Estimating Cutting Forces in Micromilling by Input Estimation from Closed-loop Data

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Outline

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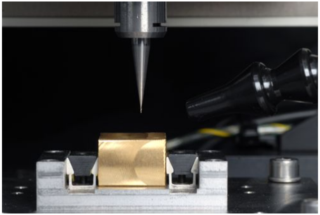
Micromilling with Active Magnetic Bearing spindles

Micromilling milling with tools with diameter $< 0.5mm$

Applications medical purposes, micro-electronics, etc.

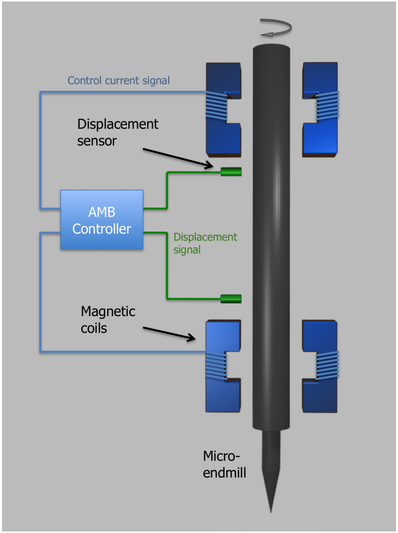
Challenge Use active nature of AMB spindles for Process Monitoring and Control

Comparison of micro-endmill (0.2mm) and normal endmill (1.0mm)



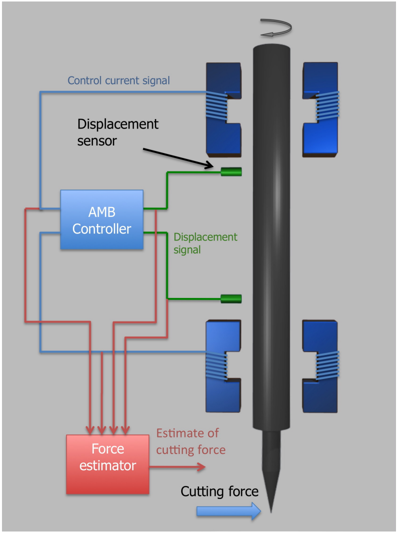
Close-up of micromilling machine

Challenge

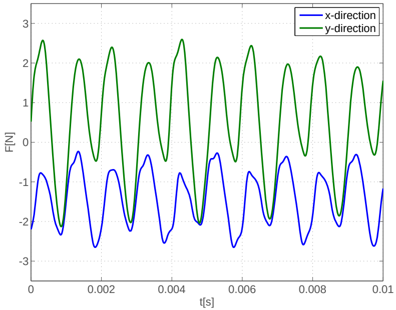


Control current signal
Displacement sensor
AMB Controller
Displacement signal
Magnetic coils
Micro-endmill

Challenge



Example of cutting forces in micromilling



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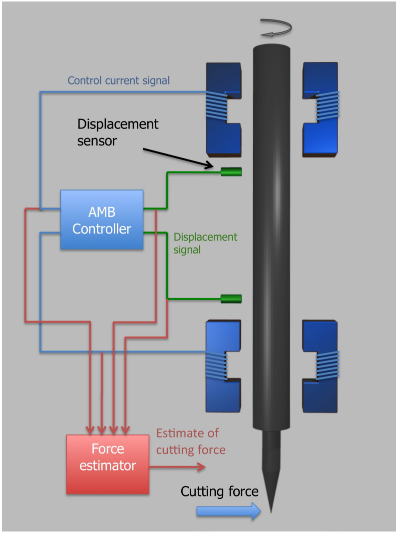
Challenge



Cutting force estimation

Observe the cutting forces that arise during micromilling from the signals of the Magnetic Bearings

