Data-driven modeling in linear dynamic networks

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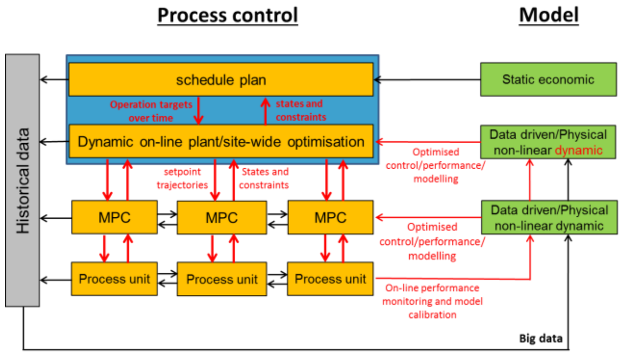
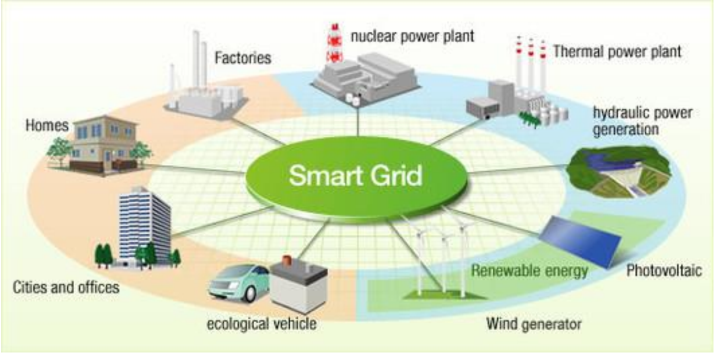
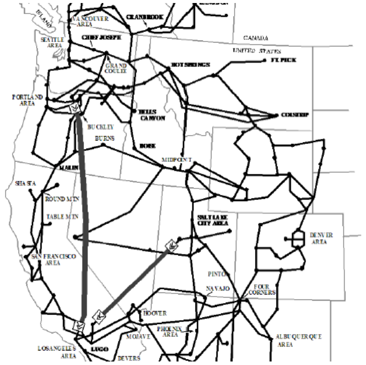
Introduction – dynamic networks

Decentralized process control



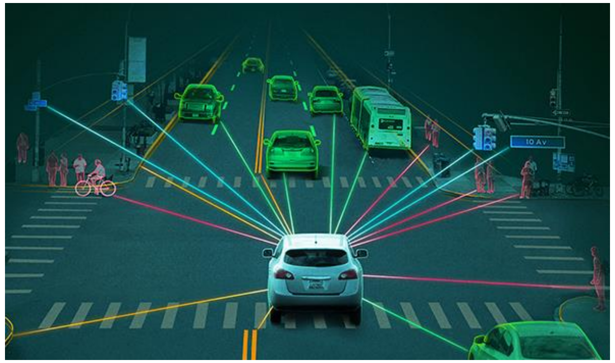
Smart power grid





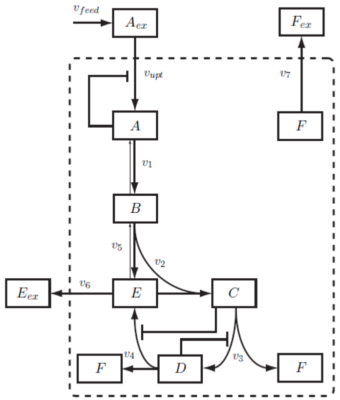
Pierre et al. (2012)

Autonomous driving



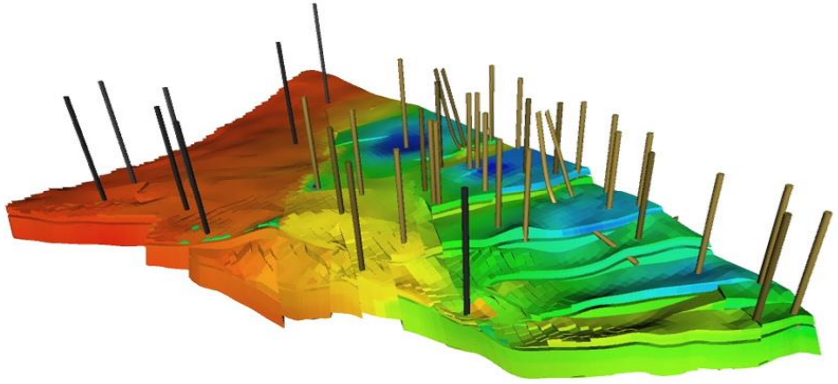
www.envidia.com

Metabolic network



Hillen (2012)

Hydrocarbon reservoirs



Mansoori (2014)

Introduction

Overall trend:

(a) The dynamic systems to be handled become (large-scale) interconnected systems of systems

(b) With hybrid dynamics (continuous / switching)

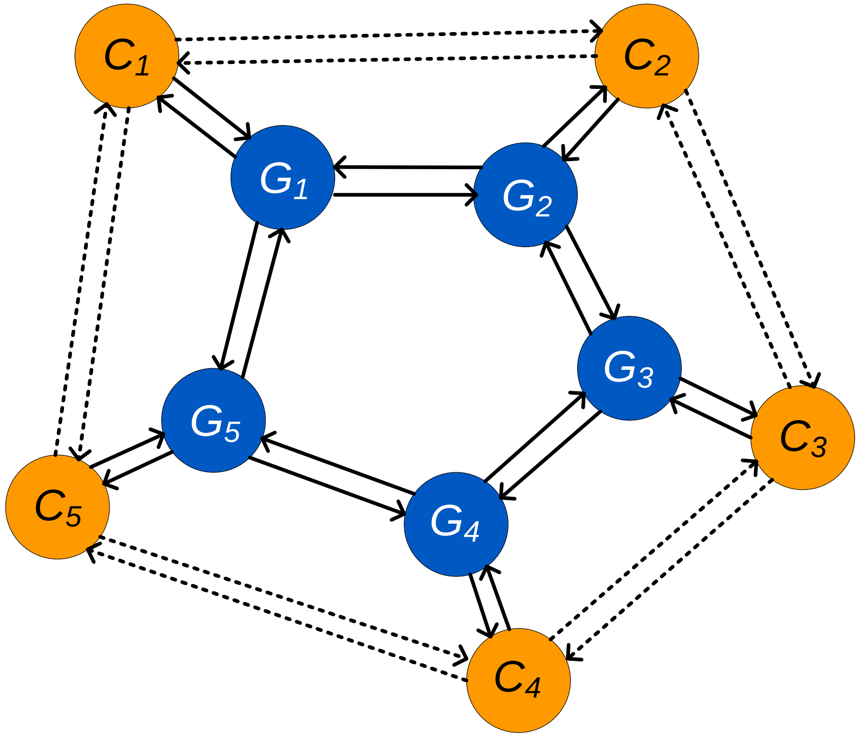
(c) The related monitoring, control and optimization problems become distributed, multi-agent type

(d) Data is “everywhere”, big data era

(e) Modelling problems will need to consider this

Introduction

Distributed / multi-agent control:



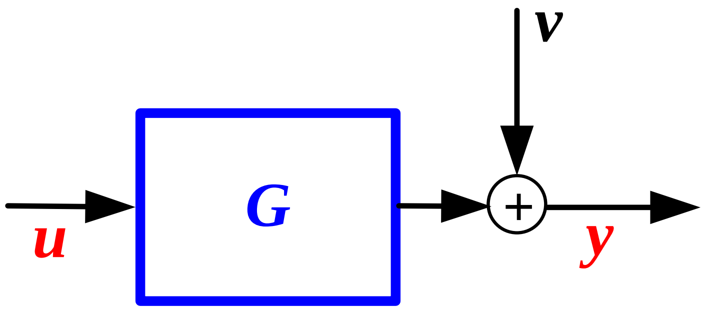
With both physical and communication links between systems G_i and controllers C_i

How to address data-driven modelling problems in such a setting?

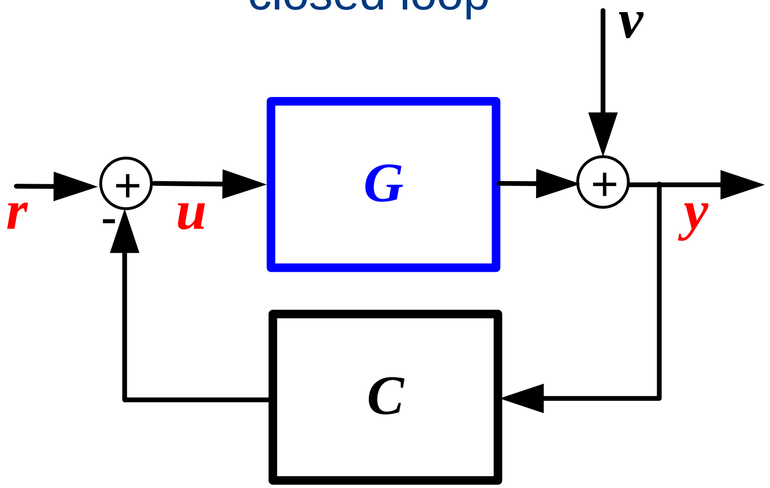
Introduction

The classical (multivariable) identification problems[1] :

open loop



closed loop



Identify a plant model $\hat{G}$ on the basis of measured signals *u, y* (and possibly *r),* focusing on *continuous LTI dynamics.*

We have to move from a fixed and known configuration to deal with and exploit ***structure*** in the problem.

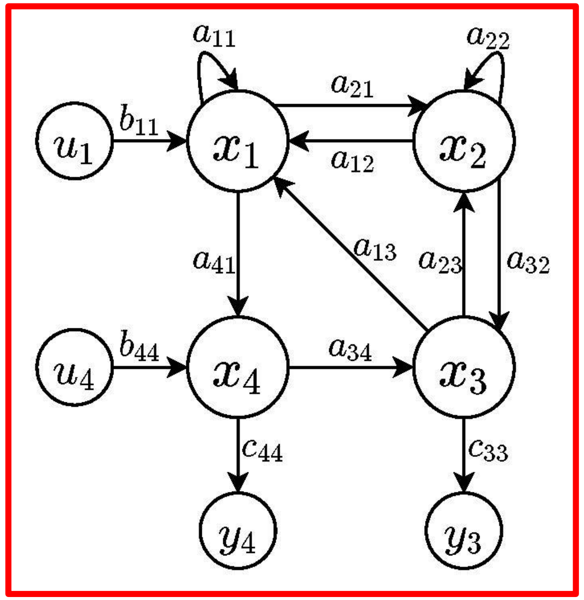
[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

Contents

- Introduction and motivation
- Dynamic networks for data-driven modeling
- Single module identification – known topology
- Network identifiability
- Extensions
- Discussion

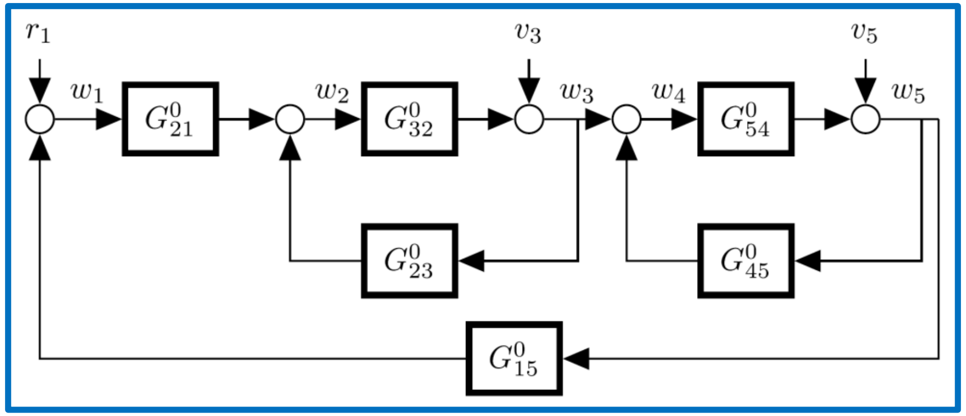
Dynamic networks for data-driven modeling

Dynamic networks



State space representations

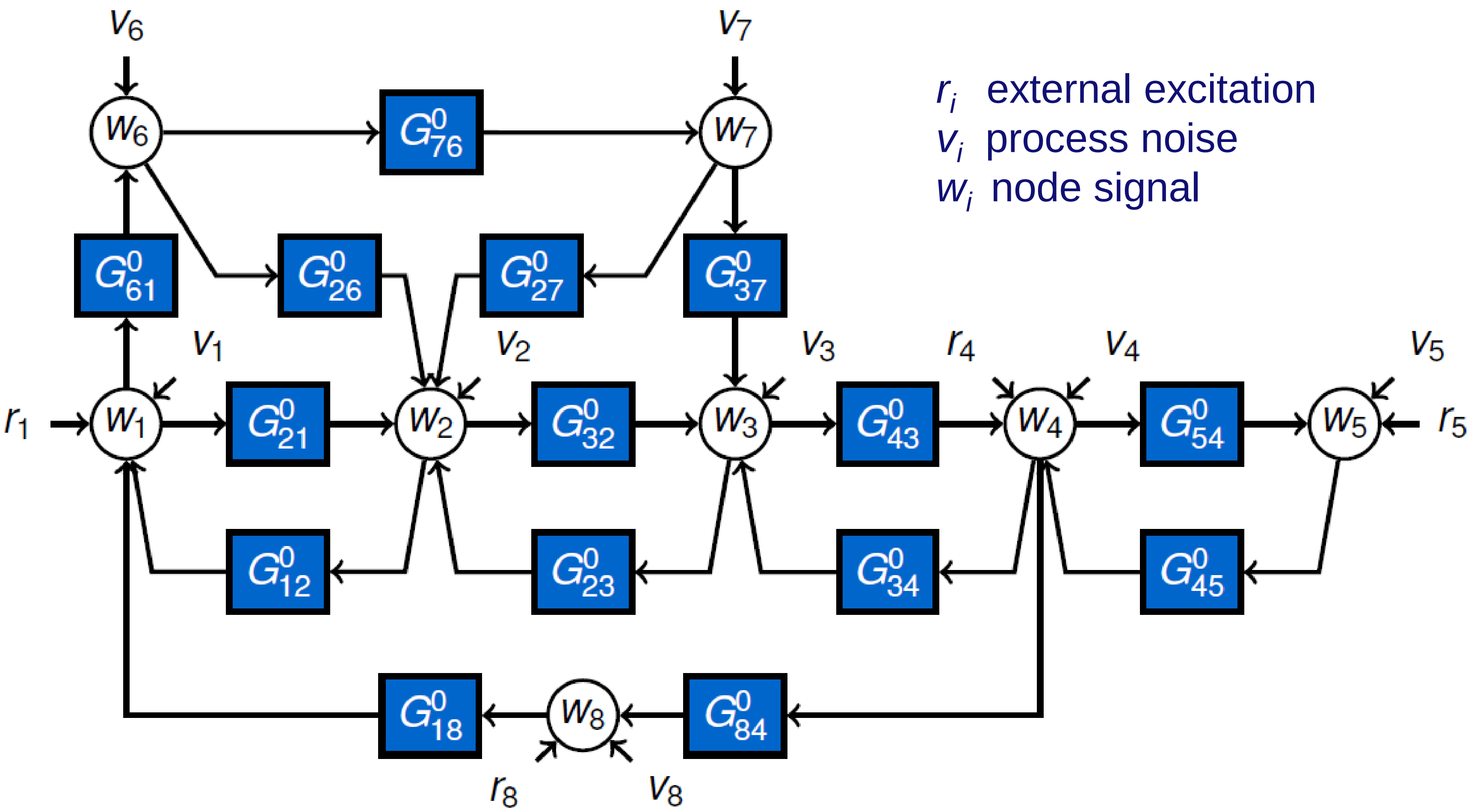
(Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…)



Module representation

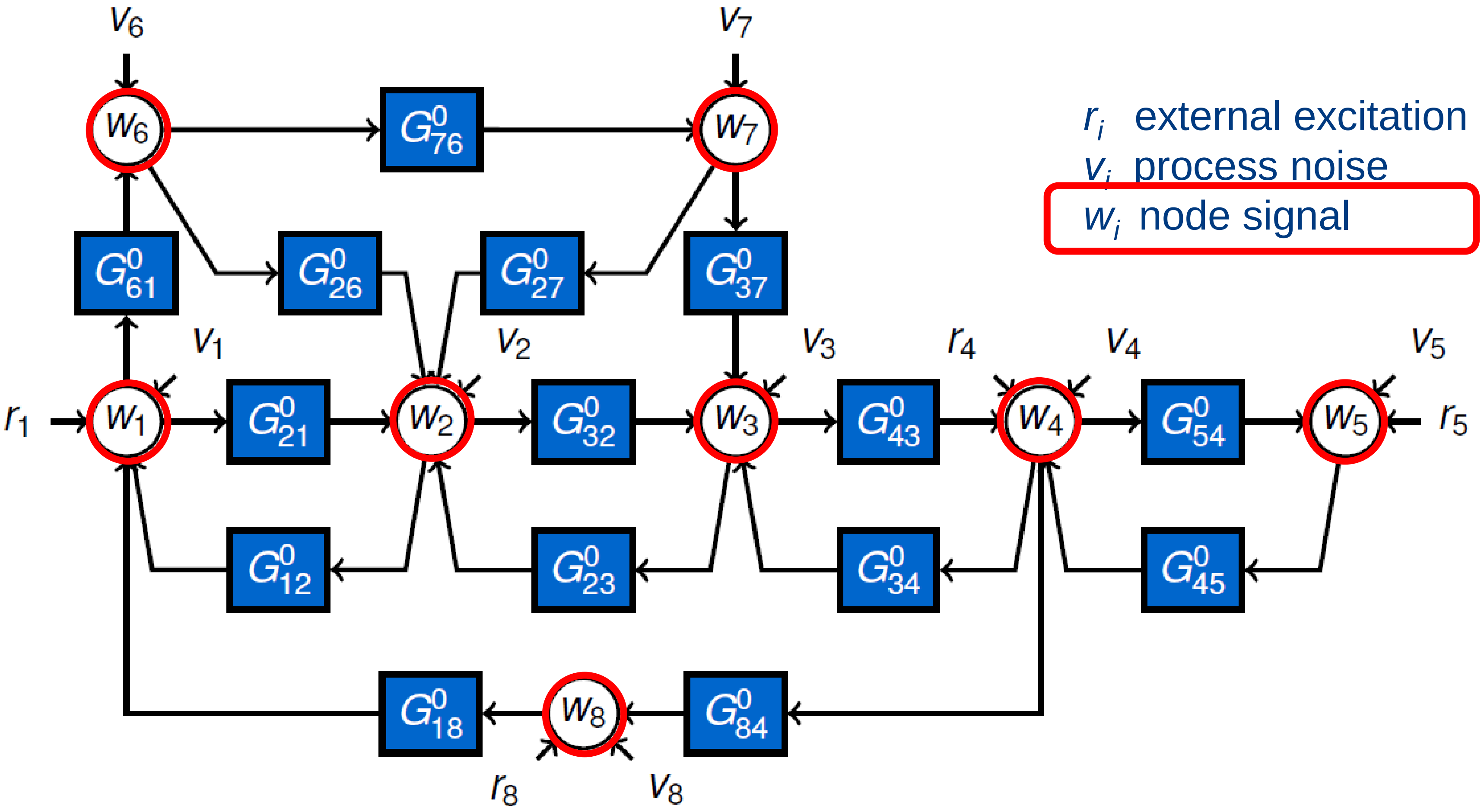
(Van den Hof, Dankers, Gevers, Bazanella,…)

