Identification for Control

A historical overview and recent results

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Symposium Four decades of data-driven modeling in systems and control
achievements and prospects
April 19, 2024

Historical perspective: A few pieces of the puzzle (= Paul’s contributions in the 1990s)

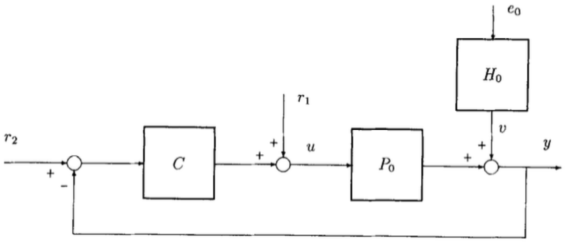
A ‘small-scale’ multivariable application

Putting the ideas together

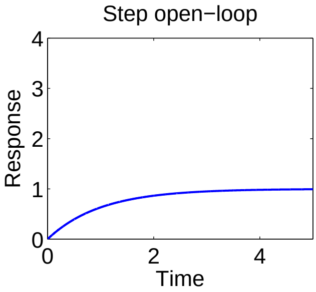
Why is this important?

Final remarks

Back to a very specific 'network' Van den Hof & Schrama (1995)



Illustrative identification example



Identification: determine a model that gives a good prediction of the output

- G_o: true system

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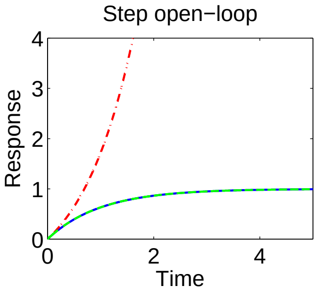

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- $\hat{G}$: model 1
- $\hat{G}$: model 2

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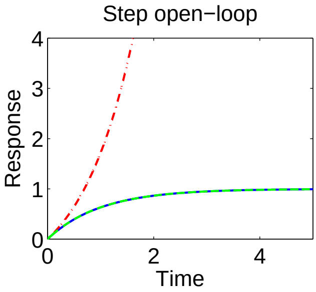

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Best model?

Idea 1: obtaining good models for control

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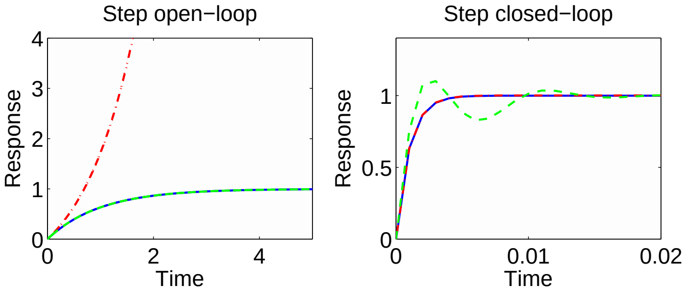

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Best model?
... bad model can be good! (and *vice versa*...)

Idea 1: obtaining good models for control

Original idea Van den Hof & Schrama (1995):
important dynamics are revealed when you get closer to the 'optimal' controller

In general terms the model and the controller are obtained according to (indexes refer to step number in the iteration):

$$\hat{P}_{i+1} = \arg\min_{\tilde{P}} \|J(P_0, C_i) - J(\tilde{P}, C_i)\| \quad (12)$$

$$C_{i+1} = \arg\min_{\tilde{C}} \|J(\hat{P}_{i+1}, \tilde{C})\| \quad (13)$$

where $\tilde{P}$, $\tilde{C}$ vary over appropriate model/controller classes, and in the control design one takes account of the constraint:

$$\|J(P_0, C_{i+1}) - J(\hat{P}_{i+1}, C_{i+1})\| << \|J(\hat{P}_{i+1}, C_{i+1})\|. \quad (14)$$

There are a couple of important observations to make here.

- The identification criterion that is reflected in (12), is completely determined by the control performance function $J(P, C)$ and the chosen norm $\|\cdot\|$, thus leading to a really control-oriented identification. The mismatch between plant and model is measured in terms of the control performance costs of plant and model, when controlled by the controller C_i.

Idea 1: obtaining good models for control

When is a model good?

