EINDHOVEN UNIVERSITY OF TECHNOLOGY

# Single module identifiability in linear dynamic networks

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18 December 2018

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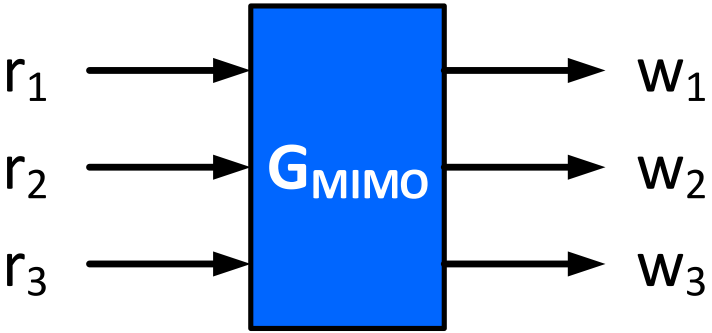
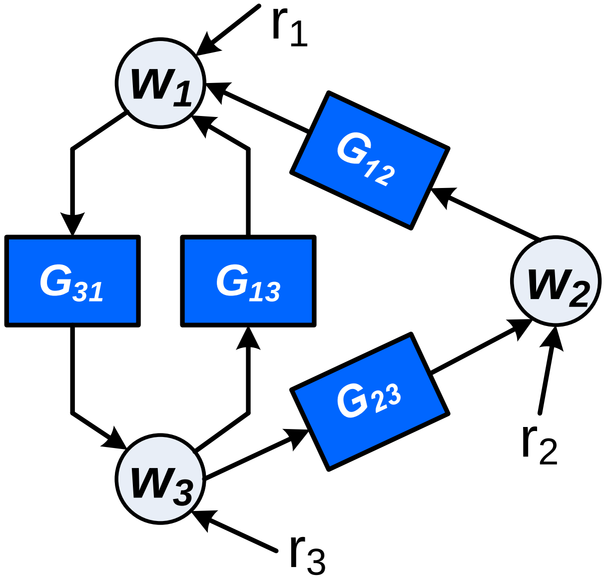
European Research Council

# Dynamic models that include structure

From input/output models towards dynamic networks




Objective: Identification of a module, where to excite?

When can we distinguish between modules?

Topology based conditions?

# Outline

- **The dynamic network**
- Single module identifiability
- Topology based conditions

# Flexible experimental setup

Choose number and location of external excitations

External variables / references

Measured nodes

Process noise

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G_{12} & \cdots & G_{1L} \\ G_{21} & 0 & \ddots & G_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G_{L1} & G_{L2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_L \end{bmatrix}$$

$$w = Gw + Rr + v$$

Modules

This talk:

Strictly proper to avoid technicalities

Stable & well-posed

Noise may be correlated

Rational transfer functions

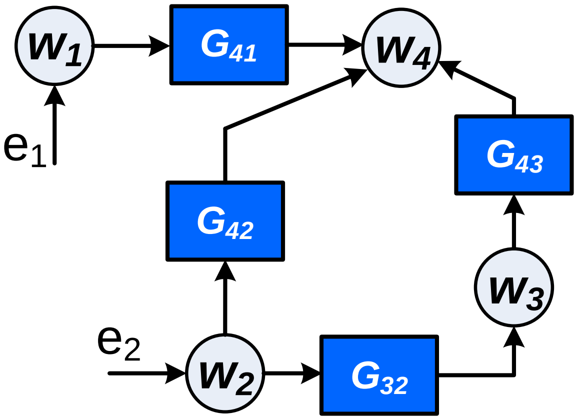
# Example

Can modules that map into node 4 be identified?

$$w = Te, \quad T = (I - G)^{-1}H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \\ G_{41} & G_{42} + G_{32}G_{43} \end{bmatrix}$$

$$(I - G(\theta))T = H$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -G_{32}(\theta) & 1 & 0 \\ -G_{41}(\theta) & -G_{42}(\theta) & -G_{43}(\theta) & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \\ G_{41} & G_{42} + G_{32}G_{43} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$




$$-G_{41}(\theta) + G_{41} = 0$$

Identifiable

2 unknowns,

not identifiable

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# Outline

- The dynamic network
- **Single module identifiability**
- Topology based conditions

# Identifiability of a single module

Embed restrictions in model set

- Topology
- Disturbance correlations
- Known dynamics
- Where excitation enters

$$\begin{cases} M(\theta) = (G(q,\theta), R(q,\theta), H(q,\theta), \Lambda(\theta)) \\ \mathcal{M} = \{M(\theta), \theta \in \Theta\} \end{cases}$$

