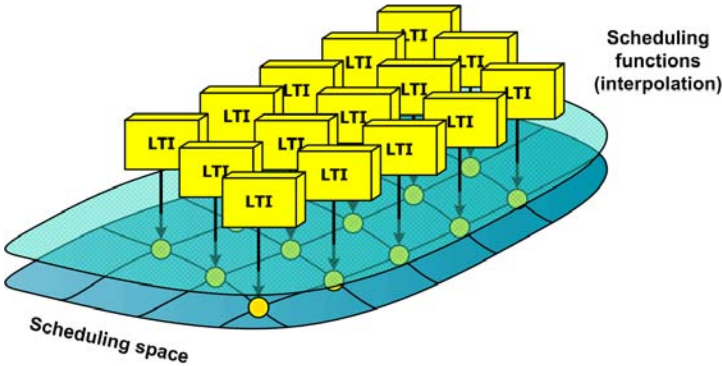
- Automotive systems
- Microfactory
- AFM nano-positioning
- Smart optics systems
- Electron microscopes
- Robotics

Traffic and Transportation:

- Discrete event and hybrid systems
- Distributed multi-agent systems
- Optimal adaptive traffic control
- Advanced driver assistance systems

Sustainable Industrial Processes:

- Increase of scale in process operation
- More flexibility in operation
- Economic optimization under operating constraints
- Process intensification
- Towards model-based process management
- hydrocarbon reservoir optimization; crystallization



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Contents

- Questions related to LPV identification
- Model structures for LPV systems
- An orthonormal basis function approach
- Example

