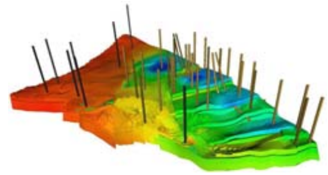

Model-Based Control and Optimization Challenges in Reservoir Engineering

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THE VALUE OF SMARTNESS

ISAPP

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Contents

- Introduction – closed-loop reservoir management
- Model estimation
- Identifiability and model structure approximation
- Simple example
- Extension to data driven approaches: two-stage procedure
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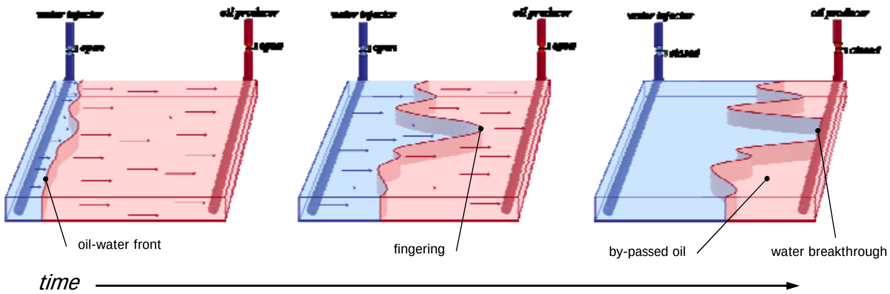
Introduction

Water flooding

- Involves the injection of water through the use of injection wells
- Goal is to increase reservoir pressure and displace oil by water
- Production to be optimized by manipulating injector and producer valves over life-cycle

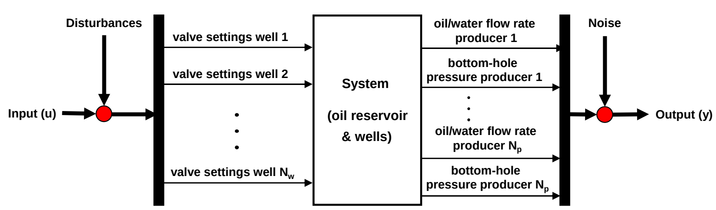


[D.R. Brouwer, 2004]

The Models

System involves the reservoir, wells and sometimes surface facilities

- Inputs: control valve settings of the wells (injectors and producers)
  - Smart wells: multiple (subsurface) valves
- Outputs: (fractional) flow rates and/or bottomhole pressures
  - Smart wells: multiple (subsurface) measurement devices



Governing differential equations

isothermal two-phase (oil-water) flow

Mass balance:

$$\nabla(\rho_i u_i) + \frac{\partial}{\partial t}(\phi \rho_i S_i) = 0 \qquad i = \{o, w\}$$

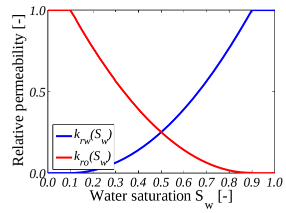
Momentum (Darcy's law):

$$u_i = -k\frac{k_{ri}}{\mu_i}\nabla p_i \qquad i = \{o, w\}$$

Variables: p_o, p_w, S_o, S_w

Saturations satisfy: $S_o + S_w = 1$

Simplifying assumptions, a.o.: $p_o = p_w$



Discretization in space and time

State space model:

$$\begin{aligned} V(x_t)\dot{x}_t &= T(x_t)x_t + q_t; \qquad x_0 \\ y_t &= h(x_t) \end{aligned}$$

$$\begin{aligned} y^T &= [p_{well}^T \; q_{well,o}^T \; q_{well,w}^T] \\ x^T &= [p_o^T \; S_w^T] \end{aligned}$$

