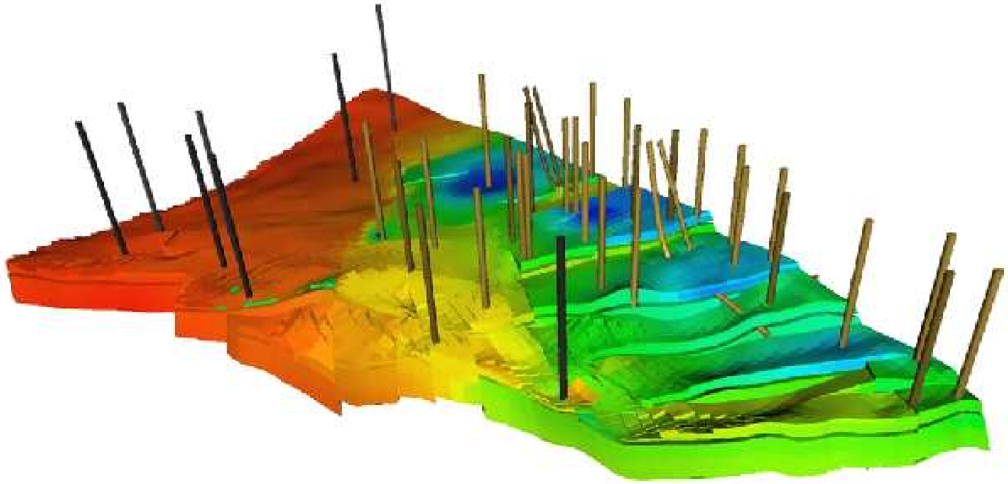
Determining Identifiable Parameterizations for Large-scale Physical Models in Reservoir Engineering



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Outline

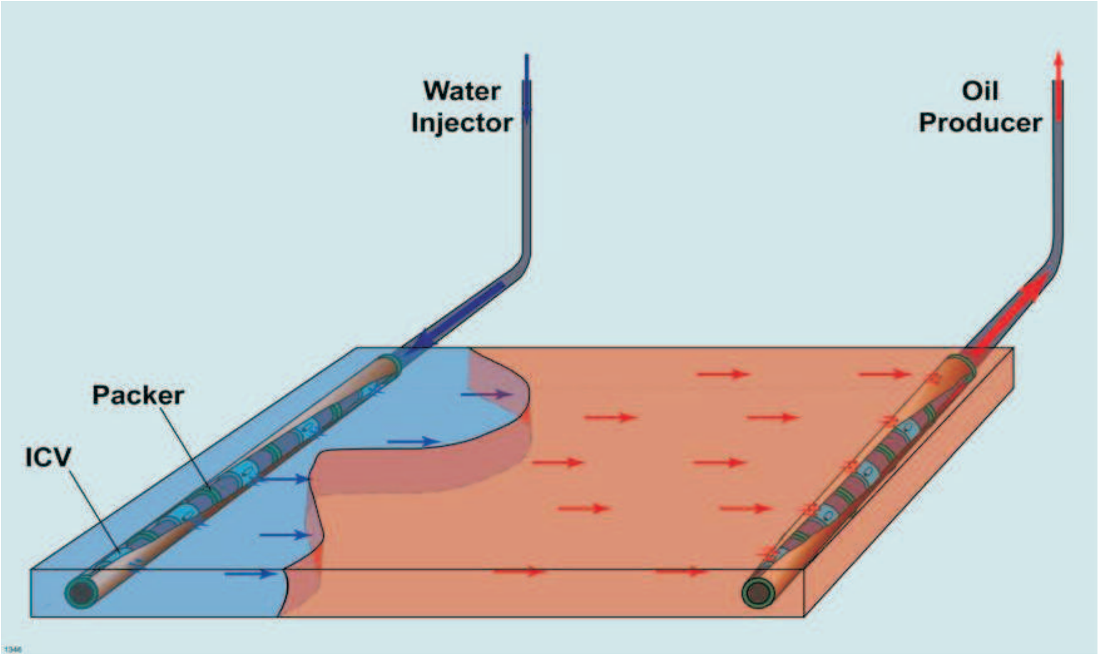
- Reservoir engineering and parameter estimation
- Structural identifiability
- Determining identifiable parameterizations
- Example with grid block permeability
- Summary

