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A PRACTICALLY FEASIBLE PROCEDURE FOR IDENTIFICATION OF $\mathcal{H}_\infty$ ERROR BOUNDS

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Abstract

A multistep procedure is presented for identification, on the basis of measured time sequences of input and output data, of a system uncertainty set in terms of a low order nominal model, together with an upper bound on a weighted $\mathcal{H}_\infty$ model error. The procedure is aimed at arriving at a feasible solution with limited conservatism, where all steps are directed to obtain an uncertainty set that allows the design of a high-performance robust controller. It contains a step of unfalsified modelling, and of system approximation in $\mathcal{H}_\infty$, and relies on simple mathematical algorithms. By starting off with a set of not-invalidated systems, it can handle both unknown-but-bounded and stochastic type of priors on the disturbance signals.

1 Introduction

In this paper we discuss and present a procedure for identification of a nominal model and an $\mathcal{H}_\infty$ error bound reflecting the uncertainty concerning the data generating system, aimed at application of the obtained results as a basis for model-based robust control design.

Identification of models and model uncertainties in view of a consecutive model-based (robust) control design is an interesting and practically relevant research topic, that has gained considerable attention lately. The identification of control-relevant -nominal- models has been subject of research, and hes led to the introduction of iterative schemes of closed-loop identification and control design, see e.g. the survey papers [6, 21]. In this approach the question is addressed how to arrive at nominal linear models that best capture that part of the plant dynamics that is essential for arriving at a high-performance controlled plant.

A second line of research has been to pay specific attention to the identification of model uncertainty bounds. This means that besides a nominal plant model, an accurate specification of the model uncertainty is obtained, that can be handled in a robust control design. Among other things this has led to an area denoted as worst-case identification or $\mathcal{H}_\infty$ identification, being based on an unknown-but-bounded assumption on the noise disturbance. The problem has been mostly handled in terms of frequency response data; in the case of time-domain data, a deterministic/worst-case approach is often referred to as set-membership identification [17] or as parameter bounding identification. Several norms are used to outerbound the obtained uncertainty sets, as e.g. in terms of frequency response magnitude [23], $\mathcal{H}_\infty$-norms [15], and ℓ_1-norms [16], An interesting approach in terms of model invalidation, characterizing a set of unfalsified models, is presented in [19].

An important draw-back of the worst-case approach is that due to the worst-case character of the assumed disturbances, the obtained system uncertainty regions generally are very conservative, if this worst-case disturbance does not actually occur [14, 18]. Additionally, uncertainty bounding procedures often lack the specification of a (low order) nominal model, that is generally imperative for consecutive (robust) control design.

Approaches that consider disturbance signals to be stochastic, but that also account for undermodelling are given in [1, 7, 5]. Here the methods of [1, 5] require periodic excitation in order to be applicable.

In this paper we will present an approach to the problem, directed towards the use of time-domain data, without imposing additional requirements on the experimental setup. We will allow the specification of either unknown-but-bounded or stochastic priors on disturbance signals, and we will aim at computationally feasible solutions to the problem. This problem will be the construction of a set of identified models, induced by a low order nominal model and an error bound on a specifically weighted $\mathcal{H}_\infty$ model error.

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The procedure is designed in such a way as to reduce the amount of uncertainty finally obtained, striving towards an identification result that provides uncertainty bounds that can indeed be used as a basis for control design.
The method presented in this paper will only be discussed in main lines. For details the reader is referred to [8].

2 Problem formulation

We consider a data generating system:

$$y(t) = G_0(q)u(t) + v(t) \tag{1}$$

with $\{u(t), y(t)\}_{t=1,\cdots N}$ available samples from scalar input and output signals, $G_0 \in \mathcal{A}$ the class of proper bounded-input bounded-output (ℓ_∞-stable) causal linear time-invariant systems, q the forward shift and v a disturbance signal.
It can be verified that $\mathcal{A} \subset \mathcal{H}_\infty$. The presented framework and solution strategy will allow the consideration of several types of disturbance signals, either stochastic or (worst-case) norm-bounded.