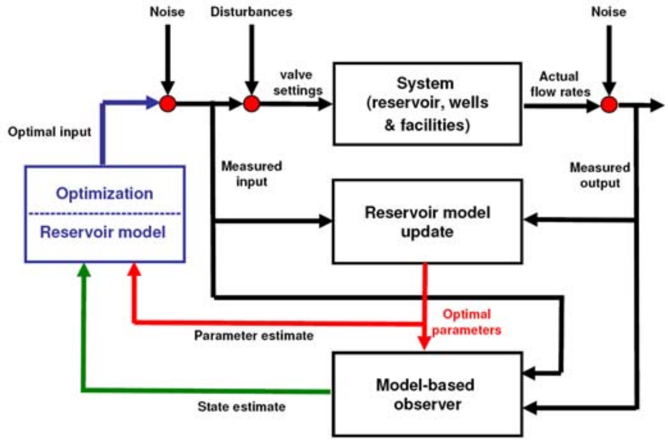
After discretization in space (and time):

$$g(x_{k+1}, x_k, u_k, \theta) = 0 \qquad dim(x) \approx 10^4 - 10^6$$

$$y_k = h(x_k)$$

and θ typically the permeabilities in each grid block

Closed-loop Reservoir Management



Closed-loop strategy:

- Natural and standard strategy in process technology (feedback strategy) and elsewhere
- Allows to reduce sensitivity with respect to uncertainties in the data (disturbances), and uncertainties in the model (robustness)
- Through the on-line use of the model, generally favouring models of limited complexity
- For models with linear dynamics, the understanding of the control-relevant parts of the models is well developed

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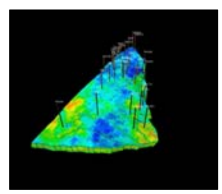
Model estimation



Is there a problem with the online use of highly complex first-principles models?

Not if:

- Computation time is not a problem, and
- They exactly describe reality

In case of model uncertainty, one would like to use measurement data to retrieve information.

Model estimation

Estimating models from experimental data can be done in different ways:

1. Estimating the dynamics between input and output in a generic (black box) model structure, using as few parameters as possible (to be determined by the data)

   Relatively "easy" for linear dynamics;

   Harder for nonlinear reservoirs
   (nonlinear behaviour dependent on front-location, single batch process, experimental limitations)

Model estimation

Estimating models from experimental data can be done in different ways:

2. Estimating the parameters (permeabilities) in a physics-based model structure possible using prior information on their numerical values

   number of parameters has to be observed → identifiability

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Identifiability

- Consider nonlinear model structure $\hat{\mathbf{y}} = \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0)$ with $\hat{\mathbf{y}}$ being a prediction of the measured outputs $\mathbf{y} := [y_1^T \cdots y_N^T]^T$

The model strucure is

- Locally identifiable in $\boldsymbol{\theta}_m$ for given $\mathbf{u}$ and $\mathbf{x}_0$ if in neighbourhood of $\boldsymbol{\theta}_m$:

$$\{\mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_1; \mathbf{x}_0) = \mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_2; \mathbf{x}_0)\} \Rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$$

[Grewal and Glover 1976]

Global properties are generally very hard to analyze (nonlinear)

- Notion of *identifiability* is instrumental in analyzing model structure properties
- It determines whether it is feasible at all to relate unique values to the physical parameter variables, on the basis of measured data

Testing local identifiability in model estimation

- Consider quadratic identification criterion based on prediction errors

$$V(\boldsymbol{\theta}) := \frac{1}{2}\epsilon(\boldsymbol{\theta})^T \mathbf{P}_v^{-1}\epsilon(\boldsymbol{\theta}), \quad \epsilon(\boldsymbol{\theta}) = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0),$$

- Hessian given by

$$\frac{\partial^2 V(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} = \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T + \mathbf{S}$$

- Local identifiability test in $\hat{\boldsymbol{\theta}} = \arg\min V(\boldsymbol{\theta})$: Hessian > 0

- With quadratic approximation of cost function around $\hat{\boldsymbol{\theta}}$:
  Hessian given by $\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T$

Testing local identifiability in identification

- Rank test on Hessian through SVD

$$\left. \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

- If $\boldsymbol{\Sigma}_2 = \mathbf{0}$ then lack of local identifiability

- SVD can be used to reparameterize the model structure through

$$\boldsymbol{\theta} = \mathbf{U}_1 \boldsymbol{\rho}, \quad \dim(\boldsymbol{\rho}) << \dim(\boldsymbol{\theta})$$

  in order to achieve local identifiability in $\boldsymbol{\rho}$

- Columns of $\mathbf{U}_1$ are basis functions of the identifiable parameter space

Testing local identifiability in identification

$$\left.\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

- What if $\boldsymbol{\Sigma}_2 \neq \mathbf{0}$ but contains (many) small singular values ?