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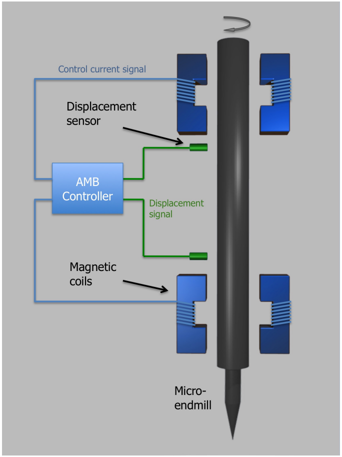
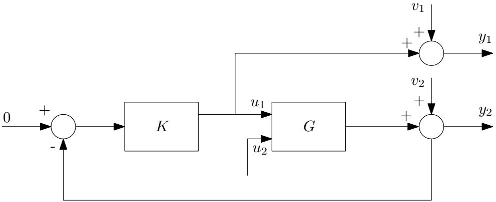
Problem statement





- G: Model of the open-loop AMB spindle dynamics (MIMO, unstable)
- K: Controller and current amplifier
- u_1: Currents through the magnetic coils
- u_2: Cutting forces acting on tooltip
- $y_{1,2}$: Measurements of currents, displacements
- $v_{1,2}$: Measurement noise on currents, displacements (white)

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Problem statement

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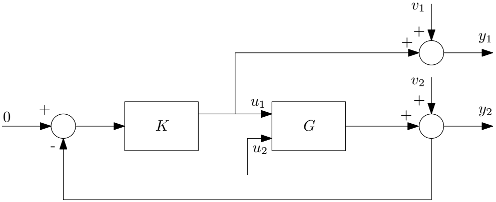
Objective: Using

- model of plant G
- information on spectrum of the unknown input u_2

design linear filter F on $y_{1,2}$ to create $\hat{u}_2(t)$ such that

$$\mathbb{E}|\hat{u}_2(t) - u_2(t-N)|^2$$

is minimized for fixed lag $N \geq 0$.



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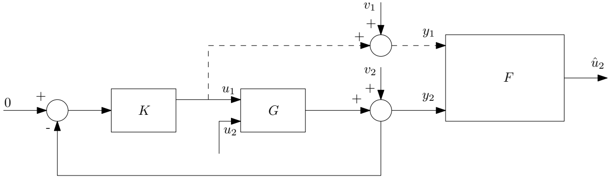
Input estimation from closed-loop data, known $K(z)$



If $K(z)$ is known, input y_1 is not needed

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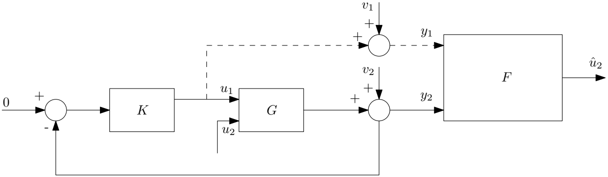

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Denote:

- Φ_{u_2}: spectrum of u_2
- Φ_{y_2}: the spectrum of y_2
- $\Phi_{u_2 y_2}$: the cross spectrum between u_2 and y_2

The causal Wiener filter

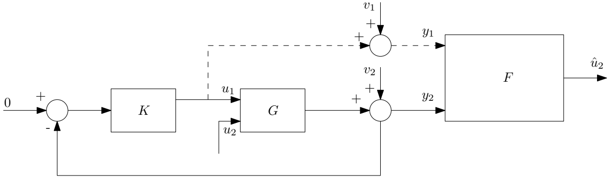
Let the canonical spectral factorization of Φ_{y_2} be given by

$$\Phi_{y_2} = MRM^*$$

with M minimum phase.
The filter causal F that minimizes $\mathbb{E}|Fy_2 - u_2(t-N)|^2$ is given by

$$F = \{z^{-N}\Phi_{u_2 y_2} M^{-*}\}_+ R^{-1} M^{-1}$$

Input estimation from closed-loop data, known $K(z)$



- $y_2 = S(G_2 u_2 + v_2)$
- $S = (I + G_1 K)^{-1}$

It is easily derived that:

- $\Phi_{u_2 y_2} = \Phi_u G_2^* S^*$
- $\Phi_{y_2} = S(G_2 \Phi_u G_2^* + R_{v_2}) S^*$