The problem to be treated in this paper can be formulated as:

Problem formulation: Given measurement data $\{u(t), y(t)\}_{t=1,\cdots N}$ from the process G_0, according to (1), and given some prior information concerning this data generating system, determine an uncertainty set

$$\mathcal{G}(\widehat{G}, W, b) = \{\widetilde{G} \in \mathcal{A} \mid \|W(z)[\widehat{G}(z) - \widetilde{G}(z)]\|_\infty \leq b\} \tag{2}$$

such that (it is very likely that) $G_0 \in \mathcal{G}(\widehat{G}, W, b)$.

In this problem formulation, $\widehat{G} \in \mathcal{A}$ is a (nominal) model of reduced complexity (small order), W is a user-defined stable and stably-invertible weighting function that shapes the uncertainty structure, and $b \in \mathbb{R}$. Note that the question whether the expression $G_0 \in \mathcal{G}(\widehat{G}, W, b)$ is absolute (100% certainty) or probabilistic is completely dependent on the type of assumption that is made on the noise disturbance v.
In our considerations concerning the system G_0 we will regularly employ a representation of this system in the form of a series expansion in terms of specific basis functions. In this formulation we restrict G_0 to belong to either of the spaces $\mathcal{H}_2$ or $\overline{\mathcal{RH}_\infty}$, the latter space being the closure of $\mathcal{RH}_\infty$ with respect to the $\mathcal{H}_\infty$-norm. Let $\{F_k(z)\}_{k=0,\cdots}$ be a pre-chosen sequence of basis functions in either $\overline{\mathcal{RH}_\infty}$ or $\mathcal{H}_2$, then there exists a unique series expansion:

$$G_0(z) = \sum_{k=0}^{\infty} g_0(k) F_k(z). \tag{3}$$

In terms of $\mathcal{H}_2$, the basis functions $F_k(z)$ can simply be chosen as the classical basis z^{-k}; however more generalized choices such as Laguerre, Kautz and generalized versions like the recently introduced Hambo functions [13, 22] can fruitfully be used here. Generalized versions of basis functions can lead to a substantial increase of the speed of convergence in (3). As prior information concerning the data generating system we will consider the following:

- For a specific choice of basis functions $\{F_k(z)\}_{k=0,\cdots}$, $g_0(k)$ is assumed to be bounded by

$$\begin{aligned} |g_0(k)| &\leq \bar{g}(k) \text{ for all } k > k^* && (4)\\ \text{with } \bar{g}(k) &\leq M\rho^k && (5)\end{aligned}$$

  for some given $k^* > 0$, $M > 0$ and $\rho < 1$.
- $|u(t)| \leq \bar{u} \;\; \forall t \leq 0$
- $v(t)$ is either norm bounded, e.g. $|v(t)| \leq \bar{v}$, $\bar{v} \in \mathbb{R}$, or is a zero-mean stationary stochastic process. This prior will be specified later on.

Given the measurement data and the prior information concerning the data generating system, there exists a subset $\mathcal{F} \subset \mathcal{A}$ of systems that are unfalsified (not invalidated) by the data. This means that on the basis of the available information, all input-output systems within the set $\mathcal{F}$ could have generated the data. The first problem to treat will be to describe in some parametrized form, this set of unfalsified models $\mathcal{F}$ as accurate as possible.
Next an uncertainty set $\mathcal{G}(\widehat{G}, W, b)$ as in (2) has to be constructed based on appropriate choices of $\widehat{G}$, W and b, such that it covers $\mathcal{F}$, i.e. $\mathcal{F} \subset \mathcal{G}(\widehat{G}, W, b)$. However, in this latter step the embedding of $\mathcal{F}$ in $\mathcal{G}(\widehat{G}, W, b)$ should be as tight as possible. The larger the set $\mathcal{G}(\widehat{G}, W, b)$ will be, the more conservatism is added to the obtained uncertainty set, and the less powerful the result will be. Note that because of the specific structure of $\mathcal{G}(\widehat{G}, W, b)$, it is unavoidable that conservatism is added, and so there will always be systems in $\mathcal{G}(\widehat{G}, W, b)$ that are not an element of $\mathcal{F}$.
In terms of the control relevance of the obtained uncertainty set $\mathcal{G}(\widehat{G}, W, b)$, it is advantageous to have the conservatism added to the set in those system-properties where it has only a small impact on the control objective. In other words, if possible only little conservatism should be added in the closed-loop relevant frequency range, such as near the cross-over frequency. A larger than necessary upper bound on the model error in those frequencies would unnecessarily retrict the achievable robust control performance, and as such the achievable control performance for the system G_0.
The approach sketched above leads to the handling of two more or less separate problems:

