Design of optimal identification experiments

DISC course: System Identification for control

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1. Reminders and introduction

Consider a real-life system $\mathcal{S}$ operated in open loop:

$$y(t) = G_0(z)u(t) + H_0(z)e(t) = G(z,\theta_0)u(t) + H(z,\theta_0)e(t)$$

with θ_0 the unknown true parameter vector (z^{-1} delay operator)

Consider also a full-order model structure $\mathcal{M}$ for this true system $\mathcal{S}$ ($\mathcal{S} \in \mathcal{M}$):

$$\mathcal{M} = \{\ G(z,\theta);\ H(z,\theta)\ \mid\ \theta \in \mathrm{R}^k\ \}$$

Prediction error identification of $\mathcal{S}$

An input signal $u(t)$ $(t = 1..N)$ is applied to $\mathcal{S}$ and the corresponding output $y(t)$ is collected

$$Z^N = \{y(t)\ u(t) \mid t = 1...N\}$$

Based on these input-output data, prediction error identification can be used to obtain a consistent estimate $\hat{\theta}_N$ of θ_0:

$$\hat{\theta}_N = arg\ \min_{\theta}\ \frac{1}{N}\sum_{t=1}^{N}\left(\underbrace{H(z,\theta)^{-1}\left(y(t) - G(z,\theta)u(t)\right)}_{=\epsilon(t,\theta)}\right)^2$$

Disturbance $v(t) = H_0(z)e(t) \Longrightarrow \hat{\theta}_N \neq \theta_0$

Question: **how can we evaluate the accuracy of** $\hat{\theta}_N$ **and relate it to the experimental conditions?**

Choice of experimental conditions

1. **the number** N **of data that will be collected**
2. **the signal** $u(t)$ **that will be applied**

Properties of the identified parameter vector $\hat{\theta}_N$

$\hat{\theta}_N$ is (asymptotically) normally distributed around θ_0

$$\sqrt{N}(\hat{\theta}_N - \theta_0) \sim \mathcal{N}(0, P_\theta)$$

$$P_\theta = \sigma_e^2 \; \left(\bar{E}\psi(t,\theta_0)\psi^T(t,\theta_0) \right)^{-1}$$

with $\psi(t,\theta) = -\frac{\partial \varepsilon(t,\theta)}{\partial \theta}$

The covariance matrix $cov(\hat{\theta}_N)$ of $\hat{\theta}_N$ given by $\frac{P_\theta}{N}$ is a measure of accuracy for $\hat{\theta}_N$

The variance $cov(\hat{\theta}_N)$ in the parameter space induces a variance $cov(G(e^{j\omega}, \hat{\theta}_N))$ in the transfer function space

$$cov(G(e^{j\omega}, \hat{\theta}_N)) = \Lambda_G^*(e^{j\omega}, \theta_0)\ \frac{P_\theta}{N}\ \Lambda_G(e^{j\omega}, \theta_0)$$

with $\Lambda_G(z, \theta) = \frac{\partial G(z,\theta)}{\partial \theta}$

$cov(G(e^{j\omega}, \hat{\theta}_N))$ is a measure of accuracy for the identified model $G(z, \hat{\theta}_N)$

Bounding the modeling error using $cov(G(e^{j\omega}, \hat{\theta}_N))$

Using $cov(G(e^{j\omega}, \hat{\theta}_N))$, we can build a confidence region for the modeling error at each frequency:

$$|G(e^{j\omega}, \hat{\theta}_N) - G(e^{j\omega}, \theta_0)| < \alpha \sqrt{cov(G(e^{j\omega}, \hat{\theta}_N))}$$

The bound holds with an user-chosen probability that can be tuned via $\alpha \in \mathrm{R}$ (e.g. the probability is 0.95 if $\alpha = 2.45$)

Remark. $cov(G(e^{j\omega}, \hat{\theta}_N))$ can be estimated after the identification experiment based on Z^N and $\hat{\theta}_N$

$$\widehat{cov(G(e^{j\omega}, \hat{\theta}_N))} = \Lambda_G^*(e^{j\omega}, \hat{\theta}_N)\ \frac{\hat{P}_\theta}{N}\ \Lambda_G(e^{j\omega}, \hat{\theta}_N)$$

with

$$\hat{P}_\theta = \hat{\sigma}_e^2 \left(\frac{1}{N} \sum_{t=1}^{N} \psi(t, \hat{\theta}_N) \psi^T(t, \hat{\theta}_N) \right)^{-1}$$

$$\hat{\sigma}_e^2 = \frac{1}{N} \sum_{t=1}^{N} \epsilon^2(t, \hat{\theta}_N)$$

Modeling error and experimental conditions i.e. N and $u(t)$

The bound on the modeling error can be expressed as follows:

$$\alpha \sqrt{cov(G(e^{j\omega}, \hat{\theta}_N))} = \alpha \sqrt{\Lambda_G^*(e^{j\omega}) \frac{P_\theta}{N} \Lambda_G(e^{j\omega})}$$

$$with\ P_\theta = \sigma_e^2 \left(\bar{E}\psi(t,\theta_0)\psi^T(t,\theta_0) \right)^{-1}$$

$$\psi(t,\theta_0) = F_u(z,\theta_0)u(t) + F_e(z,\theta_0)e(t)$$

$$F_u(z,\theta_0) = \frac{1}{H_0} \left.\frac{\partial G(z,\theta)}{\partial \theta}\right|_{\theta=\theta_0} \ and\ F_e(z,\theta_0) = \frac{1}{H_0} \left.\frac{\partial H(z,\theta)}{\partial \theta}\right|_{\theta=\theta_0}$$

Important observations

the experimental conditions N and $u(t)$ influence (the bound on) the modeling error

the larger N, the smaller the modeling error

For a given frequency content, the most powerful $u(t)$, the smaller the modeling error

For signals $u(t)$ having the same power, different frequency contents could lead to different modeling errors

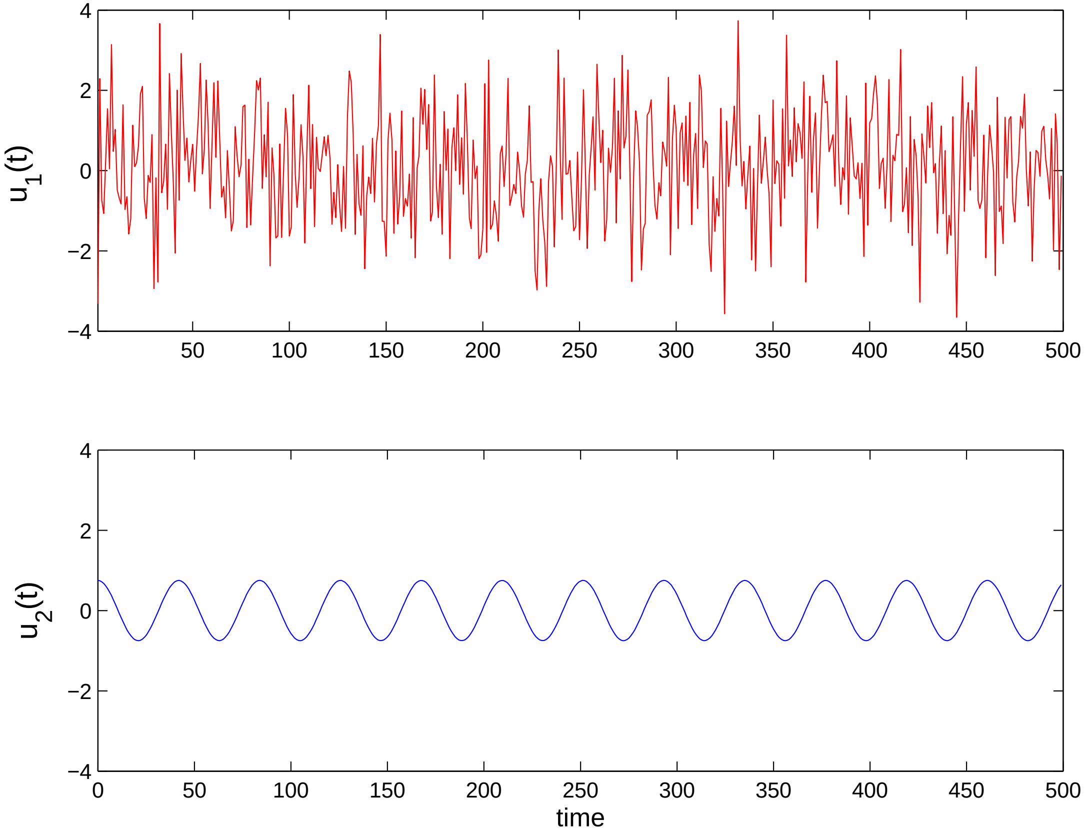
Example

$$\mathcal{S}: \quad y(t) = \frac{3.6z^{-1}}{1-0.7z^{-1}}u(t) + \left(1+0.9z^{-1}\right)e(t)$$

$$\mathcal{M}: \quad G(z,\theta) = \frac{bz^{-1}}{1-fz^{-1}} \quad H(z,\theta) = 1 + cz^{-1} \quad \theta = \begin{pmatrix} b \\ c \\ f \end{pmatrix}$$

