System Identification
Lecture 8

Closed-loop identification

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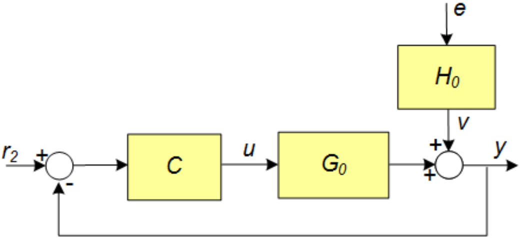
Introduction

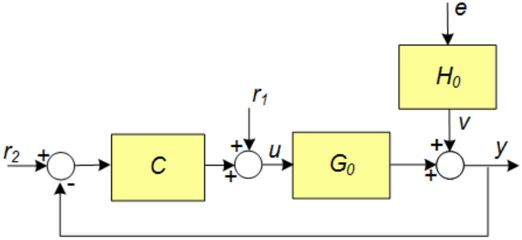


- Usual operation of many plants
- Particularly when processes are unstable
- Linearizing effect of controller

r_2 is reference or setpoint signal
Principle difference with OL situation: u and v correlated.

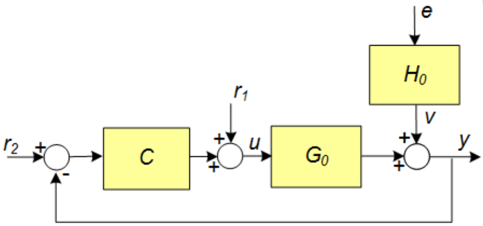
The closed-loop problem

System set-up:

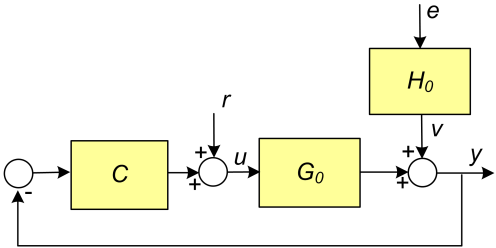


Available data:

1. $(u(t), y(t)),\ t = 1, \cdots N$ are measured
2. Additionally $r_1(t)$ or $r_2(t),\ t = 1, \cdots N$ might be measured
3. Knowledge of $C(q)$ might be used



For ease of notation:
introduce $r(t) := r_1(t) + C(q)r_2(t)$.



System's equations:

$$\begin{aligned} u &= r - Cy \\ y &= G_0 u + v \end{aligned}$$

leading to:

$$\begin{aligned} y &= \frac{G_0}{1 + CG_0} r + \frac{1}{1 + CG_0} v \\ u &= \frac{1}{1 + CG_0} r - \frac{C}{1 + CG_0} v \end{aligned}$$

Using the sensitivity function: $S_0 := \dfrac{1}{1 + CG_0}$

the system relations become:

$$\begin{aligned} y &= G_0 S_0 \; r + S_0 \; v \\ u &= S_0 \; r - CS_0 \; v \end{aligned}$$

Example ("is there a problem?") – Spectral analysis

Suppose that we make a nonparametric spectral analysis/ETFE estimate on the basis of u and y only

$$\hat{G}(e^{i\omega}) = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)};$$

$$\begin{aligned} y(t) &= S_0\left[G_0 r(t) + v(t)\right] \\ u(t) &= S_0\left[r(t) - Cv(t)\right] \end{aligned}$$

Then

$$\begin{aligned} \Phi_u(\omega) &= |S_0|^2[\Phi_r(\omega) + |C|^2\Phi_v(\omega)] \\ \Phi_{yu}(\omega) &= |S_0|^2[G_0\Phi_r(\omega) - C^*\Phi_v(\omega)] \end{aligned}$$

and so:

$$\hat{G} = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)} = \frac{G_0\Phi_r(\omega) - C^*\Phi_v(\omega)}{\Phi_r(\omega) + |C|^2\Phi_v(\omega)}$$

$$\hat{G}(e^{i\omega}) = \frac{G_0 \Phi_r(\omega) - C^* \Phi_v(\omega)}{\Phi_r(\omega) + |C|^2 \Phi_v(\omega)}$$

Limit cases:

$$\Phi_v(\omega) = 0 \quad \text{no noise} \quad \Rightarrow \quad \hat{G} = G_0$$

$$\Phi_r(\omega) = 0 \quad \text{no excitation} \quad \Rightarrow \quad \hat{G} = -1/C$$