Upstream oil industry - characteristics

- Capital intensive: well: $1 - 100 * 10^6$ US$, field: $0.1 - 10 * 10^9$ US$.
- Uncertainty: geology, oil price, limited data.
- Reservoir can only be produced once.
- Time scales stretch from days to decades.
- Trends
  - Produce more from existing reservoirs
  - Increasing knowledge- and data intensity
  - Move to model-based control

Water flooding

- Inject water to push oil to producers.
- Increase oil recovery by manipulating ICVs in wells.
- Major influence of permeability on saturation behavior.



Model equations

Model describes flow of water and oil through porous medium.

$$\begin{aligned} \frac{\partial}{\partial t}\left(\phi \rho_o \left[1 - S_w\right]\right) &= \nabla \cdot \left(\bar{k}\frac{k_{ro}}{\mu_o}\rho_o \nabla p\right) + q_o \\ \frac{\partial}{\partial t}\left(\phi \rho_w S_w\right) &= \nabla \cdot \left(\bar{k}\frac{k_{rw}}{\mu_w}\rho_w \nabla p\right) + q_w \end{aligned}$$

Discretization in space (grid blocks) and time gives nonlinear state-space model:

$$\left[\begin{array}{c} \mathbf{p}(k+1) \\ \mathbf{S}_w(k+1) \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}_{11}(\mathbf{p}(k), \mathbf{S}_w(k)) & 0 \\ \hline \mathbf{A}_{21}(\mathbf{p}(k), \mathbf{S}_w(k)) & 0 \end{array}\right] \left[\begin{array}{c} \mathbf{p}(k) \\ \mathbf{S}_w(k) \end{array}\right] + \left[\begin{array}{c} \mathbf{B}_1(\mathbf{p}(k), \mathbf{S}_w(k)) \\ \hline \mathbf{B}_2(\mathbf{p}(k), \mathbf{S}_w(k)) \end{array}\right] \mathbf{u}(k)$$

- $\bar{\mathbf{k}}$ permeabilities in EACH grid block.
- Commonly $10^5 - 10^6$ grid blocks!

Reservoir simulation

- $\mathbf{u} \in \mathbb{R}^m$: injection rates, pressures in producer wells.
- $\mathbf{y} \in \mathbb{R}^p$: pressures in injector wells, production rates.

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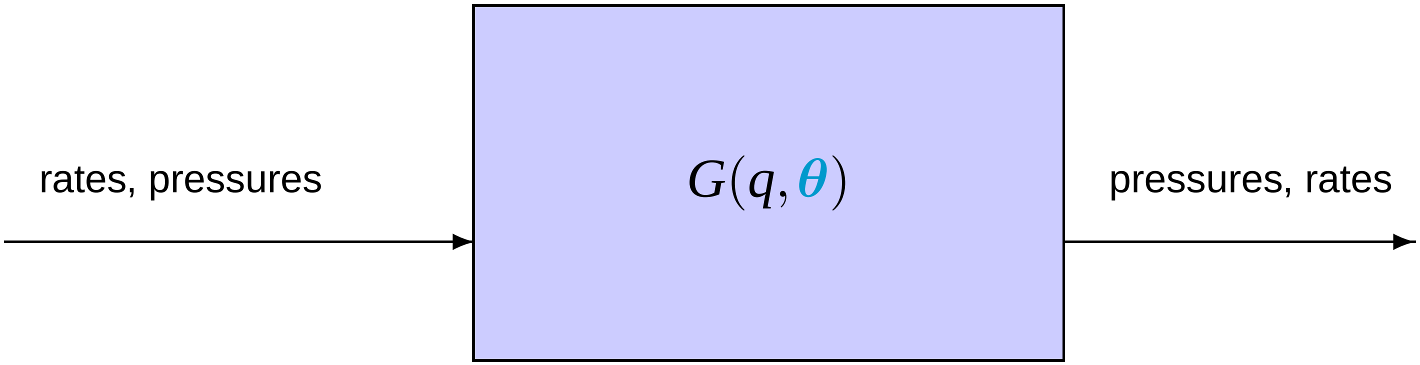
Parameter estimation