Dynamic network setup



r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup



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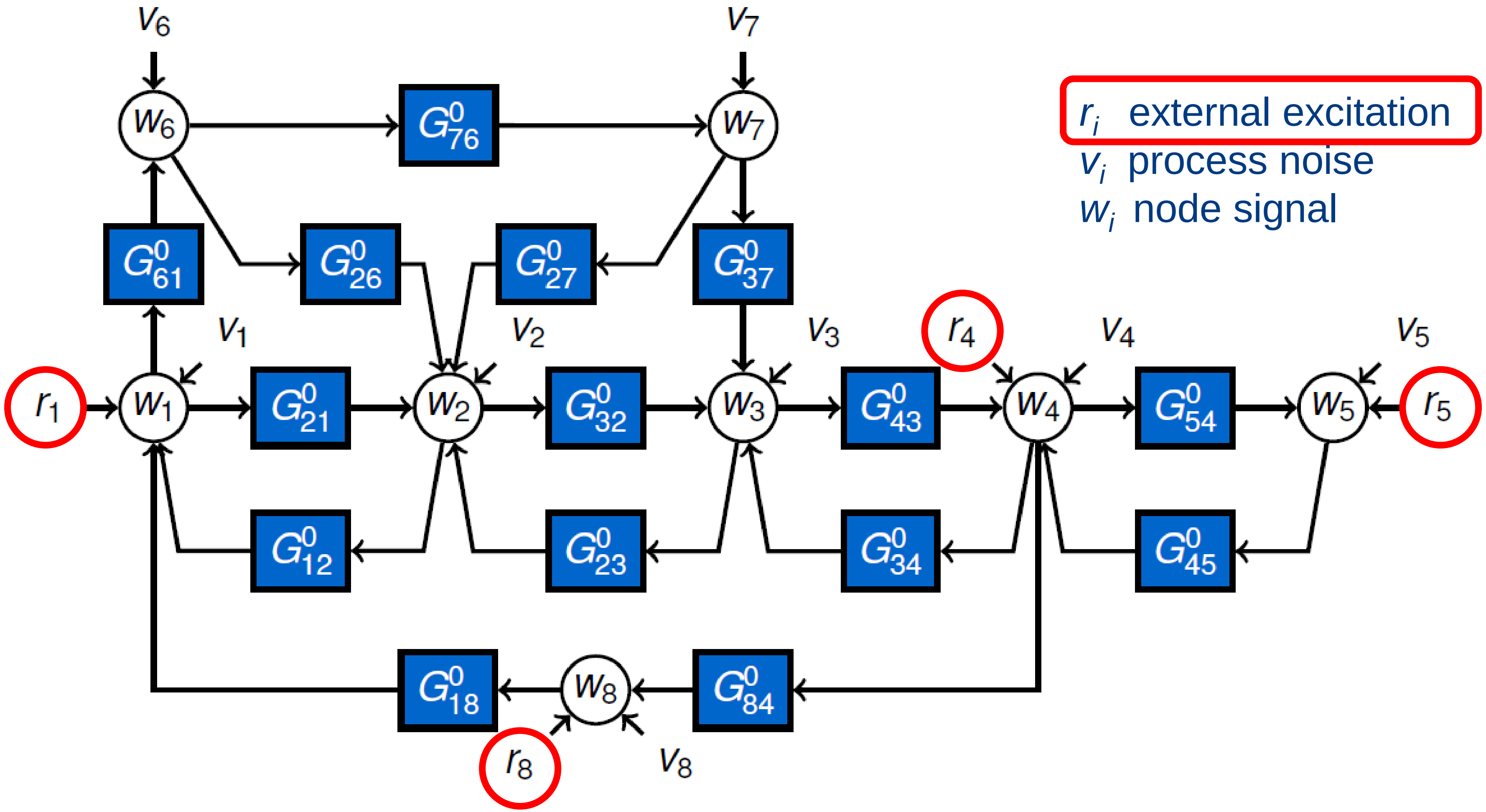

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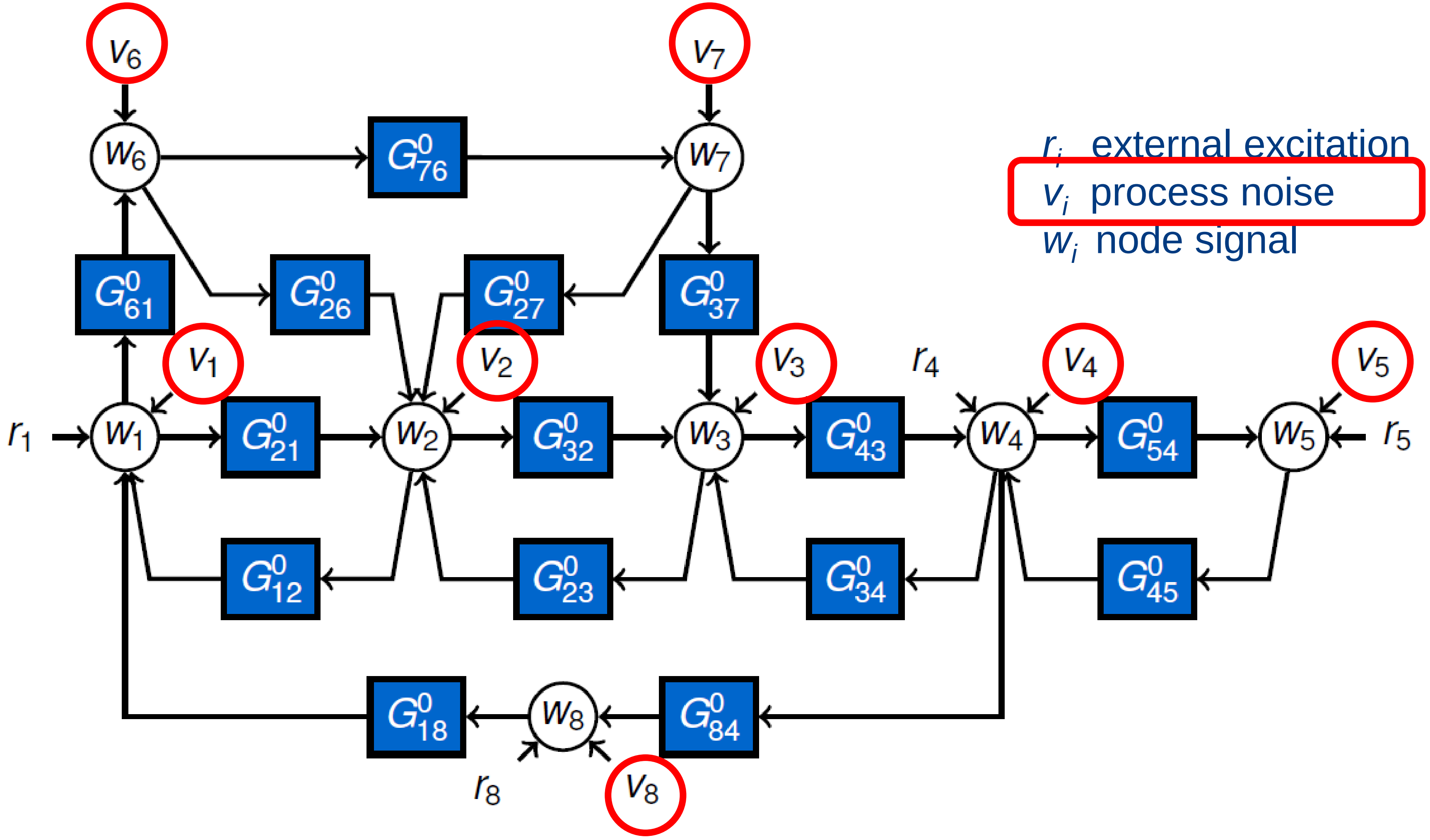

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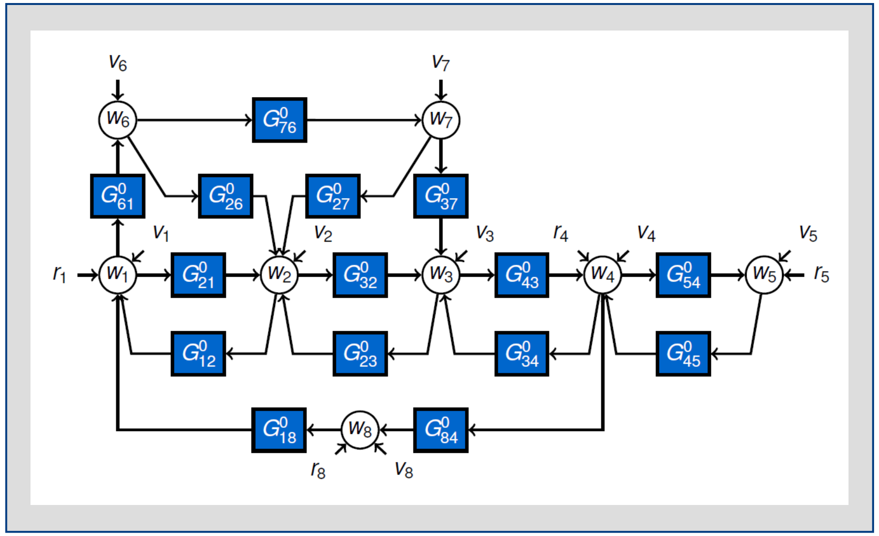
Dynamic network setup



Assumptions:

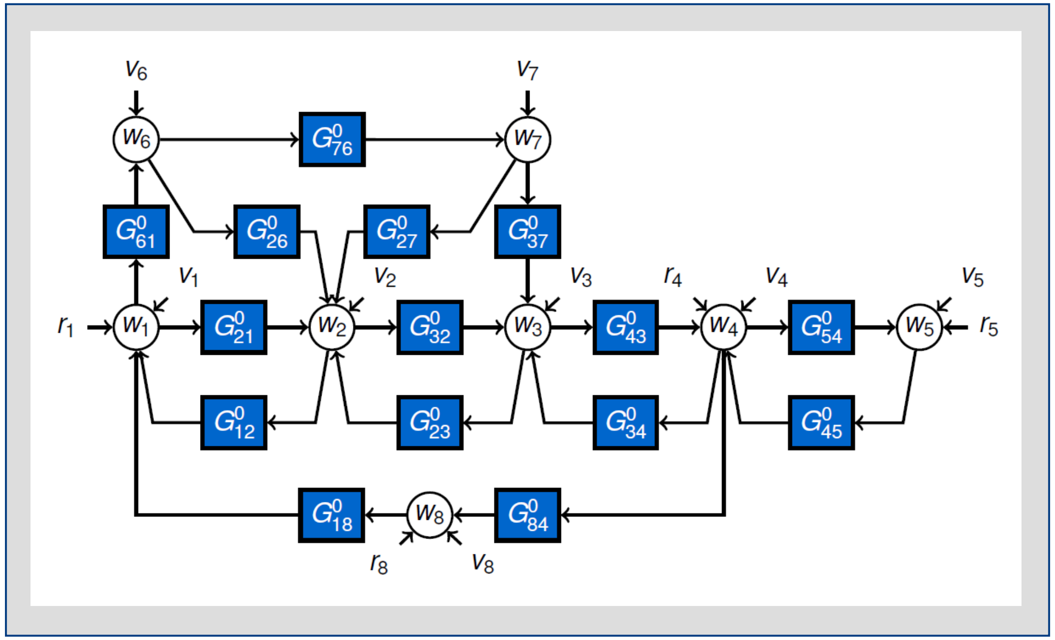
- Total of L nodes
- Network is well-posed and stable
- Modules may be unstable
- Disturbances are stationary stochastic and can be correlated

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

$$w = G^0 w + R^0 r + v \qquad \Longrightarrow \qquad w = (I - G^0)^{-1}(R^0 r + v)$$

- P.M.J. Van den Hof, A.G. Dankers, P.S.H. Heuberger and X. Bombois. *Automatica,* 2013.

Dynamic network setup



Many new identification questions can be formulated:

- Identification of a local module (known toplogy)
- Identification of the full network
- Topology estimation
- Sensor and excitation selection
- Fault detection
- Experiment design
- Distributed identification
- ....

Dynamic network setup

Early literature

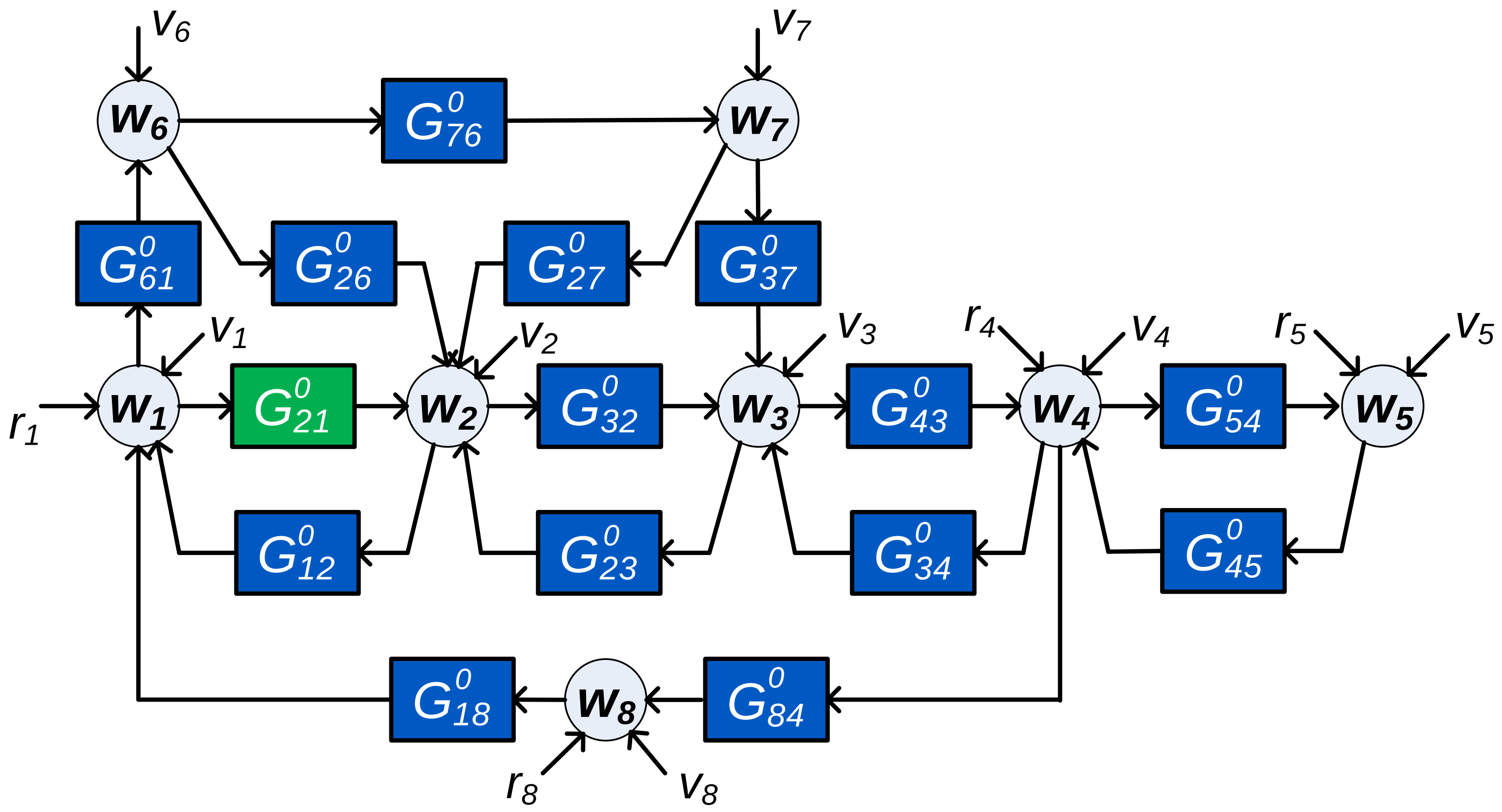
- **Topology detection**: Materassi, Innocenti, Salapaka, Yuan, Stan, Warnick, Goncalves, Sanandaji, Vincent, Wakin, Chiuso, Pillonetto exploring Granger causality, Bayesian networks, Wiener filters

- Subspace algorithms for **spatially distributed systems** with identical modules (Fraanje, Verhaegen, Werner), or non-identical ones (Torres, van Wingerden, Verhaegen, Sarwar, Salapaka, Haber)

Here: focus on **prediction error methods** and concepts for identification in generally structured (linear) dynamic networks, including **non-measured disturbances.**

Single module identification - known topology

Single module identification



For a network with known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure?

Single module identification

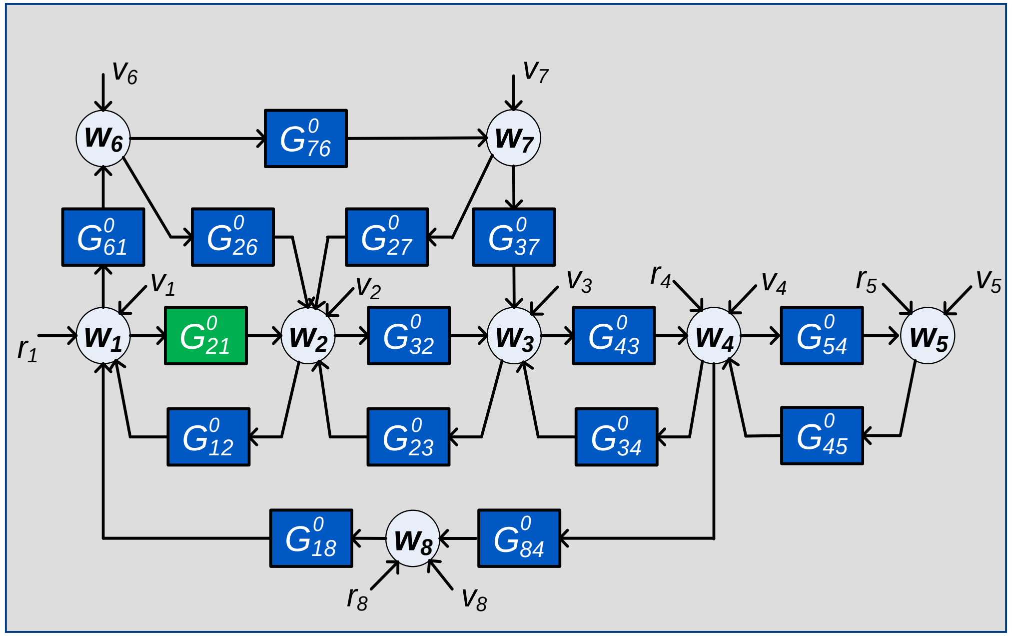
Options for identifying a module:

- Identify the full MIMO system:

$$w = (I - G^0)^{-1}[R^0 r + v]$$

from measured r and w .