- if $\hat{G}$ is exact: $\hat{G} = G_o$, then model is good for any purpose
- in practice: model errors
  - bias: model structure not flexible enough
  - variance: only finite time and noisy data available
- if $\hat{G} \neq G_o$, then quality depends on goal

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- select criterion $J(G, K) = \|N(G, K)\|$
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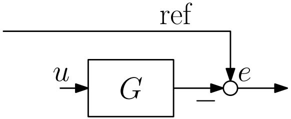
- example: sensitivity minimization



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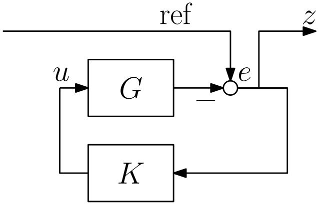

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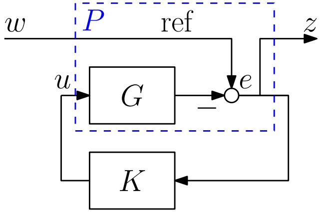

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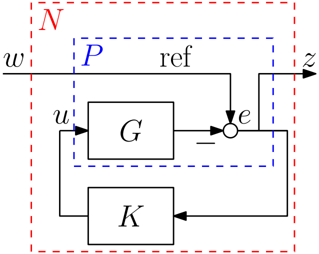

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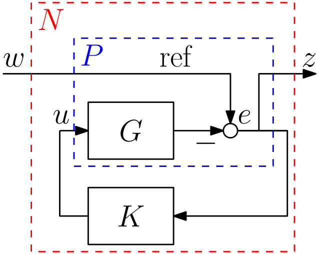
- example: sensitivity minimization : $N(G, K) = (I + GK)^{-1}$



Basic idea in identification for control [> 1990s]

- apply triangle inequality

$$\underbrace{J(G_o, K)}_{\text{achieved performance}} = \| \qquad N(G_o, K) \qquad \|$$

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$$\leq \underbrace{J(\hat{G}, K)}_{\text{model-based control}} + \underbrace{\|N(G_o, K) - N(\hat{G}, K)\|}_{\text{performance degradation}}$$

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Classical procedure

1. for a reasonable controller K^{exp}, identify $\arg\min_{\hat{G}} \|N(G_o, K^{\mathrm{exp}}) - N(\hat{G}, K^{\mathrm{exp}})\|$

   $\Rightarrow$ matches the closed-loop response

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2. model-based control $K^{\text{opt}} = \min_K(\hat{G}, K)$

The need for robustness

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Idea 2: robustness is key for feedback control

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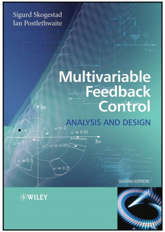

Robust control design

1. identify a model set $\mathcal{G}$, where $G_o \in \mathcal{G}(\hat{G}, \Delta)$
2. robust control: performance guarantee $J(G_o, K) \leq \sup_{G \in \mathcal{G}} J(G, K)$

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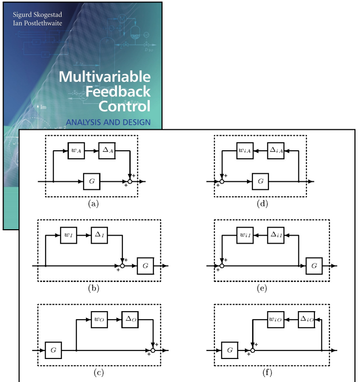

Figure 8.5: (a) Additive uncertainty, (b) Multiplicative input uncertainty, (c) Multiplicative output uncertainty, (d) Inverse additive uncertainty, (e) Inverse multiplicative input uncertainty, (f) Inverse multiplicative output uncertainty.

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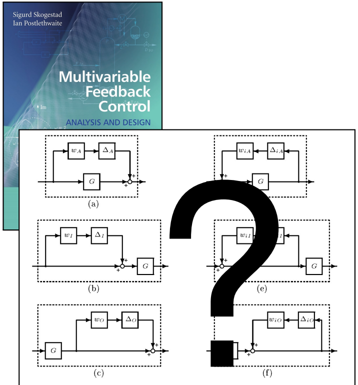
Identification of $\mathcal{G}$ for robust control [2000s - now]

- traditional structures:
  - how to guarantee $G_o \in \mathcal{G}(\hat{G}, \Delta)$? (idea 2)



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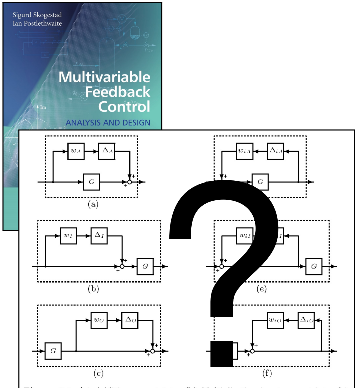
  - (Later, idea 3: $\sup_{G \in \mathcal{G}} J(G, \color{blue}{K^{\text{exp}}})$ unbounded?)