Let us compare the accuracy obtained with two input signals: a white noise and a cosine at $\omega = 0.15$

$u_1(t)$ **a white noise with variance 1.7 and** $u_2(t) = 0.75cos(0.15t)$; **both of length** $N = 500$



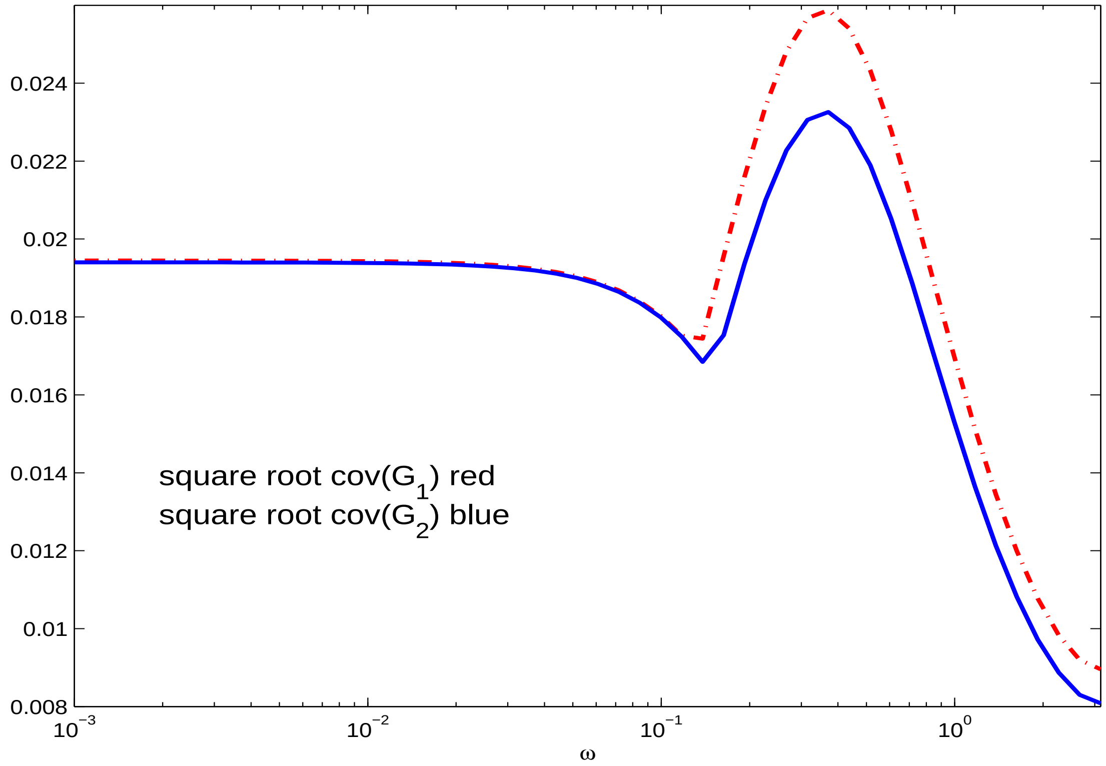
$$\mathcal{P}_{u_1} >> \mathcal{P}_{u_2}$$

Question:

Which signal leads to the most accurate model?

While $\mathcal{P}_{u_1} >> \mathcal{P}_{u_2}$, the model $\hat{G}_2$ identified with u_2 has a slightly better accuracy than the model $\hat{G}_1$ identified with u_1

$$\sqrt{cov(\hat{G}_1(e^{j\omega}))} \geq \sqrt{cov(\hat{G}_2(\omega))} \quad \forall \omega$$



A cosine at $\omega = 0.15$ seems a much better signal than a white noise to identify this true system

How can we determine the optimal input signal for a given true system?

$\rightarrow$ Optimal experiment design

2. Optimal experiment design

Suppose that, for the subsequent use of the model (e.g. for control), it is required that:

$$|G(e^{j\omega}, \hat{\theta}_N) - G(e^{j\omega}, \theta_0)| < r_{adm}(\omega) \quad \forall\omega$$

where $r_{adm}(\omega)$ is a given frequency function

The function $r_{adm}(\omega)$ can e.g. be chosen equal to $0.1|G(e^{j\omega}, \hat{\theta}_N)|$ (relative error of 10%) or can be determined using robust control considerations

$$|G(e^{j\omega}, \hat{\theta}_N) - G(e^{j\omega}, \theta_0)| < r_{adm}(\omega) \quad \forall\omega$$

How can we choose $u(t)$ and N for the identification experiment in order to guarantee that the model identified with this experiment meets the accuracy constraint?

We have thus to determine N and $u(t)$ such that:

$$\alpha\sqrt{\Lambda_G^*(e^{j\omega})\ \frac{P_\theta}{N}\ \Lambda_G(e^{j\omega})} < r_{adm}(\omega) \quad \forall\omega$$

First observation: By choosing N or the power of $u(t)$ sufficiently large, it is always possible to achieve this constraint for any given $r_{adm}(\omega)$

$\Longrightarrow$ multiple solutions !

Let us therefore

- fix the value of N

- and determine, for an experiment of that length N, the **excitation signal $u(t)$ with the least power $\mathcal{P}_u = \frac{1}{N}\sum_{t=1}^{N} u^2(t)$** that nevertheless leads to an identified model with the required accuracy

Optimization problem for the design of the excitation signal (N given)

Determine the sequence $u(t)$ $(t = 1 ... N)$ which solves:

$$\min_{u(t)\ (t=1...N)} \quad \frac{1}{N} \sum_{t=1}^{N} u^2(t)$$

$$\text{subject to} \quad \alpha \sqrt{\Lambda_G^*(e^{j\omega})\ \frac{P_\theta}{N}\ \Lambda_G(e^{j\omega})} < r_{adm}(\omega) \quad \forall \omega$$

Issues with this optimization problem

non-linear relations between $u(t)$ and $\mathcal{P}_u$ and between $u(t)$ and P_θ $\rightarrow$ convexification

chicken-and-egg problem: dependence of $cov(G(e^{j\omega}, \hat{\theta}_N))$ on θ_0 $\rightarrow$ replace θ_0 by an initial estimate θ_{init} obtained e.g. via an initial identification with white noise

infinite number of constraints i.e. at each frequency ω $\rightarrow$ frequency grid

3. Convexification of the optimization problem

Approach: use of the power spectrum $\Phi_u(\omega)$ of $u(t)$ as alternative decision variable (instead of $u(t)$ itself)

3.1 Convexification of the cost function $\mathcal{P}_u$

for large N,

$$\mathcal{P}_u = \frac{1}{N}\sum_{t=1}^{N} u^2(t) \approx \bar{E}u^2(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega)\; d\omega$$

$\mathcal{P}_u$ linear in the new decision variable $\Phi_u(\omega)$

3.2 Convexification of the constraint $\alpha\sqrt{\Lambda_G^* \, \frac{P_\theta}{N} \, \Lambda_G} < r_{adm}(\omega)$

$$P_\theta = \sigma_e^2 \, \left(\bar{E}\psi(t,\theta_0)\psi^T(t,\theta_0)\right)^{-1} \; and \; \psi(t) = F_u(z)u(t) + F_e(z)e(t)$$

$$\Longrightarrow P_\theta = \sigma_e^2 \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} F_u F_u^* \; \Phi_u(\omega) \; + \; F_e F_e^* \; \sigma_e^2 \; d\omega\right)^{-1}$$