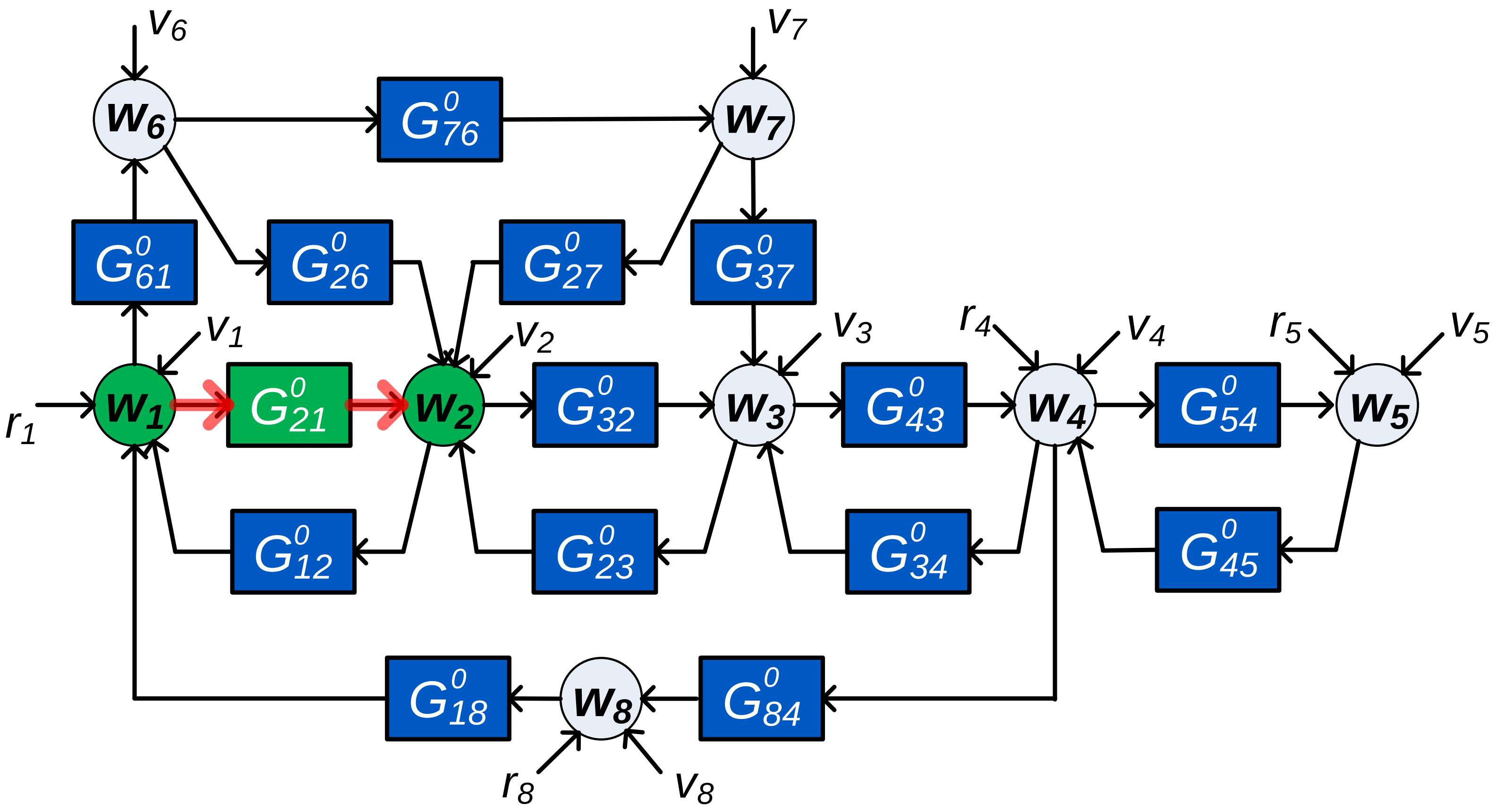
Global approach with “standard” tools



- Identify a local (set of) module(s) from a (sub)set of measured r_k and w_ℓ

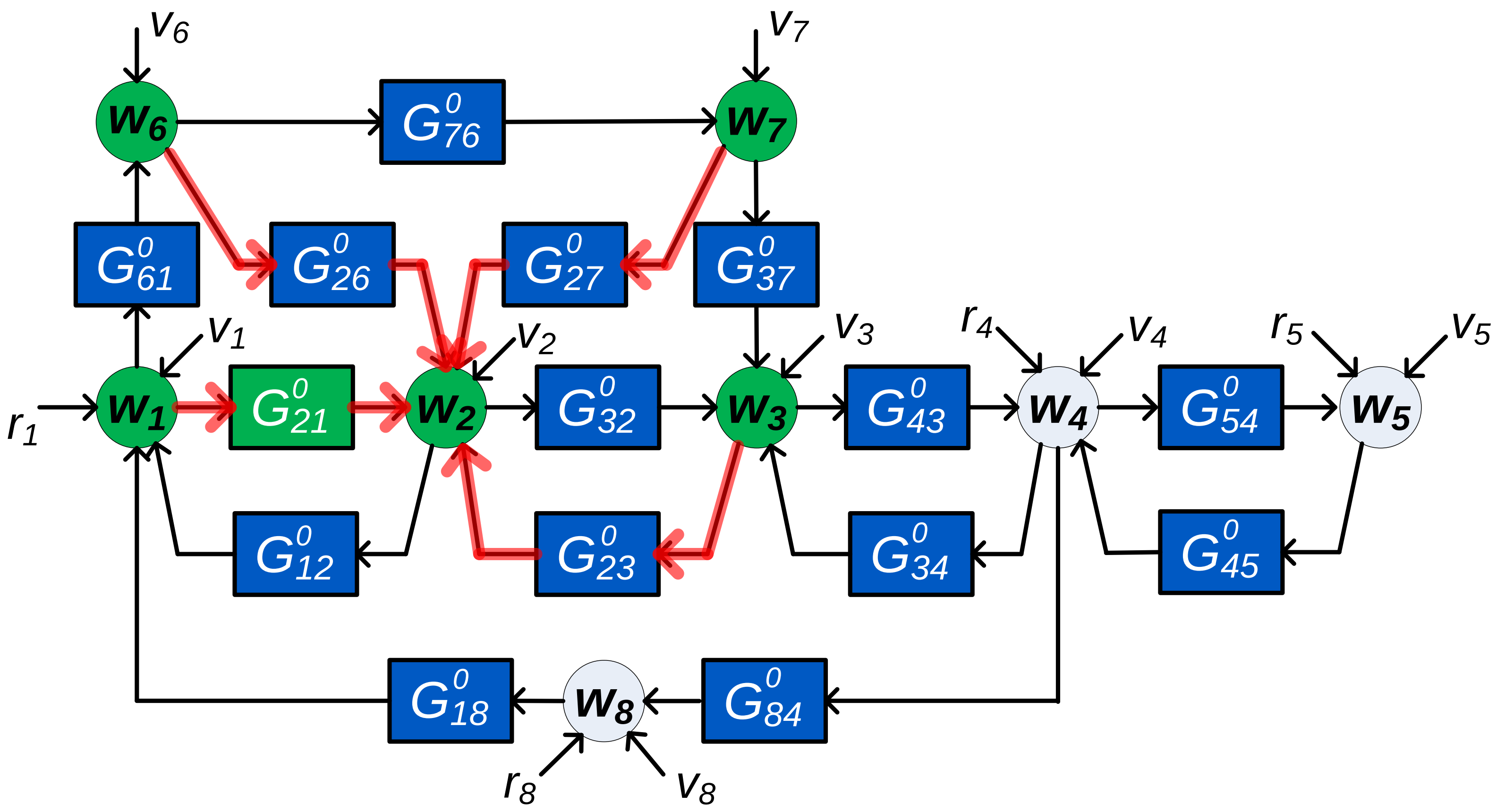
Local approach with “new” tools and structural conditions

Single module identification



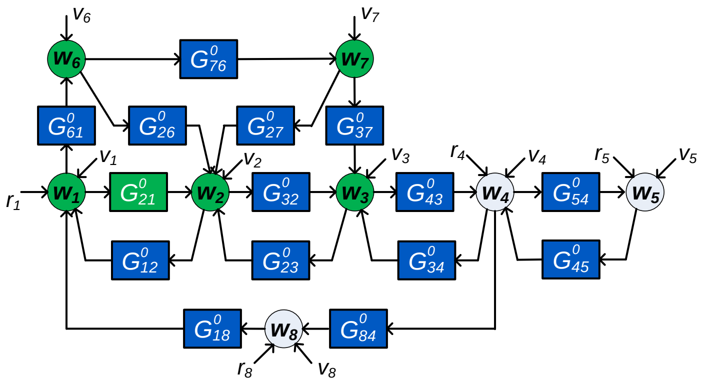
- Identifiying G^0_{21} is part of a 4-input, 1-output problem

Single module identification



- Identifiying G^0_{21} is part of a 4-input, 1-output problem

Identification methods



4-input 1-output problem
to be addressed by a
closed-loop identification method

- **Direct PE method**

$$\varepsilon(t,\theta) = H(q,\theta)^{-1}[w_2(t) - \sum_{k\in\mathcal{D}_2} G_{2k}(q,\theta)w_k(t)]$$

ML properties
Disturbances v_i uncorrelated over channels
Excitation through all signals

- **2-stage/projection/IV (indirect) method**

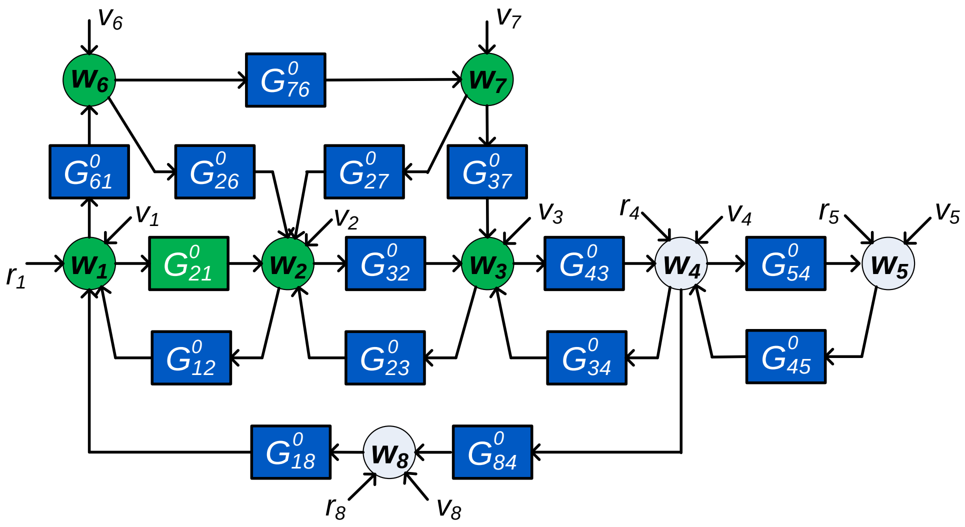
$$\varepsilon(t,\theta) = H(q,\theta)^{-1}[w_2(t) - \sum_{k\in\mathcal{D}_2} G_{2k}(q,\theta)w_k^{\mathcal{R}}(t)]$$

Consistency; no need for noise models; **no ML**
Excitation through external excitation signals only

Single module identification

4 input nodes to be measured:

Can we do with less?



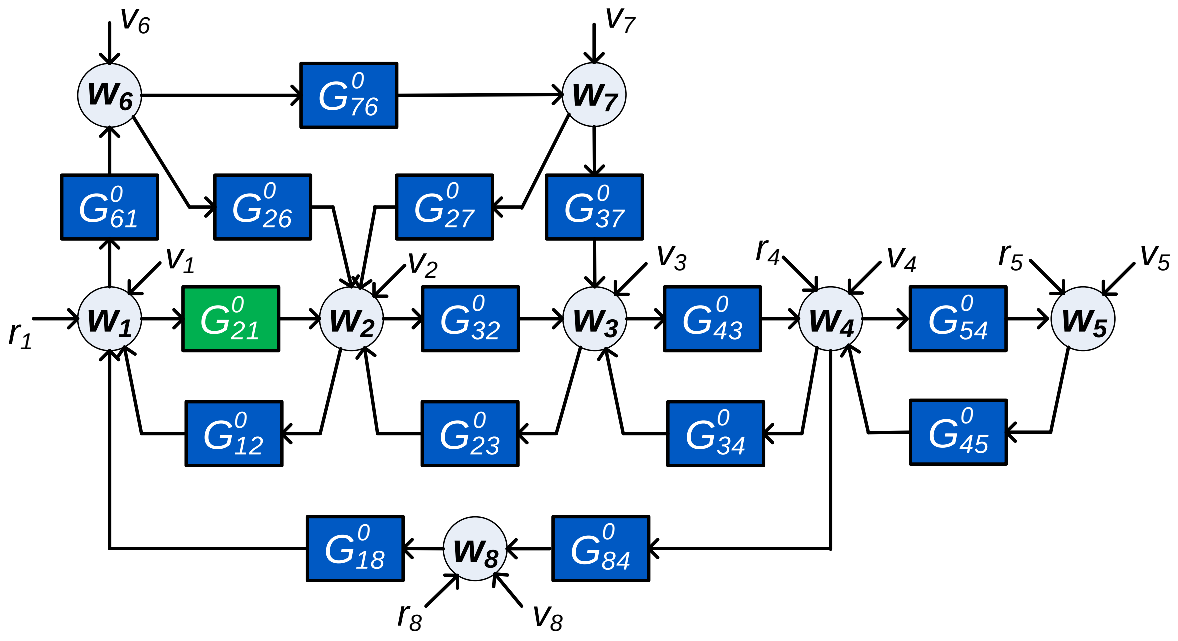
Network immersion [1]

- An **immersed network** is constructed by removing node signals, but leaving the remaining node signals invariant
- Modules and disturbance signals are adapted
- Abstraction through variable elimination (Kron reduction[2] in network theory).

[1] A. Dankers. PhD Thesis, 2014.

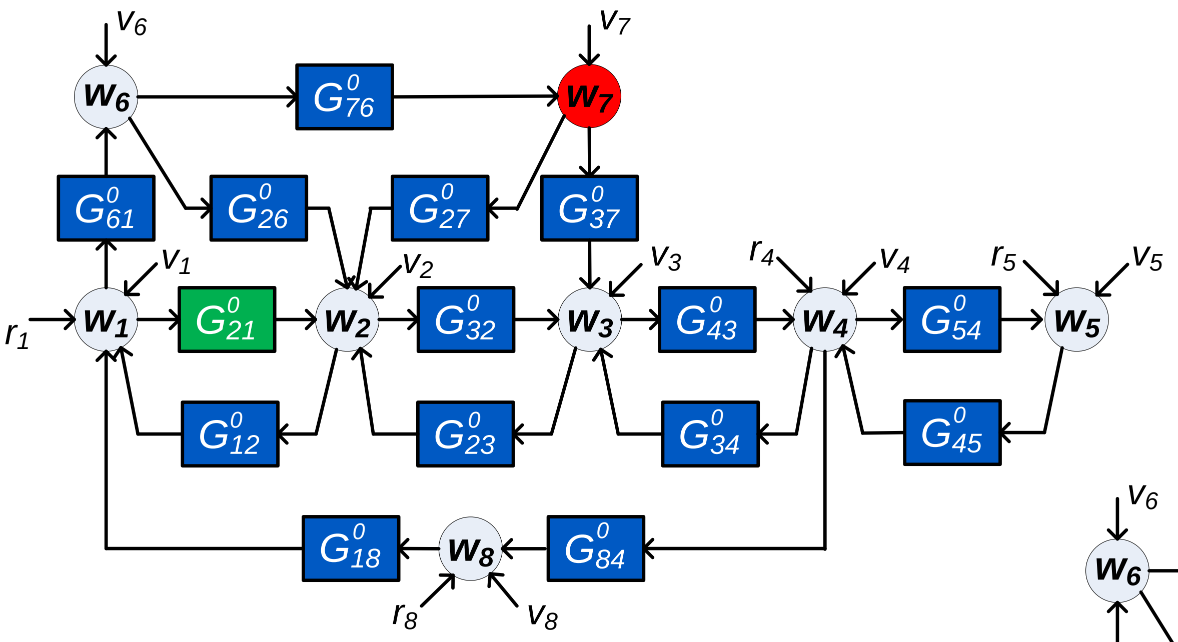
[2] F. Dörfler and F. Bullo, *IEEE Trans. Circuits and Systems I* (2013)

Immersion

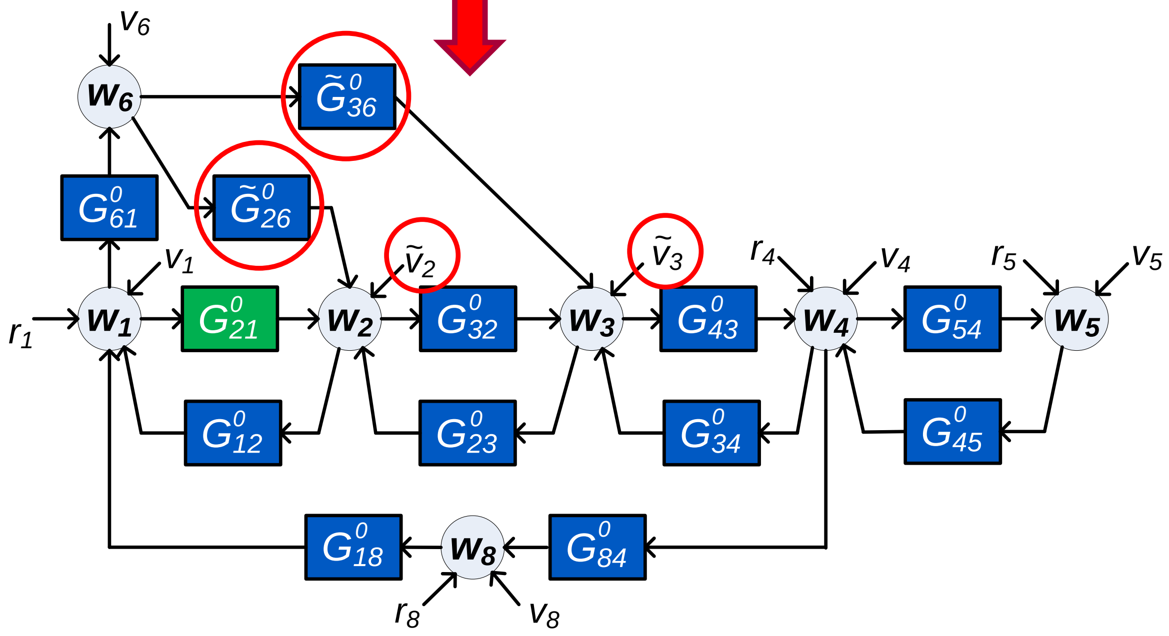


v6
v7
w6
w7
w1
w2
w3
w4
w5
w8
r1
r4
r5
r8
v1
v2
v3
v4
v5
v8
G076
G061
G026
G027
G037
G021
G032
G043
G054
G012
G023
G034
G045
G018
G084

Immersion



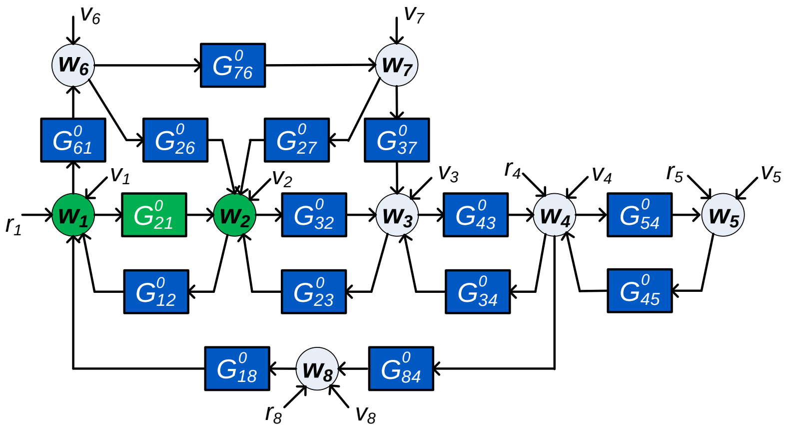
Immersing w_7



When does immersion leave G^0_{21} invariant?

Immersion

When does immersion leave G^0_{21} invariant?