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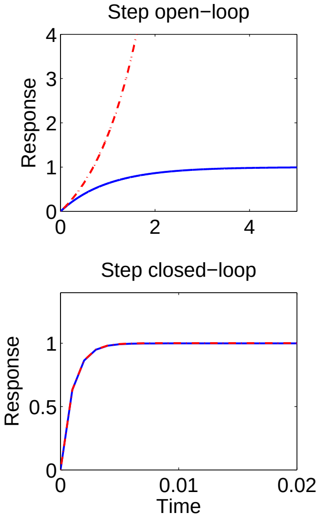
Idea 2: robustness is key for feedback control

Example revisited

- $G_o = \frac{1}{s+1}$
- $K^{\text{exp}} = 1000$ (optimal)
- $\hat{G} = \frac{1}{s-1}$ ('control-relevant')



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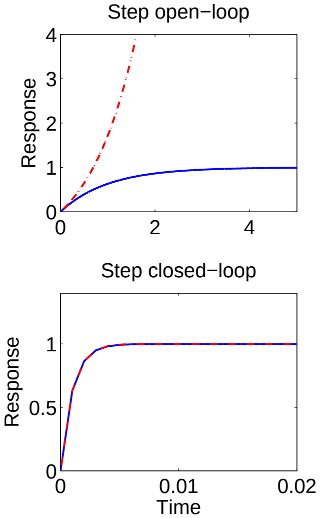

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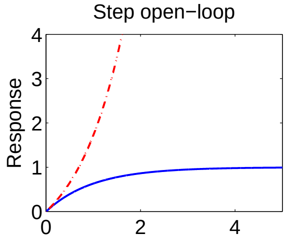
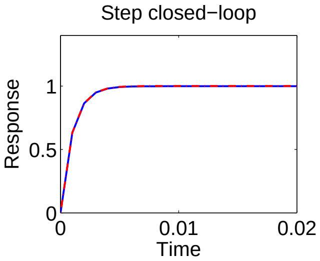
Solution: coprime factor perturbations

- $\hat{G} = \hat{N}\hat{D}^{-1}$, with $\hat{N}, \hat{D} \in \mathcal{H}_\infty$
- $G_o \in (\hat{N} + \Delta_N)(\hat{D} + \Delta_D)^{-1}$ for some stable Δ_N, Δ_D
- mechanism: now an RHP pole can be created by Δ_D
- this is fairly abstract, what does this mean?





More on coprime factor perturbations

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- think of this as two closed-loop transfer functions

$$\hat{N} = \frac{\hat{G}}{1 + \hat{G}K^{\text{exp}}}$$

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- If $K^{\text{exp}} \in \mathcal{H}_\infty$, then this is actually also a coprime factorization, since the Bezout identity

$$X\hat{N} + Y\hat{D} = 1$$

holds for $X = K^{\text{exp}}$, $D = 1$ (indeed, $S + T = 1$!)

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  holds for $X = K^{\text{exp}}$, $D = 1$ (indeed, $S + T = 1$!)
- You can easily parameterize all by $\{\hat{N}Q, \hat{D}Q\}$, with $Q, Q^{-1} \in \mathcal{H}_\infty$

Idea 2: robustness is key for feedback control

Idea Van den Hof et al. (1993):
go from control-relevant id (idea 1) to iteratively finding 'normalized' RCFs

Classical procedure

1. for a reasonable controller $K^{\exp}$, identify $\arg\min_{\hat{G}} \|N(G_o, K^{\exp}) - N(\hat{G}, K^{\exp})\|$

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...nformation Sciences

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D. C. McFarlane, K. Glover

Robust Controller Design Using Normalized Coprime Factor Plant Descriptions

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of (13), yielding

$$\min_{\substack{\theta \\ D(\theta)+CN(\theta)=F^{-1}}} \left\| W_o \begin{bmatrix} N_{o,F} - N(\theta) \\ D_{o,F} - D(\theta) \end{bmatrix} F \begin{bmatrix} C & I \end{bmatrix} W_i \right\|_{\mathcal{X}}$$