(a) Construction of (an accurate approximation of) the set of unfalsified systems $\mathcal{F} \subset \mathcal{A}$.

(b) Construction of an uncertainty set $\mathcal{G}(\widehat{G}, W, b)$ that contains $\mathcal{F}$, and adds as little conservatism as possible.

In the next section a closer look will be taken at these two problems.

3 Main lines of a feasible solution

It will be indicated how a solution can be obtained for the problems formulated in the previous section. Only the main lines will be indicated here. Further details on the respective methods will be discussed later.

Based on the measurement data and the available prior information on the data generating system it will generally be impossible to formulate a closed-form expression for the set of unfalsified systems $\mathcal{F} \subset \mathcal{A}$. However accurate approximations of this set, using specific parametrized forms, are very well possible. In our approach we will construct a system uncertainty set $\mathcal{S} \supset \mathcal{F}$ of the following form

$$\mathcal{S} = \{G \in \mathcal{A} \mid G(e^{i\omega_j}) \in \mathcal{P}(\omega_j),\ \omega_j \in \Omega, \left|\frac{dG(e^{i\omega})}{d\omega}\right| \leq \beta_1 \ \ \forall \omega \in [0,\pi]\}. \tag{6}$$

where Ω is a user-chosen frequency grid, $\Omega := \{\omega_j \in [0,\pi]\}_{j=1,\cdots n_\omega}$, and $P(\omega_j)$ determines a convex region in the complex plane for each $\omega_j \in \Omega$.

The set $\mathcal{S}$ is constructed to reflect principal uncertainty that results from the measurement data and the prior information concerning the data generating system G_0. The parametrization of $\mathcal{S}$ is quite specific. An alternative in terms of pulse response parameters with related uncertainty intervals is worked out in [8].

Now suppose a system uncertainty set $\mathcal{S}$ is available. For any given nominal model $\widehat{G}(z)$ the following bounds $\delta(\omega_j)$ can easily be established,

$$\delta(\omega_j) = \max_{G(\omega_j) \in \mathcal{P}(\omega_j)} \left|G(\omega_j) - \widehat{G}(e^{i\omega_j})\right|,$$

which is a frequency dependent upper bound on the additive model error. Now $\delta(\omega_j)$, should be translated into a parametric upper bound, represented by a filter $W(z)$ and a scalar b, the latter which can be set to 1 without loss of generality.

Approximation of $\delta(\omega_j)$ by a finite-dimensional representation, introduces conservatism, and the key point now comes down to the construction of an appropriate weighting function W. In [20] a procedure is proposed for constructing a stable, minimum-phase $W(z)$ of prespecified order, by solving the optimization problem

$$\min_{W(z)} \max_{\omega_j \in \Omega} \left| \left(\left|W^{-1}(e^{i\omega_j})\right|^2 - \delta^2(\omega_j) \right) W_1^2(\omega_j) \right|,$$

subject to the constraint that $\left|W^{-1}(e^{i\omega_j})\right| \geq \delta(\omega_j)$. This latter condition guarantees that the uncertainty description remains correct; $W_1(\omega_j)$ is a weighting function which indicates the importance of having tight upper bounds over the several frequency regions. A frequency region where little conservatism is allowed is reflected by a high value of the weighting W_1.