Proposition

Consider an immersed network where w_1 and w_2 are retained.
Then $\check{G}^0_{21} = G^0_{21}$ if

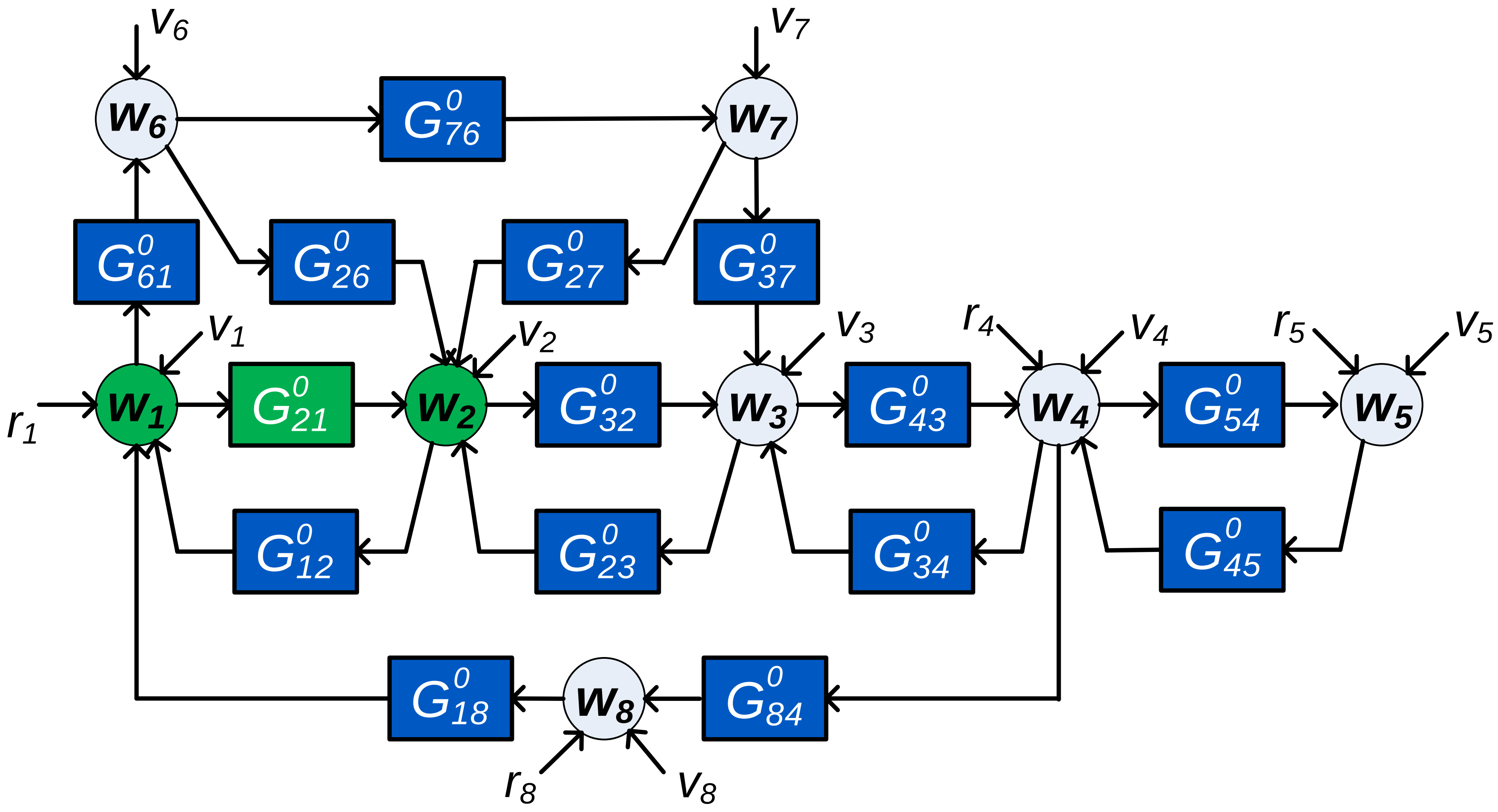
a) Every path $w_1 \to w_2$ other than the one through G^0_{21} goes through a node that is retained. (parallel paths)

b) Every path $w_2 \to w_2$ goes through a node that is retained. (loops around the output)

- A.G. Dankers, P.M.J. Van den Hof, A.G. Dankers, X. Bombois and P.S.C. Heuberger, *IEEE Trans. Automatic Control, 61*, 937-952, 2016.

