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Model and Economic Uncertainties in Balancing Short-Term and Long-Term Objectives in Water-Flooding Optimization

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Abstract

Model-based optimization of oil production has a significant scope to increase ultimate recovery or financial life-cycle performance. The Net Present Value (NPV) objective in such an optimization framework, because of its nature, focuses on the long-term gains while the short-term production is not explicitly addressed. At the same time the achievable NPV is highly uncertain due to the limited knowledge of reservoir model parameters and varying economic conditions. Different (ad-hoc) methods have been proposed to introduce short-term considerations to balance short-term and long-term objectives in a model-based approach. In this work, we address the question whether through an explicit handling of model and economic uncertainties in NPV (robust) optimization, an appropriate balance between these economic objectives is naturally obtained. A set (ensemble) of possible realizations of the reservoir models is considered as a discretized approximation of the uncertainty space, while different oil price scenarios are considered to characterize the economic uncertainty. A gradient-based optimization procedure is used where the gradient information is computed by solving adjoint equations. A robust optimization framework with an average NPV with respect to the ensemble of models and the oil price scenarios is formulated and the NPV build-up over time is studied. As robust optimization (RO) does not attempt to reduce the sensitivity of the solution to uncertainty, a mean-variance optimization (MVO) approach is implemented which maximizes the average NPV and minimizes the variance of the NPV distribution. It is shown by simulation examples that with RO, the average NPV is increased compared to the reactive strategy, with both forms of uncertainty. However, an NPV build-up over time that is considerably slower than for a reactive strategy is obtained. A faster NPV build-up compared to RO is achieved in MVO by choosing different weightings on variance in the mean-variance objective, at the price of slightly compromising on the long-term gains.

Introduction

Dynamic optimization of the water-flooding process has shown significant scope for improvement of the economic life-cycle performance of oil fields compared to a conventional reactive strategy, see e.g., Brouwer and Jansen (2004), Sarma et al. (2005), Jansen et al. (2008), and Van den Hof et al. (2012). In these studies, a financial measure, i.e., Net Present Value (NPV), is maximized.

NPV is a cumulative objective function. Together with the typical long life-cycle time for oil reservoirs, the optimization problem focuses on the long-term gains while the short-term gains are generally neglected. Furthermore, due to the high levels of uncertainty in such model-based optimization, the resulting optimal strategy and subsequently the achieved NPV are highly uncertain and unreliable. Uncertainty arises from both the modeling process of water-flooding and from varying economic conditions. The limited information contents in seismic, well logs and production data about the true reservoir parameters result in highly uncertain reservoir models and hence make the long-term predictions of these models unreliable. The NPV objective function contains economic variables such as interest rate, oil price etc., which fluctuate with time and can not be precisely predicted. The unknown time-variation of these variables becomes another source of uncertainty i.e., economic uncertainty in the NPV optimization framework.

As a result, the potential advantages of model-based economic optimization are not fully realized. On the other hand, the conventional reactive strategy is model free and has a much faster NPV time build-up (higher short-term gains) compared to the model-based optimization. Even though the realized NPV of the reactive strategy is smaller compared to the NPV of a model-based strategy, the certainty of the reactive approach gives confidence and often becomes the decision maker's choice. The question arises which steps should be taken with model-based optimization to gain confidence in it and which indicators will reflect this confidence? The straightforward answer to the first question is to explicitly incorporate and handle uncertainty in the model-based optimization framework. The second question may have different perspectives. An important indicator is the potential value of high short-term gains. Consider two production strategies which are uncertain and result in the same NPV but have different rates of NPV build-up over time. Decision makers may prefer the strategy which has a higher rate of NPV build-up (high short-term gains), which reflects an implicit attitude of not-running-into-risk of loosing revenues tomorrow. Furthermore, as the negative impact of uncertainty increases with time, a higher rate of NPV build-up may mitigate the potential loss of revenues.

In the petroleum engineering literature, both of the above questions are separately addressed. In Van Essen et al. (2009), a so called robust optimization (RO) approach is used that incorporates model uncertainty in optimization by considering an average NPV over an ensemble of geological model realizations. The ensemble gives an approximation of the geological model uncertainty space. A similar approach was used by Yeten et al. (2003). The second question is indirectly addressed in Van Essen et al. (2011), by adapting the NPV criteria to a multi-objective (short-term and long-term) approach, where the short-term gains are maximized by using NPV with a high discount factor, hence mitigating risk and improving confidence. However, this approach neglects the uncertainty which is the core reason of this risk and attacks the problem from a different angle. Chen et al. (2012) and Fonseca et al. (2014) have included geological uncertainty in the multi-objective approach, and Siraj et al. (2015) have extended the approach to include economic uncertainty. These approaches include uncertainty in the optimization framework but do not aim to reduce the sensitivity of the optimal solution to uncertainty and hence provide a poor handling of uncertainty. Capolei (2013) used a portfolio mean-variance optimization approach with geological model uncertainty to quantify risk in terms of variance of the NPV distribution. Similar approaches have been described in Yeten et al. (2003), Bailey et al. (2005) and Yasari et al. (2013). They aim to maximize the average NPV and also minimize the variance of the NPV distribution, thus provides a better handling of uncertainty.