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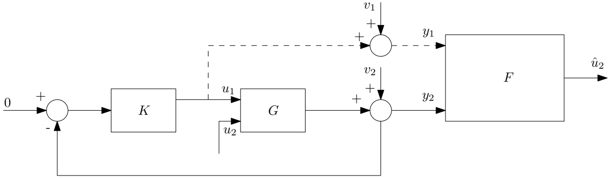
Input estimation from closed-loop data, known $K(z)$



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Causal Wiener filter for closed-loop data

Factorize $G_2 \Phi_u G_2^* + R_{v_2} = TRT^*$ such that ST minimum phase. Then:

$$\Phi_{y_2} = (ST)R(ST)^*$$

With this it follows that

$$F = \{z^{-N} \Phi_u G_2^* T^{-*}(z)\}_+ R^{-1} T^{-1}(z) S^{-1}(z)$$

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Input estimation from closed-loop data, known $K(z)$

Causal Wiener filter for closed-loop data

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How to proceed:

step 1. Find factorization $G_2 \Phi_u G_2^* + R_{v_2} = TRT^*$

step 2. Find causal part of $z^{-N} \Phi_u G_2^* T^{-*}(z)$

Step 1. Factorization of $G_2\Phi_u G_2^* + R_{v_2}$ using state space realization

- Let a realization of G be given by $\left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C & 0 & 0 \end{array}\right)$

Conditions

- A not necessarily Hurwitz
- (A, B_1) and (A, B_2) stabilizable, (A, C) detectable
- $\begin{bmatrix} A - \lambda I & B_2 \\ C & 0 \end{bmatrix}$ has full column rank for all $\lambda \in \mathbb{C}$, $|\lambda| \geq 1$

Step 1. Factorization of $G_2\Phi_u G_2^* + R_{v_2}$ using state space realization

- Let a realization of G be given by $\left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C & 0 & 0 \end{array}\right)$
- Let $\Phi_{u_2} = G_u R_u G_u^*$ be the canonical spectral factorization with minimal realization of G_u given by $\left(\begin{array}{c|c} A_u & B_u \\ \hline C_u & 0 \end{array}\right)$
- Define cascaded system $G_2 G_u$, which has realization

$$\left(\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array}\right) = \left(\begin{array}{cc|c} A_u & 0 & B_u \\ B_2 C_u & A & 0 \\ \hline 0 & C & 0 \end{array}\right)$$

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Step 1. Factorization of $G_2 \Phi_u G_2^* + R_{v_2}$ using state space realization

- With this, we can obtain the desired factorization

$$\begin{aligned} G_2 \Phi_u G_2^* + R_{v_2} &= (G_2 G_u) R_u (G_u^* G_2^*) + R_{v_2} \\ &= [C_c (zI - A_c)^{-1} L + I] R [*]^* \\ &= TRT^* \end{aligned}$$

  with
  - $L = A_c P C_c R^{-1}$,
  - $R = R_{v_2} + C_c P C_c^T$, and
  - P the unique p.d. solution of the DARE

$$P = A_c P A_c^T + B_c R_u B_c^T - LRL^T.$$

- ST is minimum phase.

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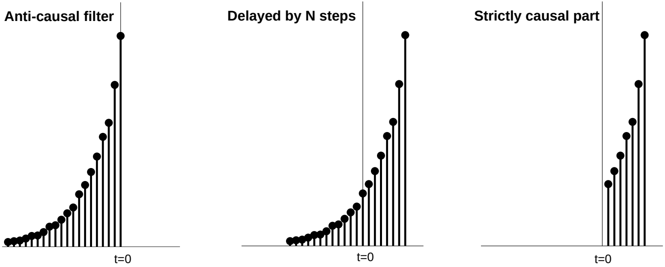
Step 2. Deriving the causal filter

$$F = \{z^{-N} \underbrace{G_u(z)R_uG_c(z)^*T^{-*}(z)}_{W(z)}\}_+ R^{-1}T^{-1}(z)S^{-1}(z)$$

