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ISAPP

Identifiability, controllability and observability in hydrocarbon reservoir models

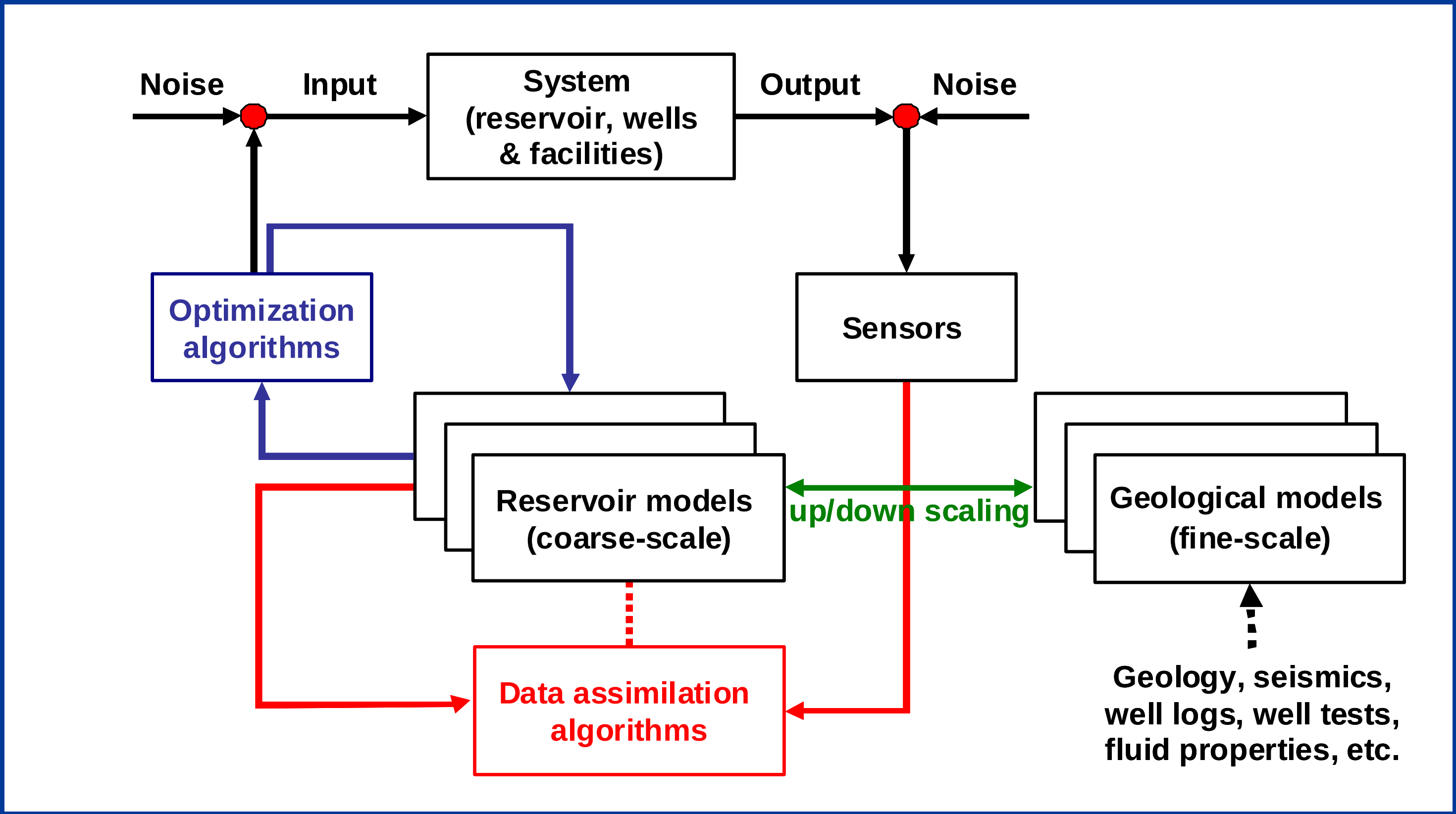
Jorn Van Doren[+], **Jan-Dirk Jansen[*#]**, Maarten Zandvliet[+#], Paul van den Hof[+], Okko Bosgra[+]*

[+] TU Delft - Delft Center for Systems and Control

[*]TU Delft - Dept. of Geotechnology

[#]Shell International E&P

Closed-loop reservoir management



- **Model-based life-cycle optimization under uncertainty**
- **Frequent model updating (history matching, data assimilation)**
- **Model-order reduction, control-relevant upscaling**