The main focus of this work is to investigate whether, by explicit handling of uncertainty in model-based economic optimization, the balance between short-term and long-term gains can be naturally obtained. As high short-term gains reflect the confidence of a strategy and mitigate risk, will, by proper handling of uncertainty, sufficiently high short-term gains be achieved to gain confidence of a decision maker? An important consideration in the long-term NPV optimization is to study the time-localized effect of uncertainty that may induce a dynamic behavior in the optimization to include short-term gains.

Both forms of uncertainty, i.e., geological uncertainty and economic uncertainty, are considered in the optimization framework. Ensembles of reservoir models and varying oil price scenarios are considered to quantify the model parametric and economic uncertainty space respectively. The averaging approach in Van Essen et al. (2009) is used with an ensemble of model realizations with fixed economic conditions to address geological uncertainty and later it is extended to be used with varying oil price scenarios with a single geological model realization to honor economic uncertainty. Because of the poor handling of uncertainty with RO, the simulation results show that it does not provide a good balancing of short-term and long-term gains. As the mean-variance approach has an intrinsic way of reducing the sensitivity of the optimal solution to uncertainty and hence better handling of the uncertainty, it is implemented with both forms of uncertainty. It is shown by simulation examples for both the cases, that a better balancing of short-term and long-term gains is achieved: short-term gains are improved at the cost of slightly compromising on the long-term gains. The weighting parameter on variance in the mean-variance objective function, proves to be a tuning parameter to balance both economic objectives.

The paper is organized as follows: In the next section, the theoretical and mathematical concepts such as the modeling process of water-flooding and model-based optimization are discussed. The definition and formulation of the robust optimization and the mean-variance approaches are also given in the Theory section. Later in the Result and Discussion section, simulation experiments with robust and mean-variance approaches considering both forms of uncertainties are presented. Conclusions of the work are drawn afterwards.

Theory

Water-flooding

In water-flooding, water is injected through the injection wells in an oil reservoir to push additional oil towards producing wells and to maintain a steady pressure in the reservoir. The dynamics of the water-flooding process can be described by the conservation of mass and momentum equations (Darcy's law); for details see, e.g., Aziz and Settari (1979) and for a description in systems and control notation, Jansen et al. (2008). A state-space form results after the discretization of governing equations in both space and time

$$\mathbf{g}_k(\mathbf{u}_k, \mathbf{x}_k, \mathbf{x}_{k-1}, \boldsymbol{\theta}) = \mathbf{0}, \quad k = 1, \ldots, K, \tag{1}$$

where subscripts refer to the discrete instants k of time, with K the total number of time steps, and where g is a non-linear vector-valued function. In isothermal reservoir simulation, the state variables vector $\mathbf{x}_\mathrm{k} \in N \subset R^n$ are typically pressure and phase saturations in each grid cell with initial conditions $\mathbf{x}_0 = \bar{\mathbf{x}}_0$. Note that, $\mathbf{x}_\mathrm{k}$ is a shortcut notation to represent $\mathbf{x}_\mathrm{k} = \mathbf{x}(t_k)$, i.e., the value of $\mathbf{x}$ at time $t = t_k$. The control vector $\mathbf{u}_k \in M \subset R^m$ can represent a combination of prescribed well flow rates, well bore pressures or valve settings. The parameter vector $\theta \in \Theta \subset R^{n_\theta}$ typically contains porosities and permeabilities in each grid cell, and other uncertain parameters such as fault transmissibility multipliers, initial fluid contacts, etc.

Typically, not all the states $\mathbf{x}$ are measurable, and only a few output variables $\mathbf{y}$ are measured, which are a function of input variables u and states variables $\mathbf{x}$ as follows:

$$\mathbf{y}_k = \mathbf{h}(\mathbf{u}_k, \mathbf{x}_k). \tag{2}$$

Based on the dynamic model defined in Eq. (1), an optimization problem is formulated in the next sub-section.

Model-based optimization

For a given well configuration, the injection flow rates and/or production valves settings can be dynamically operated over the production life-cycle, and therefore serve as control inputs for optimization. The objective is to optimize an economic function i.e., Net Present Value (NPV) for the cumulative oil and water production over a fixed time horizon which can be represented in the usual fashion as:

$$J = \sum_{k=1}^{K} \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right] \tag{3}$$

where r_o, r_w and r_{inj} are the oil price, the water production cost and the water injection cost in $\frac{\$}{m^3}$ respectively. K represents the production life-cycle i.e., the total number of time steps k and Δt_k the time interval of time step k in days. The term b is the discount rate for a certain reference time τ_t. The terms q_o, k, q_w, k and $q_{inj,k}$ represent the total flow rate of produced oil, produced water and injected 3 water at time step k in $\frac{m^3}{day}$.

The optimization problem with the objective function defined in Eq. (3) can be formulated as follows:

$$\begin{aligned} &\max_{u_k} J(\mathbf{u}_k, \mathbf{y}_k(\mathbf{u}_k)), \\ &s.t. \quad \mathbf{g}_k(\mathbf{u}_k, \mathbf{x}_k, \mathbf{x}_{k-1}, \boldsymbol{\theta}) = \mathbf{0}, \quad \mathbf{y}_k = \mathbf{h}(\mathbf{u}_k, \mathbf{x}_k), \quad \mathbf{x}_0 = \bar{\mathbf{x}}_0, \\ &\mathbf{c}(\mathbf{u}_k, \mathbf{y}_k) = \mathbf{0}, \quad \mathbf{d}(\mathbf{u}_k, \mathbf{y}_k) \leq \mathbf{0}. \end{aligned}$$