Single module identifiability when all modules are strictly proper

Module $G_{ji}$ is globally network identifiable if for all $\theta_0, \theta_1 \in \Theta$:

$$\{T(q,\theta_1) = T(q,\theta_0)\} \Rightarrow \{G_{ji}(q,\theta_1) = G_{ji}(q,\theta_0)\}.$$

# Using the network equation

$$(I - G(\theta))T = \begin{bmatrix} R(\theta) & H(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 1 & -G_{12}(\theta) & -G_{13} & -G_{14}(\theta) \end{bmatrix} T = \begin{bmatrix} 1 & 0 & H_{11}(\theta) & H_{12}(\theta) & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -G_{12}(\theta) & -G_{14}(\theta) \end{bmatrix} \check{T}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Construct $\check{T}_j$ from $T$ based on:

- If no parameters in $G_{jk}$, row $k$ from $T$ is deleted
- If parameters in $l$-th element of $[R_{j\star} \;\; H_{j\star}]$, column $l$ from $T$ is deleted.

All modules identifiable when full rank

# Conditions for single module identifiability

Relaxed condition for single module:

Construct $\check{T}_{j,(-i,\star)}$ from $\check{T}_j$ by removing row $i$

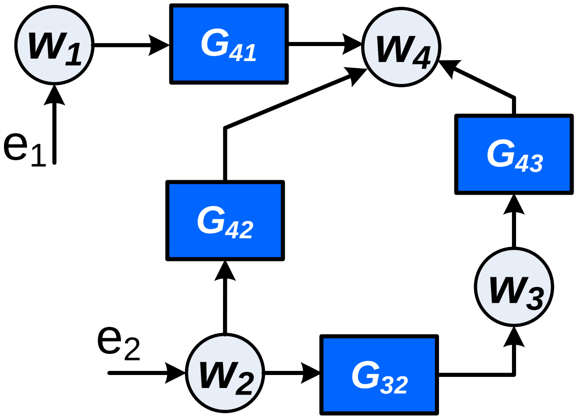
Single module identifiability conditions

Module $G_{ji}$ is globally network identifiable if and only if

$$\text{rank}\left(\check{T}_j(\theta)\right) > \text{rank}\left(\check{T}_{j,(-i,\star)}(\theta)\right)$$

for all $\theta \in \Theta$.

# Example continued




Which modules can be identified?

$$w = Te, \quad T = (I - G)^{-1}H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \\ G_{41} & G_{42} + G_{32}G_{43} \end{bmatrix}$$

$$\begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \end{bmatrix} = \check{T}_4 \qquad (\text{columns } e_1, e_2)$$

$$\begin{matrix} w_2 \\ w_3 \end{matrix} \begin{bmatrix} 0 & 1 \\ 0 & G_{32} \end{bmatrix} = \check{T}_{4,(-1,\star)} \qquad (\text{columns } e_1, e_2)$$

→ Independent row, $G_{41}$ identifiable

$$\begin{matrix} w_1 \\ w_3 \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & G_{32} \end{bmatrix} = \check{T}_{4,(-2,\star)} \qquad (\text{columns } e_1, e_2)$$

$G_{42}$ not identifiable

# Outline

- The dynamic network
- Single module identifiability
- **Topology based conditions**

# Can we formulate topology based conditions?

Ignore the dynamics? → Approach inspired by [8]

Link topology to generic rank of transfer function matrix [12]