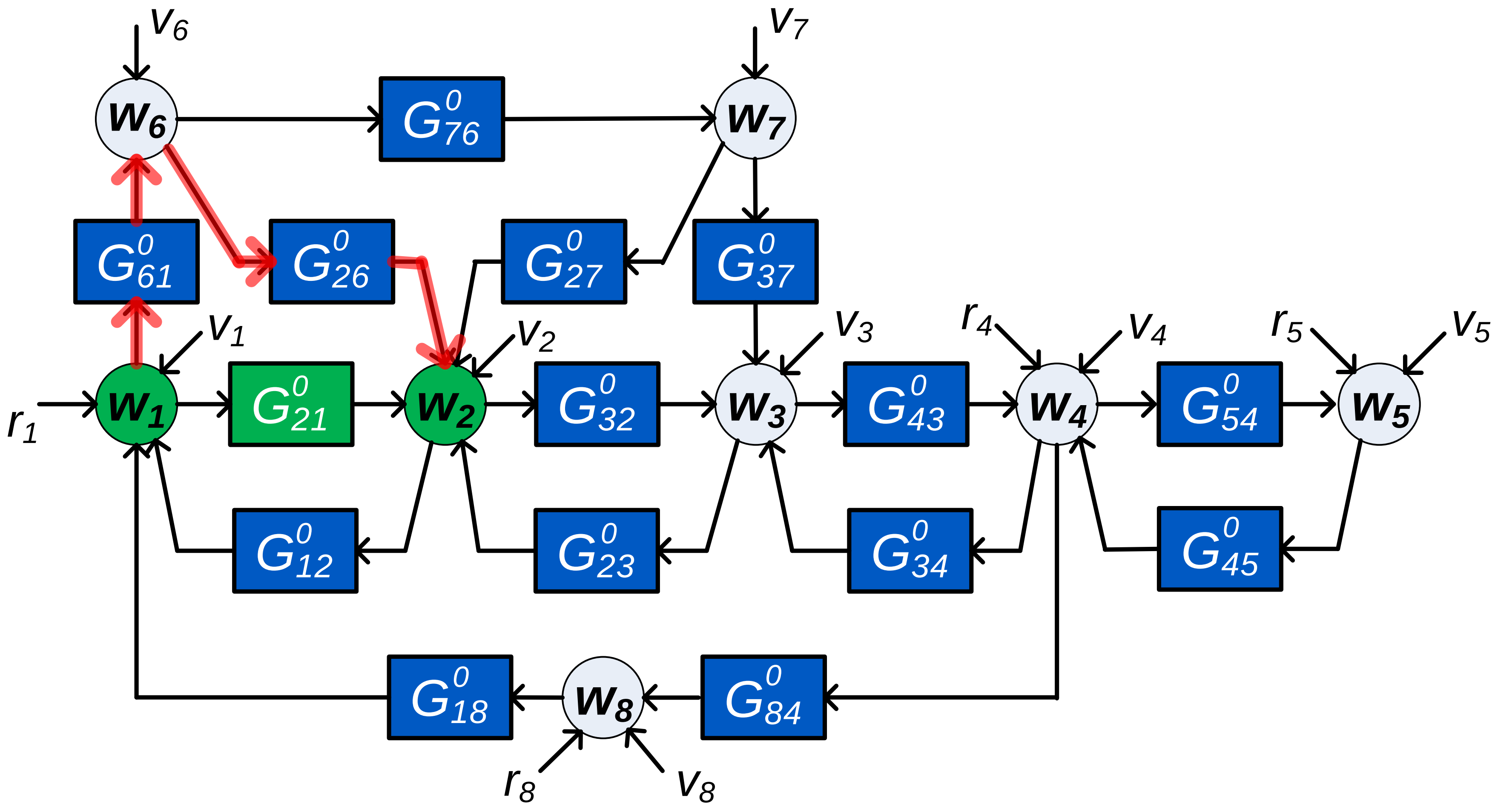
Single module identification

parallel paths, and loops around the output



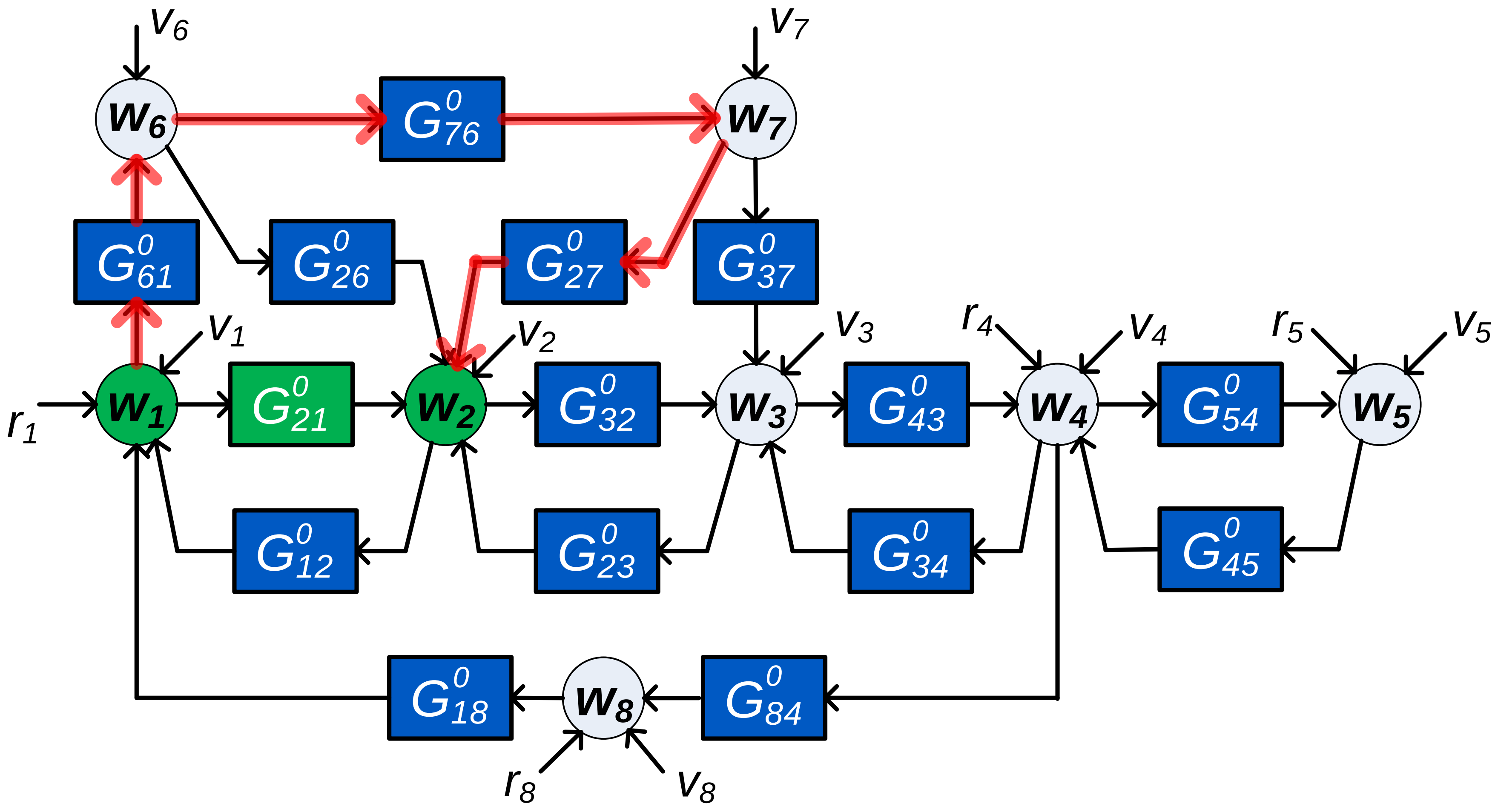
Single module identification

parallel paths, and loops around the output



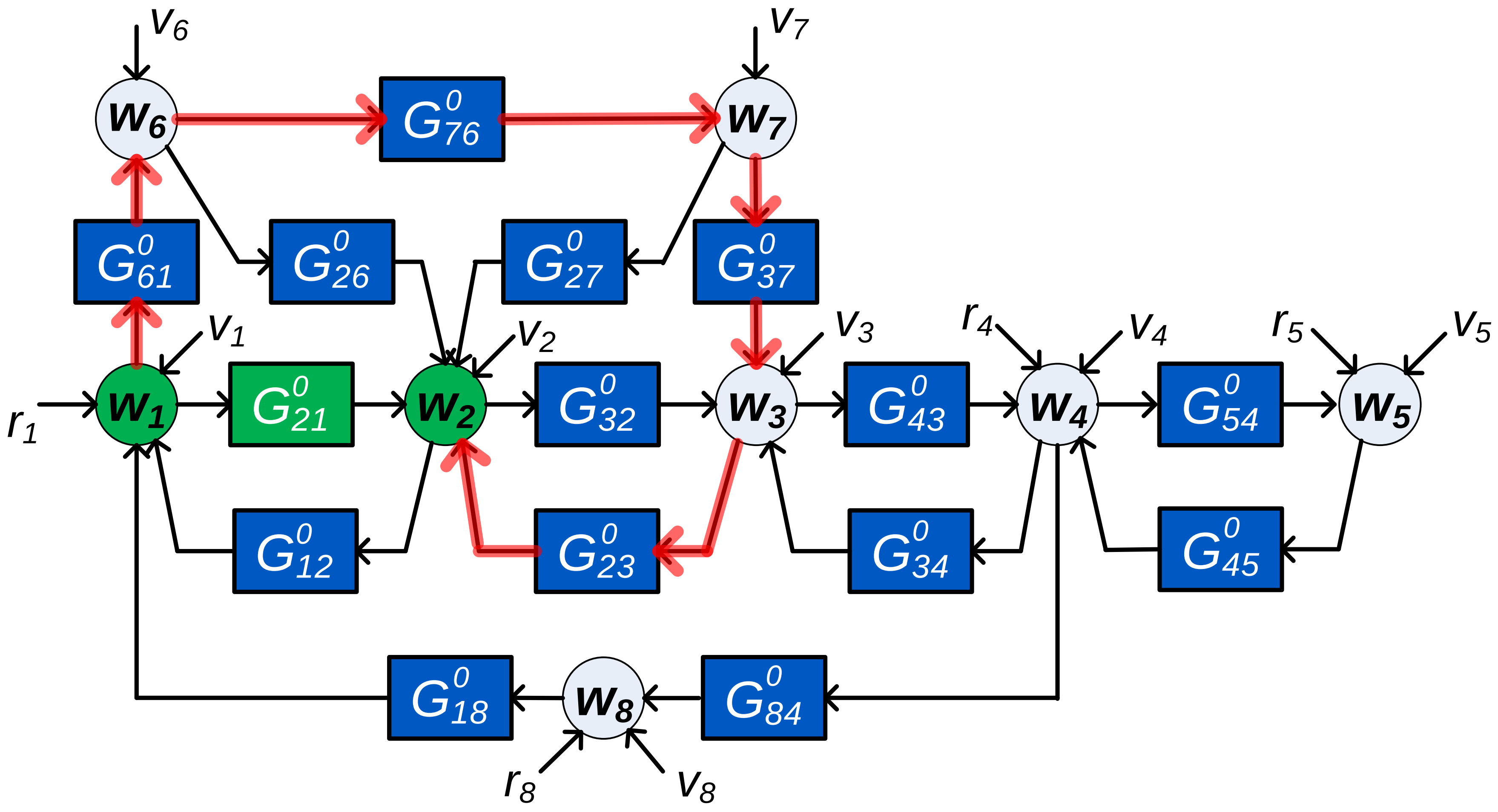
Single module identification

parallel paths, and loops around the output



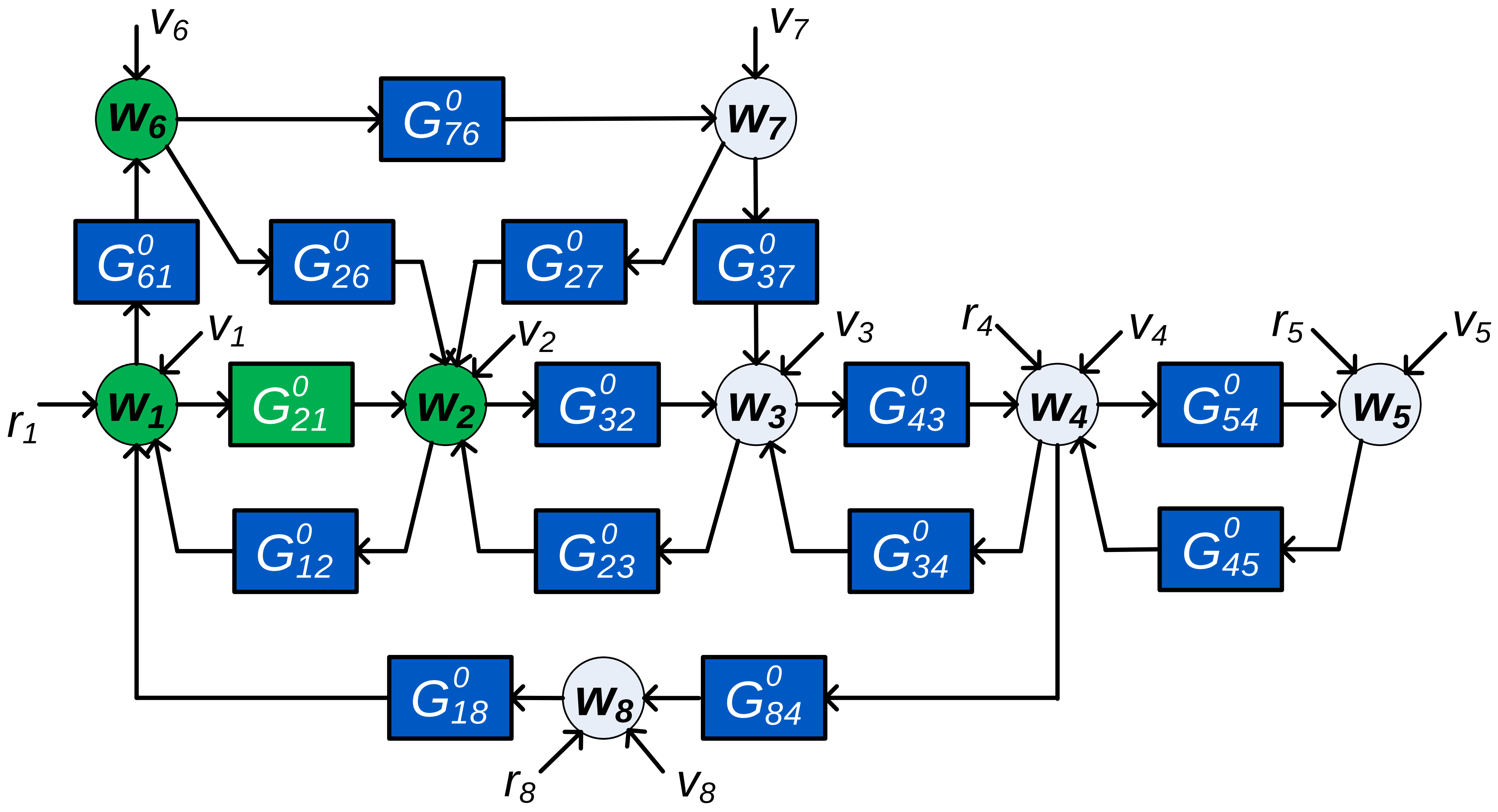
Single module identification

parallel paths, and loops around the output



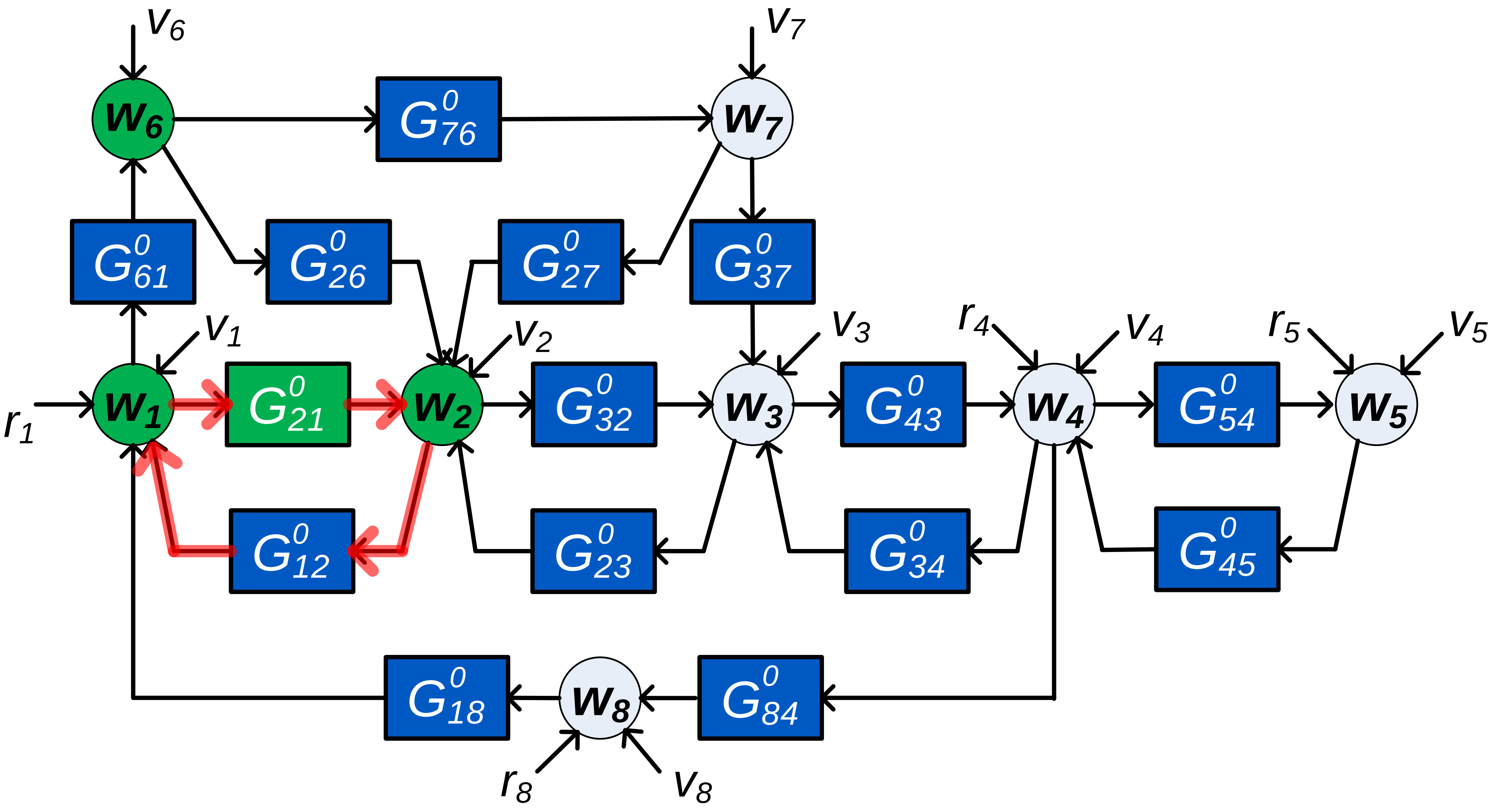
Single module identification

Choose w_6 as an additional input (to be retained)



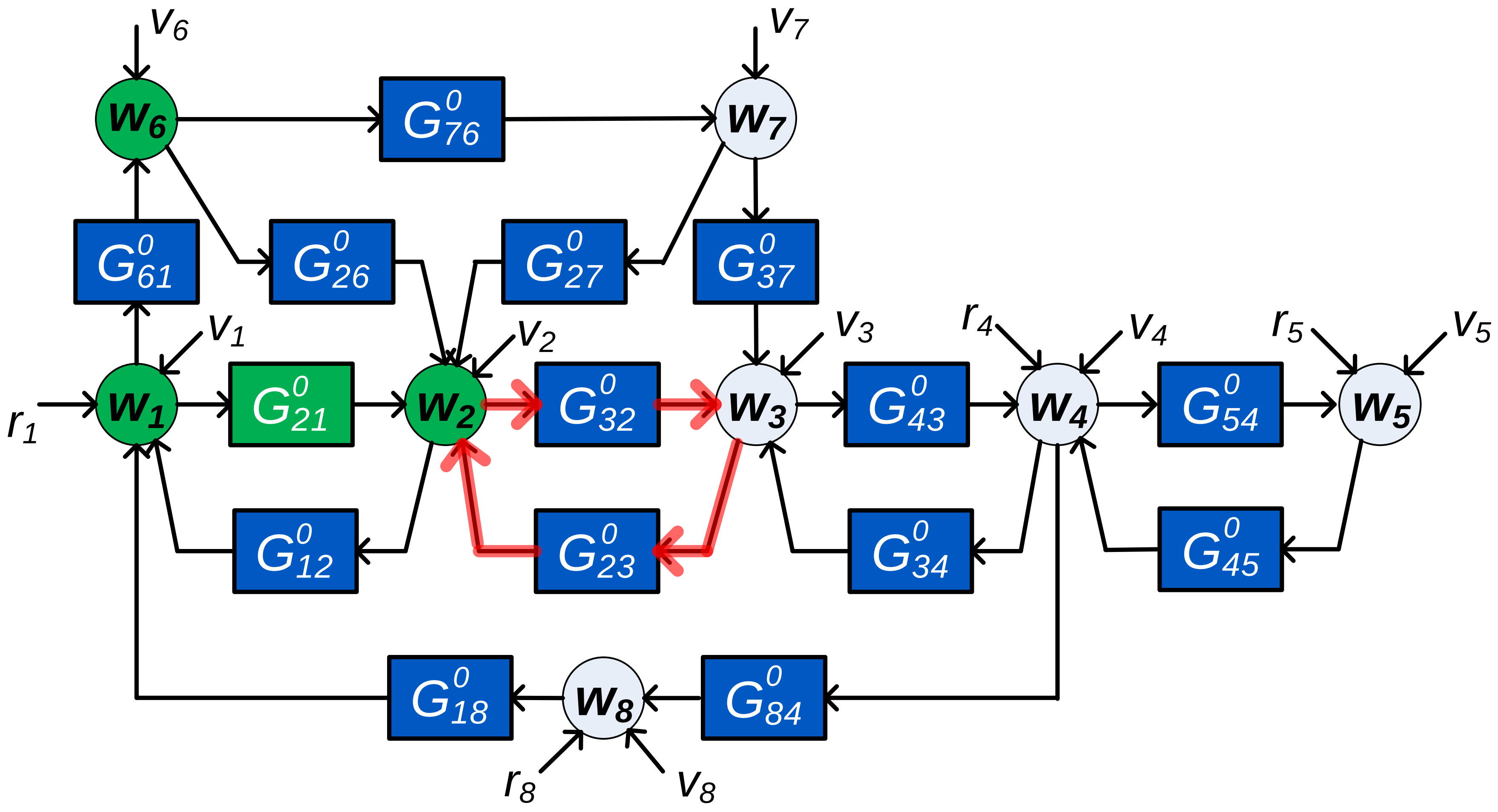
Single module identification

parallel paths, and **loops around the output**



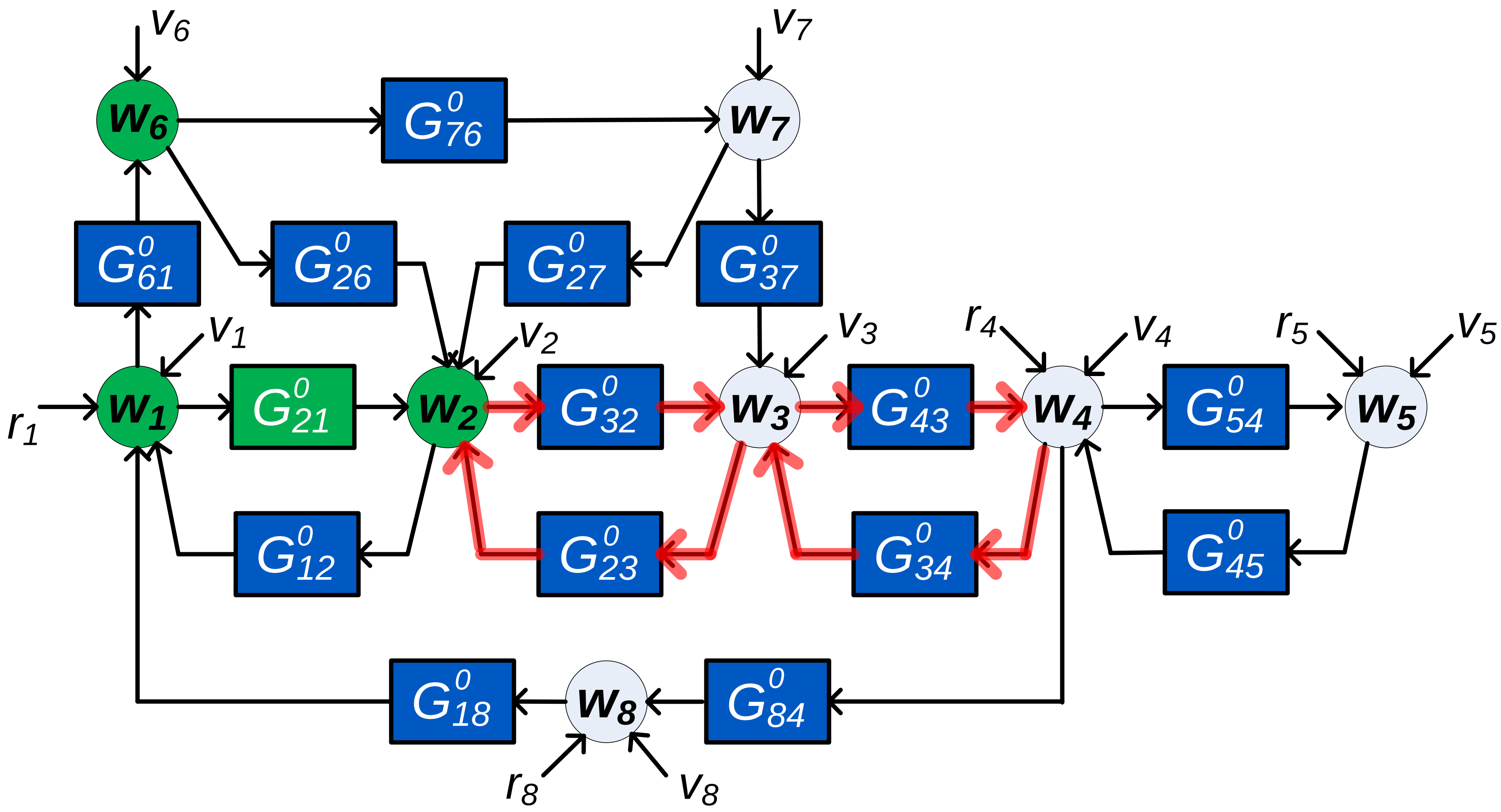
Single module identification

parallel paths, and **loops around the output**



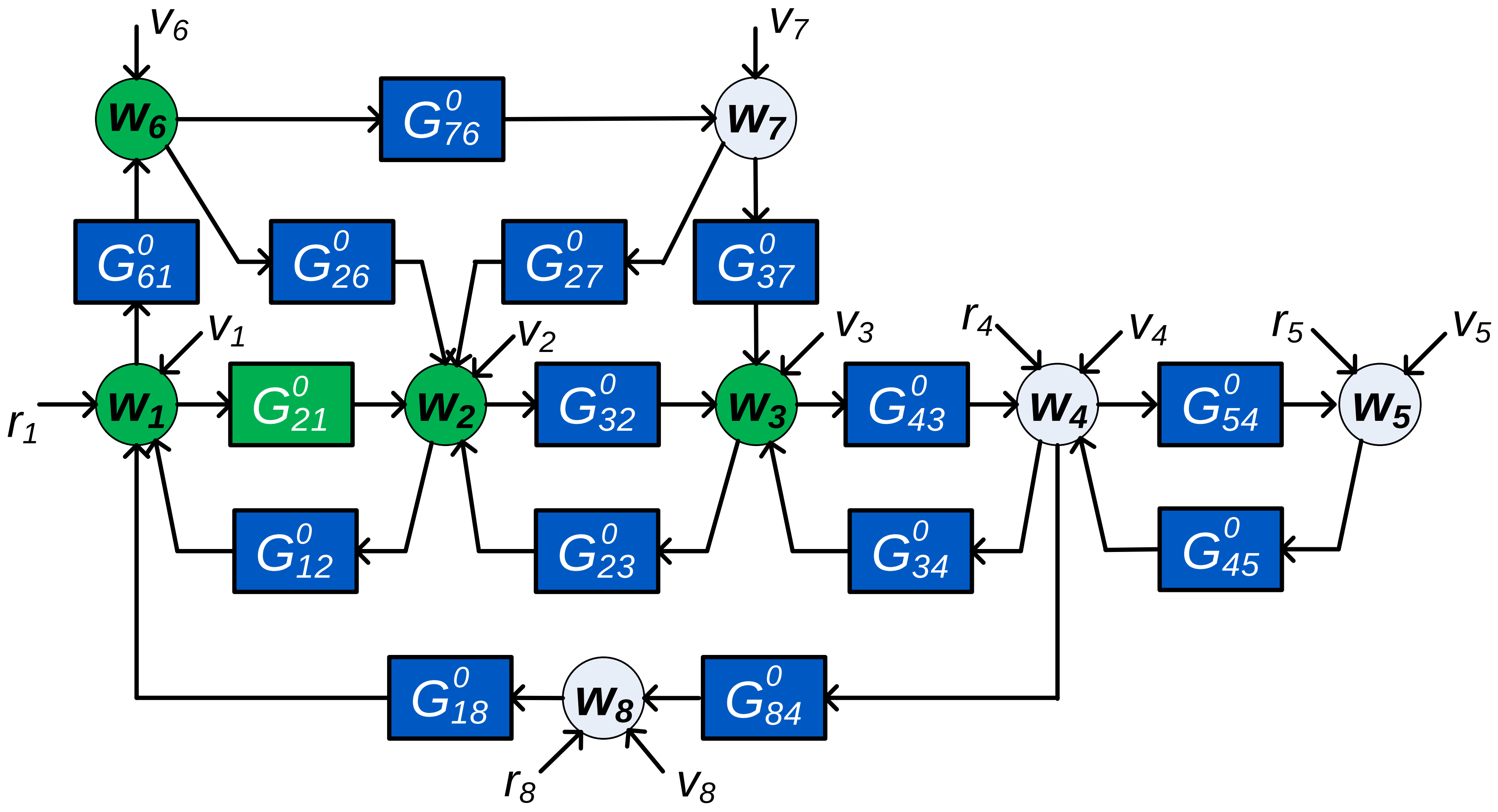
Single module identification

parallel paths, and **loops around the output**



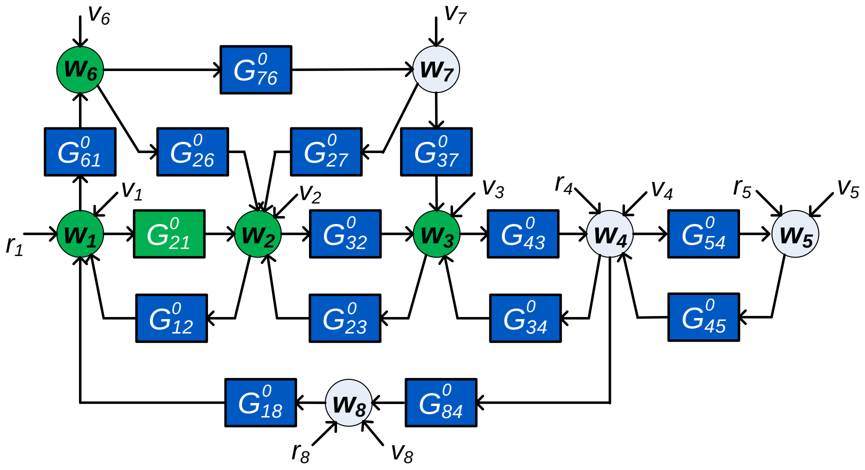
Single module identification

Choose w_3 as an additional input, to be retained



Single module identification

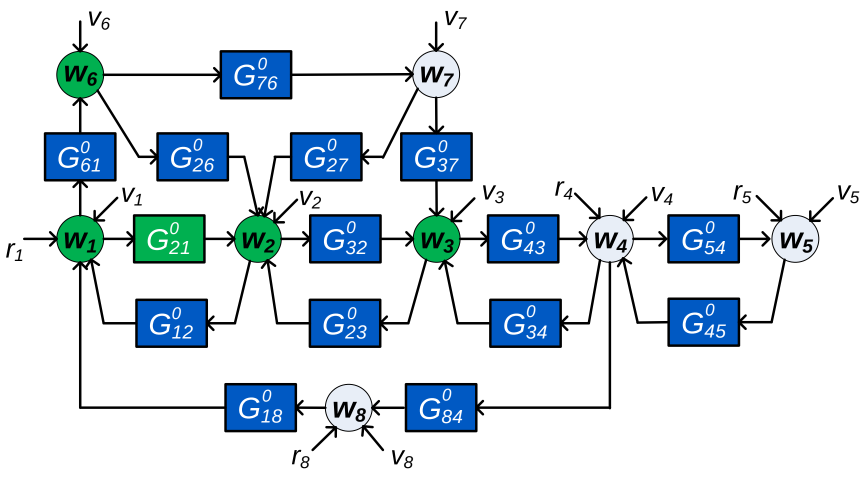
Conclusion:
With a 3-input, 1 output model we can consistently identify $\boldsymbol{G_{21}^0}$



The immersion reasoning is *sufficient* but not *necessary* to arrive at a consistent estimate, see e.g. Linder and Enqvist[1]

[1] J. Linder and M. Enqvist. *Int. J. Control*, 90(4), 729-745, 2017.

Single module identification



Conclusion:
With a 3-input, 1 output model we can consistently identify $\boldsymbol{G^0_{21}}$
with an indirect method.