where $c(\mathbf{u}_k, \mathbf{y}_k)$ and $\mathbf{d}(\mathbf{u}_k, \mathbf{y}_k)$ are equality and inequality constraints respectively. Several approaches are available in the literature to solve such large-scale dynamic optimization problems in the oil reservoir domain, see e.g., Brouwer and Jansen (2004), Ciaurri et al. (2011). In this work, a gradient-based optimization approach is used where the gradients $\left(\frac{dJ}{d\mathbf{u}_k}\right)^\top$ are obtained by solving a system of adjoint equations, see e.g., Jansen (2011), Sarma et al. (2005). The gradient information is then used in the steepest ascent algorithm to iteratively converge to the (possibly local) optimum as

$$\mathbf{u}_k^{i+1} = \mathbf{u}_k^i + \alpha \left(\frac{dJ}{d\mathbf{u}_k}\right)^\top, \quad k = 1, \cdots, K \tag{4}$$

where α is the step size of the algorithm and i is the iteration counter; see Wright and Nocedal (1999).

In the next sub-sections, the theoretical foundation of the robust optimization (RO) and the mean-variance optimization (MVO) method will be discussed in details.

Robust optimization (RO)

Van Essen et al. (2009) used a robust approach by considering an average NPV over an ensemble of possible geological realizations. The average NPV is defined as

$$J_{RO} = \frac{1}{N_{geo}} \sum_{i=1}^{N_{geo}} J_i, \tag{5}$$

where N_{geo} is the number of model realizations in an ensemble. J is the NPV as defined in Eq. (3). From this formulation, it follows that calculating the gradient of the average NPV involves a linear operation. Hence, the gradient ∇_{JRO} can be computed as:

$$\nabla J_{RO} = \frac{1}{N_{geo}} \sum_{i=1}^{N_{geo}} \nabla J_i, \tag{6}$$

where ∇J_i is the gradient of i^{th} realization in the ensemble. This approach considers the economic conditions to be fixed and certain.

RO with economic uncertainty

The above defined robust objective in Eq. (5) can easily be extended with an ensemble of oil price scenarios. With a single geological reservoir model realization, the robust objective equivalent for economic uncertainty is given as

$$J_{RO} = \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} J_i, \tag{7}$$

where N_{eco} is the number of oil price scenarios. One important point to consider here is that due to the linearity of the oil price in the NPV objective function in combination with the certainty of the geological model, the average of the individual objective functions from each realization is equal to a single objective function with an average value of all oil price realizations as shown below:

$$\frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} [J(\mathbf{u}_k, \eta_i)] = J(\mathbf{u}_k, \frac{1}{N_{eco}} \sum_{i=1}^{N_{eco}} \eta_i) \tag{8}$$

where η_i is the i^{th} oil price realization in the ensemble. The above relationship does not generally hold true for RO with geological uncertainty. Hence, unlike the case of geological uncertainty, where the gradient calculations require an extra N_{geo} simulations, the computational complexity of performing RO with economic uncertainty is relatively less involved.

RO in both cases incorporates uncertainty in the optimization framework, but it does not aim to reduce the sensitivity of the solution to uncertainty. In other words: it does not minimize the effect of the uncertainty. The uncertainty in the model parameters and in oil prices is mapped to the obtained NPV distribution by solving a robust optimization problem as in Eq. (5) and Eq. (7) respectively. Therefore, the higher the uncertainty, the bigger the spread of the NPV distribution and vice versa. This seriously limits the performance of RO to handle uncertainty.

Mean-variance optimization

Markowitz has introduced a portfolio selection approach, where a 'return' is maximized while minimizing the 'risk' associated with it, see Markowitz (1952). This approach involves a quantitative characterization of risk in terms of the variance of the returns. Based upon the investor's attitude towards risk, a risk-return profile is selected. Later, in literature various ways have been introduced to characterize risk, e.g., percentile based risk measures like Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), see Rockafellar (2007). Capolei (2013) has applied a mean-variance optimization (MVO) approach with model parametric uncertainty and with fixed economic conditions. The return is considered as the average NPV while risk is characterized with the spread or variance of the NPV distribution. The MVO reduces the spread or variance of the NPV distribution and hence it will reduce the sensitivity of the optimal solution to uncertainties. The mean-variance objective function J_{MV} for an ensemble of N_{geo} models can be defined as:

$$J_{MV} = J_M - \gamma J_V. \tag{9}$$

where J_M and J_V are the mean NPV and the variance of NPV respectively while γ is a weighting factor on the variance term. As mean and variance have different units, γ plays a dual role of scaling as well i.e., $\gamma_{weighting} \times \gamma_{scaling} = \gamma \cdot J_M$ is the same as J_{Ro} (Eqs. (5) or (7)), while the variance of the NPV is given as follows:

The gradient ∇J_{MV} can then be computed as:

$$\nabla J_{MV} = \nabla J_M - \gamma \nabla J_V, \tag{10}$$

where the gradient of the mean ∇J_M is given in Eq. (6), while the gradient of the variance ∇J_V is given as follows:

$$\nabla J_V = \frac{1}{N_{geo}-1}\sum_{i=1}^{N_{geo}}\left[\nabla(J_i - J_M)^2\right],$$

$$\nabla J_V = \frac{2}{N_{geo}-1}\sum_{i=1}^{N_{geo}}\left[(J_i - J_M)\nabla(J_i - J_M)\right],$$

$$\nabla J_V = \frac{2}{N_{geo}-1}\sum_{i=1}^{N_{geo}}\left[(J_i - J_M)(\nabla J_i - \nabla J_M)\right].$$

This approach can be extended with economic uncertainty by replacing averaging over the ensemble of model realizations N_{geo} to the averaging over the oil price scenarios N_{eco} as well.