- Physical parameters (permeabilities) estimated from production measurements with e.g. RLS, EKF or EnKF.
- Challenges:
  - Many grid block permeabilities to be estimated ($10^5 - 10^6$).
  - Limited number of wells.
- Option:
  - Determine identifiable subspace of parameter domain.
  - This leads to a map $\bar{\mathbf{k}} = \mathbf{T}\boldsymbol{\rho}$, where $\dim \boldsymbol{\rho} \ll \dim \bar{\mathbf{k}}$.
- Tool: Structural identifiability analysis.

Structural identifiability

- First stated by Bellman and Åström (1970).
- Model is structurally identifiable if its physical parameters can be uniquely determined from the input-output behavior.



$$\mathbf{x}(k+1) = \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}(\theta)\mathbf{x}(k) + \mathbf{D}(\theta)\mathbf{u}(k)$$

Local structural identifiability

- The model structure is locally structurally identifiable from the input-output behavior at $\boldsymbol{\theta}^\star$ if, for every $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$ in the neighborhood of $\boldsymbol{\theta}^\star$, $G(q,\boldsymbol{\theta}_1) = G(q,\boldsymbol{\theta}_2) \rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$.
- $G(q,\boldsymbol{\theta}) = \sum_{k=1}^{\infty} \mathbf{M}(k,\boldsymbol{\theta})q^{-k}$, where $\mathbf{M}(k) := \mathbf{C}(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta})^{k-1}\mathbf{B}(\boldsymbol{\theta})$
- Gather Markov parameters in

$$\overrightarrow{\mathbf{S}}_r(\boldsymbol{\theta}) := \begin{bmatrix} \overrightarrow{\mathbf{M}}(1,\boldsymbol{\theta}) & \overrightarrow{\mathbf{M}}(2,\boldsymbol{\theta}) & \dots & \overrightarrow{\mathbf{M}}(r,\boldsymbol{\theta}) \end{bmatrix} \in \mathbb{R}^{1\times pmr}.$$

- If $\frac{\partial \overrightarrow{\mathbf{S}}_r(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ has constant rank l in a neighborhood of $\boldsymbol{\theta}^\star$, then $\overrightarrow{\mathbf{S}}_r(\boldsymbol{\theta})$ is locally injective at $\boldsymbol{\theta}^\star$ if and only if $l = q$.

Local structural identifiability

- Test local structural identifiability by rank test on

$$I_r := \frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}} \frac{\partial \overrightarrow{\mathbf{S}}_r^T}{\partial \boldsymbol{\theta}^T}\Bigg|_{\boldsymbol{\theta}^\star} = \sum_{i=1}^{r} \sum_{j=1}^{p} \left(\frac{\partial \mathbf{M}_{j*}(i)}{\partial \boldsymbol{\theta}} \frac{\partial \mathbf{M}_{j*}^T(i)}{\partial \boldsymbol{\theta}^T} \right)\Bigg|_{\boldsymbol{\theta}^\star} \in q \times q.$$

where

$$\begin{aligned} \frac{\partial \mathbf{M}_{j*}(k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} &= \frac{\partial \mathbf{C}_{j*}}{\partial \boldsymbol{\theta}} \mathbf{A}^{k-1}\mathbf{B} + \left(\mathbf{I}_q \otimes \mathbf{C}_{j*}\mathbf{A}^{k-1}\right) \frac{\partial \mathbf{B}}{\partial \boldsymbol{\theta}} \\ &+ \sum_{l=1}^{k-1} \left(\mathbf{I}_q \otimes \mathbf{C}_{j*}\mathbf{A}^{l-1}\right) \frac{\partial \mathbf{A}}{\partial \boldsymbol{\theta}} \mathbf{A}^{k-1-l}\mathbf{B}. \end{aligned}$$

- For a given model, the expression can be calculated exactly.
- Limitation: only local linearized situation can be handled.