P_θ^{-1} **linear in the new decision variable** $\Phi_u(\omega)$

Let us thus rewrite the constraint as a linear function of P_θ^{-1}

$$\alpha \sqrt{\Lambda_G^*(e^{j\omega}) \frac{P_\theta}{N} \Lambda_G(e^{j\omega})} < r_{adm}(\omega) \quad \forall \omega$$

is equivalent (via the Schur complement) to

$$N \, P_\theta^{-1} > R_{adm}(\omega) \quad \forall \omega$$

where $R_{adm}(\omega) = \frac{\alpha^2}{r_{adm}^2(\omega)} \Lambda_G(e^{j\omega}) \Lambda_G^*(e^{j\omega})$

3.3. A convex formulation of experiment design

Reminder: initial optimization problem (non-convex)

Determine the sequence $u(t)$ $(t = 1 ... N)$ which solves:

$$\min_{u(t)\ (t=1...N)} \quad \frac{1}{N}\sum_{t=1}^{N} u^2(t)$$

subject to $$\alpha\sqrt{\Lambda_G^*(e^{j\omega})\ \frac{P_\theta}{N}\ \Lambda_G(e^{j\omega})} < r_{adm}(\omega) \quad \forall\omega$$

Convex alternative (fixed N):

Determine the power spectrum $\Phi_u(\omega)$ of the excitation signal $u(t)$ which solves:

$$\min_{\Phi_u(\omega)} \quad \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega)\, d\omega$$

subject to $\Phi_u(\omega) \geq 0 \;\; \forall\omega$ and to

$$\frac{N}{\sigma_e^2}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} F_u F_u^* \, \Phi_u(\omega) \; + \; F_e F_e^* \, \sigma_e^2 \, d\omega\right) > R_{adm}(\omega) \;\; \forall\omega$$

3.4. Parametrization of the power spectrum $\Phi_u(\omega)$

The decision variable $\Phi_u(\omega)$ has an infinite dimension $\rightarrow$ **a simpler and linear parametrization of $\Phi_u(\omega)$ is required**

Two possible approaches:

- $\Phi_u(\omega)$ for filtered white noise u
- $\Phi_u(\omega)$ for multisine u

Parametrization of the power spectrum $\Phi_u(\omega)$: filtered white noise

Suppose we want to be able to generate our input signal $u(t)$ of spectrum $\Phi_u(\omega)$ as the realization of a white noise filtered by a FIR filter $F_m(z)$ of order m:

$$u(t) = F_m(z)w(t)$$

where $w(t)$ is a white noise of variance 1 and
$F_m(z) = f_0 + f_1 z^{-1} + .. + f_m z^{-m}$ an arbitrary FIR filter

In this case, we have to restrict the structure of $\Phi_u(\omega)$ as follows:

$$\Phi_u(\omega) = |F_m(e^{j\omega})|^2 = \sum_{r=-m}^{m} c_r \; e^{j\omega \; r} \quad \text{(met } c_r = c_{-r}\text{)}$$

where c_r $(r = 0...m)$ are the new decision variables of the optimization problem

The larger m is chosen, the more flexible is the parametrization of the spectrum

Choosing $m = 0$ is equivalent to restrict attention to a flat spectrum (white noise)

Parametrization of the power spectrum $\Phi_u(\omega)$: multisine

Suppose we want to generate our input signal $u(t)$ as:

$$u(t) = \sum_{r=1}^{m} A_r \, sin(\omega_r \, t)$$

with arbitrary amplitudes A_r $(r = 1...m)$, but fixed frequencies ω_r $(r = 1...m)$ e.g. linearly distributed in the interval $[0 \; \pi]$

$$u(t) = \sum_{r=1}^{m} A_r \; sin(\omega_r \; t)$$

In this case, we have to restrict the structure of $\Phi_u(\omega)$ as follows:

$$\Phi_u(\omega) = \frac{\pi}{2} \; \sum_{r=1}^{m} A_r^2 \; \left(\delta(\omega - \omega_r) + \delta(\omega + \omega_r)\right)$$

where A_r^2 $(r = 1...m)$ are the new decision variables of the optimization problem

Here also, the larger m is chosen, the more "flexible" is the parametrization of the spectrum

A bit more details....

$$\Phi_u(\omega) = \frac{\pi}{2} \sum_{r=1}^{m} A_r^2 \ (\delta(\omega - \omega_r) + \delta(\omega + \omega_r)) \Longrightarrow$$

$$\mathcal{P}_u = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_u(\omega)\ d\omega = \sum_{r=1}^{m} \frac{A_r^2}{2}$$

$$N\ P_\theta^{-1} = \frac{N}{\sigma_e^2} \left(\left(\sum_{r=1}^{m} \frac{A_r^2}{2} M(\omega_r) \right) + R_e \right)$$

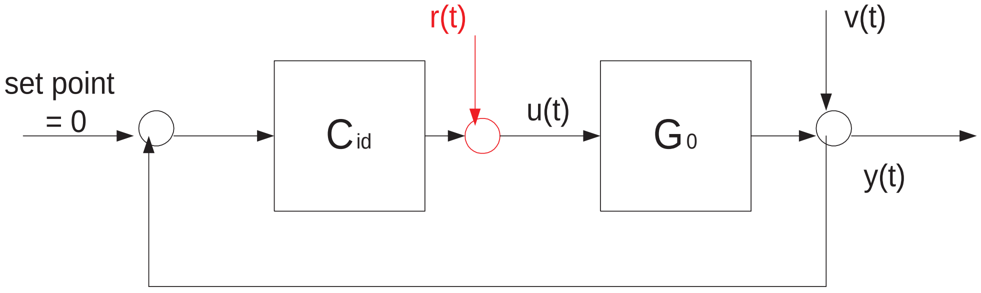
$$M(\omega_r) = Re(F_u(e^{j\omega_r}) F_u^*(e^{j\omega_r})) \quad R_e = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_e F_e^*\ \sigma_e^2\ d\omega$$

$\mathcal{P}_u$ and P_θ^{-1} are indeed affine in A_r^2 $(r = 1...m)$

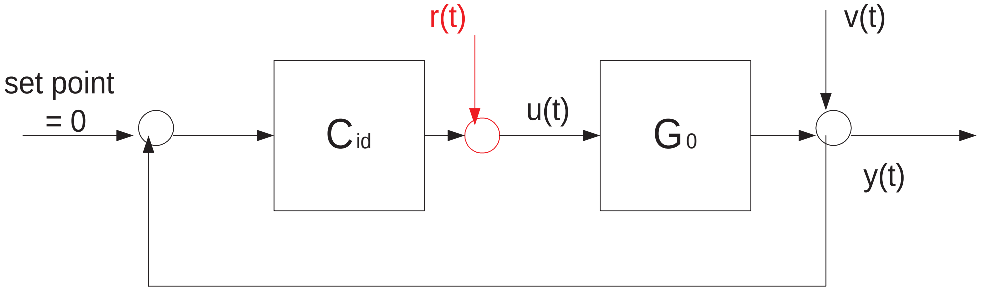
4. Some alternative experiment design problems

1) Instead of $\mathcal{P}_u$, the cost function can also be the power $\mathcal{P}_y$ of the output signal $y(t)$ (or a combination of both)

2) Optimal experiment design can also be formulated for **direct closed-loop identification**



$$Z^N = \{y(t)\ u(t) | t = 1...N\}$$ generated via $r(t)$



Decision variable: $\Phi_r(\omega)$

Expression for P_θ

$$P_\theta = \sigma_e^2 \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} F_r F_r^* \; \Phi_r(\omega) \; + \; F_v F_v^* \; \sigma_e^2 \; d\omega \right)^{-1}$$

$$F_r(z,\theta_0) = \frac{S_{id}}{H_0} \left. \frac{\partial G(z,\theta)}{\partial \theta} \right|_{\theta=\theta_0} \quad F_v(z,\theta_0) = \frac{\left. \frac{\partial H(z,\theta)}{\partial \theta} \right|_{\theta=\theta_0}}{H_0} - C_{id} S_{id} \left. \frac{\partial G(z,\theta)}{\partial \theta} \right|_{\theta=\theta_0}$$