  No lack of identifiability, but possibly very poor variance properties

- Identifiability mostly considered in a yes/no setting: qualitative rather than quantitative [Bellman and Åström (1970), Grewal and Glover (1976)]

- Approach: *quantitative* analysis of appropriate parameter space, maintaining physical parameter interpretation

Model structure approximation

- How to reduce the model structure in terms of its ***parameter space?***
  (different from "classical" model reduction, in which the model dynamics of a single model is reduced)

- **Objective:** obtain a physical parametrization (model structure) in which the parameters can be reliably estimated / validated from data.

Approximating the identifiable parameter space

Asymptotic variance analysis: $cov(\hat{\theta}) = J^{-1} = \left(\mathbf{E}\left[\frac{\partial^2 V(\theta)}{\partial \theta^2} \Big|_{\hat{\theta}} \right] \right)^{-1}$

with J = Fisher Information Matrix.

- Sample estimate of parameter variance, on the basis of $V(\theta)$:

$$cov(\hat{\theta}) = \begin{cases} \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \Sigma_1^{-2} & 0 \\ 0 & \Sigma_2^{-2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} & \text{for } \Sigma_2 > 0 \\ \infty & \text{for } \Sigma_2 = 0 \end{cases}$$

$$cov(U_1\hat{\rho}) = \mathbf{U}_1 \Sigma_1^{-2} \mathbf{U}_1^T$$

$$cov(\hat{\theta}) > cov(U_1\hat{\rho}) \quad \text{if } \Sigma_2 > 0$$

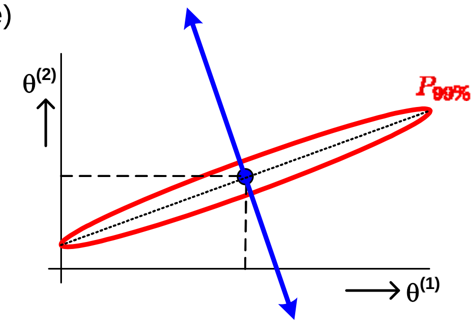
Approximating the identifiable parameter space

$$cov(\hat{\theta}) > cov(U_1\hat{\rho}) \quad \text{if } \Sigma_2 > 0$$

- Discarding singular values that are small, reduces the variance of the resulting parameter estimate
- Particularly important in situations of (very) large numbers of small s.v.'s
- Model structure approximation (local)
- Quantified notion of identifiability – related to parameter variance

Approximating the identifiable parameter space

- Interpretation:
  Remove the parameter directions that are poorly identifiable (have large variance)



- This is different from removing the (separate) parameters for which the value 0 lies within the confidence bound [Hjalmarsson, 2005]

A Bayesian approach

How does this work out in estimation through Kalman-filter type algorithms (like EnKF)?

- State vector is augmented with unknown parameters and estimated simultaneously in a recursive algorithm

These algorithms are based on a Bayesian reasoning

A Bayesian approach

- Often applied method for dealing with overdetermination in parameter space:
- Incorporate prior knowledge term (regularization) in cost function

$$V_p(\theta) := V(\theta) + \frac{1}{2}(\theta - \theta_p)\mathbf{P}_{\theta_p}^{-1}(\theta - \theta_p)$$

where θ_p is the prior parameter vector (with covariance $\mathbf{P}_{\theta_p}$).

- When model output approximated with first-order Taylor expansion, then Hessian is

$$\frac{\partial^2 V_p(\theta)}{\partial \theta^2} = \frac{\partial \mathbf{h}(\theta)^T}{\partial \theta}\mathbf{P}_v^{-1}\left(\frac{\partial \mathbf{h}(\theta)^T}{\partial \theta}\right)^T + \mathbf{P}_{\theta_p}^{-1}$$

- “Always” identifiable, since $\mathbf{P}_{\theta_p}$ full rank by construction!!