For a consistent and minimum variance estimate (direct method) there is one additional condition:

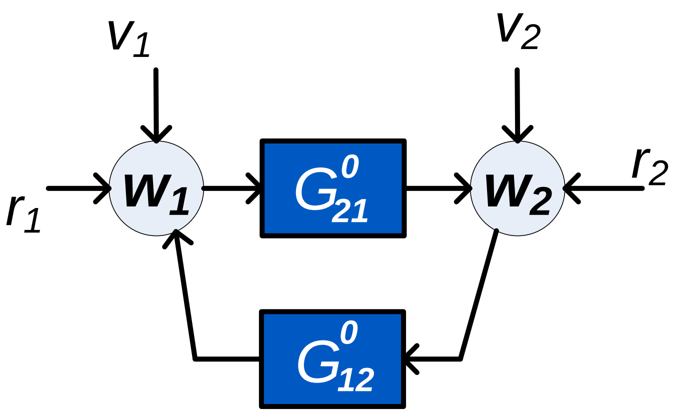
- absence of **confounding variables,** [1][2]

i.e. correlated disturbances on inputs and outputs

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009; [2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Confounding variables

Back to the (classical) closed-loop problem:



Direct identification of G^0_{21} can be consistent provided that v_1 and v_2 are uncorrelated

In case of correlation between v_1 and v_2:
joint prediction of w_1 and w_2 leads to ML results,

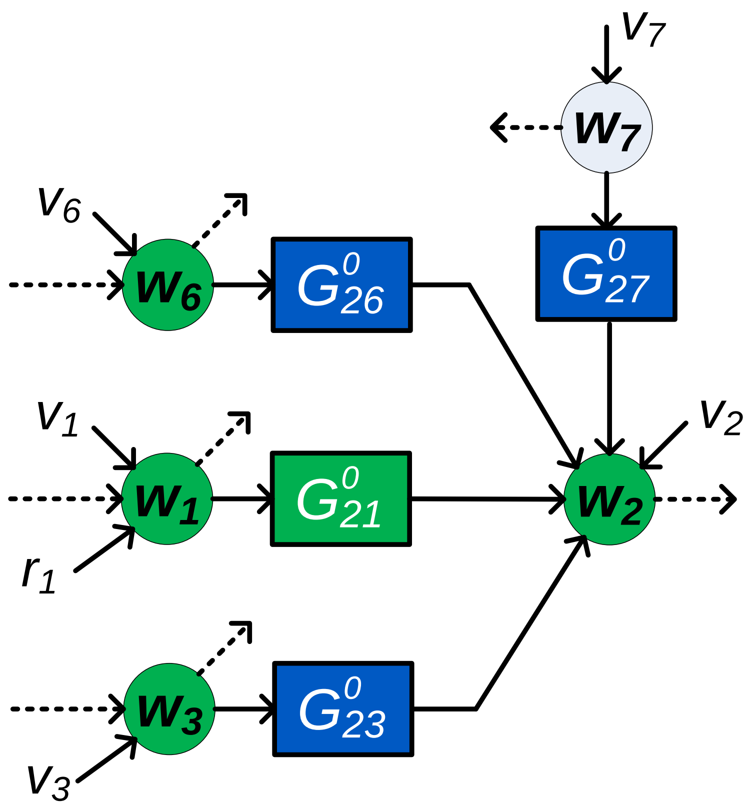
$$\begin{bmatrix} \varepsilon_1(t,\theta) \\ \varepsilon_2(t,\theta) \end{bmatrix} = H(q,\theta)^{-1} \begin{bmatrix} w_1(t) - G_{12}(q,\theta)w_2(t) \\ w_2(t) - G_{21}(q,\theta)w_1(t) \end{bmatrix}$$

Joint estimation of G^0_{21} and G^0_{12}: Joint–direct method [1,2]

related to the classical joint-io method [3,4]

[1] P.M.J. Van den Hof, A.G. Dankers, H.H.M. Weerts, *Proc. 56th IEEE CDC*, 2017; [2] H.H.M. Weerts et al., *Automatica*, 2018b, to appear.
[3] T.S. Ng, G.C. Goodwin, B.D.O. Anderson, Automatica, 1977; [4] B.D.O. Anderson and M. Gevers, Automatica 1982.

Confounding variables



- w_7 (not measured) now acts as a disturbance

- For minimum variance: **MISO direct method** loses consistency if there are confounding variables

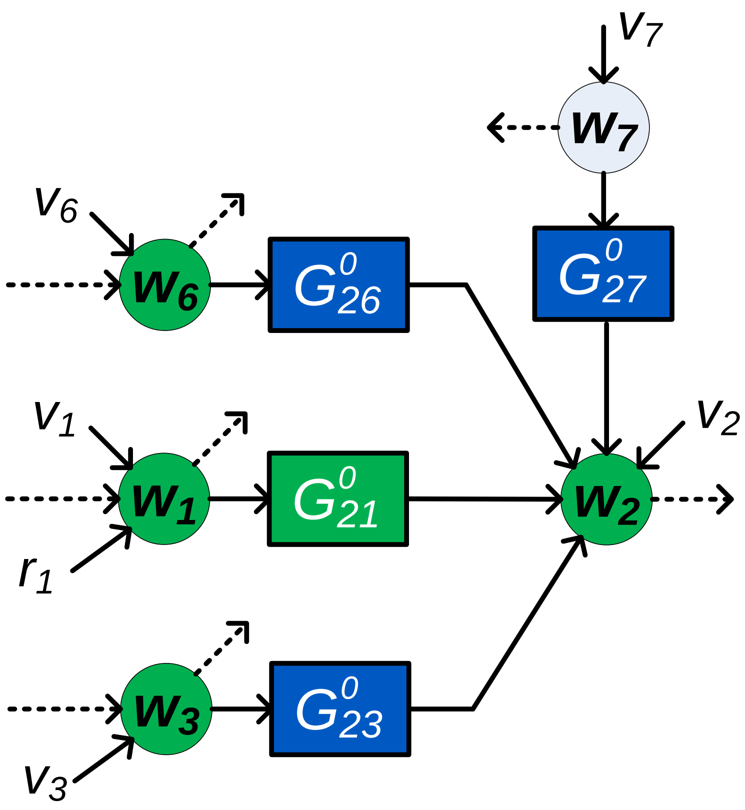
- This requires:

$$\begin{bmatrix} v_2 \\ v_7 \end{bmatrix} \text{ uncorrelated with } \begin{bmatrix} v_1 \\ v_3 \\ v_6 \end{bmatrix}$$

and no path from w_7 to an input

[1] A.G. Dankers, P.M.J. Van den Hof, D. Materassi and H.H.M. Weerts, *Proc. IFAC World Congress*, 2017.

Confounding variables



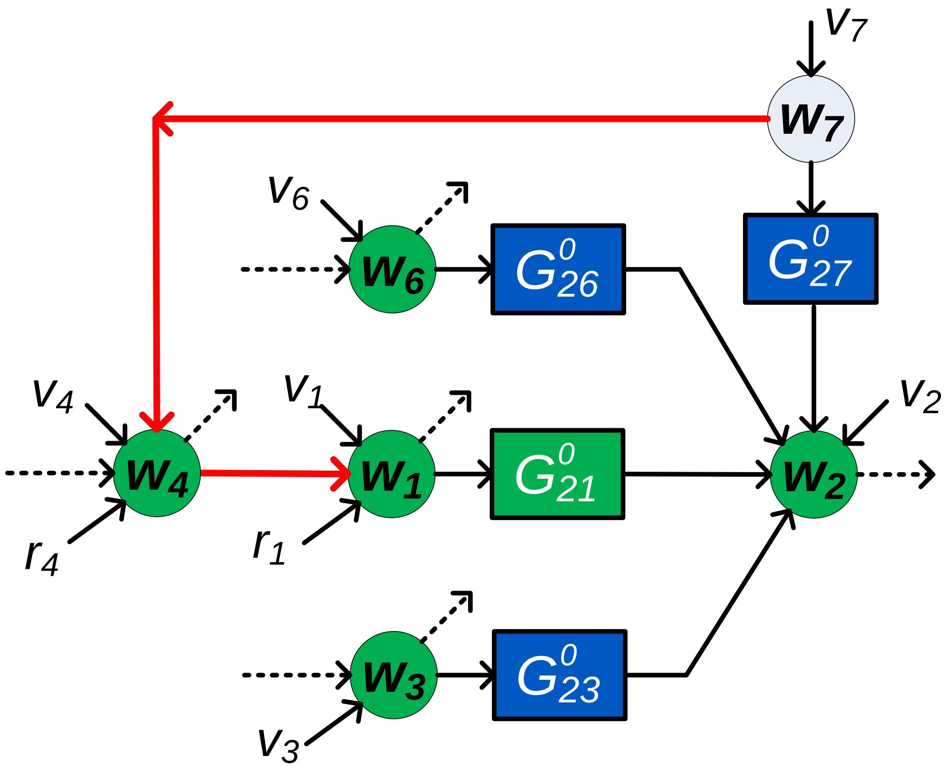
Solutions while restricting to MISO models:

(a) Including the node w_7 as additional input, or

(a) Block the paths from w_7 to inputs/ outputs by measured nodes, to be used as additional inputs.

[1] A.G. Dankers, P.M.J. Van den Hof, D. Materassi and H.H.M. Weerts, *Proc. IFAC World Congress*, 2017.

Confounding variables



Solutions:

(b) Block the paths from w_7 to input w_1 by measuring node w_4 to be used as additional inputs.

Relation with d-separation in graphs (Materassi & Salapaka)

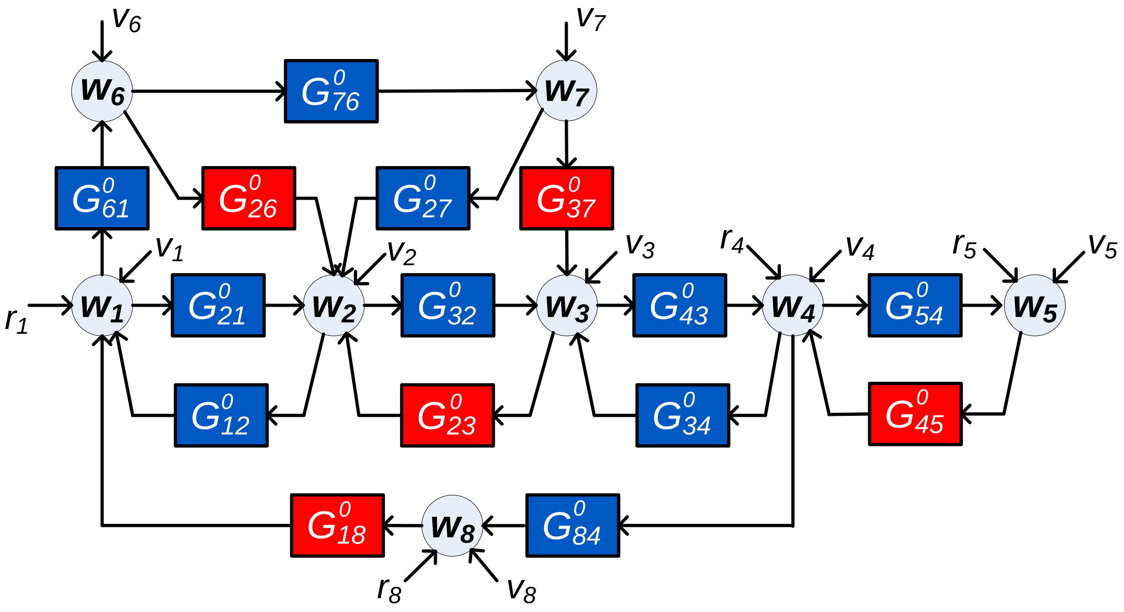
[1] A.G. Dankers, P.M.J. Van den Hof, D. Materassi and H.H.M. Weerts, *Proc. IFAC World Congress*, 2017.

Summary single module identification

- Single module identification in a network with known topology
- Methods for **consistent** and **minimum variance** module estimation
- Excitation through excitation signals only or through all external signals
- For direct method / ML results: treatment of confounding variables / correlated disturbances
- A priori known modules can be accounted for

Network Identifiability

Network identifiability



blue = unknown
red = known

Question: Can the dynamics/topology of a network be *uniquely determined* from measured signals w_i, r_i ?

Required: Can different dynamic networks be *distinguished* from each other from measured signals w_i, r_i ?

Starting assumption: all signals w_i, r_i that are present are measured

Network identifiability

Network: $w = G^0 w + R^0 r + H^0 e$ $\qquad cov(e) = \Lambda^0,$ rank *p*

$w = (I - G^0)^{-1}[R^0 r + H^0 e]$ $\qquad$ dim(*r*) = *K*

The network is defined by: $(G^0, R^0, H^0, \Lambda^0)$

a network model is denoted by: $M = (G, R, H, \Lambda)$

and a **network model set** by:

$$\mathcal{M} = \{M(\theta) = (G(\theta), R(\theta), H(\theta), \Lambda(\theta)), \theta \in \Theta\}$$

represents **prior knowledge** on the network models:

- topology
- disturbance correlation
- known modules
- the signals used for identification

Network identifiability

$$w = (I - G^0)^{-1}[R^0 r + H^0 e]$$

Denote: $w = T^0_{wr} r + \bar{v}$

$$\bar{v} = T^0_{we} e$$

$$\Phi^0_{\bar{v}} = T^0_{we}(e^{i\omega})\Lambda^0 T^0_{we}(e^{i\omega})^*$$

Objects that are uniquely identified from data r, w: T^0_{wr}, $\Phi^0_{\bar{v}}$

How to define identifiability?

Clasically:
- Property of a model set
- Unique mapping between **parameters** and models

In the **network** situation:

- Property of a model set
- Unique mapping between **models** and identified objects

Weerts et al, SYSID2015; Weerts, Van den Hof and Dankers, Automatica, 2018a;

Network identifiability

$w = T^0_{wr} r + \bar{v}$ Objects identified from data: $T^0_{wr}, \; \Phi^0_{\bar{v}}$

Denote the parametrized objects:

$$\begin{aligned} T_{wr}(q,\theta) &= (I - G(q,\theta))^{-1} R(q,\theta) \\ \Phi_{\bar{v}}(\omega,\theta) &= (I - G(\theta))^{-1} H(\theta)\Lambda(\theta)H(\theta)^*(I - G(\theta))^{-*} \end{aligned}$$

Definition

A network model set $\mathcal{M}$ is network identifiable at $M_0 = M(\theta_0)$ if for all models $M(\theta_1) \in \mathcal{M}$:

$$\left. \begin{array}{c} T_{wr}(q,\theta_1) = T_{wr}(q,\theta_0) \\ \Phi_{\bar{v}}(\omega,\theta_1) = \Phi_{\bar{v}}(\omega,\theta_0) \end{array} \right\} \Longrightarrow M(\theta_1) = M(\theta_0)$$

$\mathcal{M}$ is network identifiable if this holds for all models $M_0 \in \mathcal{M}$

Network identifiability

Let $K + p \geq L$

independent external signals $\geq$ # nodes

Proposition (full excitation case) (based on Goncalves & Warnick, 2008)

$\mathcal{M}$ is network identifiable at $M_0 = M(\theta_0)$ if there exists a nonsingular and rational $Q(q)$ such that

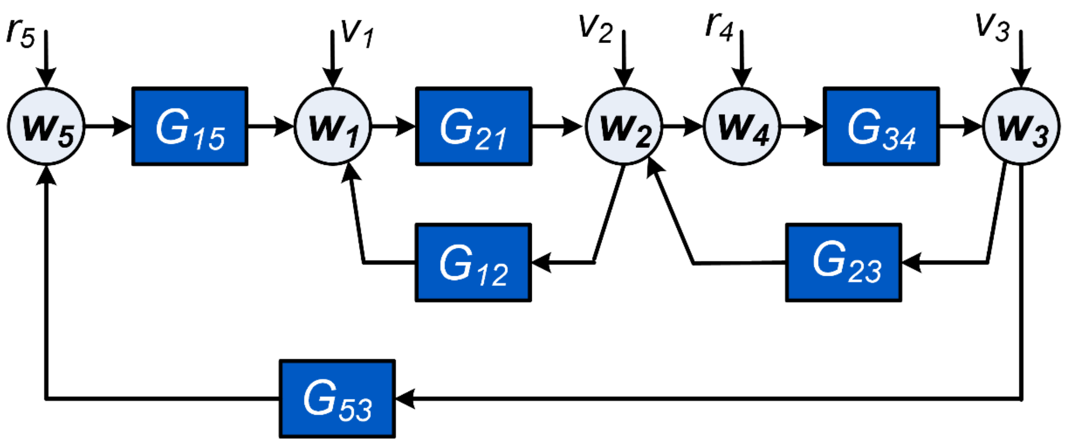
$$\begin{bmatrix} R(q,\theta) & H(q,\theta) \end{bmatrix} Q(q)$$

has a **leading diagonal matrix**,
that is full rank for all $\{\theta \in \Theta \mid T(q,\theta) = T(q,\theta_0)\}$.

Note: $G(q,\theta)$ can be fully unknown (no prior knowledge on topology)

Goncalves and Warnick, IEEE-TAC, 2008; Weerts et al, SYSID2015;
Gevers and Bazanella, 2017; Weerts, Van den Hof and Dankers, Automatica, 2018a;

Example 5-node network

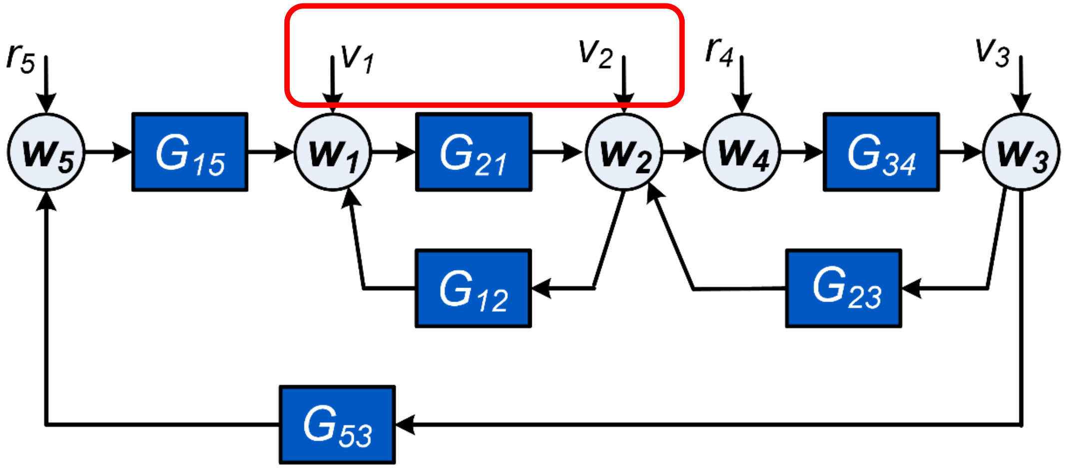


There are noise-free nodes, and v_1 and v_2 are expected to be correlated

$$\mathcal{M} \text{ with } H(\theta) = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 \\ 0 & 0 & H_{33}(\theta) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$G(\theta)$ fully parametrized

Example 5-node network



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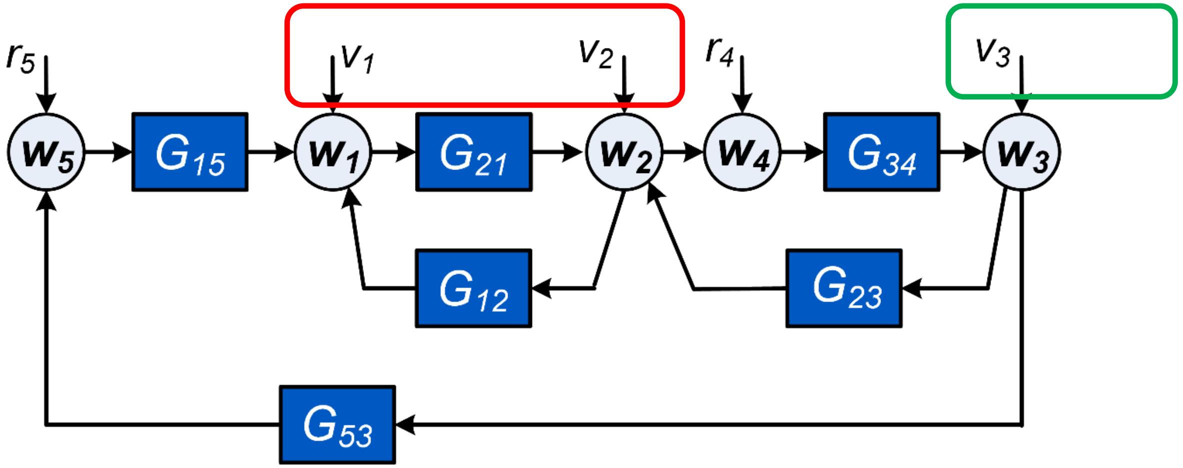
Example 5-node network