Observability and controllability

- I_r can be expressed in terms of observability and controllability
- Recall $I_r := \left. \frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}} \frac{\partial \overrightarrow{\mathbf{S}}_r^T}{\partial \boldsymbol{\theta}^T} \right|_{\boldsymbol{\theta}^\star}$
- Consider $\frac{\partial \mathbf{B}}{\partial \boldsymbol{\theta}}$ and $\frac{\partial \mathbf{C}}{\partial \boldsymbol{\theta}}$ zero, and $q = 1$, $p = 1$ and $r = 4$.
- Then $\frac{\partial \overrightarrow{\mathbf{S}}_4}{\partial \boldsymbol{\theta}} = \begin{bmatrix} 0 & \mathbf{X} \end{bmatrix}$, where

$$\mathbf{X} = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \mathbf{CA}^2 \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{A}}{\partial \boldsymbol{\theta}} & & \\ & \frac{\partial \mathbf{A}}{\partial \boldsymbol{\theta}} & \\ & & \frac{\partial \mathbf{A}}{\partial \boldsymbol{\theta}} \end{bmatrix} \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \\ & \mathbf{B} & \mathbf{AB} \\ & & \mathbf{B} \end{bmatrix}.$$

Identifiable parameterization

- Calculate I_r using $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and their analytical derivatives to $\boldsymbol{\theta}$
- Use singular value decomposition (SVD) on I_r

$$I_r = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix}^T \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix},$$

- Columns of $\mathbf{U}_1$ are structurally identifiable directions in parameter space.
- Columns of $\mathbf{U}_2$ are structurally *not* identifiable.
- Visualize each column vector (length q) by projection onto reservoir grid (q grid blocks).

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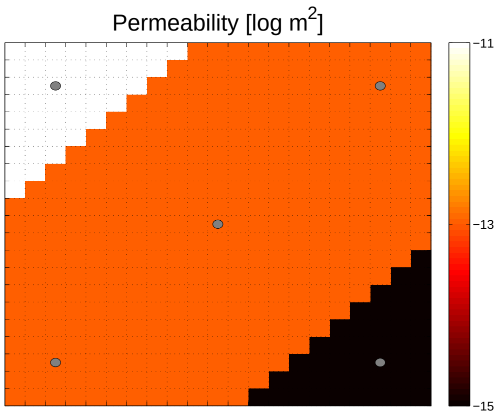
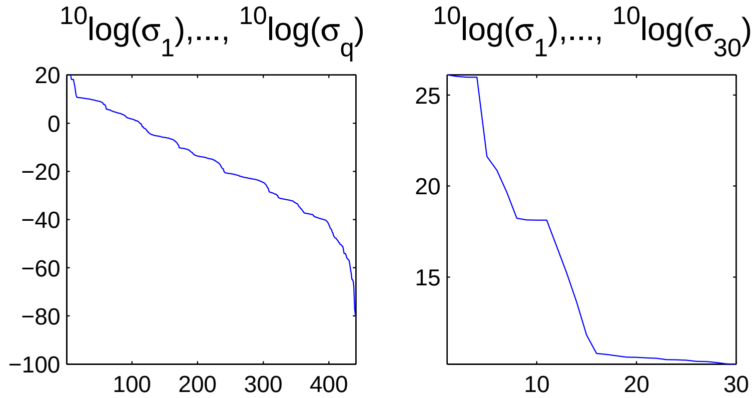
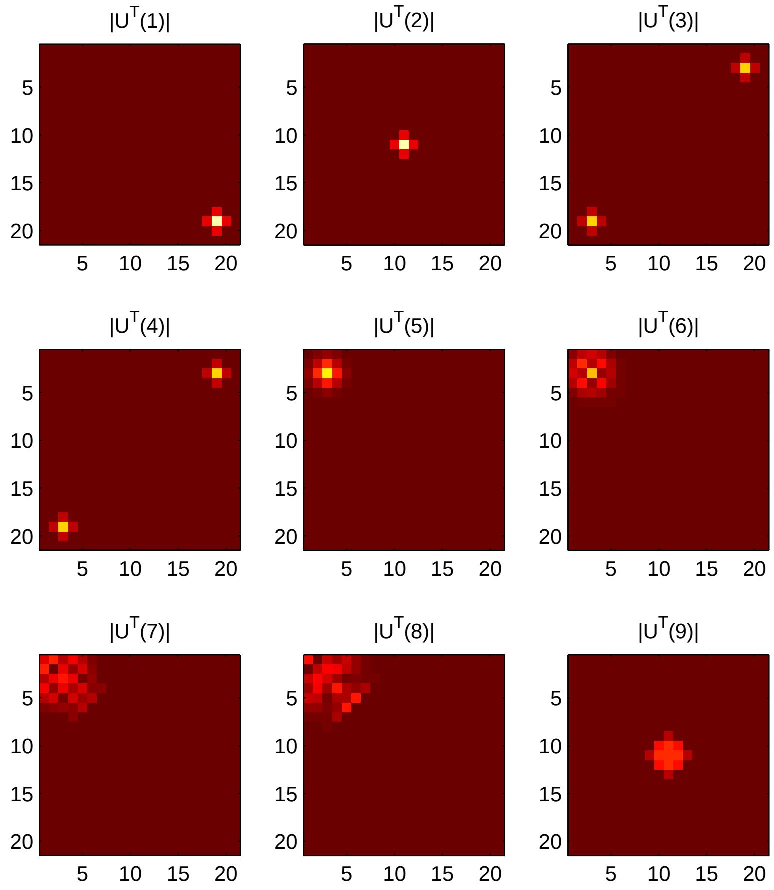
Example