A Bayesian approach

Implications

$$V_p(\theta) := \underbrace{V(\theta)}_{\text{data}} + \underbrace{\frac{1}{2}(\theta - \theta_p)\mathbf{P}_{\theta_p}^{-1}(\theta - \theta_p)}_{\text{priors}}$$

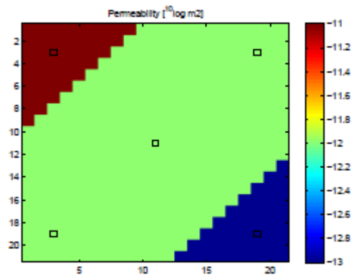
- Bayesian methods seem not to suffer from identifiability problems…….
- This includes all (extended) Kalman filter type algorithms. Where parameters are recursively estimated by augmenting the states
- Unique parameter estimates usually result, but
- In the parameter subspace that is poorly identifiable, estimated parameters will be heavily dominated by the prior information.
- Analysis of $V(\theta)$ can show identifiable directions (locally)

Simple reservoir example

[Van Doren, 2010]

2D two-phase example

(top view)

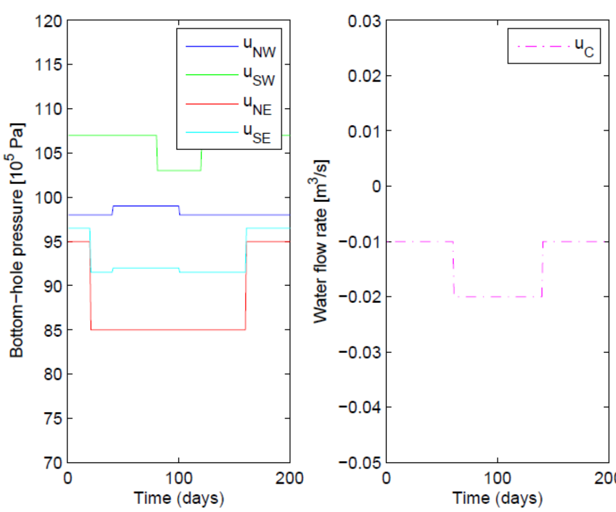


21 x 21 grid block permeabilities
5 wells; 3 permeability strokes

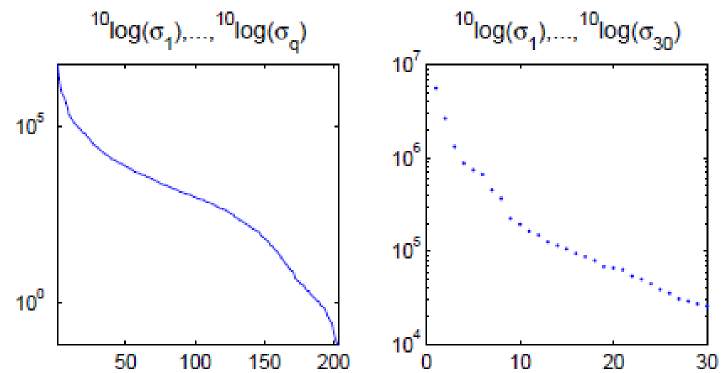
1 injector (centre)
4 producers (corners)

5 inputs: 1 injector flow-rate, and 4 bottom hole pressures
8 outputs: producer flow rates (water and oil)

Input excitation signals



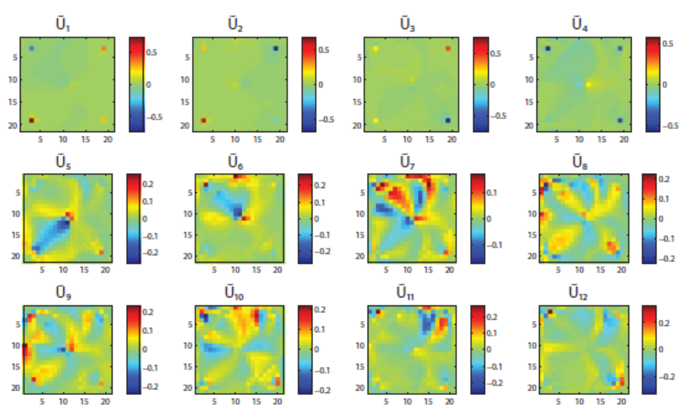
Simple reservoir example



All singular values (left) and first 30 (right)

Simple reservoir example

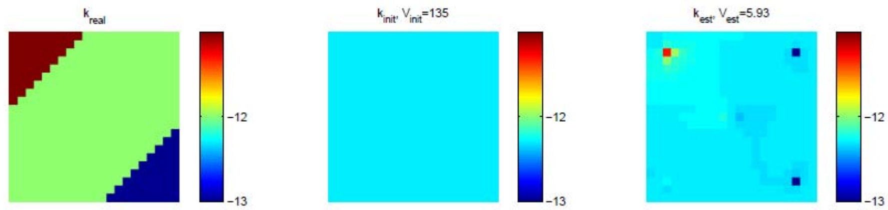
Singular vectors can be projected on the grid:



First 12 singular vectors

Simple reservoir example

Using the reduced parameter space –iteratively- in estimation:

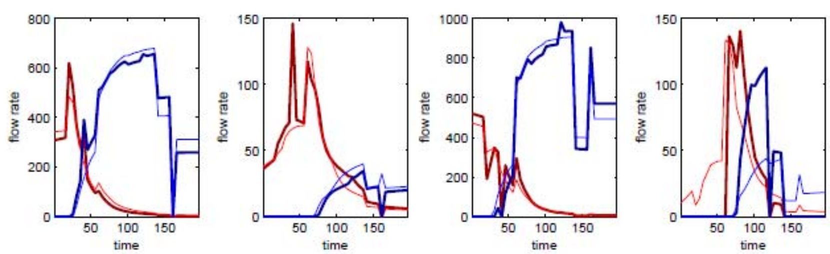


exact field | initial estimate (local point) | final estimate

Observation:

Only grid block permeabilites around well are identifiable.

Simple reservoir example



Simulated production of estimated model (thin lines) of water (blue) and oil (red) in the four producer wells

See also Vasco et al. (1997)

Simple reservoir example

- Model estimation is done in an iterative way:
  - Choosing a local identifiable parametrization
  - Estimating the parameters
  - Repeating the procedure until convergence of the cost function
- During the iterations, the quadratic cost function is reduced from 135 to 5.93.
- "Poor" model seems to be good enough for prediction of production.
- No prior info on permeability structure has been used.

General remarks

- Observation and control in the wells
  - Reservoir models will typically be poorly observable and/or poorly controllable
  - Real (local) input-output dynamics is of limited order
- Parameter estimation:
  - Physical parameters (permeabilities) determine predictive quality but one parameter per grid block leads to excessive over-parametrization (not to be validated)

Consequences of model uncertainties:

- Model predictions over large horizons have limited reliability

Suggestions to "deal" with this:

- Apply robust optimization based on an ensemble of model realizations, that reflects the model uncertainties [van Essen et al, SPE J, 2009]
- Combine long term (life cycle) performance measure with short term production objectives (hierarchical / multiobjective optimization) [van Essen et al, SPE J, 2011]
- Use data-driven models for short time horizons in combination with reservoir models for long-term strategies (separation of time scales) [van Essen, Rezapour, Van den Hof and Jansen, CDC, 2010]

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Extension: a two stage approach

Reasoning

• Optimization on the basis of nonlinear reservoir models suffers from model uncertanties

• Optimization on the basis of estimated models suffers from a lack of predictive capabilities beyond the –local- measurement interval

→ Combine the two

Extension: a two stage approach [Van Essen et al., 2010]

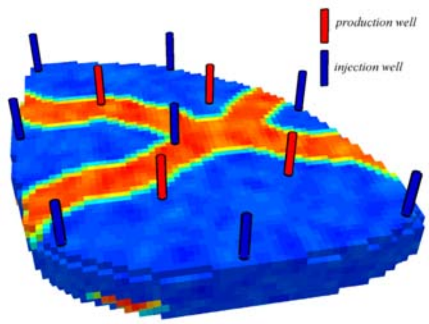
• Combine data-driven estimation of local (linear) models with tracking of long-term production targets

• Use nonlinear reservoir model (with estimated/prior chosen) grid block parameters for an (slow) ”outer loop” optimization target strategy