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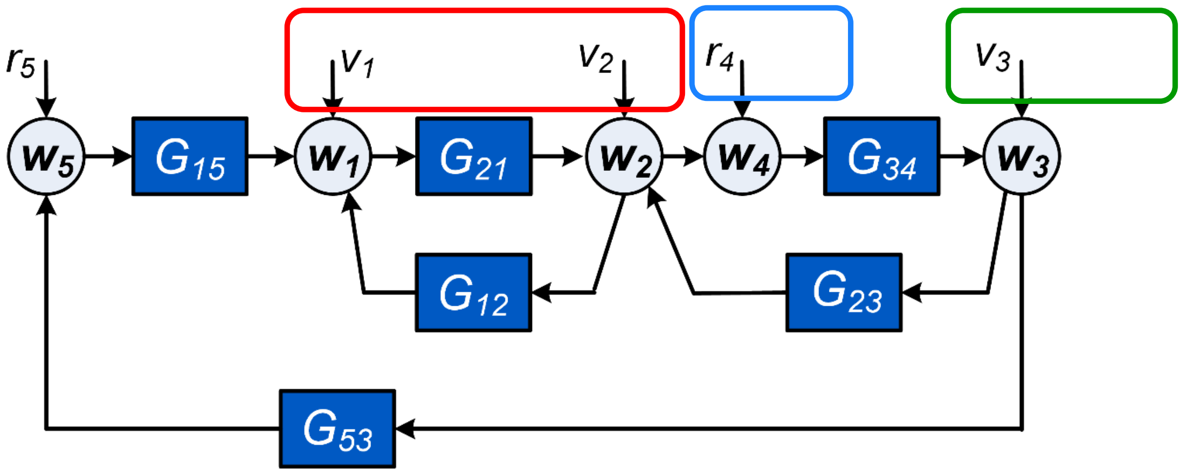

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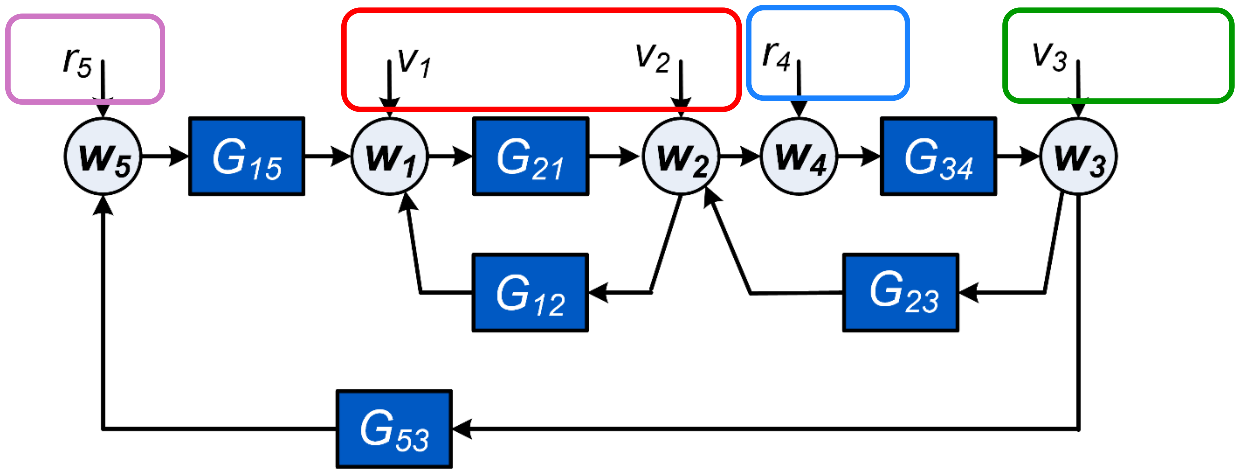
Example 5-node network



There are noise-free nodes, and v_1 and v_2 are expected to be correlated

$$\mathcal{M} \text{ with } H(\theta) = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 \\ 0 & 0 & H_{33}(\theta) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We can not arrive at a diagonal structure in $\begin{bmatrix} H(\theta) & R(\theta) \end{bmatrix}$

Network identifiability

Observations:

a) A simple test can be performed to check the condition

b) The condition is typically fulfilled if each node w_j is excited by an independent external signal (either an r_j or a v_j)

c) The result is rather **conservative**:
   1. Restricted to $\begin{bmatrix} R(q,\theta) & H(q,\theta) \end{bmatrix}$ having full row rank
   2. No account of prior knowledge in $G(q,\theta)$

Network identifiability

Theorem 3 – identifiability for general model sets

If:

a) Each unknown entry in $M(\theta)$ covers the set of all proper rational transfer functions

b) All unknown entries in $M(\theta)$ are parametrized independently

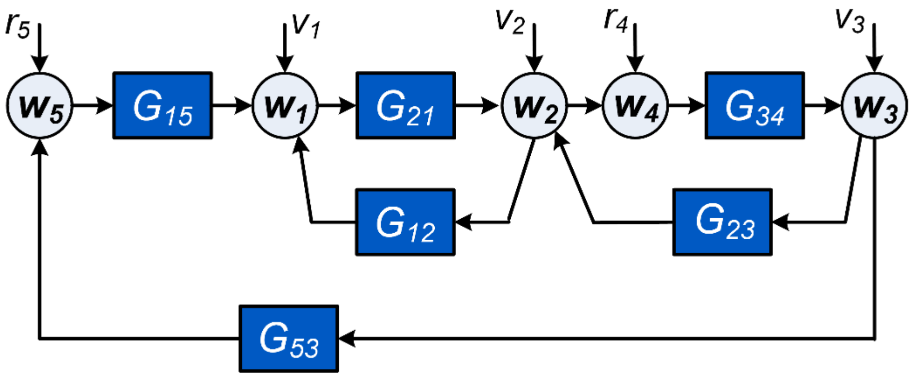
Then $\mathcal{M}$ is network identifiable at $M_0 = M(\theta_0)$ if and only if

- Each row of $[G(\theta)\ H(\theta)\ R(\theta)]$ has at most *K*+*p* parametrized entries
- For each row i the transfer matrix $\check{T}_i(q,\theta_0)$ has full row rank, with $\check{T}_i(q,\theta_0)$:

$$[G_{i*}(\theta)\ H_{i*}(\theta)\ R_{i*}(\theta)] = \begin{array}{ccccccccccccc} & & & & & & & & [v_3 & v_4 & & r_1 & r_2] \\ & & & & & & & & \downarrow & \downarrow & & \downarrow & \downarrow \\ [0 & * & 0 & * & * & | & * & * & 0 & 0 & | & 1 & 0] \\ & \downarrow & & \downarrow & \downarrow & & & & & & & & \\ & [w_2 & & w_4 & w_5] & & & & & & & & \end{array}$$

TU/e

Example 5-node network



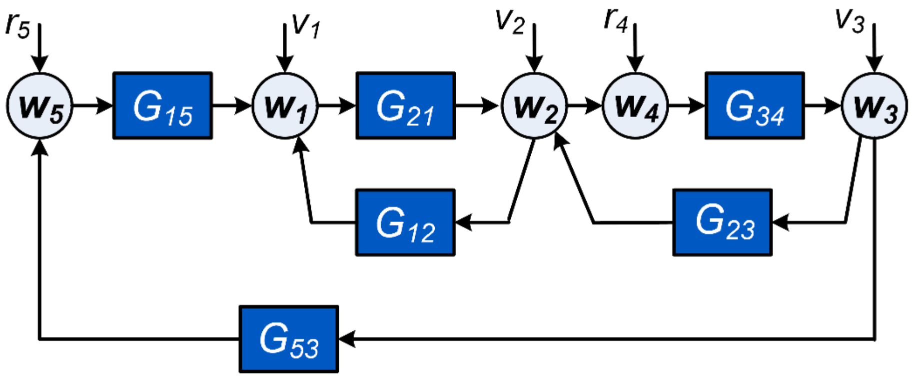
If we restrict the structure of $G(\theta)$:

$$G(\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta) \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H \;\; R] = \underbrace{\begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{K+p=5}$$

First condition:

Number of parametrized entries in each row < K+p = 5

Example 5-node network



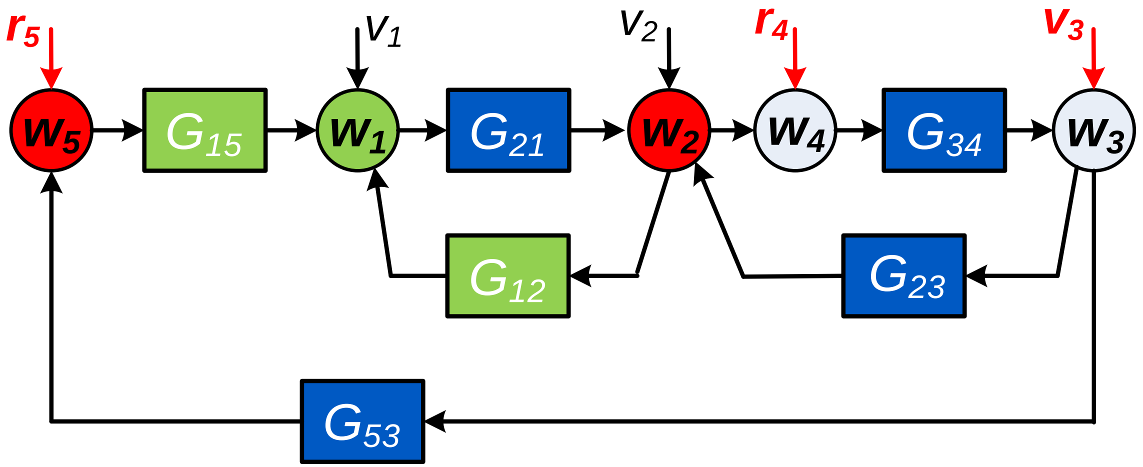
$$G(\theta) = \begin{bmatrix} \boxed{0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta)} \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H\ \ R] = \underbrace{\begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{K+p=5}$$

Rank condition:

Row 1: Full row rank of transfer: $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

Example 5-node network

Verifying the rank condition for $\breve{T}_1(q,\theta_0)$:



$i = 1:$ Evaluate the rank of the transfer matrix $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

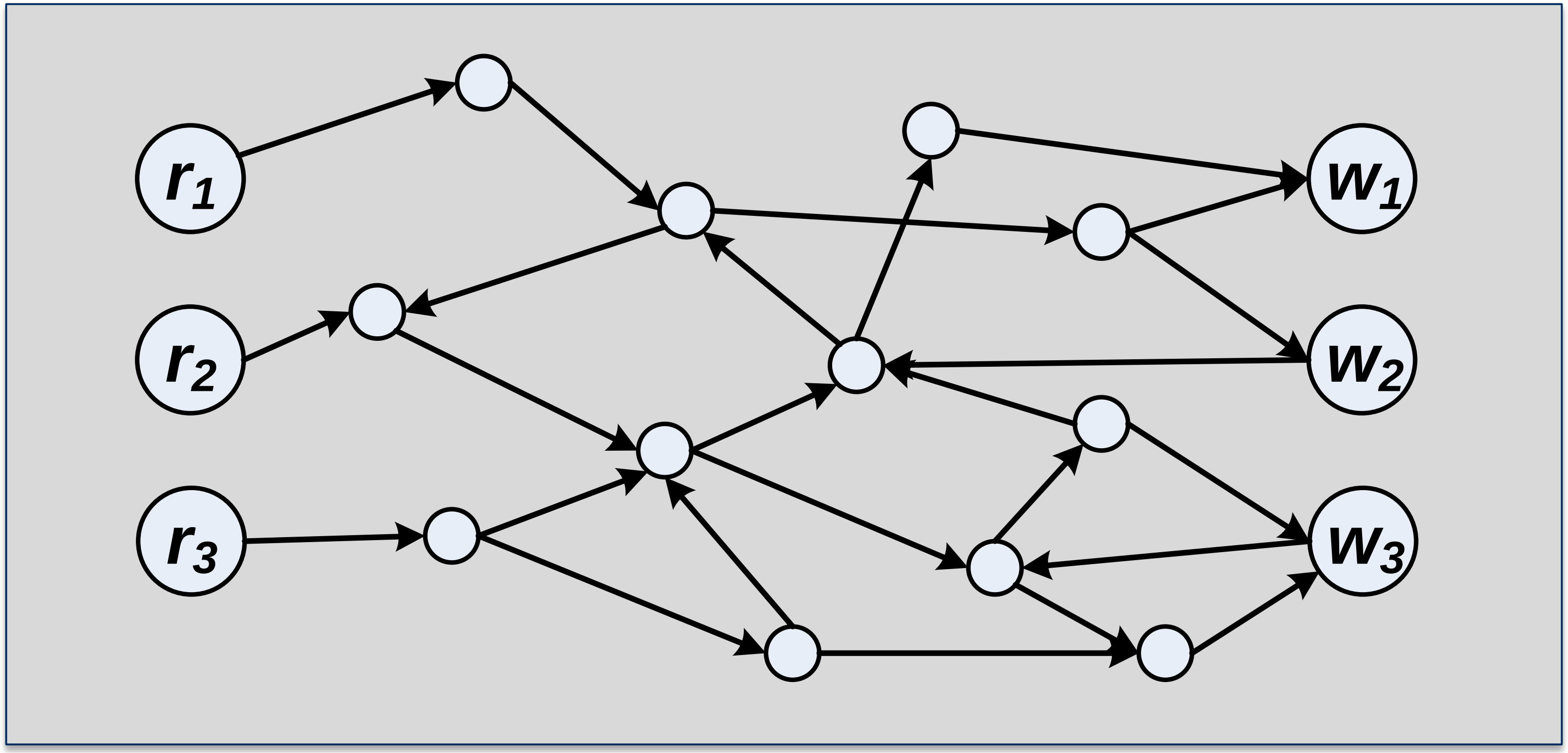
Vertex-disjoint paths

Theorem (Van der Woude, 1991; Hendrickx et al. 2017; Weerts et al., 2018)

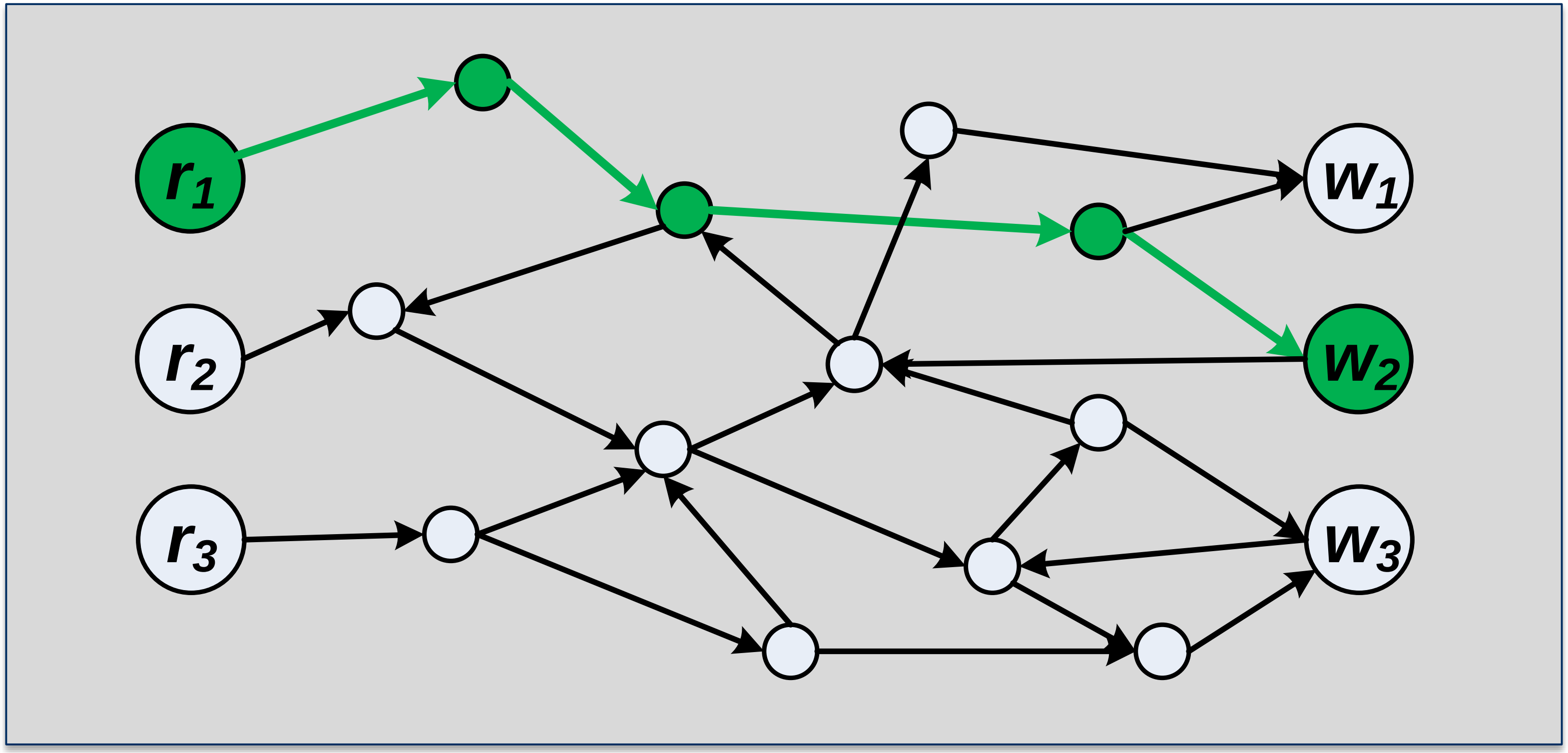
The **generic rank** of a transfer function matrix between
inputs r and nodes w
is equal to the maximum number of **vertex-disjoint paths** between the sets of inputs and outputs.

A (path-based) check on the topology of the network can decide whether the conditions for identifiabiltiy are satisfied generically.