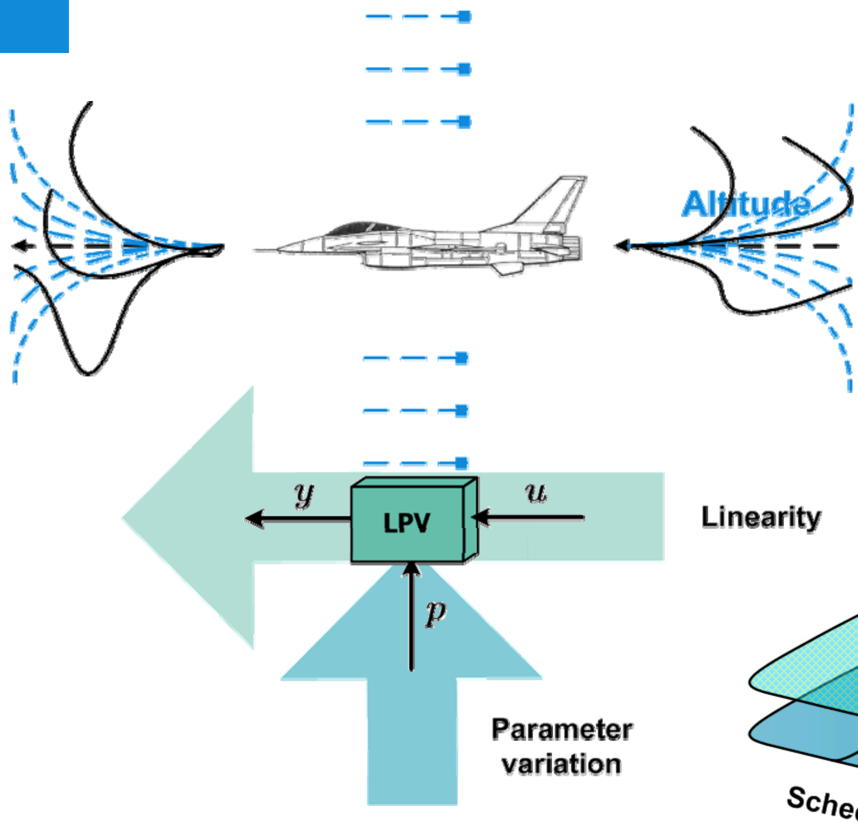
Modern control & identification

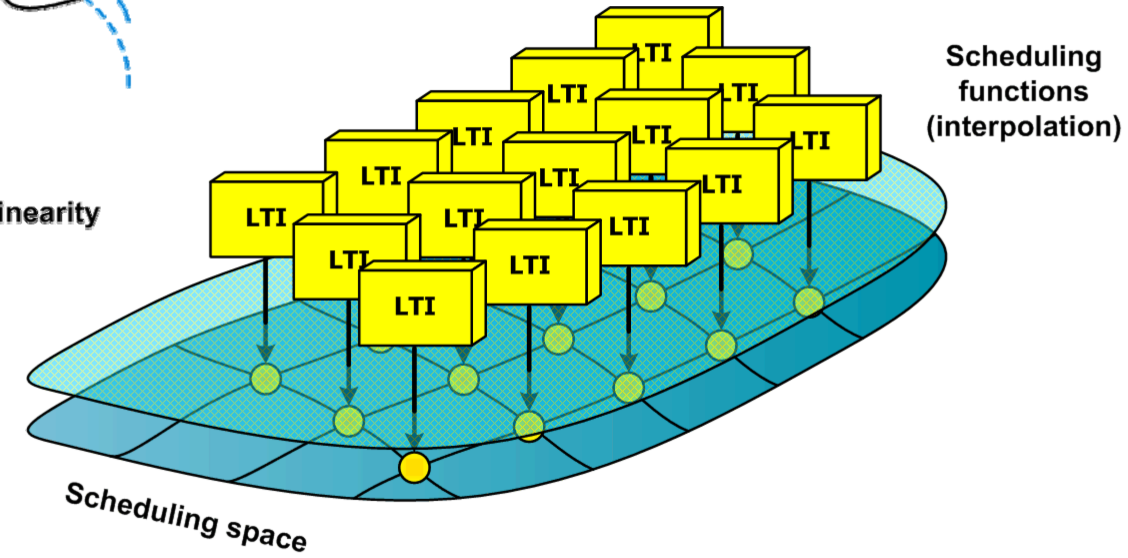
Challenges of nonlinear systems

- Expand the scope beyond the current LTI framework
- Handling systems in different operating regimes
- Attractive structure: LPV for systems with “regime”-dependent (linear) dynamics
- Advanced tools for control synthesis
- Several algorithms for LPV model identification
- LPV identification is not solved yet in a structural way

LPV systems

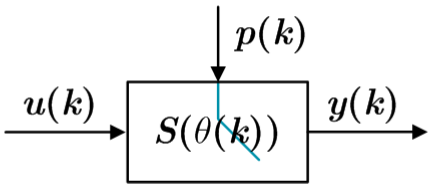
The concept