However, the above problem formulation is stated for a given nominal model $\widehat{G}$. Actually, we have to deal with a *global* objective, optimizing over both $\widehat{G}$ and W. This global problem can be read as:

$$\min_{\widehat{G},W} \max_{\omega \in [0,\pi]} \left|W^{-1}(e^{i\omega}) W_1(e^{i\omega})\right| \text{ s.t.}$$

$$\left|W^{-1}(e^{i\omega})\right| \geq \max_{G \in \mathcal{S}} \left|G(e^{i\omega}) - \widehat{G}(e^{i\omega})\right|, \ \forall \omega \in [0,\pi].$$

This global identification problem combines the problem of identifying a good nominal model and the problem of specifying a suitable weighting in the uncertainty structure. The resulting optimization problem however seems to be unfeasible. Instead we will consider two subproblems, which provide an approximate solution.

First, the nominal model $\widehat{G}(z)$ is determined according to

$$\widehat{G}(z) = \arg \min_{\widetilde{G} \in \mathcal{M}} \max_{G \in \mathcal{S}} \|W_1(z)[G(z) - \widetilde{G}(z)]\|_\infty, \tag{7}$$

where $\mathcal{M}$ is a prechosen model set with (parametrized) models of restricted complexity (order).

Next, the uncertainty structure weighting $W(z)$ is determined such that the upper bound on the worst-case model error, $\left|W^{-1}(e^{i\omega})\right|$, is minimized,

$$\min_{W(z)} \max_{\omega \in [0,\pi]} \left|W^{-1}(e^{i\omega}) W_1(e^{i\omega})\right| \text{ s.t.} \tag{8}$$

$$\left|W^{-1}(e^{i\omega})\right| \geq \max_{G \in \mathcal{S}} \left|G(e^{i\omega}) - \widehat{G}(e^{i\omega})\right|, \ \forall \omega \in [0,\pi],$$

with the nominal model $\widehat{G}(z)$ as specified by the previous step (7). This problem corresponds to the problem considered in [20] as discussed above. It will result in an upper bound $b \in \mathbb{R}$ for the model error, i.e.

$$\max_{G \in \mathcal{S}} \|W(z)[G(z) - \widehat{G}(z)]\|_\infty \leq b, \tag{9}$$

where W now reflects the minimizing argument of (8). Note that $\widehat{G}$ from (7), W from the optimization problem (8), and b from (9) constitute the resulting uncertainty set $\mathcal{G}(\widehat{G}, W, b)$.

The main line of the proposed procedure now is:

1 Construct an uncertainty set $\mathcal{S}$ as in (6).

2 Determine a nominal model $\widehat{G}$ according to (7).

3 Determine an optimal weighting W according to (8) so as to reduce conservatism.

4 Calculate upper bound b according to (9).

The several steps in the procedure sketched are all nontrivial. We aim at arriving at approximate solutions for all different steps, where the approximations should not introduce unnecessary conservatism.

In the next three sections we will pay attention to the steps 1, 2 and 4. For step 3 we refer to [20].

4 Identification of a system uncertainty region $\mathcal{S}$

The specification of the set of all systems that are consistent with measurement data and prior information can be done in many different ways. The type of problem that results is very much dependent on the type of measurement data and even more by the type of priors that are available concerning the data generating system. Actually all methods for set membership identification ([17]), and other worst-case (deterministic) identification methods as [19] can be used to provide a system uncertainty set $\mathcal{S}$. However, as most worst-case identification methods take prior assumptions on the noise disturbance v to be (only) norm-bounded, the resulting uncertainty sets are known to be very large and overly pessimistic for situations that the worst-case disturbance does not actually occur. As an alternative, methods that consider noise disturbances to be "noisy" arrive at much smaller and feasible uncertainty sets. Recently introduced methods in [5, 9] consider a combination of worst-case properties (with respect to unmodelled dynamics) and stochastic properties (with respect to noise disturbances).