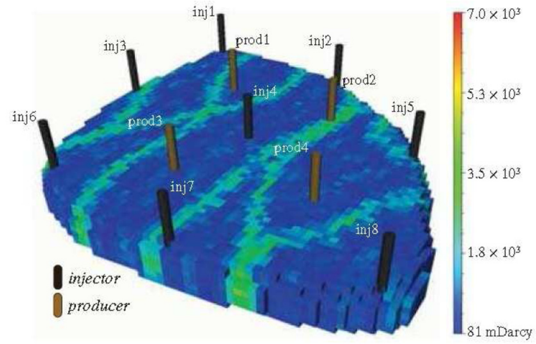


Figure 1—Permeability field and well locations of realization 1 of a set of 100 realizations (Van Essen et al. (2009)).

Results and Discussion

All the simulation experiments are performed using the Matlab Reservoir Simulation Toolbox (MRST), see Lie et al. (2012), which is a MATLAB based reservoir simulator. In the following sub-section, implementation of the RO and the MVO approaches with geological uncertainty and fixed economic conditions are presented closely following the study by Van Essen et al. (2009). Thereafter, simulation examples of the extension of these approaches with economic uncertainty and a single geological realization will be discussed in detail.

Simulation example with geological uncertainty

Reservoir models Based on the geological insight, an ensemble of 100 geological realizations of the 'standard egg model', see Jansen et al. (2013), is generated. The true permeability field is considered to constitute the set of unknown parameters, and the number of 100 realizations is assumed to be large enough to be a good representative of this parametric uncertainty space. Each model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers. The life-cycle of each reservoir model is 3600 days. The absolute-permeability field and the well locations of the first realization in the set are shown in Fig. 1.

Fig. 2 shows the permeability fields of six randomly chosen realizations of the standard egg model in an ensemble of 100 realizations. Each realization in the set is considered as equiprobable.

Economic data for NPV In this example, all economic parameters are considered as certain and fixed. An un-discounted NPV i.e., with discount factor $d = 0$ is used. Other economic parameters e.g., oil price r_o, water injection r_{inj} and production cost r_w are chosen as $126\frac{\$}{m^3}$, $6\frac{\$}{m^3}$ and $19\frac{\$}{m^3}$ respectively. We note that the oil price is much lower than the typical present day value. However we chose to use the same value as applied by Van Essen et al. (2009) to allow for a comparison of our results with theirs.

Control input The control input $\mathbf{u}_k$ involves injection flow rate trajectories for each of the eight injection wells. The minimum and maximum rate for each injection well are set as $0.2\frac{m^3}{day}$ and $79.5\frac{m^3}{day}$ respectively. The production wells operate at a constant bottom-hole pressure of $395 bar$. The control input $\mathbf{u}_k$ is reparameterized in control time intervals with input parameter vector $\boldsymbol{\varphi}$. For each of the eight injection wells, the control input $\mathbf{u}_k$ is reparameterized into twenty time periods of t_φ of 180 days during which the injection rate is held constant at value $\boldsymbol{\varphi}_i$. Thus the input parameter vector $\boldsymbol{\varphi}$ consists of $8 \times 20 = 160$ elements.

Control strategies The robust optimization (RO) approach is compared to a nominal optimization (NO) approach which is based on one realization, and to the reactive strategy. In the reactive control (RC)

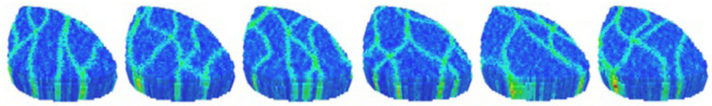
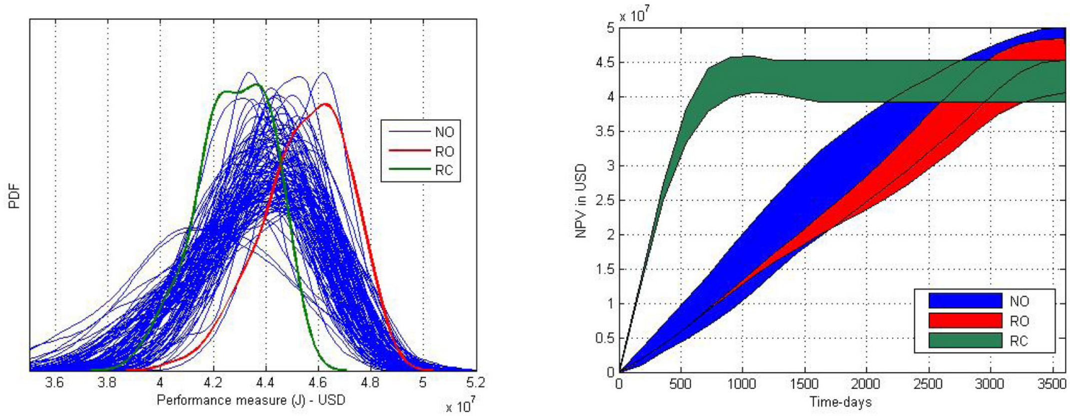
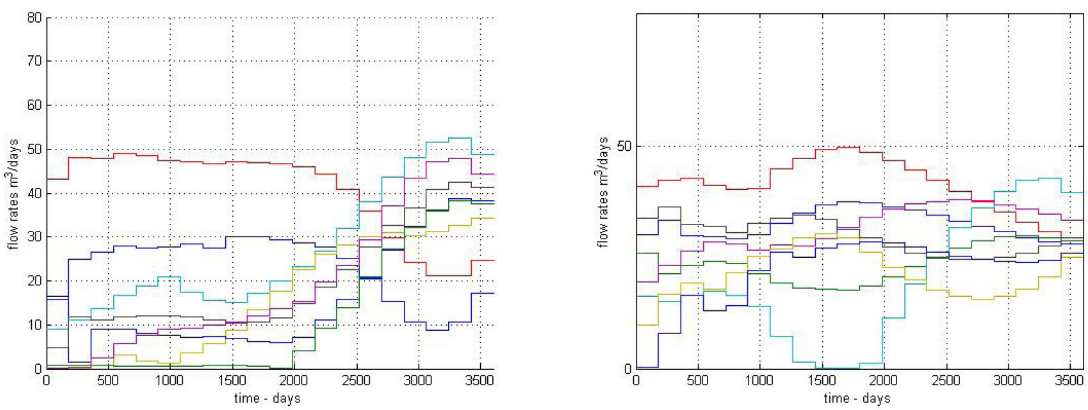