Systems and control approach

- (Linearized) reservoir model in ***state-space*** form

$$\mathbf{g}\left(\mathbf{x}_{k+1},\mathbf{x}_k,\mathbf{u}_{k+1},\mathbf{m}\right)=\mathbf{0}$$

$$\mathbf{j}\left(\mathbf{y}_{k,}\mathbf{x}_k,\mathbf{u}_k,\mathbf{m}\right)=\mathbf{0}$$

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{A}(\mathbf{m})\mathbf{x}_k+\mathbf{B}(\mathbf{m})\mathbf{u}_k, \quad \mathbf{x}_{k=0}=\mathbf{x}_0\\ \mathbf{y}_k &= \mathbf{C}(\mathbf{m})\mathbf{x}_k+\mathbf{D}(\mathbf{m})\mathbf{u}_k\end{aligned}$$

- $\mathbf{x}$ states (pressures and saturations in each grid block)
- $\mathbf{u}$ inputs (e.g. total well rates, BHPs, well positions)
- $\mathbf{y}$ outputs (predictions) (e.g. BHPs, total well rates, 4D seismics)
- $\mathbf{m}$ parameters (e.g. permeabilities in each grid block)
- k discrete time

Controllability and observability

- **Controllability: can we steer all pressures and saturations by manipulating the inputs?**
- **Observability: can we distinguish all states in the observed output?**
- **Apply rank test on C_N and O_N to evaluate.**

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \mathbf{A}^3 \\ \vdots \end{bmatrix} \mathbf{x}_0 + \begin{bmatrix} \mathbf{B} & & \\ \mathbf{AB} & \mathbf{B} & \\ \mathbf{A}^2\mathbf{B} & \mathbf{AB} & \mathbf{B} \\ & & \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \end{bmatrix}$$

$$\mathbf{C}_N = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{N\text{-}1}\mathbf{B} \end{bmatrix}$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k$$

$$\begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \end{bmatrix} \mathbf{x}_0 + \ldots$$

$$\mathbf{O}_N = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \mathbf{CA}^2 & \cdots & \mathbf{CA}^{N\text{-}1} \end{bmatrix}^T$$

Quantifying controllability and observability

In which areas of the reservoir are the pressures and saturations more controllable and observable?

- **Quantification by using Gramians**
  - Controllability Gramian $\mathbf{P} = \mathbf{C}_N \mathbf{C}_N^{\mathrm{T}}$
  - Observability Gramian $\mathbf{Q} = \mathbf{O}_N^{\mathrm{T}} \mathbf{O}_N$
- **Eigenvalues of product $\mathbf{PQ}$ determine minimum number of states required to describe dynamics**

See Zandvliet et al. (Computational Geoscience 2008)

Quantifying controllability and observability of nonlinear models

Methodology:

1. Linearize around vicinity of current state trajectory
2. Calculate LTV controllability and observability matrices
3. Split matrices into pressure and saturation part
4. Balance controllability and observability to get relevant states
5. Analyze matrices with SVD and visualize

Alternative:

1. Compute empirical controllability Gramians taking snapshots of states (related to POD)
2. Compute empirical observability Gramians taking snapshots of adjoint or dual states
3. Balance controllability and observability to get relevant states
4. Analyze matrices with SVD and visualize

Controllability/observability of pressures

homogeneous permeability (empirical Gramians)