P_θ^{-1} **linear in the decision variable** $\Phi_r(\omega)$

Possible formulation for direct closed-loop identification

$$\min_{\Phi_r(\omega)} \quad \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_r(\omega)\ d\omega$$

subject to $\Phi_r(\omega) \geq 0 \ \ \forall\omega$ and to

$$\frac{N}{\sigma_e^2}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} F_r F_r^*\ \Phi_r(\omega)\ +\ F_v F_v^*\ \sigma_e^2\ d\omega\right) > R_{adm}(\omega) \ \ \forall\omega$$

An other formulation will be presented in the sequel

3) Other (dual) paradigms for (open-loop) input design:

$$\min_{\Phi_u(\omega)} \quad det(P_\theta)$$

$$\text{subject to} \quad \Phi_u(\omega) \geq 0 \ \ \forall\omega \quad \text{and to}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u(\omega)\, d\omega < \mathcal{P}_{max}$$

with $\mathcal{P}_{max}$ the maximal power that is allowed at the input

5. Illustration 1 (see example slides 8-12)

$$\mathcal{S}: \quad y(t) = \frac{3.6z^{-1}}{1-0.7z^{-1}}u(t) + \left(1+0.9z^{-1}\right)e(t)$$

$$\mathcal{M}: \quad G(z,\theta) = \frac{bz^{-1}}{1-fz^{-1}} \quad H(z,\theta) = 1 + cz^{-1}$$

We would like to find, for an experiment of duration $N = 500$, the least powerful excitation signal $u(t)$ which leads to an identified model $G(z,\hat{\theta}_N)$ with a relative modeling error of less than 1 % at each ω

$$|G(e^{j\omega},\hat{\theta}_N) - G(e^{j\omega},\theta_0)| < \underbrace{0.01\ |G(e^{j\omega},\hat{\theta}_N)|}_{=r_{adm}(\omega)} \quad \forall\omega$$

Design of the optimal input signal

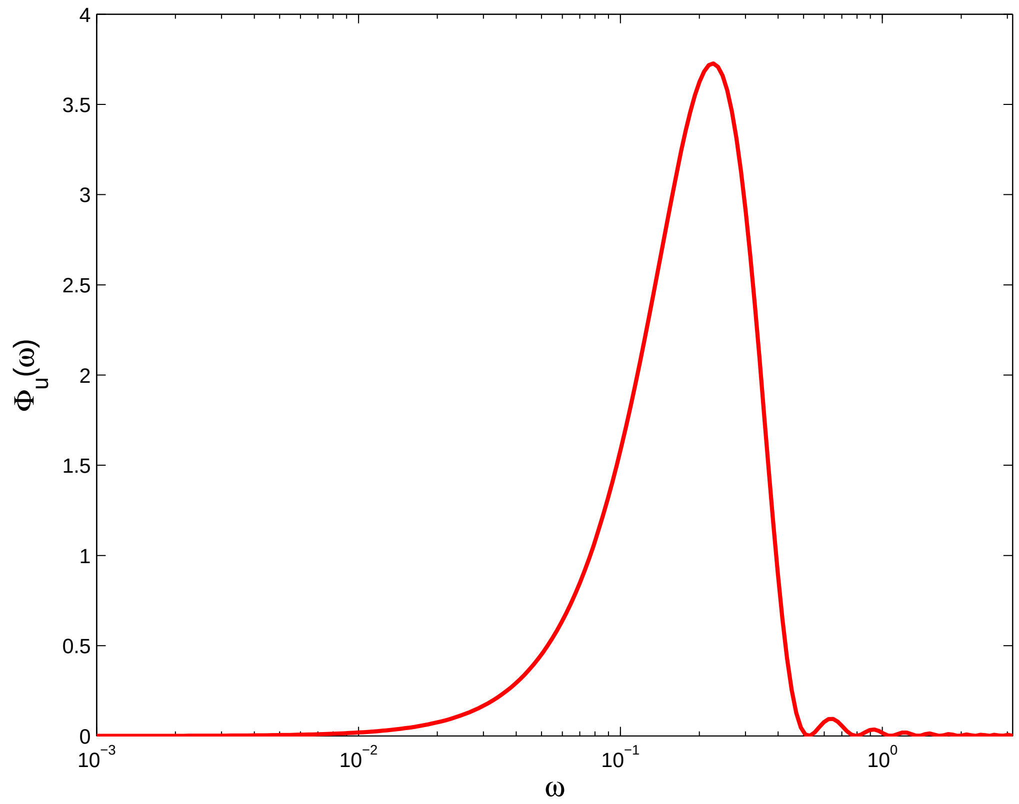
The required initial estimate θ_{init} is obtained via an initial experiment of duration $N = 100$ with white input ($\sigma_u^2 = 1$)

We solve then the convex optimization problem where θ_0 and $\hat{\theta}_N$ have been replaced by θ_{init} e.g.

$$r_{adm}(\omega) = 0.01\ |G(e^{j\omega}, \theta_{init})| \quad \forall \omega$$

Other choices: $\alpha = 2.45$ $N = 500$

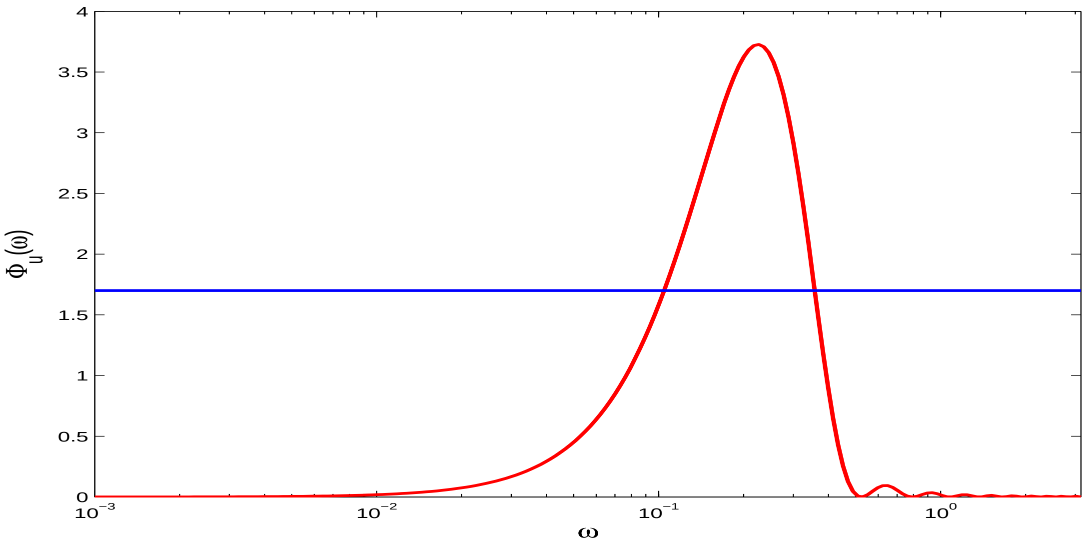
The filtered white noise parametrization of $\Phi_u(\omega)$ with $m = 20$ leads to the following optimal spectrum $\Phi_u(\omega)$:



This spectrum corresponds to a power $\mathcal{P}_{u,opt} = 0.25$

$$\mathcal{P}_{u,opt} = 0.25$$

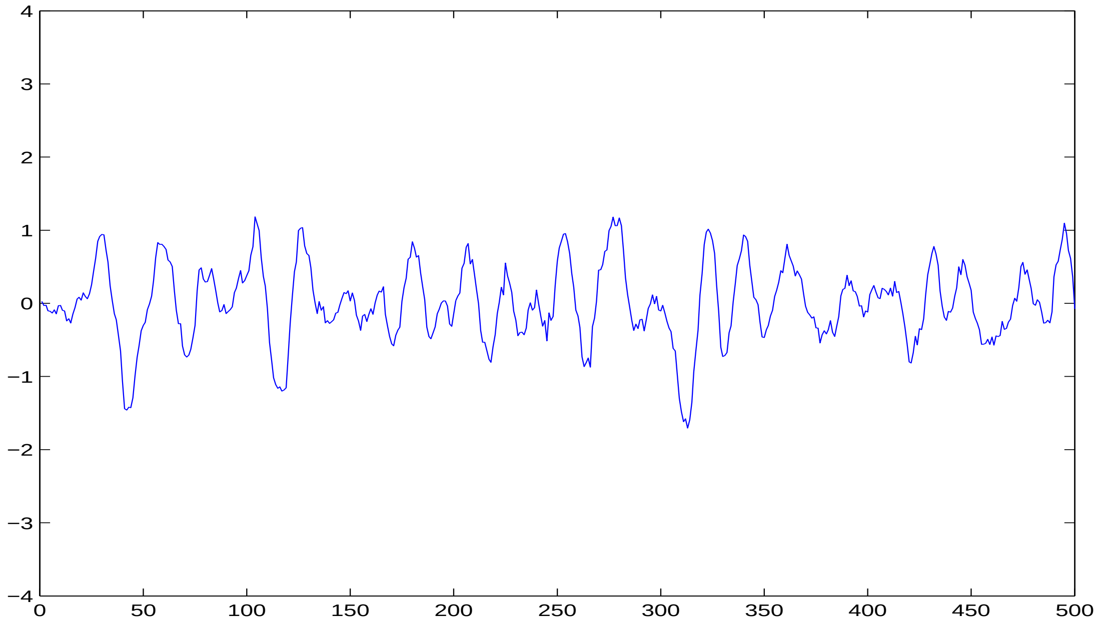
In comparison, if you want to achieve the same accuracy with a white noise ($m = 0$), you a need a white noise of power 1.7



By shaping $\Phi_u(\omega)$, less power is needed to achieve the required accuracy !

Verification

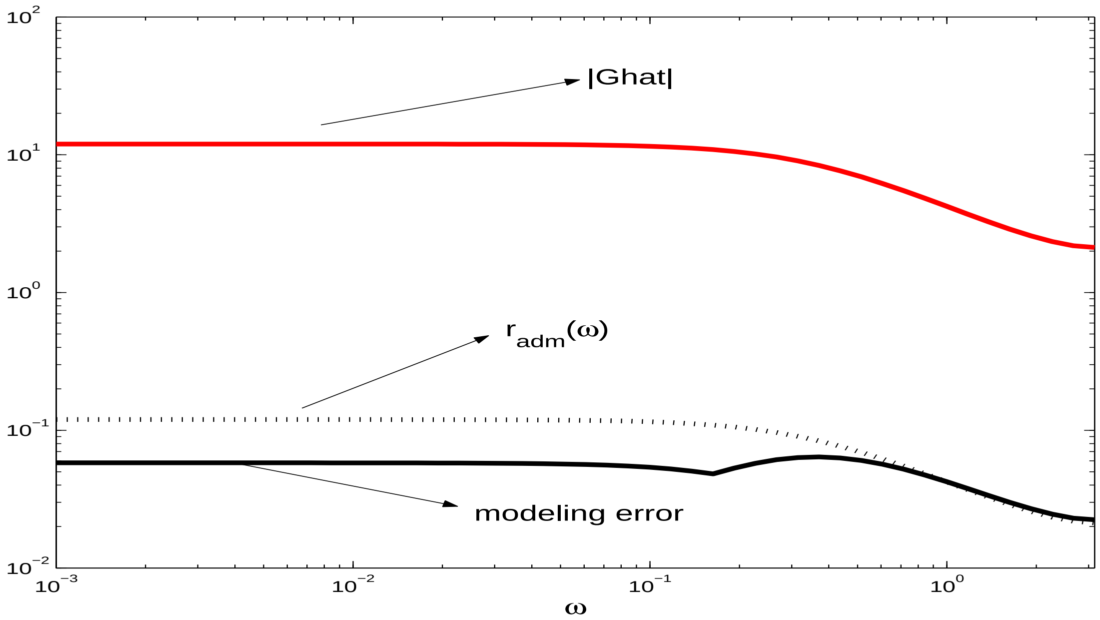
We have generated an input signal of length $N = 500$ having the optimal spectrum:



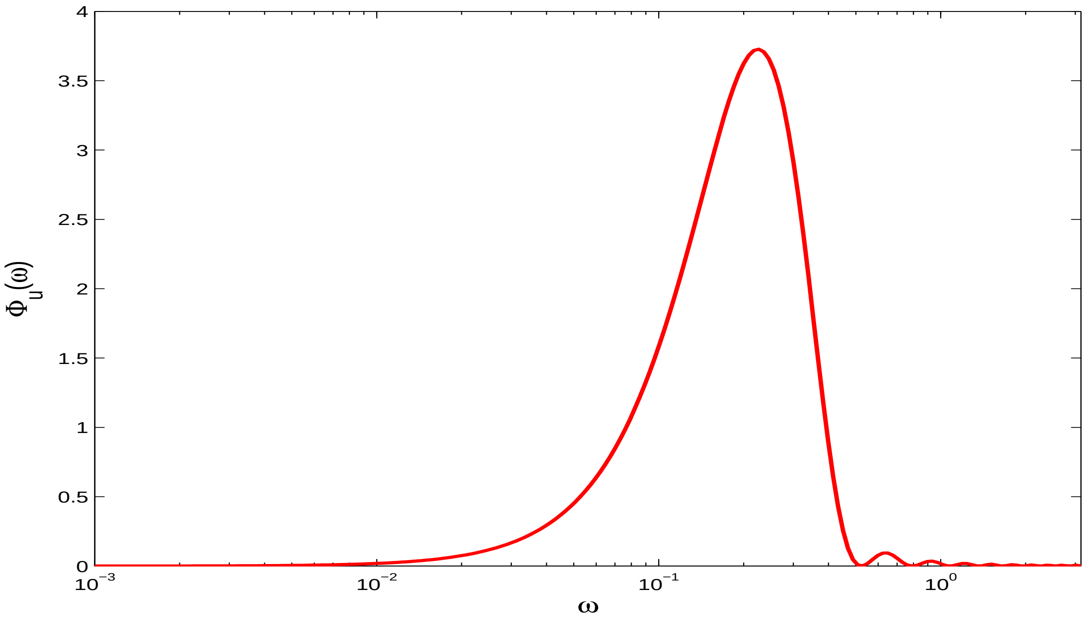
This signal is used to identify a model $G(z, \hat{\theta}_N)$ of G_0

The modeling error is computed/bounded using P_θ and we observe, as expected,

$$\alpha \sqrt{\Lambda_G^* \ \frac{P_\theta}{N} \ \Lambda_G} < r_{adm}(\omega) \quad \forall \omega$$



Analysis of the optimal spectrum $\Phi_u(\omega)$



The optimal power spectrum concentrates the power around the frequency $\omega = 0.15$ → a (co)sine at this frequency will also be appropriate

Let us thus refine the optimal spectrum with the multisine parametrization of $\Phi_u(\omega)$

This yields the optimal signal:

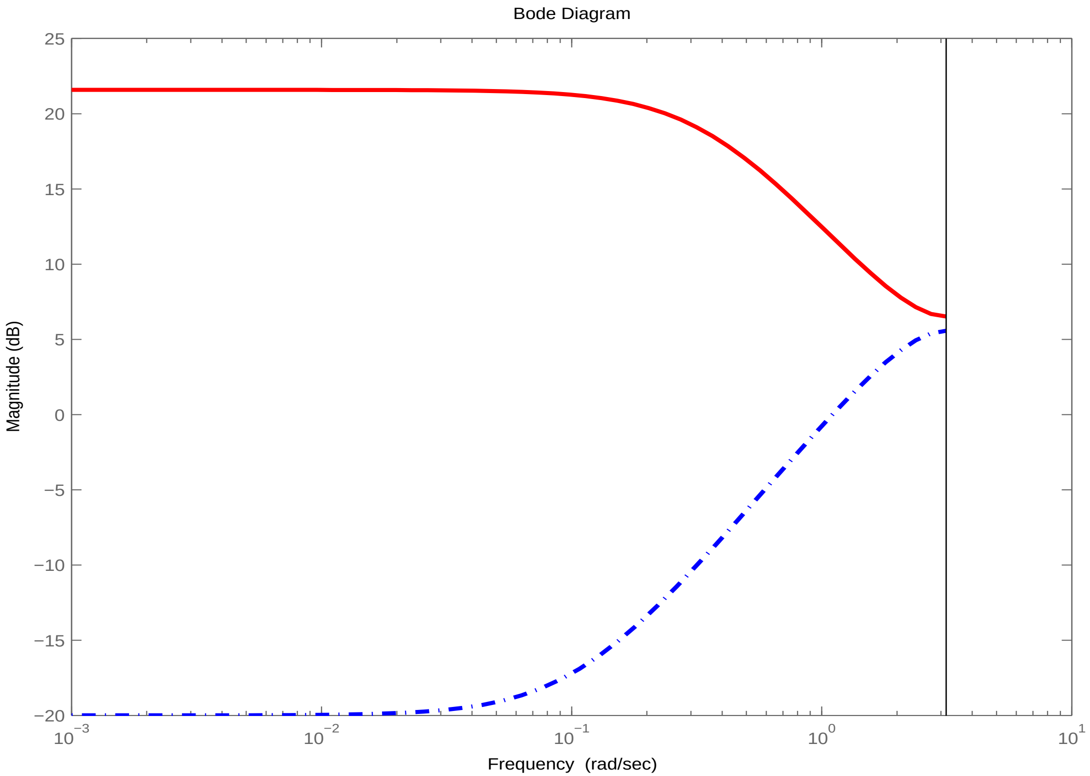
$$u(t) = 0.57 \; sin(0.15 \; t)$$

which corresponds to an even smaller $\mathcal{P}_{u,opt} = \frac{(0.57)^2}{2} = 0.16$

Remark. **If we choose a sine at $\omega = 0.01$, we would need an amplitude $A = 3$ corresponding to a power $\mathcal{P}_u = 4.5$**

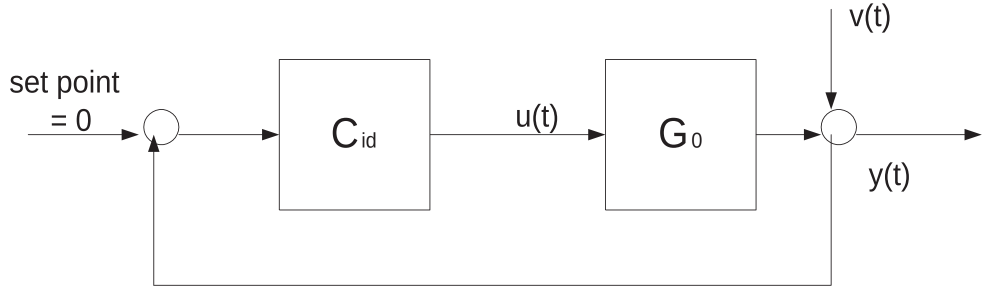
Why concentrating the power around $\omega = 0.15$?

This is the frequency range with the pole of G_0 (red) and the zero of H_0 (blue) !!!!



6. Illustration 2: least intrusive identification experiment for control

We consider a real-life system G_0 (with two resonances) operated in closed loop whose objective is to reject the disturbance $v(t) = H_0(z)e(t)$:



This closed-loop delivers a product $y(t)$

Objective: (re)-identify a model of G_0 (and H_0) to update C_{id} by a new robust controller designed with the model

Objective: (re)-identify a model of G_0 (and H_0) to update C_{id} by a new robust controller designed with the model

$$\rightarrow \text{ a constraint } \alpha\sqrt{\Lambda_G^* \, \frac{P_\theta}{N} \, \Lambda_G} < r_{adm}(\omega) \ \forall\omega$$

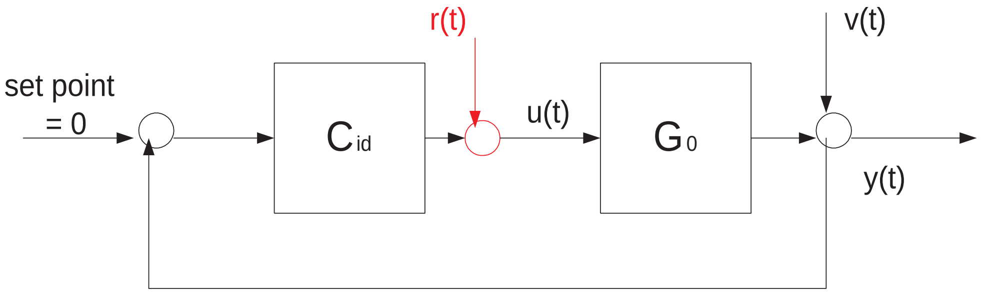
Extra requirement: in order to be able to continue to produce a good $y(t)$, we would like to achieve this objective with the least intrusive identification experiment

How can we evaluate the effects of an identification experiment on the production quality??

How can we evaluate the effects of an identification experiment on the production quality??

Normal operation vs. Identification experiment operation

$$\begin{cases} y(t) = S_{id}v(t) \\ u(t) = -C_{id}S_{id}v(t) \end{cases} \quad vs. \quad \begin{cases} y(t) = \overbrace{G_0 S_{id} r(t)}^{y_r(t)} + S_{id}v(t) \\ u(t) = \overbrace{S_{id} r(t)}^{u_r(t)} - C_{id}S_{id}v(t) \end{cases}$$



The perturbations y_r and u_r are present during the experiment duration N

Their presence reduce the production quality

For fixed N, we can measure the performance degradation due to the application of a signal with power spectrum $\Phi_r(\omega)$ by:

$$\begin{aligned}\mathcal{J}_r &= \alpha_y \mathcal{P}_{y_r} + \alpha_u \mathcal{P}_{u_r} \\ &= \alpha_y \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{y_r}(\omega)\ d\omega \right) + \alpha_u \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_r}(\omega)\ d\omega \right)\end{aligned}$$

e.g. $\alpha_u = \alpha_y = 1$

$\Longrightarrow$ Least intrusive identification experiment for control

$$arg \quad \min_{\Phi_r(\omega)} \quad \overbrace{\mathcal{P}_{u_r} + \mathcal{P}_{y_r}}^{\mathcal{J}_r}$$

under the constraint that $N \; P_\theta^{-1} \geq R_{adm}(\omega) \quad \forall\omega$

Reminder: $N \; P_\theta^{-1} > R_{adm}(\omega)$ **is equivalent to:**

$$\alpha\sqrt{\Lambda_G^* \, \frac{P_\theta}{N} \, \Lambda_G} < r_{adm}(\omega)$$