Figure 2—Permeability fields of 6 randomly chosen realizations (Van Essen et al. (2009)).



(a) PDF based on nominal, reactive and robust strategies (b) Max and min (band) for time-evolution of NPV

Figure 3—Results comparison for the three control strategies i.e., RO, NO and RC approaches with geological uncertainty



(a) Optimal inputs for RO (b) Optimal inputs for MVO

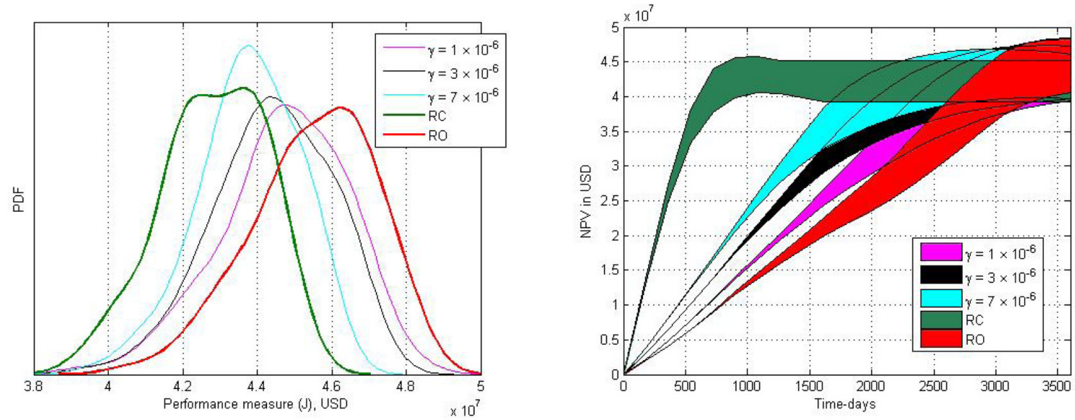
Figure 4—Optimal input trajectories for each of eight injection wells with geological uncertainty

strategy, water is injected at the maximum rate and each production well is simply shut-in when the production is no longer profitable. Here the profitability threshold corresponds to a water-cut of 87%.

Results for RO

The simulation example in Van Essen et al. (2009) is reproduced. A gradient-based optimization procedure as discussed in the model-based optimization sub-section is used with a line search to find the optimal step size along the direction of the greatest ascent. The optimal flow rates for all injection wells are shown in Fig. 4a.

Fig. 3a (similar to the one in Van Essen et al. (2009)) depicts the probability distribution function (PDF) of the NPV resulting from 100 nominal optimizations, RO and RC strategies. An important point to note is that the uncertainty in the model parameters is mapped to the RO NPV distribution. A larger uncertainty in the parameters will result in a higher variance of the distribution and vice versa. Therefore, RO does incorporate the uncertainty in the optimization framework and shows its effect on the optimized strategy, but it does not reduce sensitivity of the solution to the uncertainty.



(a) **PDF based on robust, reactive and mean-variance strategies.**

(b) **Max and min (band) for time-evolution of NPV**

Figure 5—Results comparison for the three control strategies i.e., MVO, RO and RC approaches with geological uncertainty

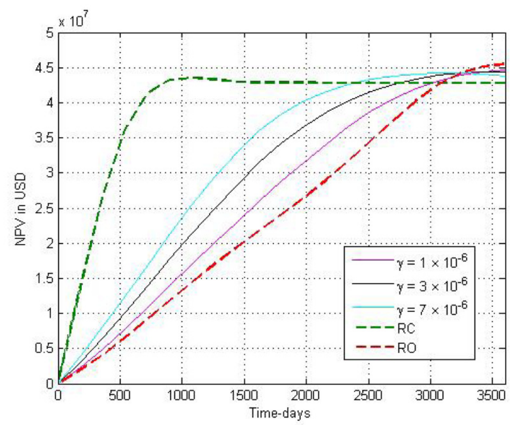


Figure 6—Average values for the time-evolution of NPV

The time evolution of NPV for all three strategies is compared in Fig. 3b. The 100 NO strategies from each model are applied to themselves, while the RO and RC strategies are applied to the set of 100 models. The maximum and minimum values of time-evolution of NPV will form a band. The width of the band shows the variability of a strategy over the ensemble of model realizations in terms of the time evolution of NPV.