Proof: With $W_1 = W_o$, $W_2 = F[C\ I]W_i$ the argument of $\|\cdot\|_{\mathcal{X}}$ in (11) equals the argument of $\|\cdot\|_\infty$ in (5), by substituting the results of lemma 5.1. □

5.2 Minimization with restriction

In order to deal with the restriction (13) in the minimization, basically two approaches can be followed. The first approach is to parametrize $N(\theta)$ and $D(\theta)$ in such a way

...ifferent rcf's of $P(\theta)$ give the same value of the objective function. This gives us an additional freedom in the construction of the "fixed" filter F, characterized by: $F_i = [D(\hat{\theta}_{i-1})Q + CN(\hat{\theta}_{i-1})Q]^{-1}$, where Q is just any stable transfer function.

In order to reduce this additional freedom, which is required for obtaining convergence of the iterative scheme, the filter F will be updated by restricting the rcf of the model to be normalized. In this way

$$F_i = (\bar{D}_{i-1} + C\bar{N}_{i-1})^{-1}$$

where $(\bar{N}_{i-1}, \bar{D}_{i-1})$ is a normalized rcf of the model $P(\hat{\theta}_{i-1})$ with $P(\hat{\theta}_{i-1}) = N(\hat{\theta}_{i-1})D(\hat{\theta}_{i-1})^{-1}$, which is ...vith a unitairy matrix. ...lel estimate with nor... $= \bar{N}$ and $\bar{D}_{i-1} = \bar{D}$, ...ws.

...$\bar{N}_{i-1})^{-1}$ and simulate

...e minimization given ...parameter restriction

Proceedings of the 32nd Conference on Decision and Control
San Antonio, Texas • December 1993

FA10 - 10:10

Identification of normalized coprime plant factors for iterative model and controller enhancement

Paul M.J. Van den Hof   Ruud J.P. Schrama[†]   Okko H. Bosgra   Raymond A. de Callafon[§]

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Idea 3: non-careful robustness may induce conservatism

- $(\hat{N} + \Delta_N)(\hat{D} + \Delta_D)^{-1}$ guarantees $G_o \in \mathcal{G}$ for some $\Delta_N, \Delta_D \in \mathcal{H}_\infty$
- however: $\mathcal{J}_{\mathrm{WC}}(\mathcal{G}, K^{\mathrm{exp}})$ can become unbounded
  - no guarantees that all candidate models in $\mathcal{G}$ stabilized by K^{exp}...

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Recall: Youla parameterization (1970s)

Let K^{exp} be a stabilizing controller for $\hat{G} = \hat{N}\hat{D}^{-1}$, with $K^{\text{exp}} = N_c D_c^{-1}$. Then all stabilizing controllers for $\hat{G}$ are given by

$$K = (N_c + \hat{D}Q)(D_c - \hat{N}Q)^{-1}, \; Q \in \mathcal{H}_\infty$$

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Dual-Youla: switch role of K^{exp} and $\hat{G}$!

All models stabilized by K^{exp} are given by

$$(\hat{N} + D_c\Delta)(\hat{D} - N_c\Delta)^{-1}, \, \Delta \in \mathcal{H}_\infty$$

Idea 3: non-careful robustness may induce conservatism

Why does this lead to a robust-control-relevant model set?

- general LFT uncertainty:
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Brief Paper

Controller tuning freedom under plant identification uncertainty: double Youla beats gap in robust stability☆

Sippe G. Douma[a], Paul M.J. Van den Hof[a,*], Okko H. Bosgra[b]

Idea 3: non-careful robustness may induce conservatism

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Historical perspective: A few pieces of the puzzle (= Paul’s contributions in the 1990s)

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A 3 $\times$ 3 wafer stepper application (de Callafon & Van den Hof 2001)

IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 9, NO. 2, MARCH 2001 381

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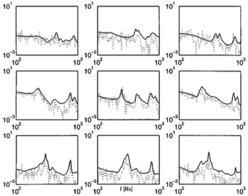

Fig. 10. Amplitude bode plot of estimated uncertainty bound $\delta(\omega)$ (—) of $\overline{\Delta}$ and frequency domain estimate of $\overline{\Delta}$ ($\cdots$).

- Uncertainty regions for frequencies in any user-chosen frequency grid are computed from bias and variance errors.