LPV model structures

Discrete time



- LPV framework (SISO), $p(k) : \mathbb{Z} \mapsto \mathbb{P}$
  - State-space models

$$\begin{aligned} x(k+1) &= A(p(k))x(k) + B(p(k))u(k) \\ y(k) &= C(p(k))x(k) + D(p(k))u(k) \end{aligned}$$

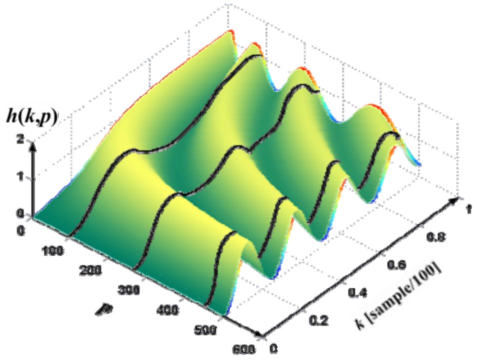
  - Input-output models, $n_{\mathrm{a}} \geq n_{\mathrm{b}} > 0$

$$y(k) = -\sum_{i=1}^{n_{\mathrm{a}}} a_i(p(k))y(k-i) + \sum_{j=0}^{n_{\mathrm{b}}} b_j(p(k))u(k-j)$$

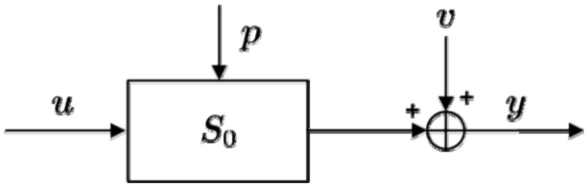
Usually, use is restricted to static (nonlinear) maps $p(k) \mapsto \theta(k)$