In order to evaluate the questions discussed in the Introduction section, RO forms the answer to the first question of explicitly incorporating uncertainty in the optimization framework. But due to its poor uncertainty handling it provides a poor balancing of short-term and long-term objectives. Due to the aggressive nature of the reactive strategy a superior performance in terms of short-term gains can be observed but at the cost of reduced long-term gains. None of the NO strategies include uncertainty and therefore they result in the typical slow build-up of NPV with better performance in terms of long-term gains than the reactive strategy.

A mean-variance (MVO) objective provides better handling of uncertainty as it not only maximizes the average NPV but also minimizes the variance of the NPV distribution as discussed in the next sub-section.

Results for MVO

The mean-variance optimization (MVO) as used in Capolei (2013) and explained by Eq. (9) is implemented with the same ensemble of model realizations, economic data and control inputs. Different values of the weighting and scaling parameter γ, i.e., $\gamma \in [1 \times 10^{-6}, 3 \times 10^{-6}, 7 \times 10^{-6}]$, are used. Fig. 4b shows the optimal inputs results from the MVO optimization with $\gamma = 7 \times 10^{-6}$. It can be observed that the optimal inputs are initially more aggressive compared to the optimal inputs obtained by RO as depicted in Fig. 4a.

MVO optimal strategies obtained using different values of γ are applied to each member of the set of model realizations, resulting in 100 NPV values for each γ. The corresponding PDFs are obtained by approximating a non-parametric Kernal density estimation (KDE) with MATLAB routine *'ksdensity'* on these NPV data values as shown in Fig. 5a. The PDFs obtained from the RO and the RC strategies are also shown in Fig. 5a. NO strategies are not compared for the sake of clarity. The first observation is that

Table 1—RESULTS FOR GEOLOGICAL UNCERTAINTY

Control strategies	Average NPV at day = 720	Increase from RO	Average NPV at day = 3600	Decrease from RO
RO	9.2 million USD	-	45.5 million USD	-
MVO for $\gamma = 1 \times 10^{-6}$	10.8 million USD	18.35%	44.7 million USD	1.71%
MVO for $\gamma = 3 \times 10^{-6}$	13.9 million USD	51.92%	44.4 million USD	2.42%
MVO for $\gamma = 7 \times 10^{-6}$	16.9 million USD	84.39%	43.7 million USD	3.78%
RC	41.3 million USD	350.09%	42.8 million USD	6.31%

the variance is reduced with increasing values for weighting parameter γ: the higher the value of γ, the lower the variance. The lower variance is achieved at the cost of compromising the average NPV. As the uncertainty is mapped to the spread of the NPV distribution, the reduction of variance reflects the reduction of sensitivity of the strategy to the uncertainty. Hence MVO aims to mitigate the negative effect i.e., risk of the uncertainty. Beside this variance as a risk measure, there are other percentile based risk measures, e.g., Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), that provide better uncertainty handling and risk management. However, as the main theme of this work is not the shaping of the NPV distribution, only variance is considered as the risk measure.

The time-evolution of NPV with all three strategies i.e., MVO, RO and RC are compared in Fig. 5b. MVO has an intrinsic way of reducing the sensitivity of the solution to uncertainty and hence provides better handling of uncertainty. It can be observed that, compared to the RO strategy, all MVO strategies provide a faster build-up of NPV over time (high short-term gains) but at the cost of compromising long-term gains. It can also be observed that the value of γ affects the rate of NPV build-up with a corresponding reduction of long-term gains. In portfolio optimization, the selection of γ provides a way to choose a return-risk profile as per investor's interest. Here it also plays a role of a tuning parameter to balance short-term and long-term economic according to the investor choice. Hence the MVO approach forms both the answers of incorporating uncertainty in the optimization framework and also with the higher short-term gains indicates high confidence on the strategy. From another perspective, the higher short-term gains also reflects the mitigation of risk by improving the rate of oil production. For the sake of clarity of results, the average values for the time-evolution of NPV for all three strategies are compared in Fig. 6.

The results are also summarized in the Table. 1. The table shows the average NPV obtained for all the control strategies at 720 days of oil production and the average NPV obtained at the end of the simulation period, i.e., 3600 days. An increase of average NPV at day 720 and a decrease at the end of the simulation period compared to RO are shown. The first observation is that RO has the lowest average NPV at day 720, while it results in the maximum average NPV at the end of the simulation period compared to other strategies. Reactive control has a maximum increase in the short-term gains but with a decrease of 6.31% compared to RO. For the MVO strategy, as discussed before, γ provides an explicit way of balancing short-term and long-term objectives and hence becomes a tuning parameter for balancing both objectives. A higher value of γ will result in a faster build-up of NPV at the cost of compromising the final NPV.

In the next sub-section, the effect of economic uncertainty in balancing short-term and long-term objectives are studied with a simulation example.

Simulation example with economic uncertainty

Economic variables such as oil prices, interest rates etc., are involved in different ways to quantify the economic value of oil and gas reserves. These variables fluctuate with time and can not be precisely predicted. This economic uncertainty has a time-varying dynamic nature. As oil price has a dominant effect on the NPV compared to other uncertain economic variables, only varying oil price scenarios are considered as uncertainty.