Pressure

Saturation

Controllable pressures

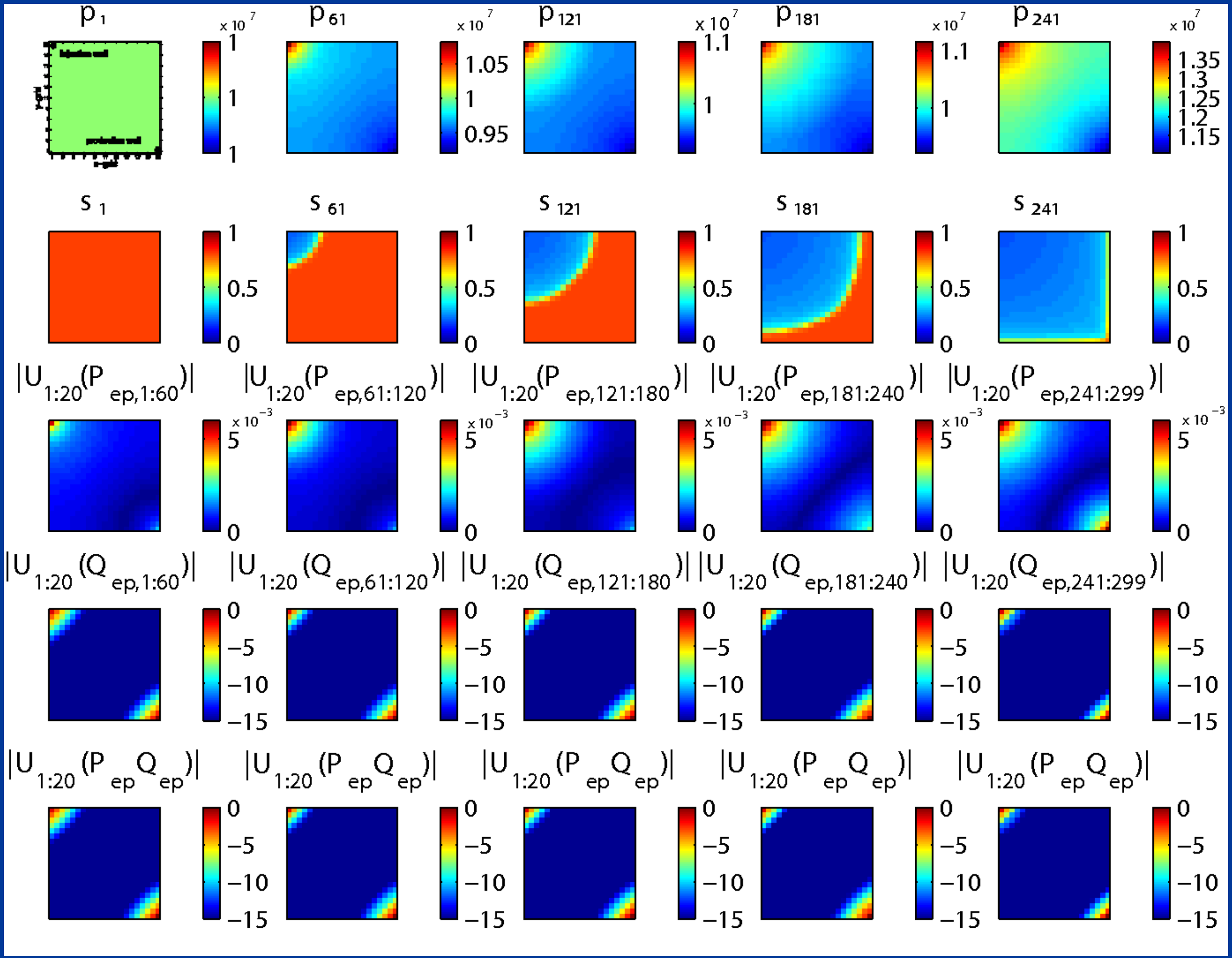
!!! Logarithmic scale

Observable pressures

!!! Logarithmic scale

Relevant pressures

!!! Logarithmic scale



Controllability/observability of saturations

homogeneous permeability (empirical Gramians)