There are two dynamical relationships between u and y (forward and backward)

A linear combination of the two is estimated dependent on the signal to noise ratio

Example: identifiability / lack of input excitation

Consider $\mathcal{S} \in \mathcal{M}$ for an ARX model set:

$$y(t) + ay(t-1) = bu(t-1) + e(t).$$

Controller: $u(t) = f \cdot y(t)$ (without external excitation)
Prediction error:

$$\begin{aligned} \varepsilon(t,\theta) &= y(t) + ay(t-1) - bu(t-1) \\ &= y(t) + (a - bf)y(t-1) \end{aligned}$$

and all models $(\hat{a}, \hat{b})$ such that $\hat{a} = a_0 + \gamma f, \ \hat{b} = b_0 + \gamma$ give the same prediction error for any $\gamma \in \mathbb{R}$. Knowledge of f does not help.

No unique model can be estimated (not enough excitation)

Possible way out when external excitation r is present, follows from system equations:

$$
\begin{aligned}
y &= G_0 S_0 \; r + S_0 \; v \\
u &= S_0 \; r - C S_0 \; v
\end{aligned}
$$

When r and v are uncorrelated, the two equations actually reflect two open-loop systems (with input r)

Note that

$$G_0 = \frac{G_{yr}}{G_{ur}}$$

where G_{yr} and G_{ur} can be estimated either parametrically or nonparametrically.

Two principle approaches:

- Direct identification (based on u and y only)
  Which part of the (open-loop) theory can still be used?
- Indirect identification (based on u, y and (r and/or C))

Which results are we going to focus on:

(a) Consistency of $(\hat{G}, \hat{H})$ (situation $\mathcal{S} \in \mathcal{M}$)

(b) Consistency of $\hat{G}$ (situation $G_0 \in \mathcal{G}$)

(c) Asymptotic variance properties

Direct identification method

Leading principle:

Use measured y, u and identify a -standard- model by "neglecting" the presence of C

Analysis results

Convergence result (Chapter 5) is valid also under closed-loop conditions:

$$\hat{\theta}_N \to \theta^* \quad \text{w.p. 1 as } N \to \infty$$

with

$$\theta^* = \arg\min_{\theta} \bar{V}(\theta) = \arg\min_{\theta} \bar{\mathbb{E}}\varepsilon^2(t, \theta)$$

Principle question:
Does $\min \bar{V}(\theta)$ lead to models with desired properties?

Combining:

$$\begin{aligned} y(t) &= G_0 u(t) + H_0 e(t) \\ u(t) &= r(t) - Cy(t) \\ \varepsilon(t,\theta) &= H(\theta)^{-1}[y(t) - G(\theta)u(t)] \end{aligned}$$

delivers:

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1 + CG(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon e}(q,\theta)} e(t)$$

If CG_0 and $CG(\theta)$ are strictly proper (i.e. the products contain a delay), then

$T_{\varepsilon e}(q,\theta)$ is *monic*

This requires that there is no algebraic loop in the system: $\lim_{z\to\infty} C(z)G_0(z) = 0$.

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1 + CG(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon e}(q,\theta) \text{ monic}} e(t)$$

Minimum of $\bar{E}\varepsilon(t,\theta)^2$ is achieved for

- $T_{\varepsilon r}(q,\theta^*) = 0$
- $T_{\varepsilon e}(q,\theta^*) = 1$

(provided that r is persistently exciting)

$$T_{\varepsilon r}(q,\theta^*) = 0 \Rightarrow \frac{(G_0 - G(\theta^*))}{H(\theta^*)(1 + CG_0)} = 0$$
$$\Rightarrow G(q,\theta^*) = G_0(q)$$

This, together with

$$T_{\varepsilon e}(q,\theta^*) = 1 \Rightarrow \frac{H_0(1 + CG(\theta^*))}{H(\theta^*)(1 + CG_0)} = 1$$

implies that $H(q,\theta^*) = H_0(q)$.

Consistency result of direct method

If $\mathcal{S} \in \mathcal{M}$, r is sufficiently exciting, and there are no algebraic loops in closed-loop system and parametrized models, then

$$G(q,\theta^*) = G_0(q); \qquad H(q,\theta^*) = H_0(q)$$

i.e. $G(q,\hat{\theta}_N)$ and $H(q,\hat{\theta}_N)$ are consistent estimates.