- Single-phase reservoir model with 21×21 grid blocks.
- 5 wells (5 inputs, 5 outputs) measuring BHP and total well flow rate.
- Determine identifiable parameterization of permeability $\bar{\mathbf{k}} \in \mathbb{R}^{441}$.





Identifiable parameterization



First 9 column vectors of $|U_1|$ projected on reservoir simulation grid.

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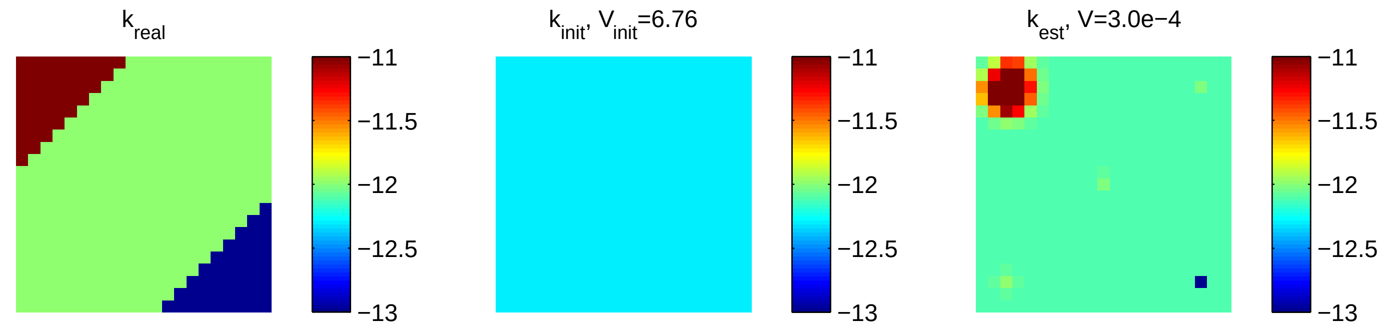
Estimation

- Use U_1 to go from high-dimensional $\bar{\mathbf{k}}$ to low-dimensional $\boldsymbol{\alpha}$.
- Reparameterize permeability as $\bar{\mathbf{k}} = \mathbf{U}_1\boldsymbol{\alpha}$.
- Iteratively estimate $\boldsymbol{\rho} = \arg\min_{\boldsymbol{\rho}} V(\mathbf{U}_1\boldsymbol{\rho})$.
- Advantage: only those (combinations of) grid block permeabilities are estimated that are relevant for the input-output behavior.



Conclusions

- Developed analytical expression to test structural identifiability.
- Can be expressed in observability, sensitivity and controllability matrix.
- Determined with SVD most identifiable parameterization with strongly reduced number of parameters.
- Used parameterization to estimate permeability in reservoir model.