Pressure

Saturation

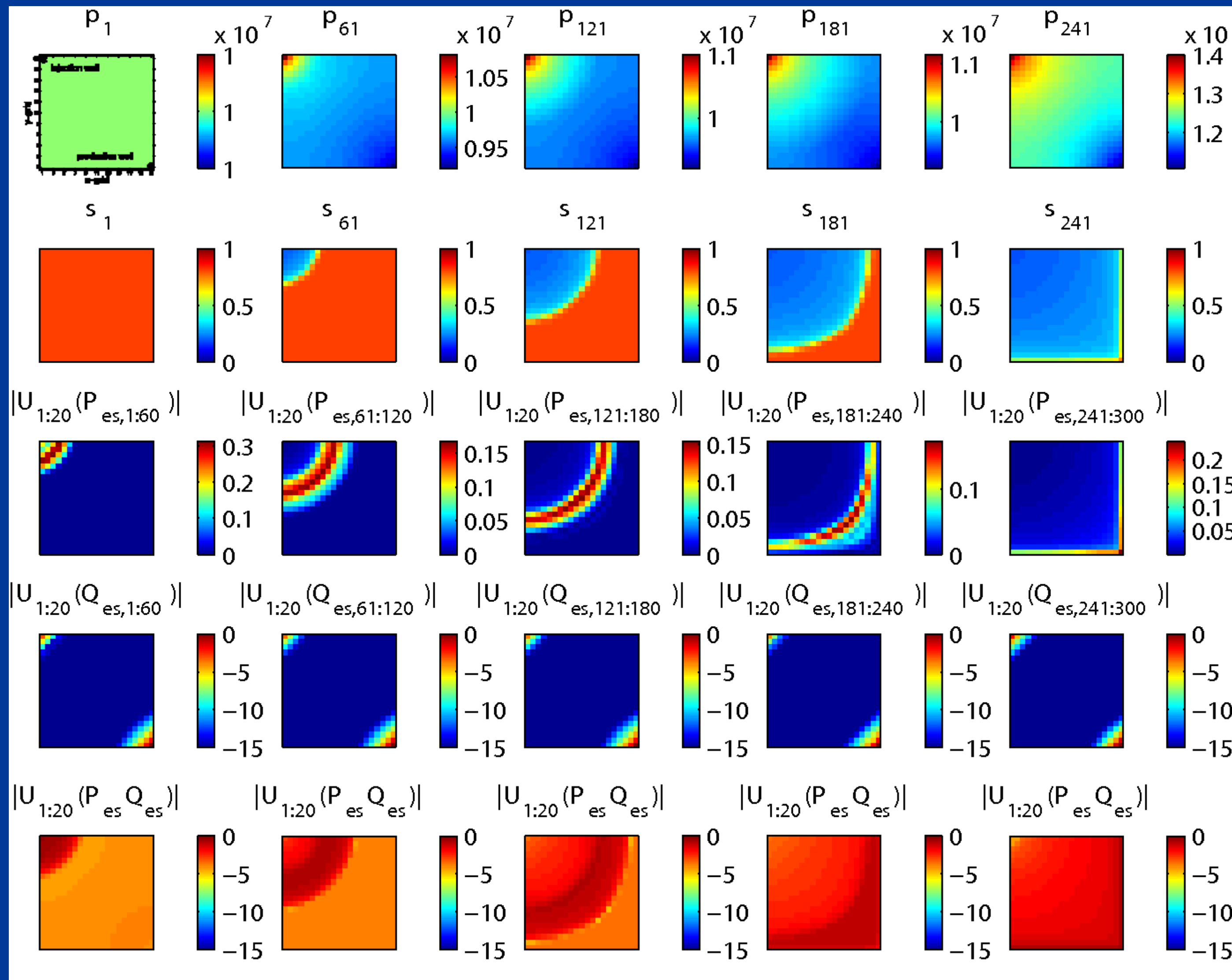
Controllable saturations

Observable saturations

!!! Logarithmic scale

Relevant saturations

!!! Logarithmic scale



Controllability/observability of saturations
heterogeneous permeability (empirical Gramians)