Let $G_\Sigma$ be the graph corresponding to the state-space system

$$\dot{x} = Ax + Bu, \quad y = Cx.$$

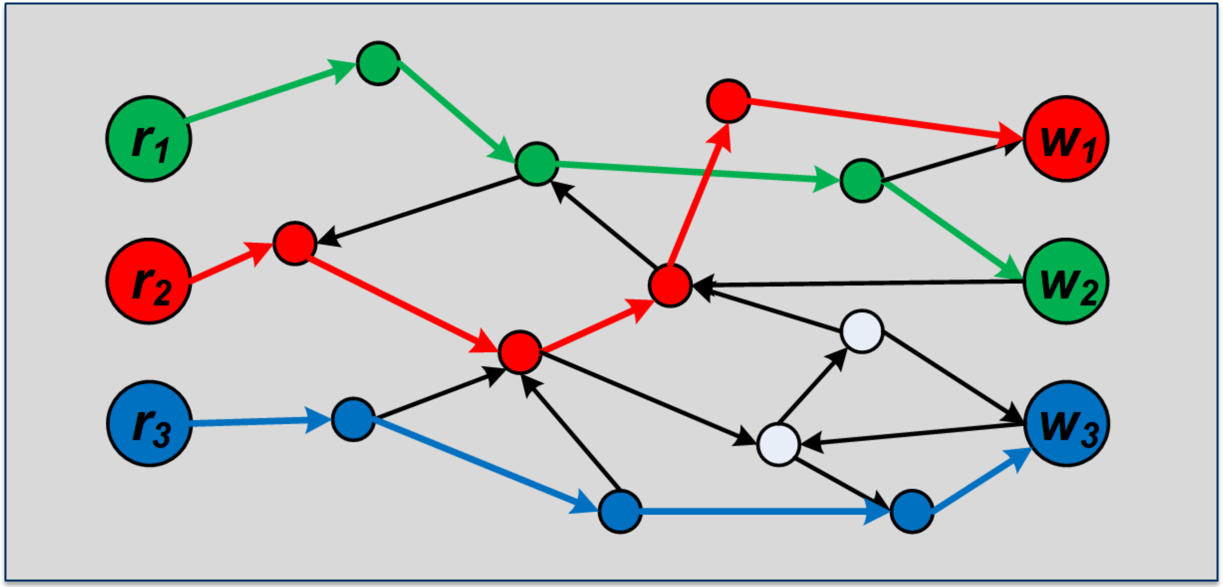
The maximum number of vertex disjoint paths in $G_\Sigma$ from signals in $u$ to signals in $y$ equals the generic rank of $T_{ss} = C(sI - A)^{-1}B$.

[8] Bazanella, Gevers, et al. *CDC,* 2017

[12] J. van der Woude. *Mathematics of Control, Signals, and Systems,* 1991.

# Vertex disjoint paths

Vertex disjoint paths do not share nodes



# More explanations

Generic rank of $\check{T}_j(\theta)$ is the rank for all parameters, except those in a set of measure zero.

Dynamic network represented as structured state-space

→ Nodes appear as states

→ Paths also appear in state-space

# Adapt result for network models

Define set $\mathcal{Y}$ as nodes corresponding to rows of $\check{T}_j$

Define set $\mathcal{U}$ as external excitations and noises corresponding to columns of $\check{T}_j$

The maximum number of vertex disjoint paths in the network from signals in $\mathcal{U}$ to signals in $\mathcal{Y}$ equals the generic rank of $\check{T}_j$.



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# Topology based identifiability

Modified identifiability definition

Module $G_{ji}$ is **generically** network identifiable if for all $\theta_1 \in \Theta$ and for **almost** all $\theta_0 \in \Theta$:

$$\{T(q,\theta_1) = T(q,\theta_0)\} \Rightarrow \{G_{ji}(q,\theta_1) = G_{ji}(q,\theta_0)\}.$$

Modified rank conditions

Module $G_{ji}$ is generically network identifiable if and only if

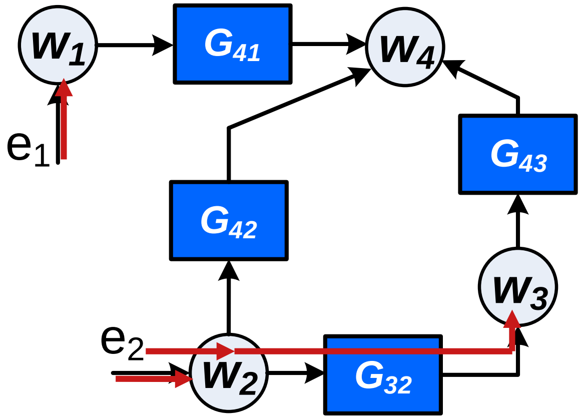
$$\text{generic rank}\left(\check{T}_j(\theta)\right) > \text{generic rank}\left(\check{T}_{j,(-i,\star)}(\theta)\right).$$

Generic rank can be checked based on topology

# Example continued

Which modules can be identified?

$$w = Te, \quad T = (I - G)^{-1}H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \\ G_{41} & G_{42} + G_{32}G_{43} \end{bmatrix}$$




$$\check{T}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & G_{32} \end{bmatrix}$$

$$\underbrace{e_1, e_2}_{\mathcal{U}} \rightarrow \underbrace{w_1, w_2, w_3}_{\mathcal{Y}}$$

$$\check{T}_{4,(-1,\star)} = \begin{bmatrix} 0 & 1 \\ 0 & G_{32} \end{bmatrix}$$

$$\underbrace{e_1, e_2}_{\mathcal{U}} \rightarrow \underbrace{w_2, w_3}_{\mathcal{Y}}$$

# Take home messages

**Identifiability of a module can conveniently be checked based on just the network topology**

Tool for determining where to excite a network

Topological conditions are easy to check

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