- Split $W(z)$ in a causal and anti-causal part: $W(z) = \underbrace{W_1(z)}_{\text{causal}} + \underbrace{W_2(z)}_{\text{anti-causal}}$
- Then $\{z^{-N}W(z)\}_+ = z^{-N}W_1(z) + \{z^{-N}W_2(z)\}_+$
- $\{z^{-N}W_2(z)\}_+$ can be found by truncating the Laurent expansion of $W_2(z)$



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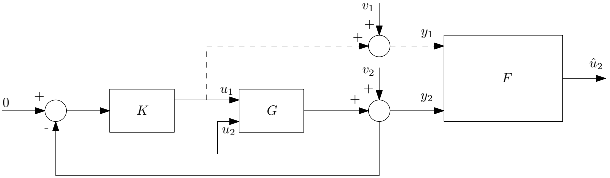
No explicit information on $K(z)$



- S is the only factor in F that depends on $K(z)$

$$\hat{u}_2 = \left(z^{-N}W_1(z) + \{z^{-N}W_2\}_+\right) R^{-1}T^{-1}(z)S^{-1}y_2$$

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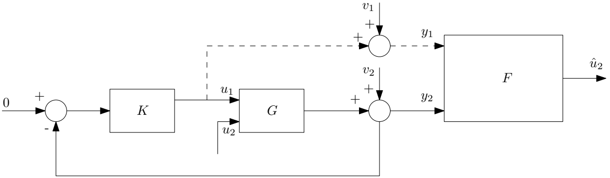

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$$\hat{u}_2 = \left(z^{-N} W_1(z) + \{z^{-N} W_2\}_+\right) R^{-1} T^{-1}(z) (I + G_1 K) y_2$$

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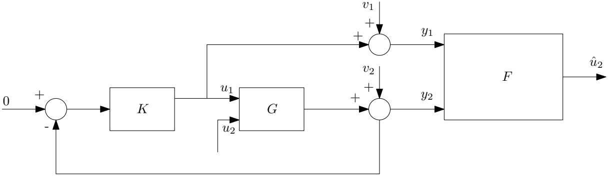

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- If v_1 negligible, $y_1 = -K(z) y_2$. With this:

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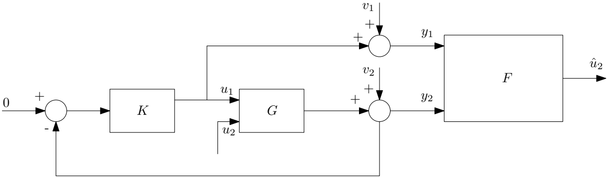

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- This filter is optimal for any $K(z)$

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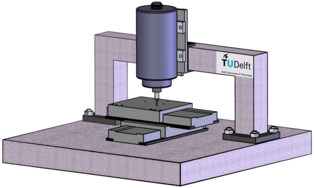
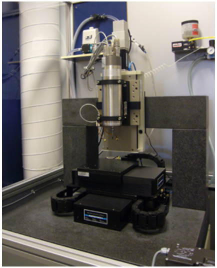
Results: Simulation with Simulink

- Simulation performed based on parameters of micromilling setup in laboratory
- Open loop AMB system modeled using first principles
- Cutting forces are simulated using model from the micromilling literature





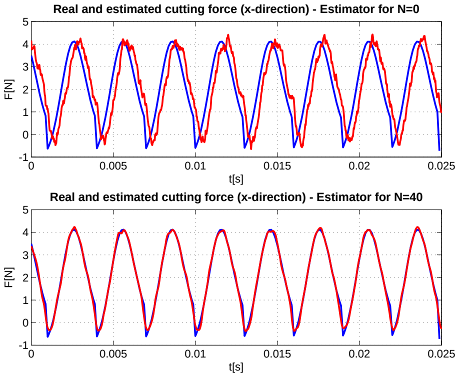
Results: Simulation with Simulink

First test:

- Rotational speed is 10,000 rpm, $T = 25\mu s$
- Random walk model for input spectrum
- Input estimator constructed for $N = 0$ and $N = 40$

Results:

- Filter for $N = 40$ outperforms filter for $N = 0$
- Estimator for $N = 0$ appears to yield delayed estimation results