- Open-loop consistency result seems to be fully valid for the closed-loop case!

However:

Note: There is no consistency result for only $G_0 \in \mathcal{G}$.

$$\varepsilon(t,\theta) = \underbrace{\frac{(G_0 - G(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon r}(q,\theta)} r(t) + \underbrace{\frac{H_0(1 + CG(\theta))}{H(\theta)(1 + CG_0)}}_{T_{\varepsilon e}(q,\theta)\ \text{monic}} e(t)$$

- If $H(\theta) \neq H_0$, then bringing $T_{\varepsilon e}(q,\theta)$ "close to monic" needs to be comprised with making $T_{\varepsilon r}(q,\theta)$ "close to 0".
- This is due to the presence of $G(\theta)$ in $T_{\varepsilon e}(q,\theta)$.

The excitation condition on r can actually be relaxed.

It is sufficient for consistency in the case $\mathcal{S} \in \mathcal{M}$, that for any two models $M_1 = (G_1, H_1)$, $M_2 = (G_2, H_2)$ in $\mathcal{M}$ with predictor filters

$$W_i = [H_i^{-1} G_i \quad 1 - H_i^{-1}] \quad i = 1, 2$$

it holds that

$$\bar{\mathbb{E}} \left\{ \left[[W_1(q) - W_2(q)] \begin{pmatrix} u(t) \\ y(t) \end{pmatrix} \right]^2 \right\} = 0 \implies W_1 = W_2$$

Notion: Data set is informative w.r.t. $\mathcal{M}$

It guarantees that $\bar{V}(\theta) = \bar{V}(\theta_0)$ implies $\theta = \theta_0$

Rewriting the "power is 0" condition to the frequency domain:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{W}(e^{i\omega})\Phi_z(\omega)\tilde{W}^T(e^{i\omega})d\omega = 0$$

with $\tilde{W} = W_1 - W_2$, and

$$\Phi_z(\omega) = \begin{bmatrix} \Phi_u(\omega) & \Phi_{uy}(\omega) \\ \Phi_{yu}(\omega) & \Phi_y(\omega) \end{bmatrix}$$

it follows that informativity is guaranteed if

$$\Phi_z(\omega) > 0 \quad \forall\, \omega$$

This can be achieved by

- Presence of an exciting r, or
- A controller of sufficiently high order, or
- A time-varying or nonlinear controller

Question:

Where did the ARX-example from slide 9 go wrong?

Question:

Can we use identification techniques that rely on periodic input signal u?

Unstable plants

For all presented results is required:

- Predictor

$$\varepsilon(t,\theta) = H(\theta)^{-1}G(\theta)u(t) + (1 - H(\theta)^{-1})y(t)$$

  is (uniformly) stable.

For unstable G_0 this can be satisfied if system can be modelled in an ARX or ARMAX structure.
Then unstable dynamics in $G(q,\theta)$ is cancelled out in $H(q,\theta)^{-1}G(q,\theta)$.

Summary direct identification method

- Consistent estimates in the situation $\mathcal{S} \in \mathcal{M}$, under excitation conditions
- Excitation conditions can be realized by either presence of r or through input excitation through the noise process
- No consistency when only $G_0 \in \mathcal{G}$
- No "free" excitation of input u; periodic excitation of u is not feasible
- Unstable plants can be modelled only with particular model structures (ARX,ARMAX)

Indirect identification methods

Main step with respect to direct methods:

- Additional use of measured r and/or controller C

Several indirect methods are all closely related:

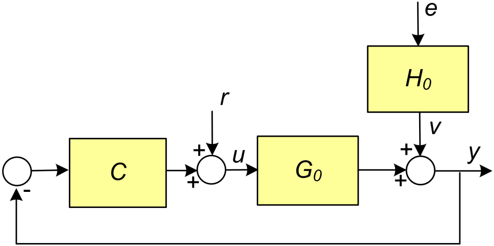
- Nonparametric / Coprime factor method
- Projection methods (Two-stage and Instrumental variable (IV)-method)
- Classical indirect
- Dual-Youla method

1. Non-parametric / coprime factor method

$$\hat{G} = \frac{\Phi_{yr}(\omega)}{\Phi_{ur}(\omega)}$$

i.e. based on nonparametric frequency response estimates of

$$G_{yr} = \frac{G_0}{1 + CG_0} \text{ and } G_{ur} = \frac{1}{1 + CG_0}.$$



1. Non-parametric / coprime factor method

- Using reference excitation to identify two "open-loop" systems, and take quotient
- Any desired excitation (e.g. periodic) can be used for r.