Reservoir model As the purpose of this simulation example is to show the effect of only economic uncertainty on the optimal strategy, a single realization i.e., the first realization of the ensemble of the standard egg model as shown in Fig. 1 is used as the reservoir model.

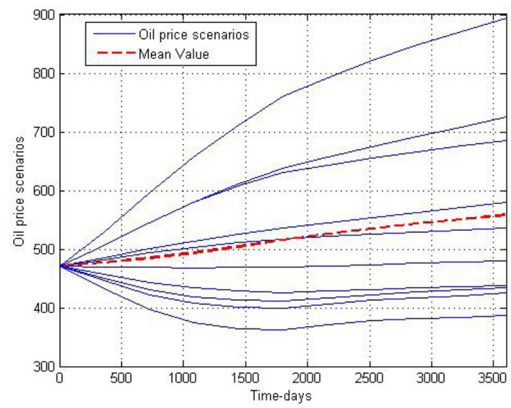


Figure 7—Oil price scenarios (solid colored) with the mean oil price values (dashed red).

Economic data An un-discounted NPV i.e., with discount factor $d = 0$ is used. Other economic parameters e.g., water injection r_{inj} and production cost r_w are chosen and kept fixed at $23\frac{\$}{m^3}$ and $72\frac{\$}{m^3}$ respectively. According to today's oil price, a more realistic base line oil price value of $471\frac{\$}{m^3}$ is used. There are various ways to predict the future values of changing oil prices, but for this example a simplified auto regressive-moving-average (ARMA) model, see Ljung (1999), is used to generate oil price time-series. The ARMA model is:

$$r_{o_k} = a_0 + \sum_{i=1}^{6} a_i r_{o_{k-i}} \tag{11}$$

where α_i are randomly selected coefficients.

A total of 10 scenarios i.e., $N_{eco} = 10$ are generated as depicted in Fig. 7.

Control input The control input u_k is the same as used in the previous simulation example except that it is reparamterized into ten time periods of t_φ of 600 days instead of twenty time steps as before. Thus the input parameter vector φ in this example consists of $8 \times 10 = 80$ elements.

Control strategies Nominal optimization (NO) based on one oil price scenario is not considered in this example. The simulations are performed with MVO approach and compared with RO and the RC strategies.

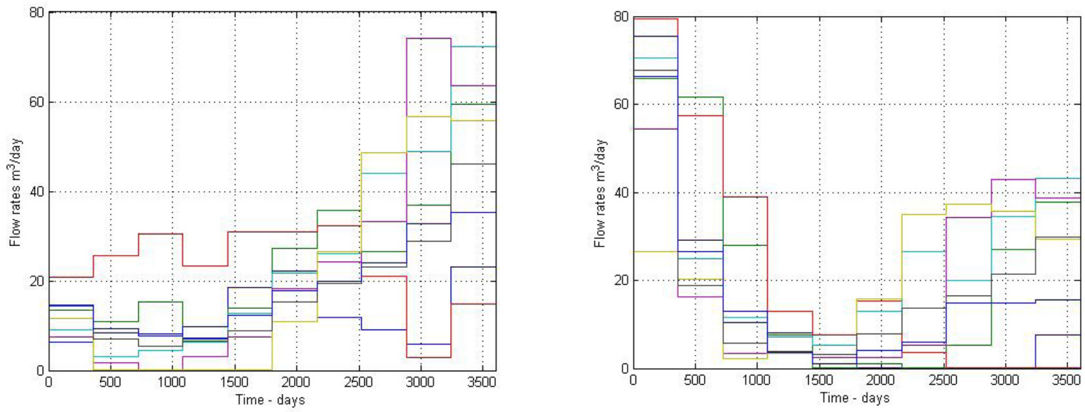
Results for RO

The extended robust optimal control strategy with economic uncertainty is determined using the same gradient-based optimization procedure as mentioned in the Model-based optimization section. The optimal flow rates for all injection wells are shown in Fig. 8a.

Results for MVO

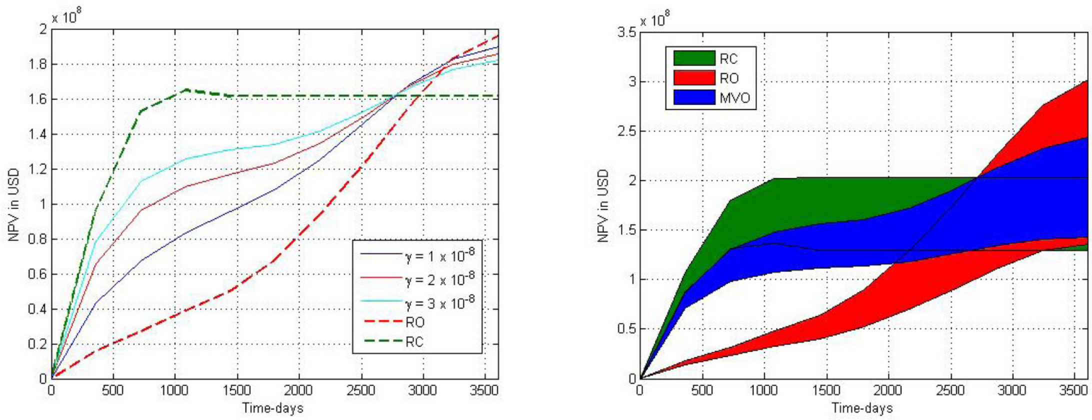
The mean-variance optimization approach with economic uncertainty is implemented with 10 different oil price scenarios as shown in Fig. 7. Reservoir model realization and the control inputs are the same as used in the RO case. Different values of γ, i.e., $\gamma \in [1 \times 10^{-8}, 2 \times 10^{-8}, 3 \times 10^{-8}]$ are used. Fig. 8b shows the optimal inputs resulting from the MVO optimization with $\gamma = 3 \times 10^{-8}$. It can be observed that the inputs are initially more aggressive compared to the optimal inputs obtained by RO as in Fig. 8a