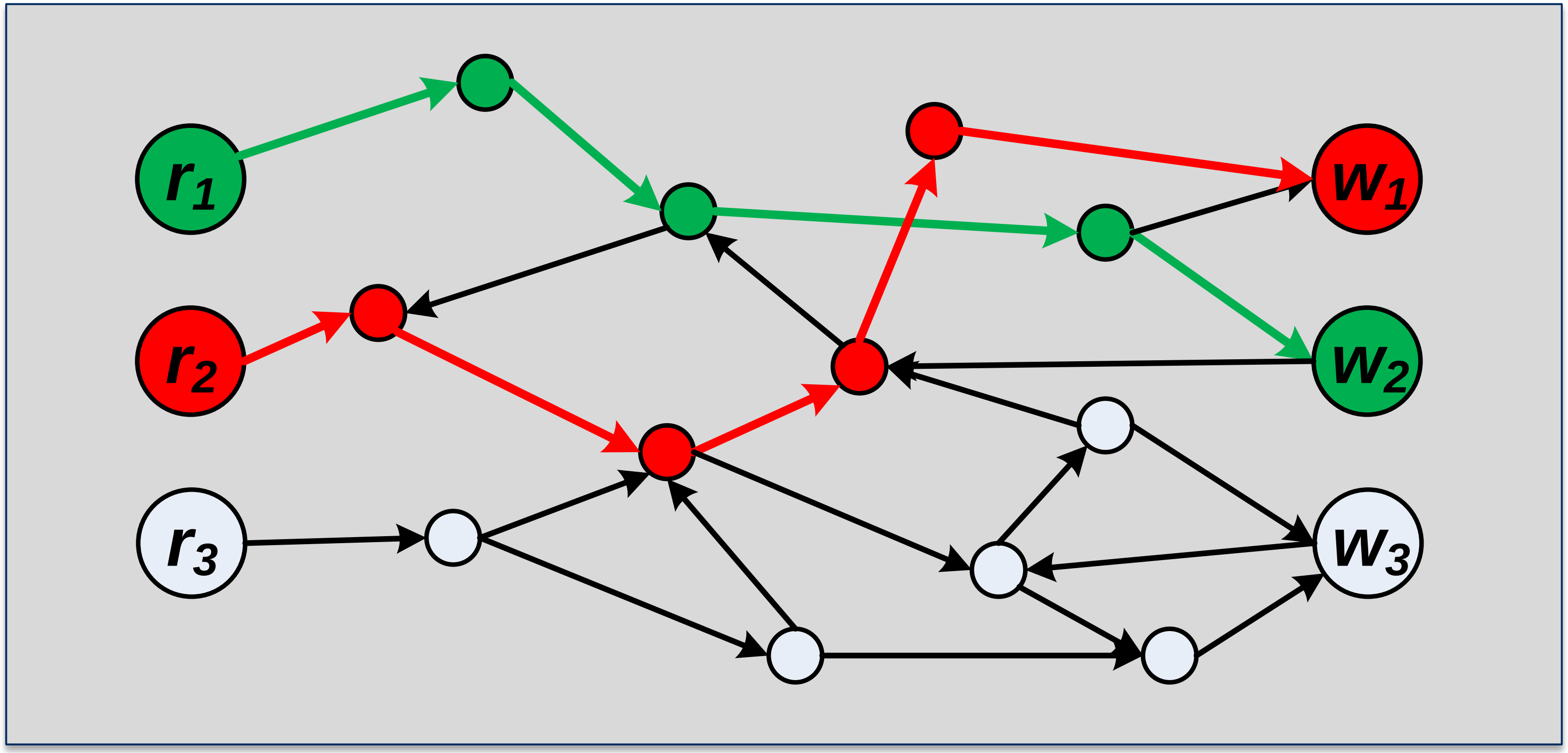
Vertex-disjoint paths



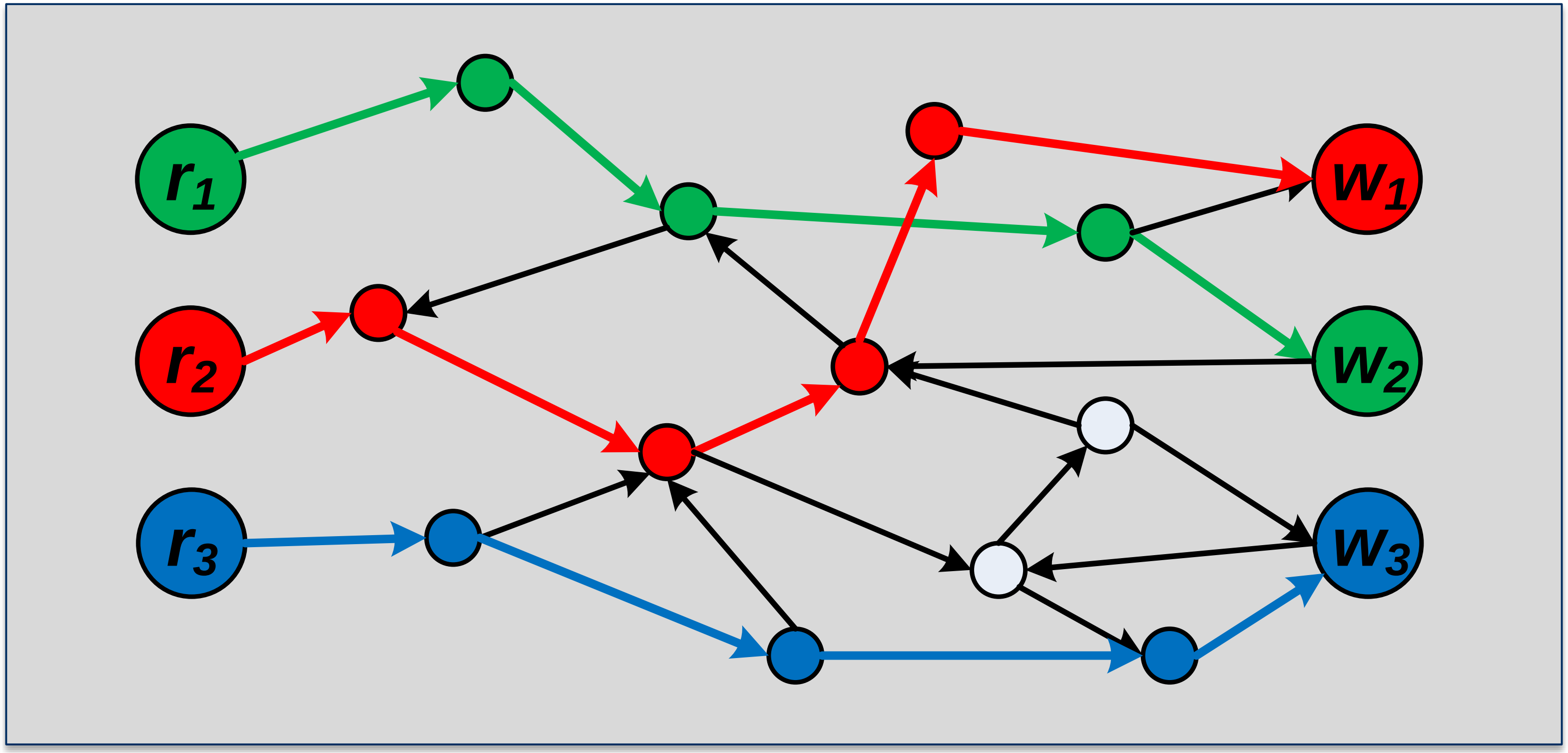
Vertex-disjoint paths



Vertex-disjoint paths



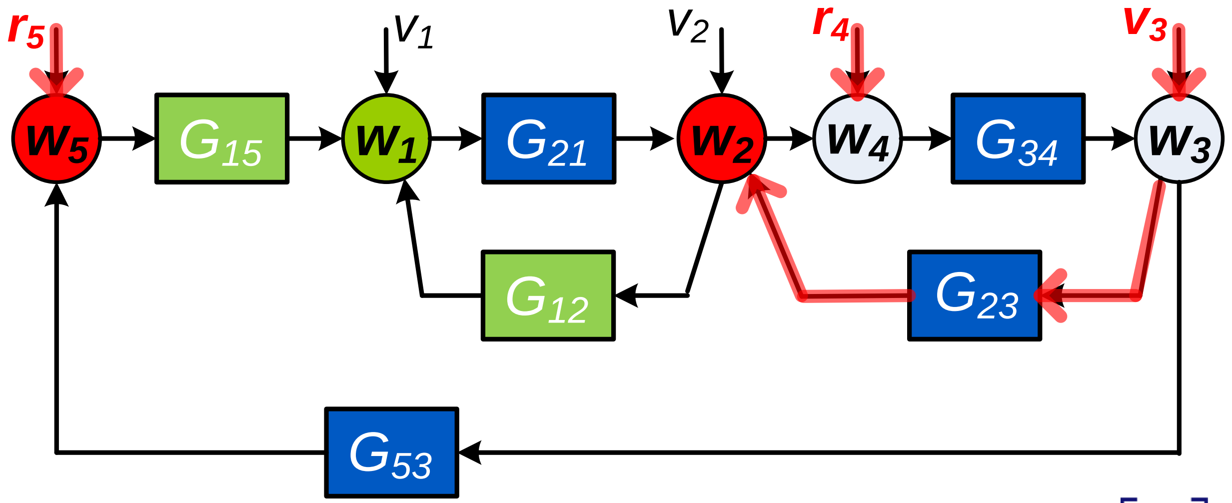
Vertex-disjoint paths



Generic rank = 3

Example 5-node network

Verifying the rank condition for $\check{T}_1(q, \theta_0)$:

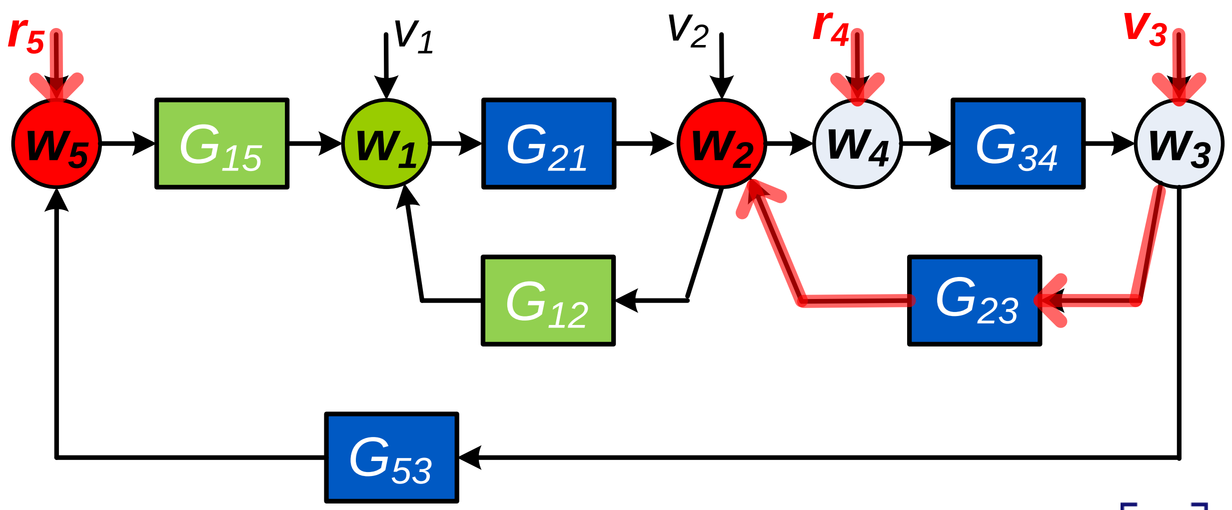


$i = 1:$ Evaluate the rank of the transfer matrix from $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix}$ to $\begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

2 vertex-disjoint paths → full row rank 2

Example 5-node network

Verifying the rank condition for $\check{T}_1(q,\theta_0)$:



$i=1:$ Evaluate the rank of the transfer matrix from $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix}$ to $\begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

For each row i :

unknown modules $G_{ik}(q,\theta) \leq$ # external signals uncorrelated with v_i

Summary identifiability

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Prior knowledge on modules

So far:

- All node signals assumed to be measured
- Fully applicable to the situation $p < L$ (i.e. reduced-rank noise)
- Identifiability of the full network model – conditions per row/output node

Summary identifiability

Extensions:

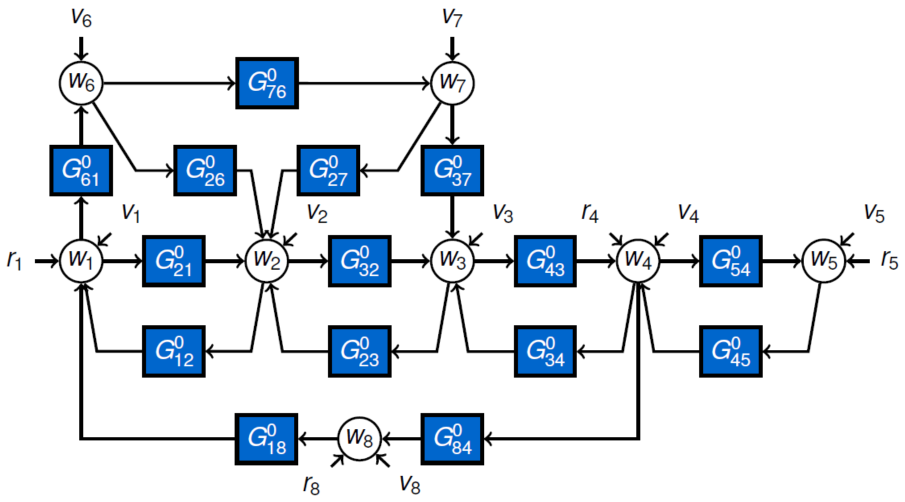
1. Relation back to state space structures,
   (Hayden, Yuan, Goncalves, TAC 2017)

2. When topology is known and every node is excited, which nodes need to be measured for unique identification of a **particular module** on the basis of T^0_{wr} ?
   (Hendrickx, Gevers & Bazanella, CDC 2017, ArXiv 2018)

3. When all nodes are measured, what are conditions for identifiability of a **particular module** in a network model set?
   (Weerts, Van den Hof, Dankers, CDC 2018)

Extensions

Reduced rank noise



$$\begin{bmatrix} v_1(t) \\ \vdots \\ v_L(t) \end{bmatrix} = H^0(q) \begin{bmatrix} e_1(t) \\ \vdots \\ e_p(t) \end{bmatrix}$$

Question:
How to identify (parts of) a dynamic network, when the process noise is of reduced rank *($p < L$)*?

Typically considered in dynamic factor models[1]

[1] M. Deistler, W. Scherrer and B.D.O. Anderson, 2015

Reduced rank noise

Typical: multi-output situation

Weighted LS criterion:

$$\hat{\theta}_N^{WLS} = \arg\min_{\theta\in\Theta} \frac{1}{N}\sum_{t=1}^{N} \varepsilon^T(t,\theta)\; Q\; \varepsilon(t,\theta) \qquad\qquad Q > 0$$

Properties:

- Consistent estimate under regularity conditions,
- But for minimum variance an optimal Q has to be chosen

Typical choice, leading to minimum variance estimator:

$$Q = [cov(\breve{e})]^{-1} = (\breve{\Lambda}^0)^{-1}$$

but in our situation $\breve{\Lambda}^0$ is singular

Reduced rank noise

Solution: Parametrize dependencies in innovation process, and include them as constraints:

Identification criterion becomes a
constrained quadratic problem
with ML properties for Gaussian noise.

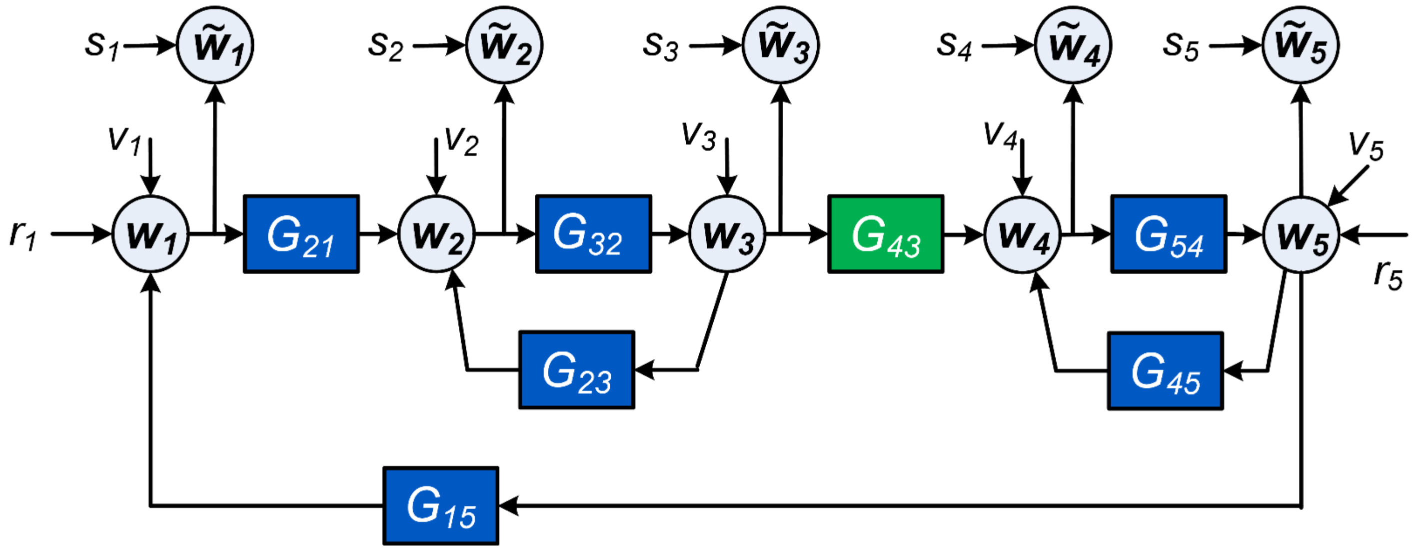
Reformulation of Cramer-Rao bound

Some parameters can be estimated variance-free.

H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers, *IFAC World Congress*, 2017; Automatica, 2018b to appear.

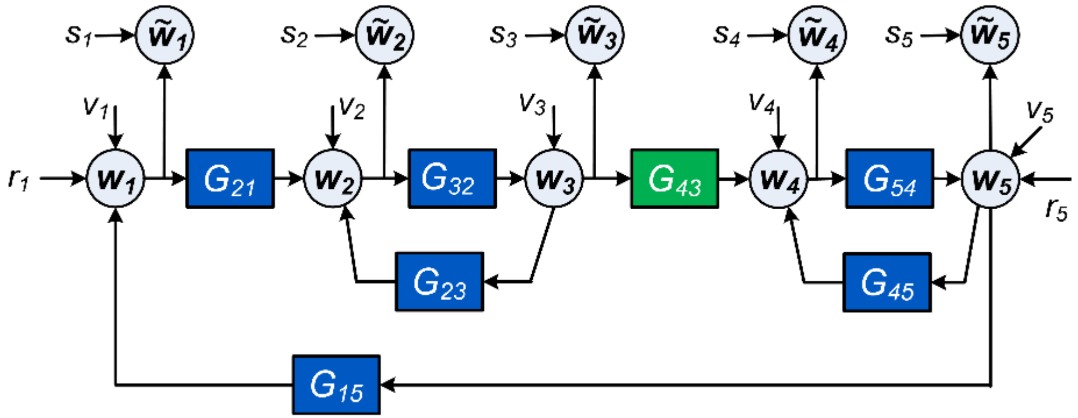
Sensor noise

Identification of a single module under the influence of sensor noise:



- Typical tough problem in open-loop identification (errors-in-variables)
- In dynamic networks this may become *more simple* due to the presence of multiple (correlated) node signals

Sensor noise



Solution strategies:

1. Apply a correlation (IV) approach to mitigate the effect of sensor noise
   → choosing instrument signals (based on topology)

2. Combine:
   1. a correlation (IV) technique to mitigate sensor noise, and
   2. BJ noise models for addressing process noise.

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Discussion

- **Dynamic network identification:**
  intriguing research topic with many open questions

- The (centralized) LTI framework is only just the beginning

- From classical PE methods to regularized (kernel-based) approaches
  (Everitt, Bottegal, Hjalmarsson, 2018; Ramaswamy et al, CDC 2018)
- Further move towards data-aspects related to distributed identification and control

- Algorithmic aspects for large-scale data handling

- Move towards including nonlinear dynamics (M. Schoukens, SYSID2018)

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Jonas Linder
Sean Warnick
Alessandro Chiuso
Hakan Hjalmarsson
Miguel Galrinho

The End

Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica,* Vol. 49, no. 10, pp. 2994-3006.

- A. Dankers (2014). System identification in dynamic networks. Dr. Dissertation, TU Delft.

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