Can be done with parametric models (coprime factors of G_0).

- $\hat{G} = \frac{\hat{G}_{yr}}{\hat{G}_{ur}}$
- Hard to control the order of G_0 (because of quotient)
- Using filtered versions of r as input, can help in low-order representations of G_0

2. Projection methods (two-stage / IV)

Approach

- Decompose the input signal u into

$$u(t) = u^r(t) + u^e(t)$$

  i.e. the components of u that result from r and e respectively. Since r and e are uncorrelated this implies that both u-components are uncorrelated too.

- Then

$$y(t) = G_0 u^r(t) + \underbrace{G_0 u^e(t) + H_0 e(t)}_{\text{disturbance terms}}.$$

- Identify G_0 (and possibly a noise model), based on input u^r and output y. This is basically an open-loop problem.

2. Projection methods (two-stage / IV)

- Typically G_0 is identified with a parametric model
- The model order of $\hat{G}$ can directly be prespecified (contrast with coprime factor method)

How to construct (an estimate of) u^r?

- Identify the transfer function G_{ur} on the basis of r and u (open-loop problem)
- Simulate:

$$\hat{u}^r(t) = \hat{G}_{ur}(q)r(t)$$

- Use $\hat{u}^r(t)$ as input signal in the identification of G_0.

2. Projection methods (two-stage / IV)

Instrumental variable (IV) method:

Alternative identification criterion:

$$\hat{\theta}_N = \text{sol}_\theta \{ \sum_{t=1}^{N} \zeta(t) \varepsilon(t, \theta) = 0 \}$$

with $\zeta(t) \in \mathbb{R}^d$, the instrument vector.

LS-ARX method is IV with $\zeta(t) = \varphi(t) =$

$$[-y(t-1)\ -y(t-2)\ \cdots -y(t-n_a)\ u(t)\ u(t-1)\ \cdots u(t-n_b+1)]^T$$

Indeed for LS-ARX:

$$\sum_t \varphi(t)\varepsilon(t, \hat{\theta}_N) = \sum_t \varphi(t)[y(t) - \varphi^T(t)\hat{\theta}_N] = 0$$

2. Projection methods (two-stage / IV)

For consistency of $G(,q,\hat{\theta}_N)$, ζ should typically satisfy:

- ζ is correlated to the input and output signals of the system to be modelled;
- ζ is uncorrelated to the output noise

If ζ chosen as delayed versions of the reference signal (uncorrelated with v), then it satisfies the conditions.

So e.g.:

$$\zeta(t) = [r(t)\ r(t-1)\ r(t-2)\ \cdots\ r(t-d)]^T$$

2. Projection methods (two-stage / IV)

IV is usually applied with an ARX (linear regression) model structure:

$$\varepsilon(t,\theta) = y(t) - \varphi^T(t)\theta$$

Then

$$\hat{\theta}_N^{IV} = \left[\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\varphi^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\zeta(t)y(t)\right].$$

If the real system satisfies: $y(t) = \varphi^T(t)\theta_0 + w(t)$, then

$$\hat{\theta}_N^{IV} = \theta_0 + \left[\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\varphi^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\zeta(t)w(t)\right]$$

3. Classical indirect method

Approach

- Identify $G_{yr} = \frac{G_0}{1+CG_0}$ on the basis of r and y
- Use the knowledge of C to derive an estimate of G_0: Choose $\hat{G}$ such that

$$\hat{G}_{yr} = \frac{\hat{G}}{1 + C\hat{G}}$$

i.e.

$$\hat{G} = \frac{\hat{G}_{yr}}{1 - C\hat{G}_{yr}}.$$

- Like with the coprime factor method, the model order of $\hat{G}$ is a result of the manipulations

4. Dual Youla method

Main principle:
Parametrize the plant model $G(\theta)$ within the class of all linear plant models that are stabilized by a given (and known) controller C.