Issues in LPV identification

- Approaches to the identification problem
  - Local approach
    - Identify local linear models (for fixed scheduling $p(k) = \bar{\mathrm{p}}_i$)
    - Use global data to interpolate into an LPV model



  - Global approach
    - Determine a global LPV model structure
    - Use global data to estimate an LPV model



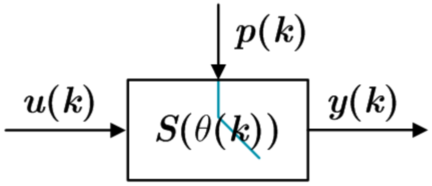
Both PE and subspace approaches can be followed

Issues in LPV identification

(cont'd)

- What are the appropriate model structures?
- How can they be defined?
- What are the criteria to select them?
- Many more questions related to
  - Estimation accuracy
  - Experiment design
  - Validation
  - etc.

LPV model structures



- State-space models

$$
\begin{aligned}
x(k+1) &= A(p(k))x(k) + B(p(k))u(k) \\
y(k) &= C(p(k))x(k) + D(p(k))u(k)
\end{aligned}
$$

- Input-output models, $n_\mathrm{a} \geq n_\mathrm{b} > 0$

$$
y(k) = -\sum_{i=1}^{n_\mathrm{a}} a_i(p(k))y(k-i) + \sum_{j=0}^{n_\mathrm{b}} b_j(p(k))u(k-j)
$$

- Question: are these structures equivalent (as in the LTI case)?
- Answer: In general not, if you restrict $p \mapsto \theta$ to be static; dynamic p–dependencies are generally required (Tóth et al., ECC 2007)

LPV model structures

Example:

- State-space model

$$
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(k+1) = \begin{bmatrix} 0 & a_1(p(k)) \\ 1 & a_2(p(k)) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(k) + \begin{bmatrix} b_1(p(k)) \\ b_2(p(k)) \end{bmatrix} u(k)
$$

$$
y(k) = x_2(k).
$$

- Input-output model,

$$
\begin{aligned}
y(k) &= a_2(p(k-1))y(k-1) + a_1(p(k-2))y(k-2) \\
&\quad + b_2(p(k-1))u(k-1) + b_1(p(k-2))u(k-2),
\end{aligned}
$$

LPV model structures

- Consequence 1
  - Mapping estimated IO models to SS or vice versa, while retaining a static dependence of the scheduling functions introduces (substantial) error.
- This points to the need to either
  - Estimate the LPV model in the same model structure where information on the (static) effect of p is available, or
  - Include a dynamic map $p \mapsto \theta$ in the model structure

LPV model structures

- Consequence 2
  - We need appropriately defined notions of equivalence between LPV systems (and definitions of LPV systems as a start)

> Note: transfer function is not available for this purpose as the systems is time-varying

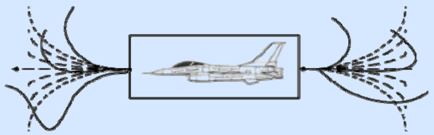
> Notion of equivalence is well-defined in terms of $\mathfrak{B}$

- Solution: through Willems' behavioral framework:

Time axis, $\mathbb{T} = \mathbb{Z}$

Behavior, $\mathfrak{B} \subseteq (\mathbb{W} \times \mathbb{P})^{\mathbb{T}}$

$$\mathcal{S} = (\mathbb{T}, \mathbb{P}, \mathbb{W}, \mathfrak{B})$$



Scheduling space, $\mathbb{P} \subseteq \mathbb{R}^{n_{\mathrm{P}}}$

Signal space, $\mathbb{W} = \mathbb{R}^{n_{\mathrm{W}}}$

LPV model structures

$$r(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) = \frac{1 + \mathrm{x}_3}{1 - \mathrm{x}_2}$$

Unique association $\Downarrow$ $n_{\mathbb{P}} = 2$

$$(r \diamond p)(k) = \frac{1 + p_1(k+1)}{1 - p_2(k)}$$

- Generic representation of an LPV system behavior:

$$\sum_{i=0}^{n_\xi} (r_i \diamond p) q^i w = 0 \quad \text{or} \quad (R(q) \diamond p) w = 0$$

where $r_i \diamond p$ represents any quotient of homeomorphic functions of p and shifted versions of p.