A 3 × 3 wafer stepper application (de Callafon & Van den Hof 2001) - the details

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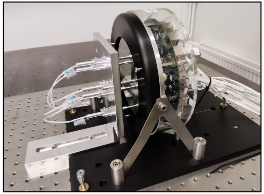
Don't even think about trying this on our 26 $\times$ 52 system (= 1352 elements!)



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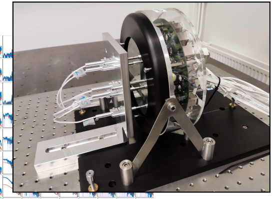

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Idea 1 and 2 connect!

Result (Oomen & Bosgra 2012): The control-relevant identification criterion is equivalent to a coprime factor identification problem:

$$\min_{\hat{G}} \|W\left(T(G_o, K^{exp}) - T(\hat{G}, K^{exp})\right)V\|_\infty = \min_{\hat{N},\hat{D}} \left\| W\left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix}\right) \underbrace{\cancel{\begin{bmatrix} \tilde{N}_{e,2} & \tilde{N}_{e,1} \end{bmatrix}}}_{\text{co-inner}} \right\|_\infty$$

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Resulting coprime factorization $\{\hat{N}, \hat{D}\}$ of $\hat{G}$

- generally not normalized: $\hat{N}^* \hat{N} + \hat{D}^* \hat{D} \neq I$
- direct identification from data:
  - reduces complexity: 4-block $\Rightarrow$ 2-block
  - frequency domain identification algorithm
- use of non-normalized coprime factorizations also appearing in robust control theory (Lanzon & Papageorgiou 2009)

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connects Δ and criterion $\mathcal{J}$: avoids multivariable & frequency dependent weighting

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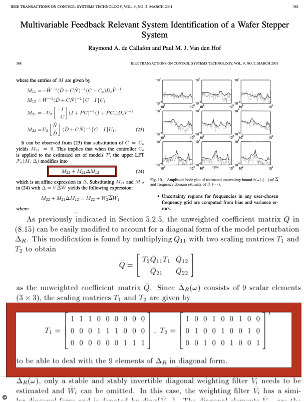

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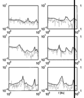

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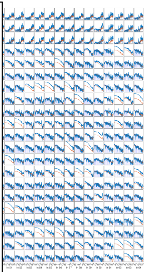
Packard and Doyle (1993)

TABLE 1. GUARANTEED EQUALITY BETWEEN μ AND THE UPPER BOUND

	F=0	F=1	F=2	F=3	F=4
S=0		YES	YES Section 9.1	YES Section 9.3	NO Section 9.2
S=1	YES	YES Section 9.4	NO Section 9.6	NO	NO
S=2	NO Section 9.5	NO	NO	NO	NO



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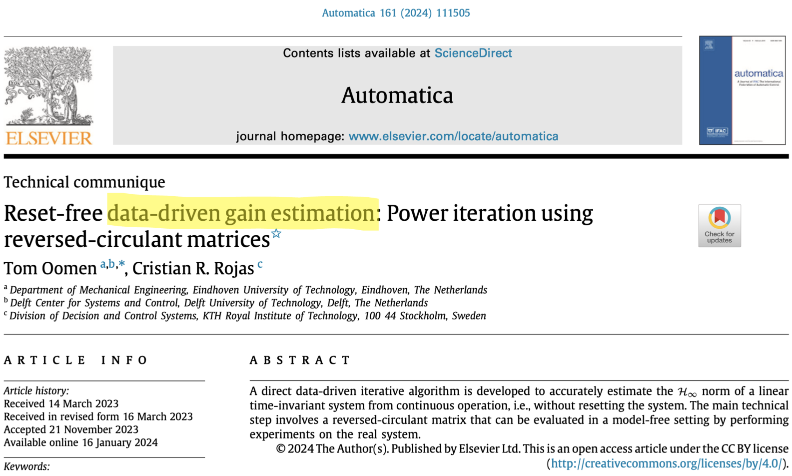
Always μ-simple, so nonconvervative D-K iteration, independent of input-output dimension!

Data-driven gain estimation (no multivariable/frequency scaling!)