4. Dual Youla method

Coprime factorization over $\mathbb{R}\mathcal{H}_\infty$ (Vidyasagar, 1985).
Let G_0 be a (possibly unstable) system, and let N, D be stable rational transfer functions in $\mathbb{R}\mathcal{H}_\infty$. Then the pair (N, D) is a (right) coprime factorization (rcf) of G_0 over $\mathbb{R}\mathcal{H}_\infty$, if

(a) $G_0 = ND^{-1}$, and

(b) there exist stable transfer functions $X, Y \in \mathbb{R}\mathcal{H}_\infty$ such that $XN + YD = I$ □

Coprime factors do not have unstable zeros that cancel in the quotient.

4. Dual-Youla method

Dual-Youla Parametrization

Hansen, Franklin (1989), Lee et al. (1993), Schrama (1991)

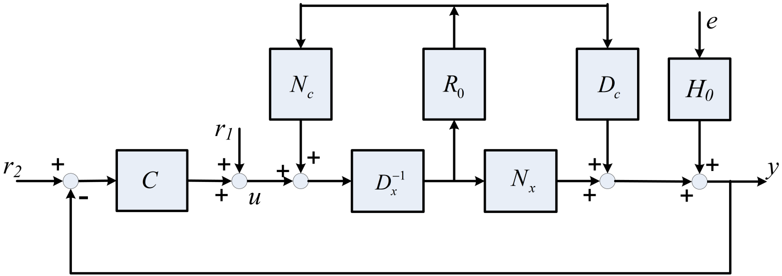
Dual-Youla parametrization

Let $C = N_c/D_c$ be a coprime factorization of an LTI controller C, and let N_x/D_x be a coprime factorization of any plant G_x that is stabilized by C. Then an LTI plant G_0 is stabilized by C if and only if there exists a stable transfer function R_0 such that

$$G_0 = \frac{N_x + D_c R_0}{D_x - N_c R_0}$$

$R(\theta)$: parametrization of all LTI plants that are stabilized by a given controller.

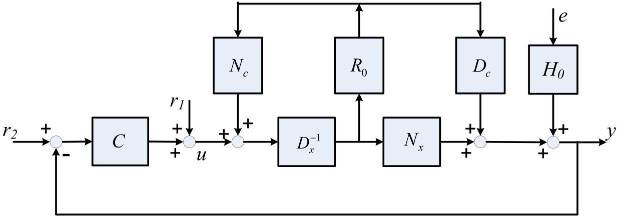
4. Dual Youla method

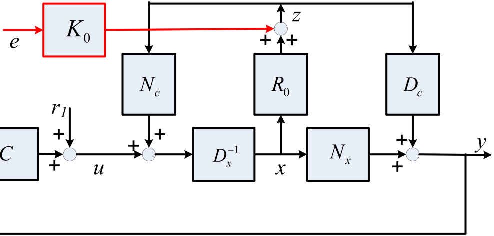


G_0 is replaced by

$$\frac{N_x + D_c R_0}{D_x - N_c R_0}$$

4. Dual Youla method





Equivalent in view of u and y, when $K_0 = \frac{H_0}{D_c(1+CG_0)}$.

4. Dual Youla method

- With $K_0 = \frac{H_0}{D_c(1+CG_0)}$ it appears that the block diagram is equivalent to the standard one, in view of the signals y, u, r and e.
- The indicated signals z and x satisfy

$$\begin{aligned} x(t) &= (D_x + CN_x)^{-1}[u(t) + C(q)y(t)] \\ z(t) &= (D_c + G_x N_c)^{-1}[y(t) - G_x(q)u(t)] \end{aligned}$$

  and therefore can be calculated on the basis of measured data y, u, and knowledge of C and G_x.
- Since $u + Cy = r_1 + Cr_2 = r$, x and e are uncorrelated.

4. Dual Youla method

As a result we can write "new" system equations, taking the form:

$$z(t) = R_0(q)x(t) + K_0(q)e(t)$$

with x and e uncorrelated, and

$$R_0 = \frac{(G_0 - G_x)D_x}{D_c(1 + CG_0)} \qquad K_0 = \frac{H_0}{D_c(1 + CG_0)}.$$

R_0 and K_0 can be identified as in an open-loop situation.