Result: LPV system equivalence, canonical forms in SS and IO form, etc., Taking account of dynamic phenomena in p and w .

(Tóth et al., ECC 2009)

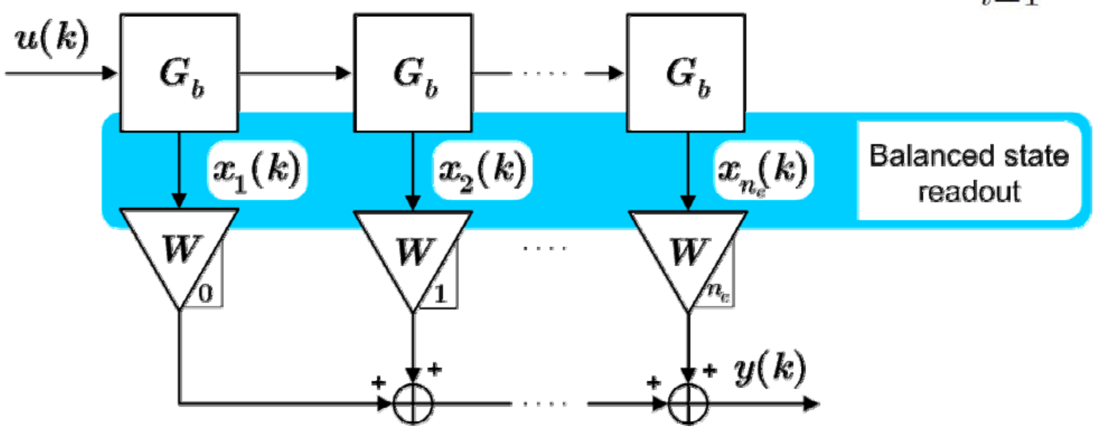
LPV model structures

- Additional aspects:
  - In linear PE identification we benefit from linearity-in-the-parameters;
    Can this be maintained?
  - Interpolating local linear state space models is hard when McMillan degree varies over local models;
    Can we accommodate this?

An OBF approach

Orthonormal basis functions

- For local linear models: $F(z) \approx \sum_{i=0}^{n_\mathrm{f}} w_i \phi_i(z)$
- Generation of the OBFs
  - By a set of stable poles: $\Xi_n = \{\lambda_1, \ldots, \lambda_n\} \subset \mathbb{D}$
  - By a stable all-pass (inner) function: $G_\mathrm{b}(z) = \prod_{i=1}^{n} \frac{1 - z\lambda_i^*}{z - \lambda_i}$



(Heuberger et al., Springer, 2005)

Choice of poles determines rate of convergence of the series expansion

An OBF approach

Opportunities for LPV models

$$y = \sum_{i=0}^{n_\mathrm{f}} w_i(p)\phi_i(q)u$$

- Scheduling of coefficients w_i retains linearity-in-the parameters
- If basis can be chosen (globally) fixed, interpolation of local models becomes interpolation $\{w_i(\bar{\mathrm{p}}_j)\}_{j=1,\dots,n_l}$

$\bar{\mathrm{p}}_j$ constant scheduling point belonging to local model j

- No problem with interpolation of models with different McMillan degrees

Question: How to choose the global basis functions $\phi_i(q)$?

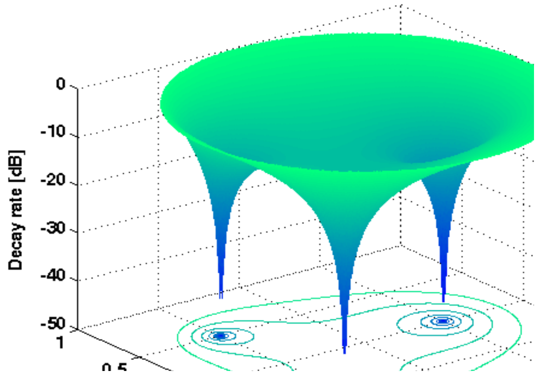
An OBF approach

Selection of basis functions

- Identify a number of local linear models in several in different regimes $\bar{\mathrm{p}}_j$
- Plot all identified poles in the complex plane
- Cluster the poles in groups and find optimal cluster centers (basis poles)
- So as to minimize a distance measure that is relevant for the (worst case) length of the resulting series expansions