Pressure

Saturation

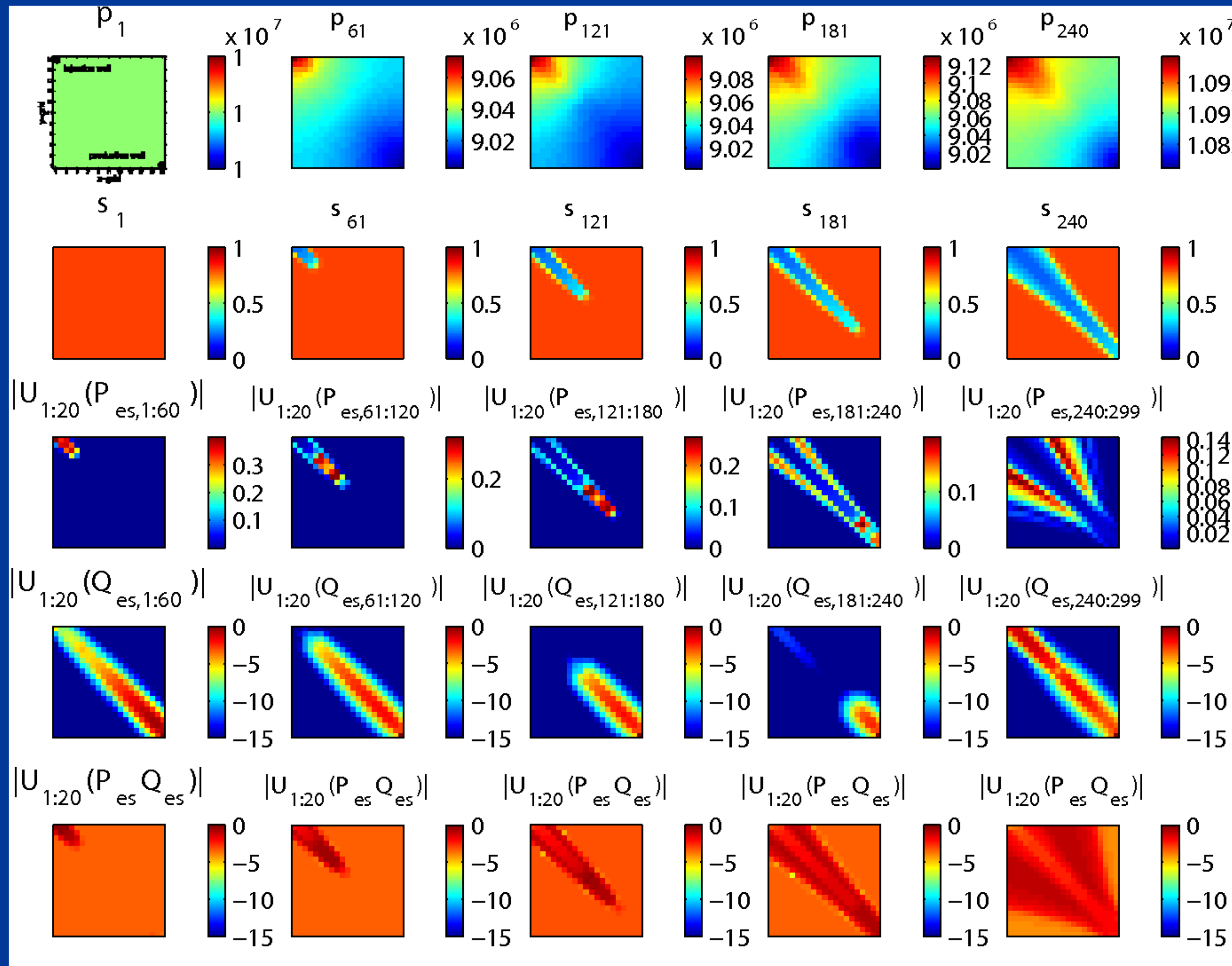
Controllable saturations

Observable saturations

!!! Logarithmic scale

Relevant saturations

!!! Logarithmic scale



Identifiability in reservoir engineering

- **Different parameters lead to same i/o behavior**
- **This means that models are not *identifiable***
- **Problematic because different parameters lead to different predictions of the future**

- **Solutions:**
  - Apply regularization to enforce unique solution
    (Bayesian approach, e.g. Gavalas et al. 1976)
  - Approximate model structure leading to parameterizations with less parameters

Parameterizations in reservoir engineering

- **Zonation, grad zones, adaptive multiscale methods** (Jacquard & Jain 1965, Jahns 1966, Bissell et al. 1994, Grimstad 2003)
- **Subspace algorithm** (Abacioglu et al., 2001)
- **Wavelets** (Sahni and Horne, 2005)
- **PCA/POD of permeability field** (Sarma, 2007)
- **Discrete cosine transform** (Jafarpour and McLaughlin, 2007)
- **Gradual deformation, pilot point method** (Hu, RamaRao)
- **Identifiable parameterization based on sensitivity matrix**

  $(\partial y(m)/\partial m)$ (Shah, 1978)

Identifiability

- Identifiability implies that in the neighborhood of $\mathbf{m}^*$ there are no models with distinct parameters that have the same input-output behavior, for a given input and initial condition

- If this is not the case, we approximate the model structure at $\mathbf{m}^*$ and compute an identifiable parameterization. This leads to a map

$$\boldsymbol{\rho} = \mathbf{T}\mathbf{m} \quad \text{with} \quad \dim \boldsymbol{\rho} \ll \dim \mathbf{m}$$

Identifiable parameterization

- **Cost function**

$$J(\mathbf{m}) = (\mathbf{d}-\mathbf{y})\mathbf{P}_d^{-1}(\mathbf{d}-\mathbf{y}) + (\hat{\mathbf{m}}-\mathbf{m})P_m^{-1}(\hat{\mathbf{m}}-\mathbf{m})$$

- **Identifiable parameterization**
  - Non-Bayesian: $\Gamma_{\mathbf{m}} \frac{\partial \mathbf{y}(\mathbf{m})}{\partial \mathbf{m}} \mathbf{P}_d^{\frac{1}{2}}, \quad \Gamma_{\mathbf{m}} = diag(\mathbf{m}).$