Automatica 161 (2024) 111505

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Technical communique

Reset-free data-driven gain estimation: Power iteration using reversed-circulant matrices☆

Tom Oomen [a,b,*], Cristian R. Rojas [c]

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[b] Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands
[c] Division of Decision and Control Systems, KTH Royal Institute of Technology, 100 44 Stockholm, Sweden

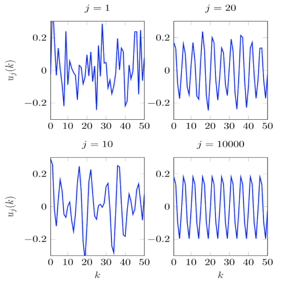
A R T I C L E I N F O

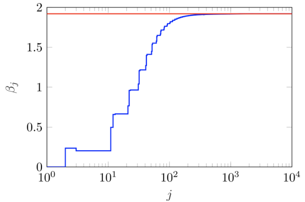
Article history:
Received 14 March 2023
Received in revised form 16 March 2023
Accepted 21 November 2023
Available online 16 January 2024

Keywords:

A B S T R A C T

A direct data-driven iterative algorithm is developed to accurately estimate the $\mathcal{H}_\infty$ norm of a linear time-invariant system from continuous operation, i.e., without resetting the system. The main technical step involves a reversed-circulant matrix that can be evaluated in a model-free setting by performing experiments on the real system.





$P = (\hat{N} + D_c\Delta)(\hat{D} - N_c\Delta)^{-1}, \Delta \in \mathcal{H}_\infty \ldots$???

$P = (\hat{N} + D_c\Delta)(\hat{D} - N_c\Delta)^{-1}, \Delta \in \mathcal{H}_\infty \ldots ???$

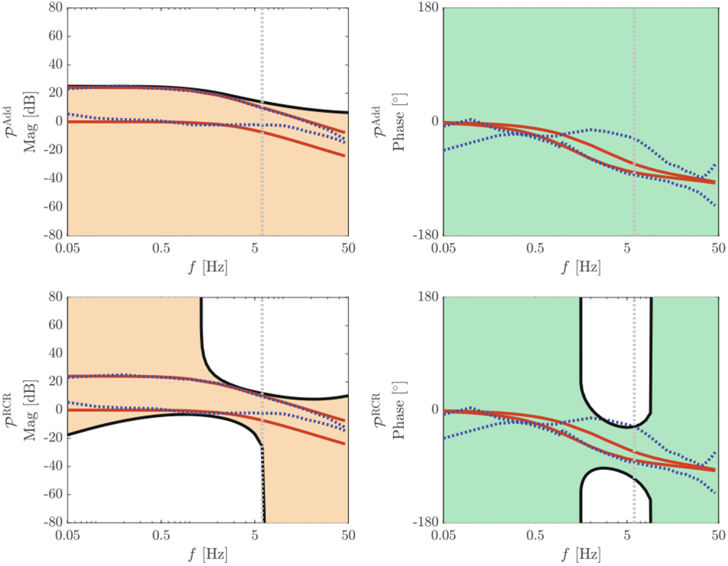
Received: 2 August 2022 | Revised: 16 May 2023 | Accepted: 23 June 2023
DOI: 10.1002/rnc.6866

RESEARCH ARTICLE

WILEY

Comparing multivariable uncertain model structures for data-driven robust control: Visualization and application to a continuously variable transmission

Paul Tacx[1] | Tom Oomen[1,2]



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- three important and original ideas that are essential pieces of a larger puzzle (3x Paul in 1990s)
- they connect: control-relevant (idea 1) and coprime-factor identification (idea 2):

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- thanks Paul! For all the fantastic interactions, I learned a lot! (Including the initial 4,5 hour scientific discussion (June 29, 2009, Delft), invitation to ERNSI, etc. etc. etc.)

References I

de Callafon, R. A. & Van den Hof, P. M. J. (2001), 'Multivariable feedback relevant system identification of a wafer stepper system', *IEEE Transactions on Control Systems Technology* **9**(2), 381–390.

Lanzon, A. & Papageorgiou, G. (2009), 'Distance measures for uncertain linear systems: A general theory', *IEEE Transactions on Automatic Control* **54**(7), 1532–1547.

Oomen, T. & Bosgra, O. (2012), 'System identification for achieving robust performance', *Automatica* **48**(9), 1975–1987.

Van den Hof, P. M. J. & Schrama, R. J. P. (1995), 'Identification and control - closed-loop issues', *Automatica* **31**(12), 1751–1770.

Van den Hof, P. M. J., Schrama, R. J. P., Bosgra, O. H. & de Callafon, R. A. (1993), Identification of normalized coprime plant factors for iterative model and controller enhancement, *in* 'Proceedings of the 32nd Conference on Decision and Control', San Antonio, Texas, United States, pp. 2839–2844.