An OBF approach

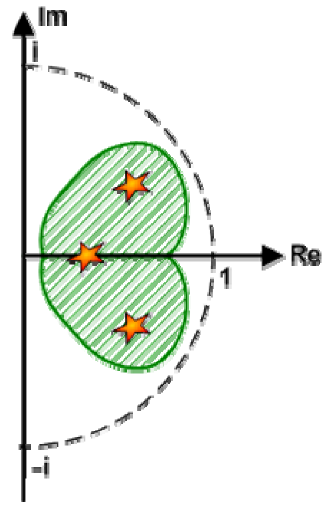
Kolmogorov n-width theory



- Worst-case modeling:
  - Result (Oliveria e Silva, 1996):
    - $G_\mathrm{b}(z)$ an inner function
    - Let K be the set of systems with poles in the region

$$\{z \in \mathbb{D} \mid |G_\mathrm{b}(z^{-1})| < \rho\}$$

    - The OBFs, generated by $G_\mathrm{b}(z)$ are optimal for K in the n-width sense



An OBF approach

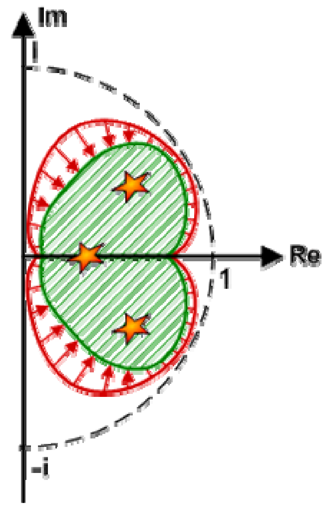
Kolmogorov n-width theory

- The inverse n-width concept:
  - Given a region of poles: Ω
  - Try to approximate it as

$$\Omega \approx \Omega(\Xi_n, \rho) = \{z \in \mathbb{D} \mid |G_\mathrm{b}(z^{-1})| < \rho\}$$

$$\rho = \text{ decay rate of the expansion}$$

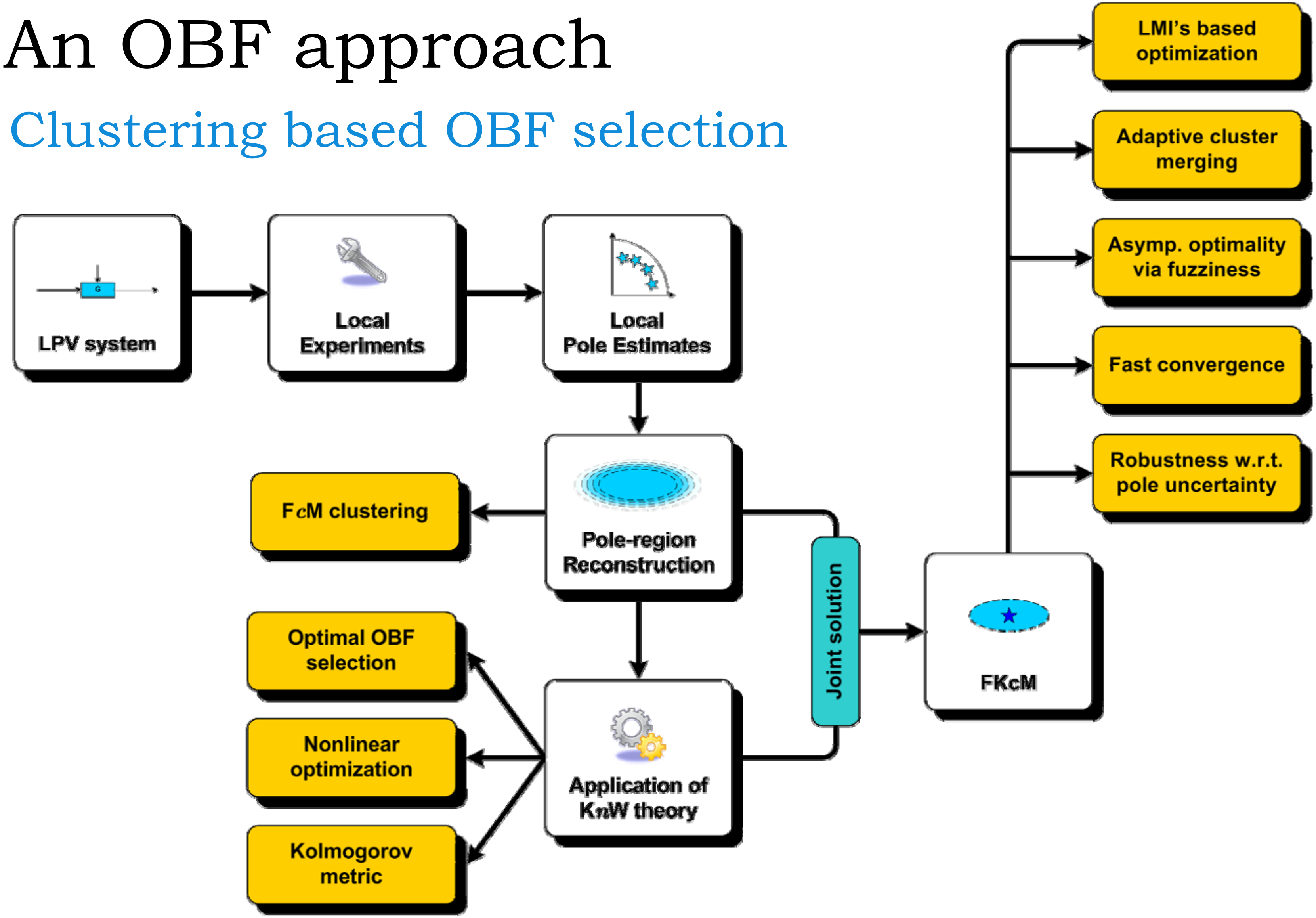
  - The n optimal OBFs are obtained through (Kolmogorov measure minimization)