RO and RC strategies are applied to the chosen model realization with each oil price scenario resulting in 10 different NPVs. The time evolution of the NPV with these strategies, i.e., RO and RC, are compared in Fig. 9b. The maximum and minimum values of the time-evolution of NPV form a band. The width of the bands clearly shows that the economic uncertainty, i.e., the varying oil prices, has a very profound effect on the obtained NPV. The large uncertainty in the oil price scenarios is mapped to the large spread of the NPV bands. All three MVO strategies, i.e., with $\gamma \in [1 \times 10^{-8}, 2 \times 10^{-8}, 3 \times 10^{-8}]$ are also applied to the single model realizations with 10 different oil price scenarios. This results in three different bands for each MVO strategy, but for the sake of clarity only one time evolution band, corresponding to the NPV for $\gamma = 3 \times 10^{-8}$ is shown in Fig. 9b.



(a) **Optimal inputs for RO** (b) **Optimal inputs for MVO**

Figure 8—Optimal input trajectories for each of eight injection wells with economic uncertainty



(a) **Average values for the time-evolution of NPV** (b) **Max and min (band) for time-evolution of NPV**

Figure 9—Results comparison for the three control strategies i.e., MVO, RO and RC approaches with economic uncertainty

Table 2—RESULTS FOR ECONOMIC UNCERTAINTY

Control strategies	Average NPV at day = 720	Increase from RO	Average NPV at day = 3600	Decrease from RO
RO	26.9 million USD	-	195.8 million USD	-
MVO for $\gamma = 1 \times 10$-^{8}	67.6 million USD	151.3%	189.9 million USD	3.0%
MVO for $\gamma = 2 \times 10$-^{8}	96.2 million USD	257.6%	185.8 million USD	5.1%
MVO for $\gamma = 3 \times 10$-^{8}	112.6 million USD	318.5%	181.9 million USD	7.1%
RC	152.7 million USD	467.6%	161.2 million USD	17.6%

Fig. 9a shows the average NPV values of the bands. The results for the MVO strategy for three different γ, i.e., $\gamma \in [1 \times 10^{-8}, 2 \times 10^{-8}, 3 \times 10^{-8}]$ are also shown in Fig. 9a. With MVO approaches, an improvement in the short-term gains compared to the RO case can be observed. Similar to the case of geological uncertainty, γ becomes a tuning parameter to balance short-term and long-term gains.

The results are also summarized in the Table. 2. Similar to Table. 1, the table shows the average NPV obtained for all the control strategies at 720 days of oil production and the average NPV obtained at the end of the simulation period of the reservoir model, i.e., 3600 days, with percentage increase and decrease compared to RO. As with the geological uncertainty, RO has a lowest average NPV at day 720 with the maximum average NPV at the end of the simulation period compared to the other strategies. Reactive control has a maximum increase in the short-term gains, but with a decrease of 17.6% compared to RO, RC reaches its maximum NPV, i.e., 161.2 million USD after approximately 2 years of production. Economic uncertainty has a profound effect on NPV optimization compared to the geological uncertainty, as with the MVO strategy with $\gamma = 3 \times 10^{-8}$ an increase of 318.5% on short-term gains can be achieved at the cost of a 7.1% decrease on final NPV, compared to a reactive strategy with increase of 467.6% on the short-term with a significant drop of 17.6% on long-term gains. Here again, γ provides an explicit way of balancing short-term and long-term objectives and hence provides decision makers a tuning parameters for balancing both objectives.

Conclusions

Model-based NPV optimization suffers from high levels of uncertainty in the models that are being used. Even when the optimization is performed over an ensemble of geological realizations (robust optimization (RO)), the optimized trajectory shows a relatively slow build-up of NPV over time. This phenomenon does not help in convincing production staff to follow this (initially cautious) strategy. Different ad-hoc strategies have been proposed to include short-term gains in the long-term NPV optimization, such as hierarchical and multiobjective approaches. In this paper we have evaluated whether a more pronounced handling of model and economic uncertainty in the optimization will automatically lead to an appropriate balance between long-term and short-term economic objectives, and put more emphasis on a faster build-up of NPV over time. As a start a robust approach with an average NPV over the ensemble of model realizations is considered. RO incorporates uncertainty in the optimization framework but does not minimize the effect of uncertainty. Therefore as an improvement, a mean-variance optimization framework is implemented. With the maximization of average NPV, MVO also reduces the variance of the NPV distribution. The MVO approach shows a slight increase of short-term gains but not at a convincing level yet. Later on economic uncertainty is introduced, where an ensemble of oil price scenarios characterize the economic uncertainty. The MVO approach with economic uncertainty results in a significant improvement in the short-term gains at the cost of slightly compromising the long-term gains.

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