In this paper, we will employ the procedure introduced in [9], where the signal v is considered to be a realization of a stationary stochastic process.

Consider a high-order linearly parametrized model set:

$$G(z,\theta) = \sum_{k=0}^{n} g(k,\theta)F_k(z) \qquad (10)$$

where the coefficients $\{g(k,\theta)\}_{k=0,\cdots n}$ constitute the unknown parameter vector θ. Based on the measurement data an auxiliary model $\overline{G} = G(z,\widehat{\theta}_N)$ is estimated by least squares (open loop situation) or instrumental variable techniques (closed loop situation). Then it can be found that for all ω_j in a user chosen frequency grid Ω, we can bound the difference between $\overline{G}$ and G_0 asymptotically ($N \rightarrow \infty$) by

$$\begin{aligned} &\left|Re\left(\overline{G}(e^{i\omega_j}) - G_0(e^{i\omega_j})\right)\right| \leq \\ &\quad f_1(e^{i\omega_j},\alpha,R_v) + f_2(e^{i\omega_j},\bar{g},n) + \\ &\quad f_3(e^{i\omega_j},\bar{g},\bar{u}) + f_4(e^{i\omega_j},\bar{g},n) \;\; w.p. \geq \alpha, \end{aligned} \qquad (11)$$

In this result the different terms refer to the following mechanisms:

- f_1 Effect of noise disturbance on the data (variance error); $R_v(\tau)$ is the covariance function of the noise process v;
- f_2 Effect of the unmodelled dynamics (neglected tail in the series expansion (10)) on the parameter estimate $\widehat{\theta}_N$;
- f_3 Effect of unknown initial conditions;
- f_4 Effect of the unmodelled dynamics (neglected tail in (10) itself).

All different terms on the right hand side of (11) can be calculated from data and prior information for each separate $\omega_j \in \Omega$, except for the covariance function $R_v(\tau)$ of the noise v in f_1. In [11, 12] a procedure is presented to estimate this quantity from data such that the corresponding f_1-term is overbounded, and thus the expression (11) remains valid when R_v is substituted by its estimate $\widehat{R}_v$. Expressions similar to (11) are available for bounding the imaginary part of the frequency functions, as well as for a simultaneous bound on both real and imaginary parts, leading to ellipsoidal confidence regions for each separate ω_j. These results directly lead to either rectangular or ellipsoidal uncertainty regions $\mathcal{P}(\omega_j)$ as denoted in (6).

Note that the expression (11) clearly shows and distinguishes between the several sources of uncertainty. As a result it also points to specific actions to take by the experimenter for improved experiment design, in case the obtained uncertainty region is too large. This situation is similar to results obtained in [3, 5].

The derivative of the system's frequency response as formulated in (6), can be bounded by

$$\beta_1 = \sum_{k=0}^{\infty} \bar{g}(k) \| \frac{d\ F_k(z)}{dz} \|_\infty \qquad (12)$$

being available from prior knowledge. For computational aspects we refer to [8].

In this procedure for constructing $\mathcal{S}$, conservatism of the uncertainty region is limited by focusing on direct computational solutions for each separate ω_j rather than on closed-form expressions. However, because of the stochastic nature of the disturbance signal, the upper bound (11) will be valid with a prespecified probability $\alpha < 1$. In our approach we set α to a high level of probability, and consider the resulting $\mathcal{S}$ as an approximation of the set of unfalsified systems. A counterpart procedure on the basis of worst-case unknown-but-bounded disturbance signals is analysed in [8]. Moreover, any other method that delivers a set of unfalsified models structured as (6) can be substituted.