(Van Doren et al., 2009)

  - Bayesian: $\mathbf{P}_{\mathbf{m}}^{\frac{1}{2}} \frac{\partial \mathbf{y}(\mathbf{m})}{\partial \mathbf{m}} \mathbf{P}_d^{\frac{1}{2}}$

(Zhang et al., 2001)

- **SVD gives patterns in parameter space that can be best identified from measurements, and not identified at all**

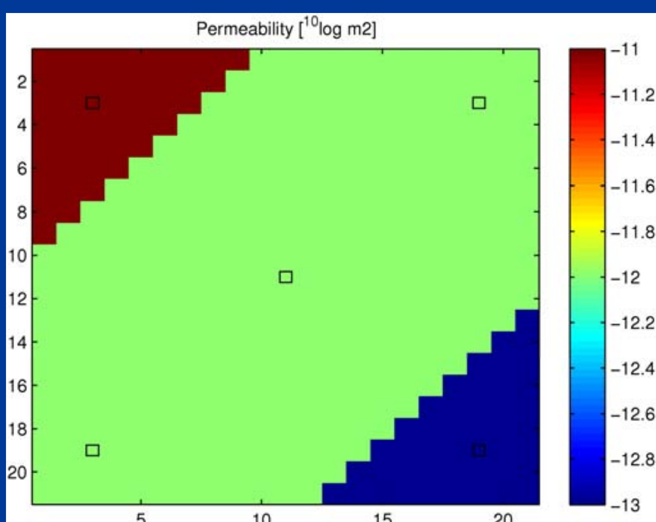
$$\begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix}^T \begin{bmatrix} \mathbf{\Sigma}_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

Identifiable parameterization

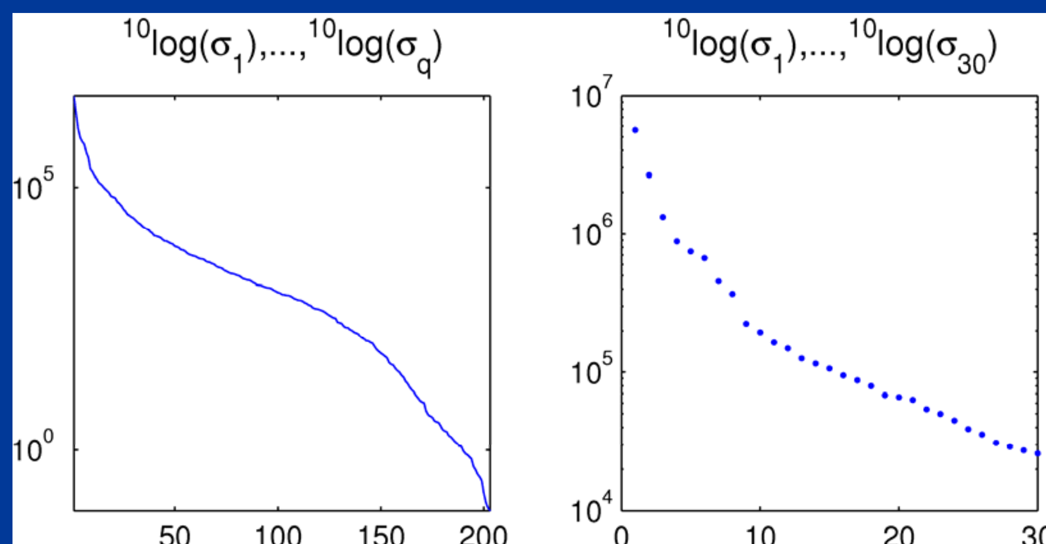
- Insight into information content of measurements
- Insight into which parameters can and cannot be identified
- Most identifiable ‘directions’ can be seen as basis functions to solve estimation problem
- Analysis can be applied to any parameterization, also geological parameterizations

Example permeability two-phase

- Two-phase reservoir model with 21 × 21 grid blocks
- One injector in center with flowrate as input, BHP output
- Four producers in the corners with BHP as input, oil and water flow rates output
- Determine identifiable parameterization of permeability
- Analyse sensitivity matrix with SVD