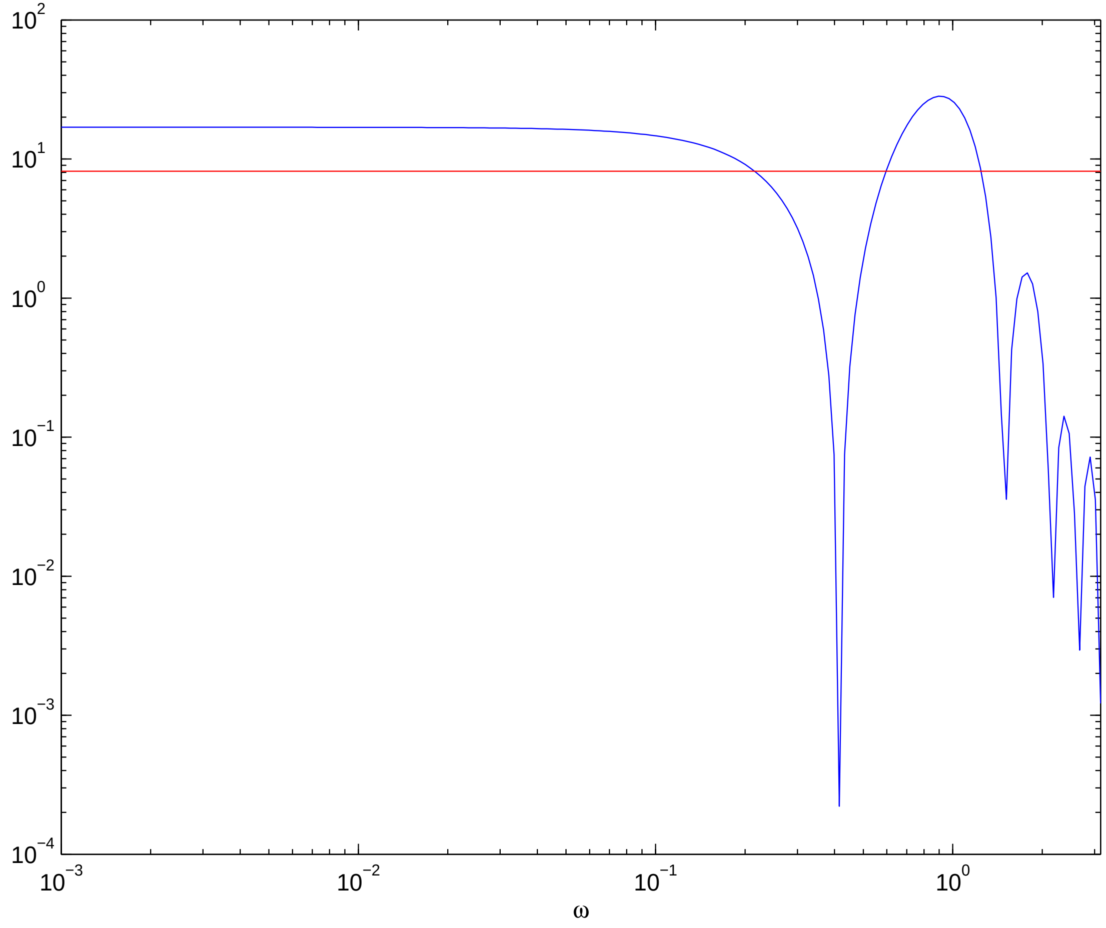
Here, $r_{adm}(\omega)$ **has been chosen based on robust control considerations and the duration of the experiment is fixed to** $N = 500$.

$$arg \min_{\Phi_r(\omega)} \mathcal{P}_{u_r} + \mathcal{P}_{y_r} \quad subject\ to\ N\ P_\theta^{-1} \geq R_{adm}(\omega)\ \forall\omega$$

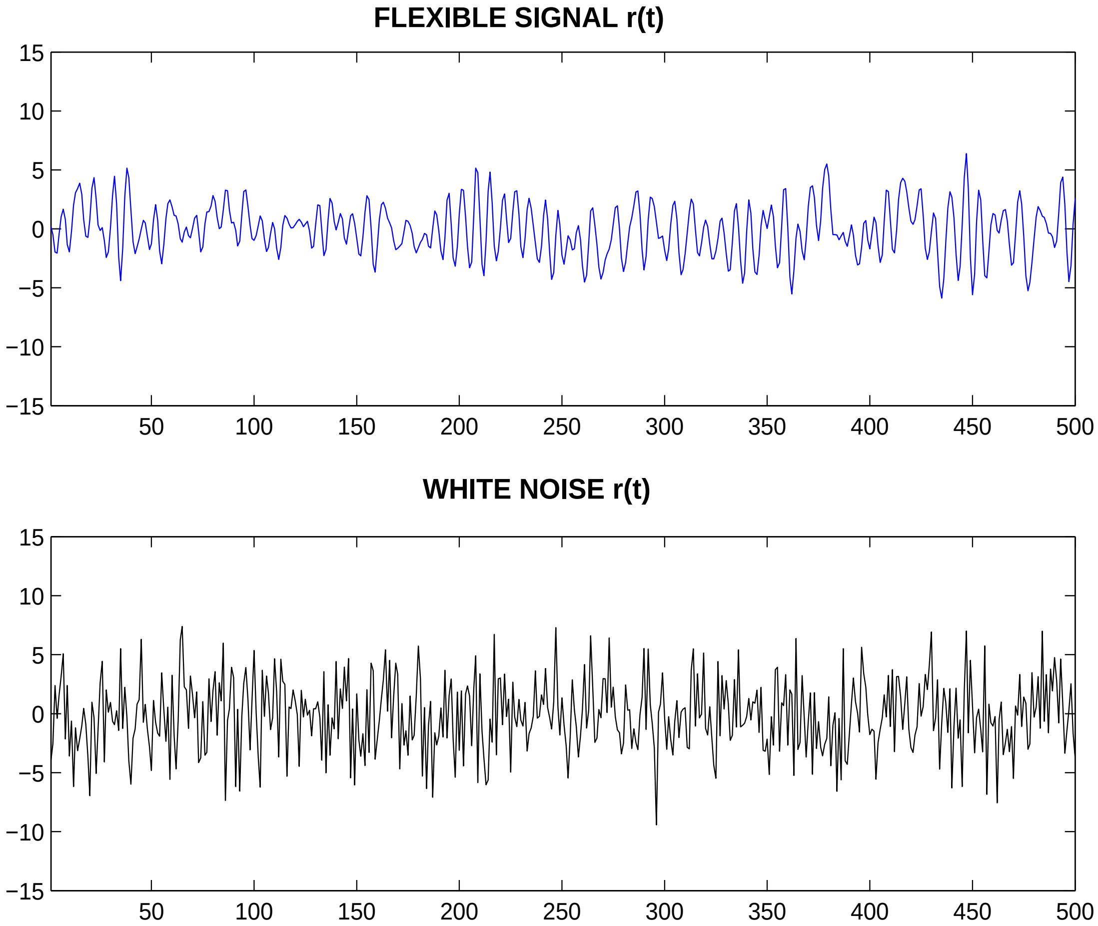
We consider two cases for the optimization on $\Phi_r(\omega)$ (filtered white noise parametrization):

- **the first one constrains $r(t)$ to be a white noise ($m = 0$)**
- **the second one allows $r(t)$ to have more flexible $\Phi_r(\omega)$ ($m = 10$)**

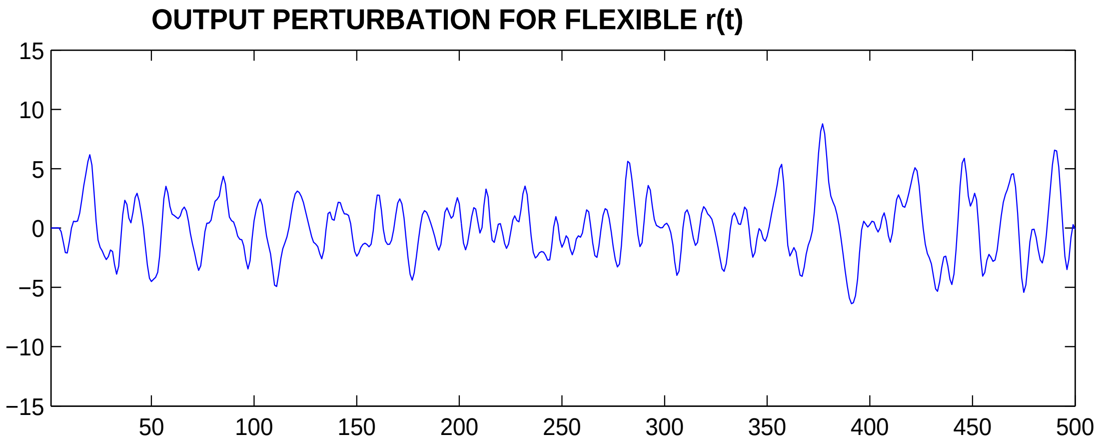
The two optimizations deliver the following spectra: white noise (red) and flexible spectrum (blue):

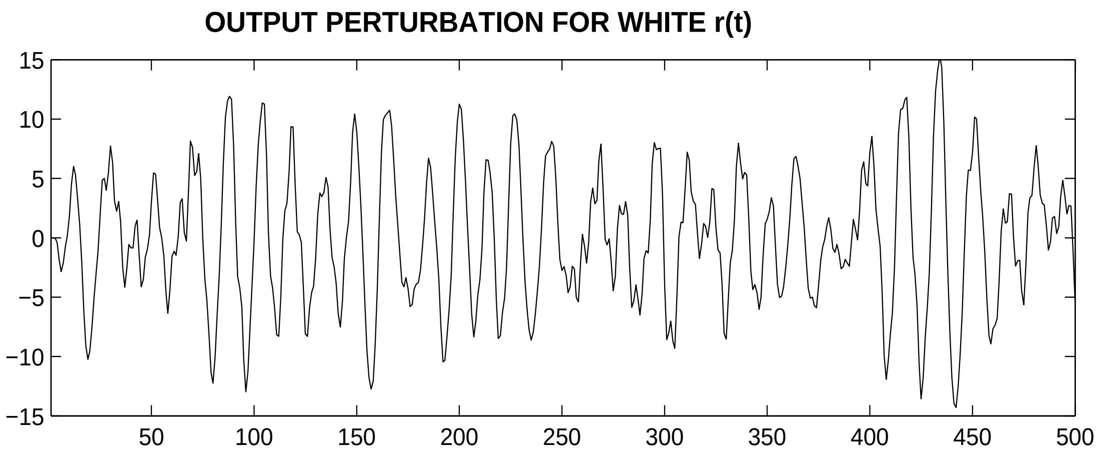


Let us generate a realization of length $N = 500$ of those two spectra...