5 Identification of a nominal model

The second step in our procedure is the construction of a nominal model according to

$$\widehat{G}(z) = \arg \min_{\widetilde{G} \in \mathcal{M}} \max_{G \in \mathcal{S}} \|W_1(z)[G(z) - \widetilde{G}(z)]\|_\infty, \qquad (13)$$

where $\mathcal{M}$ is an appropriate set of models. This concerns a worst-case system approximation problem in an $\mathcal{H}_\infty$-setting, for which no tractable solutions are available yet. Instead we will consider an upper bound for the $\mathcal{H}_\infty$-norm to be minimized, leading to a tractable solution. Note that

$$\begin{aligned} &\max_{G \in \mathcal{S}} \|W_1(z)[G(z) - \widetilde{G}(z)]\|_\infty = \\ &\qquad \max_{G \in \mathcal{S}} |W_1(e^{i\omega^*})[G(e^{i\omega^*}) - \widetilde{G}(e^{i\omega^*})]| \end{aligned} \qquad (14)$$

with ω^* the (unknown) frequency for which the maximum is achieved. Using a triangle inequality, it can be verified that

$$\max_{G\in\mathcal{S}} \|W_1(z)[G(z)-\widetilde{G}(z)]\|_\infty \leq$$
$$|W_1(e^{i\omega^*})| \max_{G\in\mathcal{S}} |G(e^{i\omega^*})-G_c(\omega^*)| +$$
$$+\max_{\omega\in[0,\pi]} \left|W_1(e^{i\omega})\left(G_c(\omega)-\widetilde{G}(e^{i\omega})\right)\right|, \quad (15)$$

for any complex-valued function $G_c(\omega)$.

Both terms on the right hand side of the inequality can be minimized when we restrict attention to the finite frequency grid Ω. Concerning the first term we can determine:

$$\widehat{G}_c(\omega_j) = \arg \min_{G_c(\omega_j)\in\mathbb{C}} \max_{G(e^{i\omega_j})\in\mathcal{P}(\omega_j)} |G(e^{i\omega_j})-G_c(\omega_j)| \quad (16)$$

for all $\omega_j \in \Omega$. This refers to the construction of a sequence of complex numbers $\{\widehat{G}_c(\omega_j), \omega_j \in \Omega\}$ that minimize the worst-case amplitude error for each frequency point in Ω separately. This problem comes down to finding the centers of the rectangular or ellipsoid regions $\mathcal{P}(\omega_j)$ that constitute the uncertainty region $\mathcal{S}$.

Subsequently the second term on the right hand side of (15) is handled by

$$\widehat{G}(z) = \arg \min_{\widetilde{G}\in\mathcal{M}} \max_{\omega_j\in\Omega} \left|W_1(e^{i\omega_j})\left(\widehat{G}_c(\omega_j)-\widetilde{G}(e^{i\omega_j})\right)\right|, \quad (17)$$

where $\widehat{G}_c(\omega_j)$ is determined by (16).

The worst-case system approximation problem has been rewritten into a combination of the construction of a central estimate (16) and a system approximation problem in terms of a weighted maximum amplitude criterion (17). As the uncertainty regions $\mathcal{P}(\omega_j)$ obtained in the previous section are either rectangular or ellipsoidal, the construction of their centers is straightforward. For the second problem (17), an efficient solution is available in [10] in terms of a curve fit procedure, employing linear and nonlinear programming techniques and restricting the model set to contain only stable models.

In (17) we have restricted attention to the finite frequency grid Ω, and this of course does not necessarily lead to good quality of the model for frequencies outside this grid. However, in [8] it is shown that for any model $\widetilde{G}$ in the model set $\mathcal{M}$, with the transfer functions parametrized in a numerator/denominator form, it follows that

$$\max_{\omega\in[0,\pi]} \left|W_1(e^{i\omega})[\widehat{G}_c(\omega)-\widetilde{G}(e^{i\omega})]\right| \leq$$
$$\leq \max_{\omega_j\in\Omega} \left|W_1(e^{i\omega_j})[\widehat{G}_c(\omega_j)-\widetilde{G}(e^{i\omega_j})]+\gamma\right| \quad (18)$$