Error analysis

Observe that the estimation error for $N = 0$ consists of two terms:

$$e = \hat{u}_2 - u_2 = (F_2 G_2 - I)u_2 + F_2 v_2$$

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- $|F_2 G_2| \approx I$ for lower frequencies
- Then $F_2 G_2$ acts as a pure delay for those frequencies if the group delay

  $$\tau_g = -\frac{d}{d\omega} arg\ F_2 G_2(e^{j\omega})$$

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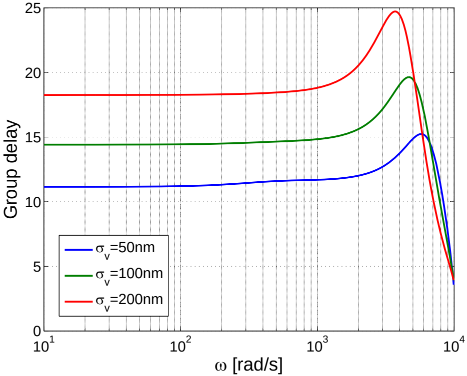

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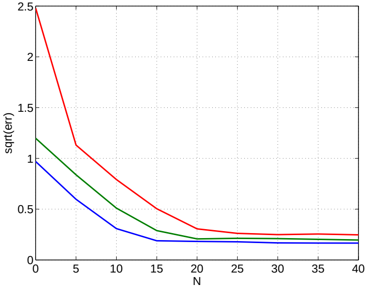
Error analysis

Estimation error for $N > 0$:

$$e \quad = \quad \hat{u}_2 - z^{-N} u_2 = (F_2 G_2 - z^{-N} I) u_2 + F_2 v_2$$



Plot of $\sqrt{\frac{1}{k}\sum_k e^2}$ for increasing N

- Estimation error descreases for increasing N
- Key question: do both terms of e decrease for increasing N?

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Error analysis

Estimation error for $N > 0$:

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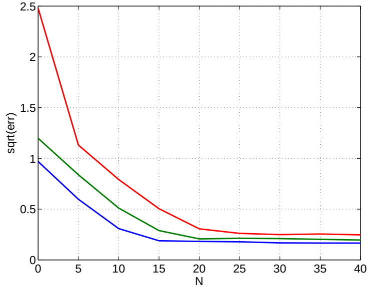

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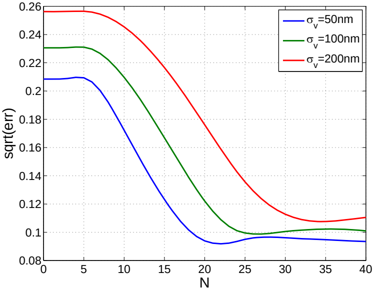

Plot of $\|\sigma_v F_2\|$ for increasing N

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The effect of delay

Observations

- Specifying a filter without lag ($N = 0$) results in a filter with a lag
- Specifying a filter with this lag, results in a filter with better performance
- Specifying a filter with a larger lag, improves the results even further

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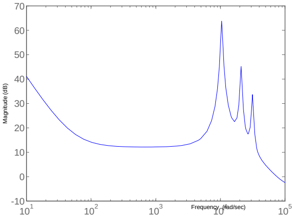
Results: Improved spectral model of input

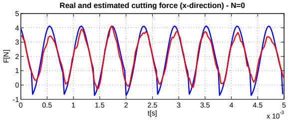


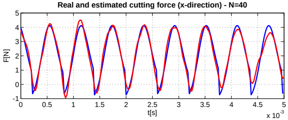
Second test:

- Rotational speed 50,000 rpm;
- Φ_u has high density at frequencies related to the rotational speed;
- Filter derived for $N = 0$ and $N = 40$.

Results:

- Filter with lag again performs better





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Conclusions

- An optimal input estimator was developed to estimate the cutting forces in micromilling from AMB signals
- No additional sensors are needed
- No knowledge on the AMB controller is needed, if exact measurements of the control currents are available
- The estimator has an adjustable delay allowing to trade off the estimation error against the lag
- There exists a minimum delay that can be attained
- Estimation results can be improved by using a priori information on the spectral content of the cutting forces.