This leads to the following perturbations $y_r(t)$





$\Phi_r(\omega)$	required $\mathcal{J}_r$ for $N = 500$
white	22.5
flexible	9.9

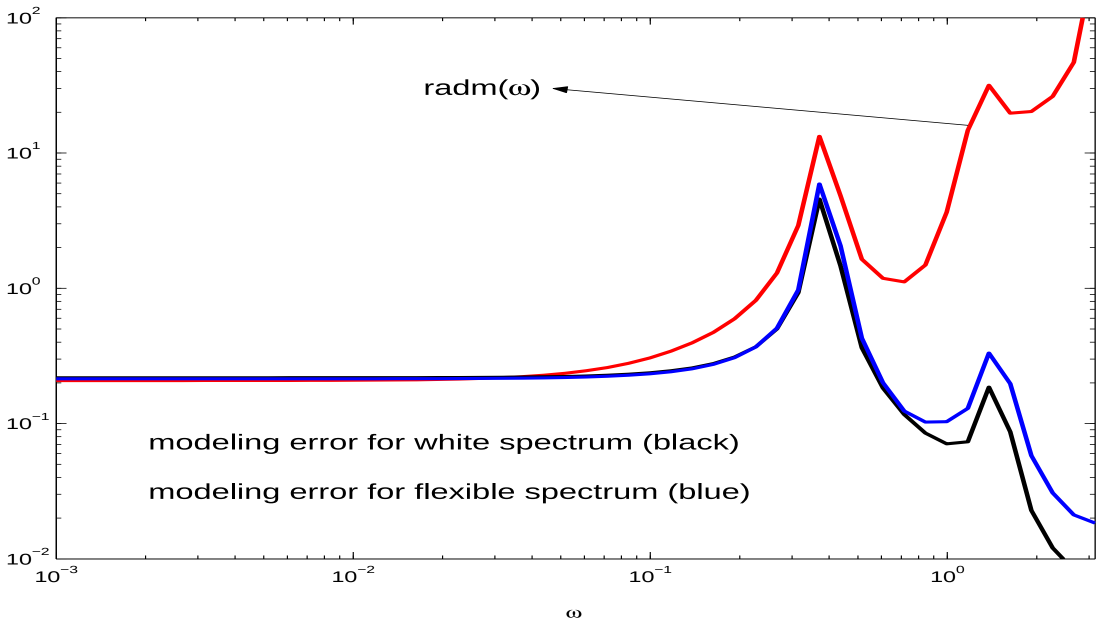
Conclusions:

- both experimental conditions deliver a model accurate enough for robust control (see next slide)

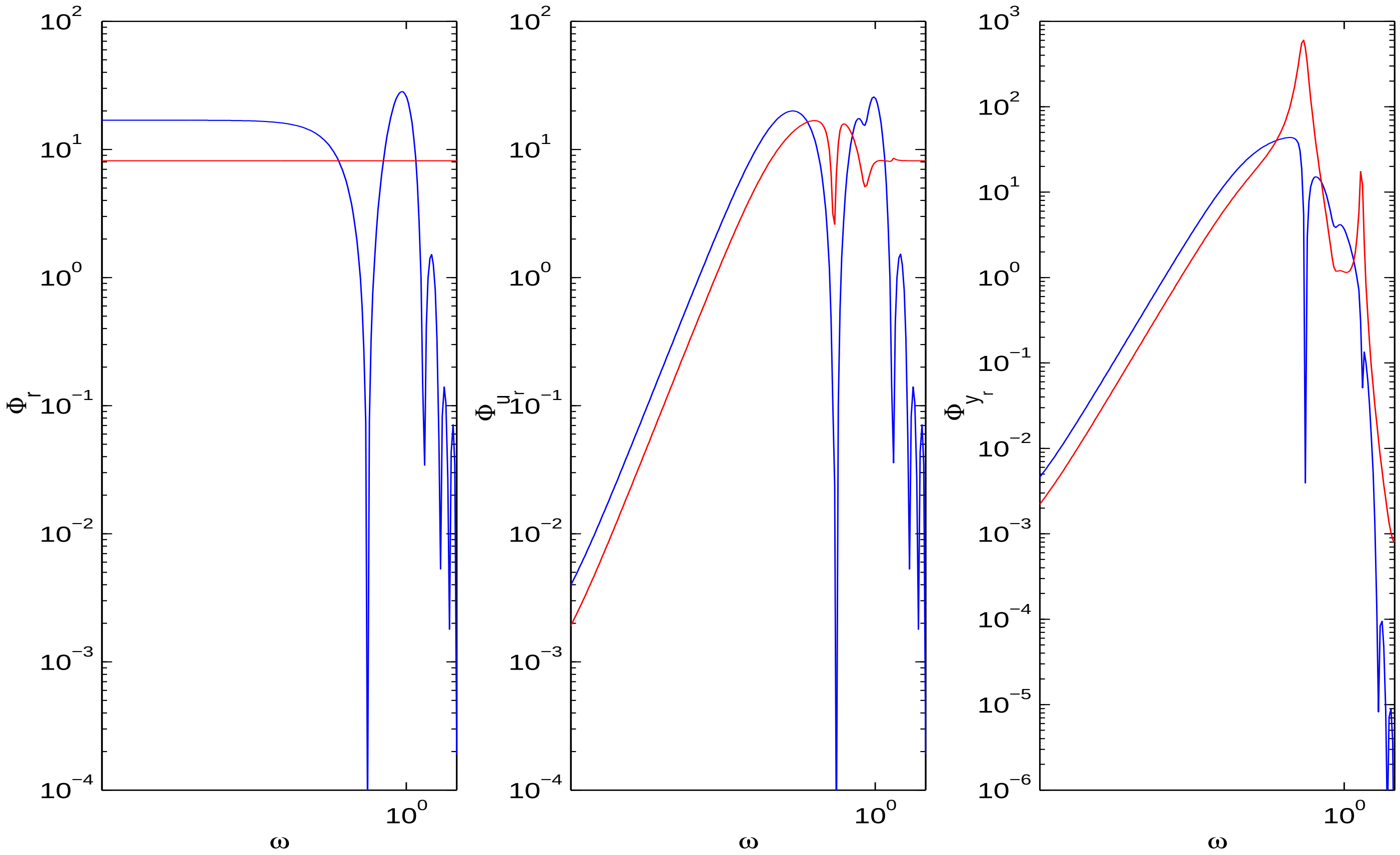
- $\mathcal{J}_r = \mathcal{P}_{u_r} + \mathcal{P}_{y_r}$ is two times larger with the RBS than with the frequency-dependent signal (i.e. 22.5 vs. 9.9)

That both the flexible and the white spectra deliver a model accurate enough for robust control can be seen by verifying in both cases that

$$\underbrace{\alpha\sqrt{\Lambda_G^* \, \frac{P_\theta}{N} \, \Lambda_G}}_{modeling\ error} < r_{adm}(\omega) \quad \forall\omega$$



Explanation of the better $\mathcal{J}_r$ for the flexible signal:



$\Phi_r(\omega)$, $\Phi_{u_r}(\omega)$ and $\Phi_{y_r}(\omega)$ for the white $r(t)$ (red) and for the flexible $r(t)$ (blue)

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