where $\gamma \geq 0$ is bounded, and

$$\arg \min_{\widetilde{G}\in\mathcal{M}} \max_{\omega\in[0,\pi]} \left|W_1(e^{i\omega})[\widehat{G}_c(\omega)-\widetilde{G}(e^{i\omega})]\right| \to$$
$$\arg \min_{\widetilde{G}\in\mathcal{M}} \max_{\omega_j\in\Omega} \left|W_1(e^{i\omega_j})[\widehat{G}_c(\omega_j)-\widetilde{G}(e^{i\omega_j})]\right| \quad (19)$$

as Ω becomes dense in $[0,\pi]$. Consequently the error due to the frequency sampling is bounded and the error vanishes if more and more frequencies are taken into account in Ω. Note that the choice of the frequency grid is a user's choice, independent of the experimental data available. With the procedure for constructing the uncertainty set $\mathcal{S}$ in Section 4, the frequency grid Ω can be chosen freely.

6 Quantification of a model error bound

The last step in our procedure concerns the construction of the model error bound b, as denoted in (2). To this end we consider the nominal model $\widehat{G}$ to be given, and we suppose that a stable and stably-invertible weighting function W is given. As mentioned before, an optimal choice of W according to (8) can be found by the algorithm in [20].

The worst-case model error can now be calculated for each $\omega_j \in \Omega$ separately. As for each ω_j the uncertainty region in the frequency domain is either a rectangular or an ellipsoid (see Section 4), it is not hard to specify $\delta_u(\omega_j)$ according to

$$\delta_u(\omega_j) := \max_{G(e^{i\omega_j})\in\mathcal{P}(\omega_j)} \left|W(e^{i\omega_j})[G(e^{i\omega_j})-\widehat{G}(e^{i\omega_j})]\right|. \quad (20)$$

In case $\mathcal{P}(\omega_j)$ is rectangular, this is simply done by evaluating the model error over the four vertices of the rectangular area.

As a result a model error bound $\delta_u(\omega_j)$ has been established for frequencies in the set Ω. Using an interpolation argument based on the derivative bound β_1 (12), a bound on the $\mathcal{H}_\infty$-norm of the model error can be established, similar to [1, 4],

$$\max_{G\in\mathcal{S}} \|W(z)[G(z)-\widehat{G}(z)]\|_\infty \leq$$
$$\max_{j=1,\cdots,\ell} [\delta_u(\omega_j)+\lambda_j\beta] =: b.$$

Here, λ_j is the distance between two adjacent elements in the frequency grid Ω, and β is a bounded positive real scalar that can be calculated on the basis of β_1, $W(z)$, $\widehat{G}(z)$ and $\mathcal{S}$. When Ω becomes dense in $[0,\pi]$, the above inequality becomes an equality. For technical details and proofs we refer to [8].

The contribution of the interpolation is worst-case and can be pessimistic if too few frequency points are considered. In practical situations it will often be justified to neglect the interpolation term, provided sufficient frequencies are present in Ω. This approach is also supported in [2].

7 Discussion

We have presented a four step procedure for arriving at a set of identified models induced by a low order nominal model, and a bound on a weighted $\mathcal{H}_\infty$ model error. The obtained result is in a format that is suitable for model-based robust control design. The several steps in the procedure are designed in such a way that conservatism is limited. In case stochastic priors are considered on the noise disturbance, probabilistic aspects have been concentrated in Step 1 of the procedure. Steps 2 and 3 only take the set of not-invalidated models S as a starting point, irrespective of the fact whether S is guaranteed to contain G_0 or whether this holds true with some high probability < 1. The proposed procedure results in an uncertainty model that can be calculated by feasible mathematical tools, leading to results that are not overly conservative. Thus they provide appropriate models for robust control design. Application of the procedure to uncertainty modelling and control design for a glass tube manufacturing process has been reported in [8].

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