4. Dual Youla method

Particular example

If C is a stable controller, a possible choice is:

$$C = C/1; \qquad G_x = 0/1$$

and consequently

$$R_0 = \frac{G_0}{1 + CG_0}$$
$$z(t) = y(t); \quad x(t) = r(t)$$

(classical indirect method of closed-loop ID).

- Dual-Youla framework allows handling unstable plants and controllers.
- Estimated plant model $G(\hat{\theta}_N)$ is guaranteed to be stabilized by C.

Consistency

Direct method

Consistency of (G, H) can be obtained in the situation $\mathcal{S} \in \mathcal{M}$.

Indirect methods

Consistency of G can be obtained in the situation $G_0 \in \mathcal{G}$.

Asymptotic variance

Direct method

- The variance results of the open-loop situation remain valid, provided that we have consistency ($\mathcal{S} \in \mathcal{M}$).
- This includes the Maximum Likelihood properties of the estimates (minimum variance asymptotically)
- Asymptotic-in-order-of-G result for $n, N \to \infty$, and noise model in model set:

$$var\, \hat{G}(e^{i\omega}) \sim \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u(\omega)}$$

- In the case $\mathcal{S} \in \mathcal{M}$ the full input signal u is used for identification.

Asymptotic variance

Indirect methods

- Typically the reference signal r is used as input for identification;
- Typical variance result (asymptotic in model order n and in N):

$$var\,\hat{G}(e^{i\omega}) \sim \frac{n}{N}\frac{\Phi_v(\omega)}{\Phi_{u^r}(\omega)}$$

  valid in the situation $G_0 \in \mathcal{G}$.
- Only the reference-part of the input signal contributes to variance reduction.
- Neglecting u^e as input signal contributes to a worse SN-ratio.

Asymptotic variance

Reasoning behind asymptotic variance result

Asymptotic $(n, N \to \infty)$ result is:

$$E \begin{pmatrix} \hat{G}(e^{i\omega}) - G_0(e^{i\omega}) \\ \hat{H}(e^{i\omega}) - H_0(e^{i\omega}) \end{pmatrix} \begin{pmatrix} \quad \end{pmatrix}^*$$

$$\sim \frac{n}{N} \Phi_v(\omega) \cdot \begin{bmatrix} \Phi_u(\omega) & \Phi_{eu}(\omega) \\ \Phi_{ue}(\omega) & \sigma_e^2 \end{bmatrix}^{-1}.$$

Using

$$\Phi_u = \Phi_u^r + \Phi_u^e$$

and direct use of the system's equations delivers:

$$var(\hat{G}(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u^r(\omega)}$$

Bias expressions - Approximations

Bias in direct closed-loop identification

$$\hat{\theta}_N \to \theta^* = \arg\min_{\theta} \bar{V}(\theta); \qquad \bar{V}(\theta) = \bar{\mathbb{E}}\varepsilon_f^2(t,\theta)$$

By Parsseval, $\bar{V}(\theta) =$ (see slide 13)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\left|\frac{G_0 - G(\theta)}{1 + CG_0}\right|^2 \Phi_r + \left|\frac{1 + CG(\theta)}{1 + CG_0}\right|^2 \Phi_v\right\} \frac{|L|^2}{|H(\theta)|^2} d\omega$$

No explicit (tunable) approximation criterion for $G(\theta)$,
since $G(\theta)$ appears in both terms of the integrand

Bias in indirect closed-loop identification

Bias expressions for "all" indirect alternatives

$$\theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \frac{G_0}{1 + CG_0} - \frac{G(\theta)}{1 + CG(\theta)} \right|^2 \frac{\Phi_r |L|^2}{|K|^2} d\omega$$

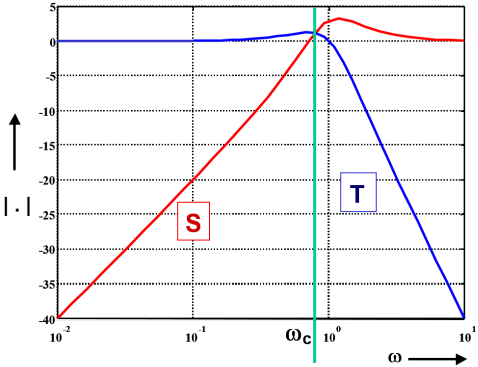
(with slight variations) and K a fixed noise model.