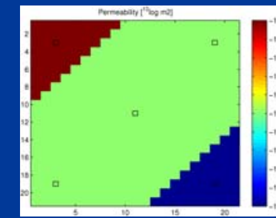


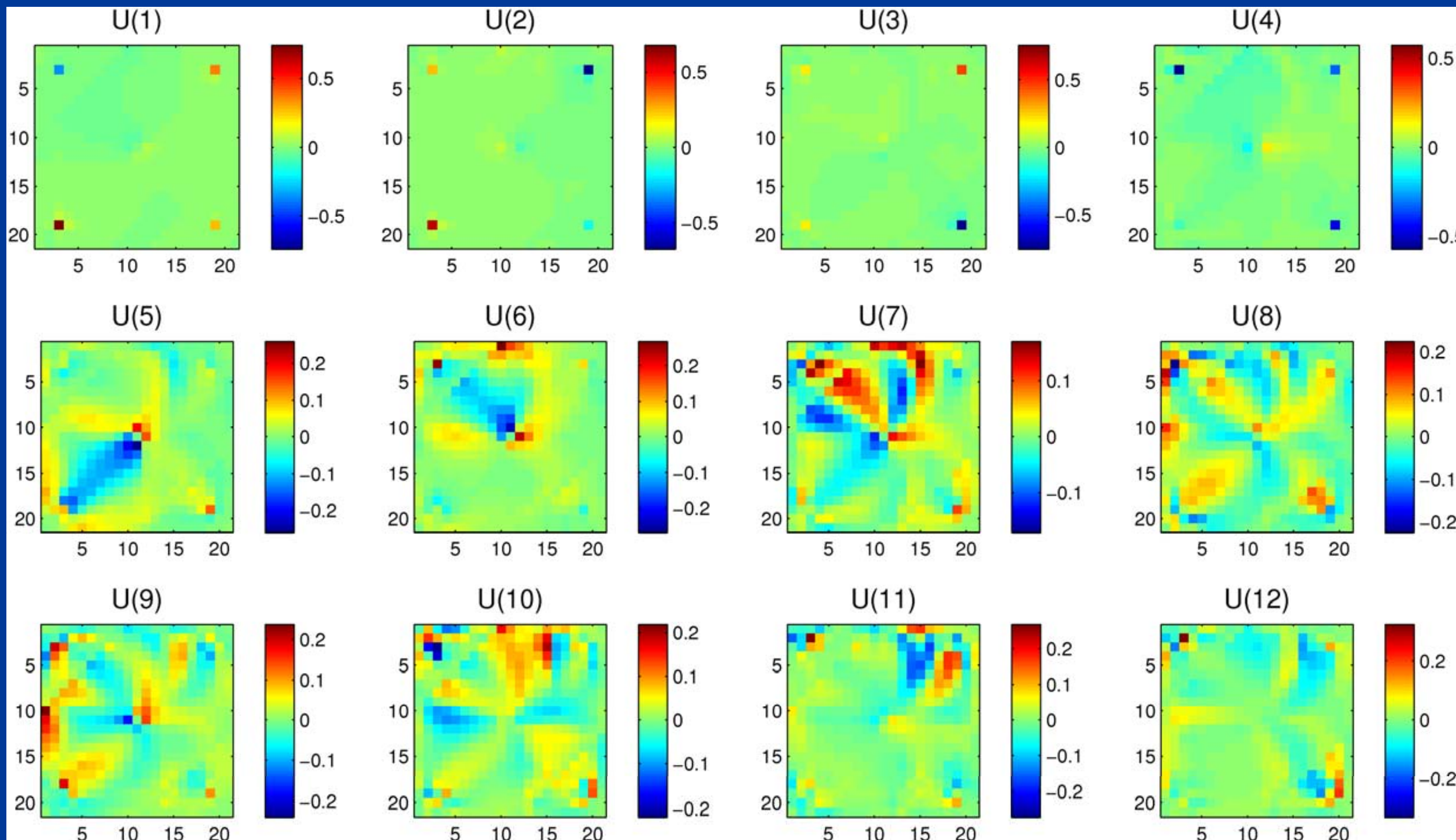
Permeability field



Singular values

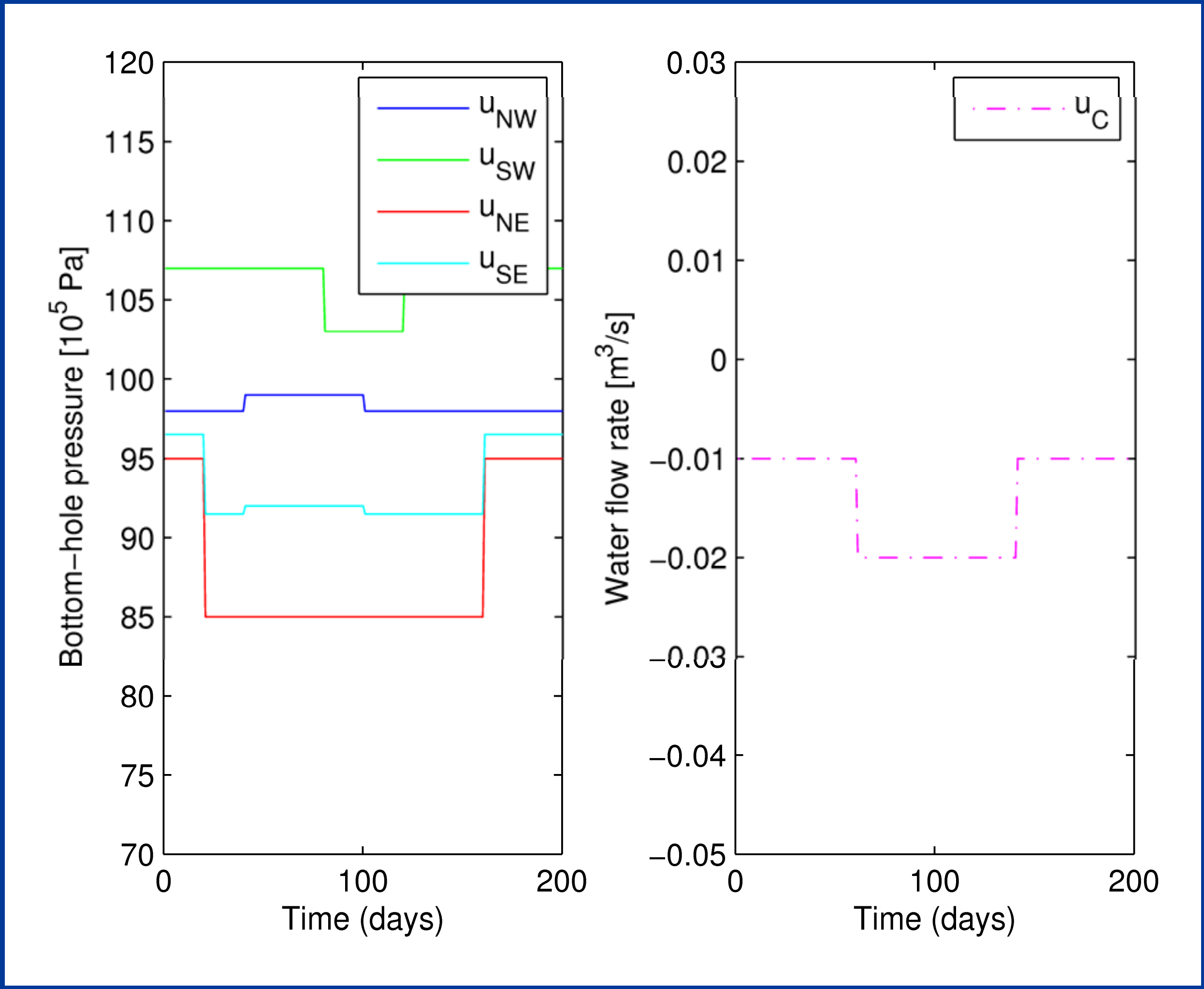
Identifiable parameterization U_1 (two-phase)





- First 12 singular vectors
- Note that patterns are input dependent

Input signals for identifiability example



Link between identifiability, controllability and observability

- **Identifiability (parameters) can be expressed in terms of controllability and observability (states):**

Identifiability = observability x sensitivity x controllability

See Van Doren et al. (SysID 2009)

- ***Maximum* number of parameters that can be identified is**

$$n_m = (n_u + n_y)k + n_u n_y$$

- **Example: 5 inputs, 5 outputs, 4th order --> 65 parameters**

Zandvliet et al. (2008)

Concluding remarks

- Pressures and saturations most observable around the wells
- Pressures most controllable around the wells
- Saturations most controllable around the oil-water front

- Analysis of identifiability can be used to approximate the model structure
- Most identifiable parameterization can be described with reduced number of parameters

- Possibible applications:
  - Parameter estimation
  - Reduced-order modeling
  - Control-relevant upscaling