$$\min_{\Xi_n \subset \mathbb{D}} \rho = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} |G_\mathrm{b}(z^{-1})| = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} \left| \prod_{i=1}^{n} \frac{z - \lambda_i}{1 - z\lambda_i^*} \right|$$

An OBF approach

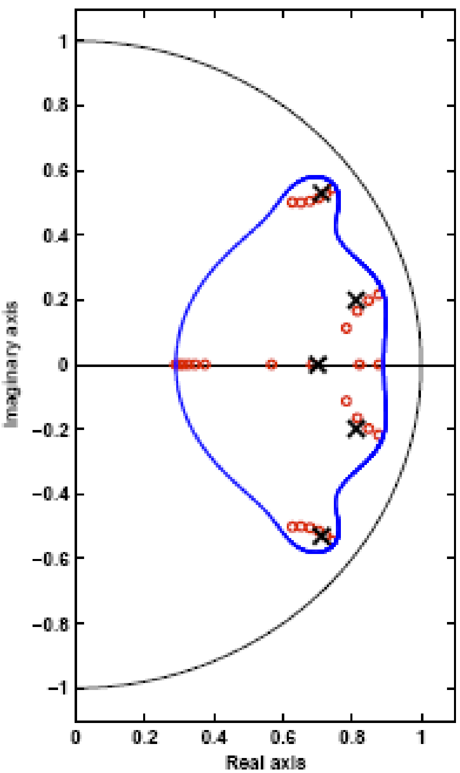
Clustering based OBF selection



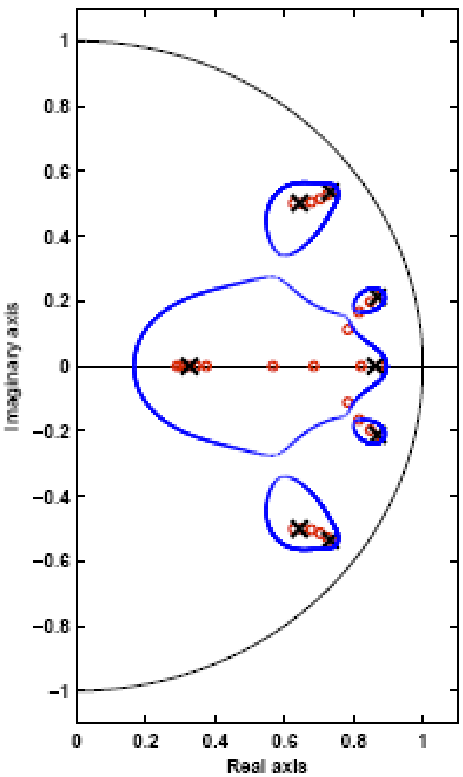
An OBF approach

Example of clustering

30 poles



(a) $m = 8, c = 5$

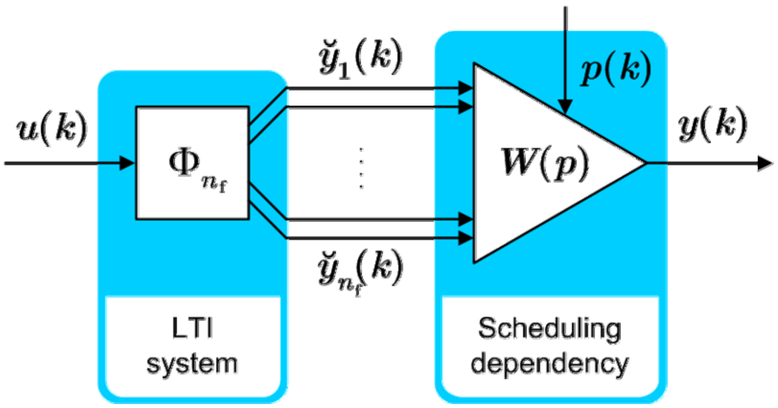


(b) $m = 8, c = 8$

An OBF approach

LPV-OBF model structures

- The following global model structure results:
  - Static p-dependence is linearly parametrized (e.g. polynomial, splines)
  - Estimation through linear regression (OE-form)

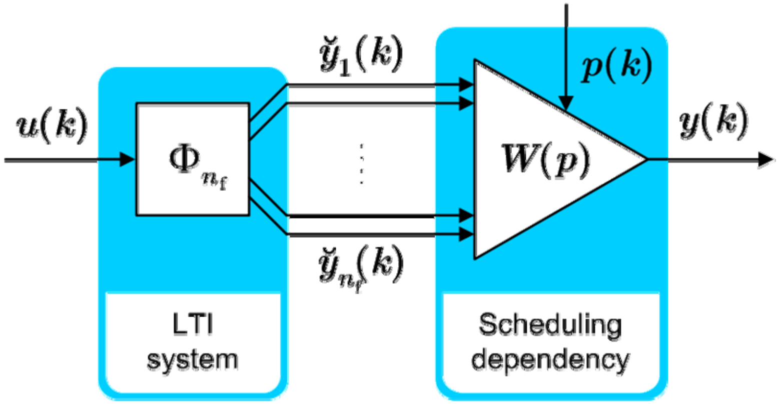


Wiener LPV model

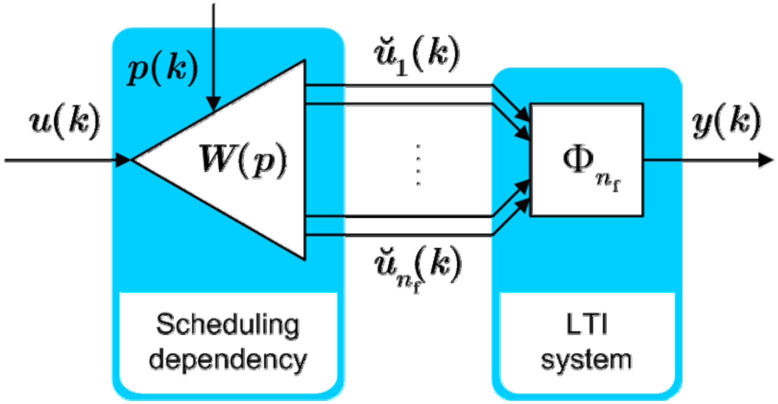
An OBF approach

LPV-OBF model structures

- Different alternatives:



Wiener LPV model



Hammerstein LPV model

Different results due to the finite expansion and the static p -dependence

Example

- LPV system $\mathcal{S}$ with I/O representation:

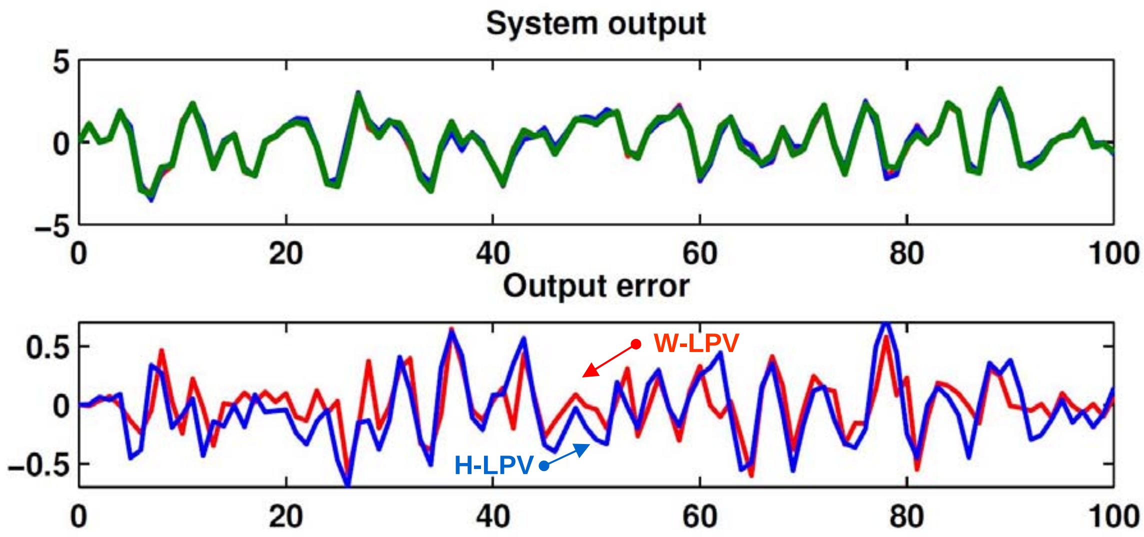
$$a_0(p(k))\,y(k) = b_1(p(k))\,u(k-1) - \sum_{l=1}^{5} a_l(p(k))\,y(k-l)$$

$$\begin{aligned}
a_0(p) &= 0.58 - 0.1p, & a_1(p) &= -\frac{511}{860} - \frac{48}{215}p^2 + 0.3(\cos(p) - \sin(p)),\\
a_2(p) &= \frac{61}{110} - 0.2\sin(p), & a_3(p) &= -\frac{23}{85} + 0.2\sin(p),\\
a_4(p) &= \frac{12}{125} - 0.1\sin(p), & a_5(p) &= -0.003, \;\; b_1(p) = \cos(p).
\end{aligned}$$

with $\mathbb{P} = [0.6, 0.8]$.

Identify $\mathcal{S}$ with W-LPV and H-LPV OBF models!

Example



Global identification of $\mathcal{S}$

model	SNR	MSE	BTF	VAF
W-LPV	15 dB	0.0572	83.69%	97.34%
H-LPV	15 dB	0.0973	78.72%	95.48%

7 OBFs

Data: $p \in \mathcal{U}(0.6, 0.8)$, $u \in \mathcal{U}(-1, 1)$

500 samples long

Noise: $v_e \in \mathcal{N}(0, 0.5)$

output additive

Conclusions

- LPV models:
  - Intermediate step between nonlinear and LTI models.
  - Effective engineering tool for dealing with nonlinear systems.
- LPV model structures for identification are studied and basic structures and phenomena have been clarified.
- OBF's provide a powerful tool for parametrizing relevant classes of LPV systems
- There is work to be done on completing the picture of a general framework for LPV identification.

Further reading

- Tóth, R., J.C. Willems, P.S.C. Heuberger, P.M.J. Van den Hof (2009). A behavioral approach to LPV systems. *In Proc. of the European Control Conf.* August, 2009, Budapest, Hungary, pp. 2015-2020.

- R. Tóth (2008). Modeling and Identification of Linear Parameter-Varying Systems – An Orthonormal Basis Function Approach. **Dr. Dissertation**, Delft University of Technology, December 2008. To be published by Springer Verlag, 2010.

- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2008). Asymptotically optimal orthonormal basis functions for LPV system identification. ***Automatica***, 45, pp. 1359-1370.