Closed-loop properties of the plant ($G_0 S_0$) are best approximated.

Note

$$\frac{G_0}{1 + CG_0} - \frac{G(\theta)}{1 + CG(\theta)} = \frac{G_0 - G(\theta)}{(1 + CG_0)(1 + CG(\theta))}$$

"Additive error" is weighted with sensitivity of plant and model.



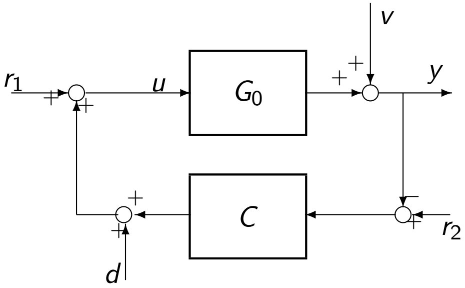
Typical curve for Bode magnitude plot of sensitivity function S_0 and related complementary sensitivity $G_0 C/(1 + CG_0)$.

Model errors are highly weighted around the cross-over frequency of the closed-loop.

Main properties of the different methods

	Direct	Cop-Fac	Two-St	Clas.Ind/DY
Consistency $(\hat{G}, \hat{H})$	$+$	$+$	$+$	$+$
Consistency $\hat{G}$	$-$	$+$	$+$	$+$
Tunable bias	$-$	$+$	$+$	$+$
Fixed model order	$+$	$-$	$+$	$-$
Variance	$+$	$-$	$-$	$-$
C assumed known	no	no	no	yes
C assumed linear	no	yes	yes	yes
$G(\hat{\theta}_N)$, C stable	no	no	no	no/yes

For noise disturbed controller output:



- If C known: use $u + Cy$ as external signal $\rightarrow$ d can effectively be used as external signal reducing the variance
- If C unknown, r measured: d acts as additional disturbance

Knowledge of C is more informative than knowledge of r

Model validation in closed-loop

- For all indirect methods:
  validation with correlation tests as in open-loop
- For direct method: Careful with test on $R_{\varepsilon u}(\tau)$.

$$\begin{aligned}\varepsilon(t,\theta) &= H(\theta)^{-1}[(G_0 - G(\theta))u(t) + H_0 e(t)] \\ R_{\varepsilon u}(\tau) &= H(\theta)^{-1}[G_0 - G(\theta)]R_u(\tau) + H(\theta)^{-1}H_0 R_{eu}(\tau).\end{aligned}$$

Can the second term influence the cross-correlation test?

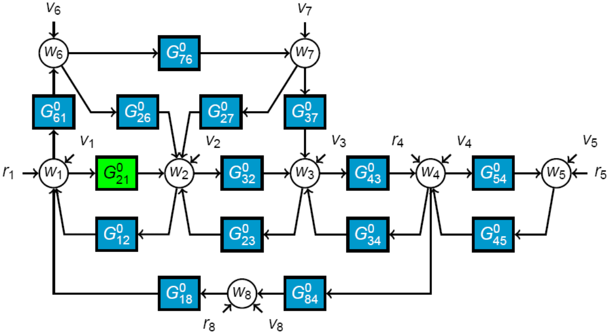
$R_{eu}(\tau)$ will have a contribution for $\tau < 0$ only.
The second term can then have a contribution for $\tau > 0$ if the filter $H(\theta)^{-1}H_0$ has dynamics, i.e. when the noise model is incorrect.

For the direct method the residual tests should not be interpreted independently (validation of $\hat{G}$ and $\hat{H}$ simultaneously).

Summary - Closed-loop identification

- Nonparametric estimation in closed-loop needs to be done in an indirect way (using a reference r)
- Parametric models can be consistently identified with a direct method
- but only through modelling G and H simultaneously ($\mathcal{S} \in \mathcal{M}$)
- Indirect methods can provide consistent estimates in the situation $G_0 \in \mathcal{G}$
- Direct methods lead to smaller variance (ML-properties)
- In closed-loop identification, the frequency area around the cross-over frequency of the closed-loop system, typically is most dominantly present in the plant data/models.

Extensions / challenges Distributed/networked systems



- Where to measure for identifying a local module?
- Can we identify the full network (identifiability)
  - topology and dynamics?
- Prior knowledge of modules