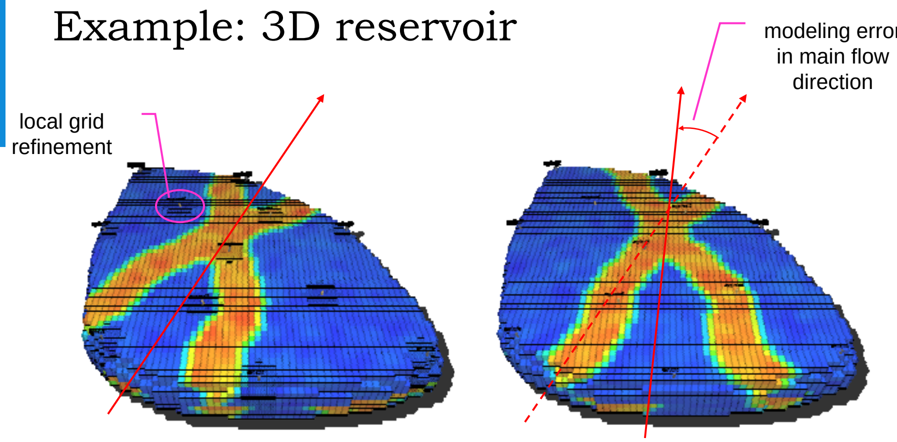
• Base short term operational decisions to follow the target strategy on a locally identified linear model, using simple black box estimation techniques, on the basis of deliberately perturbed input settings

Example: 3D reservoir

- 8 injection / 4 production wells
- High permeability channels
- Life-cycle approx. 11.5 year
- Goal: maximize NPV
- Inputs
  - Water injection rates
  - Bottom-hole pressures producers
- Outputs
  - Liquid rates producers



Example: 3D reservoir



'truth' model

time step size: 0.25 days

8 injection wells, 4 production wells

reservoir model

time step size: 30 days

Modeling error due to geological uncertainty & undermodeling of fast, local dynamics

Example: 3D reservoir

3 production strategies

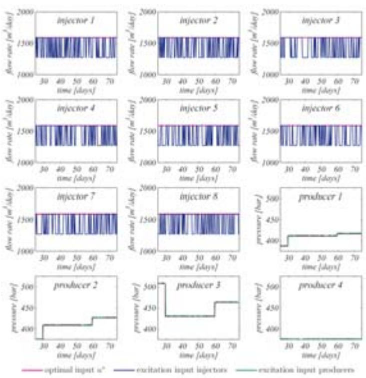
1. Reactive control
   - Maximal injection rates/minimal bottom-hole pressures
   - Shut-in wells when watercut >0.90
2. Open-loop life-cycle optimization
   - Optimize inputs based on reservoir model
   - Apply to ‘truth’ model
3. Combined dynamic optimization & MPC control
   - Life-cycle optimization on reservoir model to obtain references
   - Excitation on ‘truth’ model to identify low-order model
   - MPC on ‘truth’ model to track references

Example: Identification Experiment

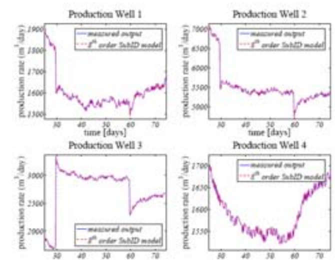
- MIMO
  - 12 inputs: 8 injection rates & 4 producer BHP’s
  - 4 outputs: 4 producer liquid rates
- First 75 days of production
  - First 25 days omitted: initial reservoir conditions
  - Approximately 5x largest time constant
- RBS signals
  - Clock period: 3x sample time (0.25 day)
  - u^* as mean, unless limited constraints
    - Maintain good (economic) performance during experiment
  - Amplitude determined using reservoir model
    - relative contribution of injection rates and BHP’s on outputs equal

Example: Identification Experiment

Input excitation for identification

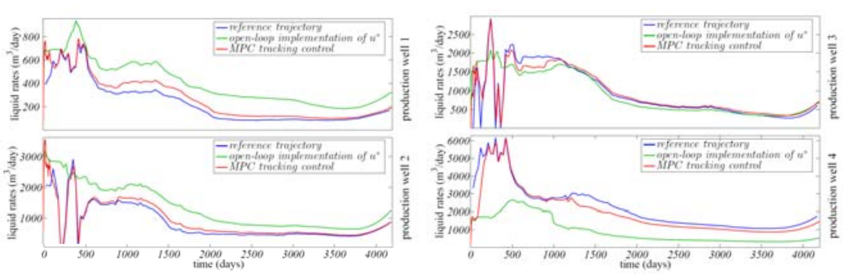


sub-space identification



Simulation fit of 8th order identified model

Example: Results



	NPV	%
Case 1: Reactive Control	550 M$	-
Case 2: Open-loop Optimization	558 M$	+1.5%
Case 3: Two-level Control	594 M$	+8.0%
Maximum based on reservoir model	596 M$	+8.4%

Remarks

- Base short term actions on short term models
- They are (locally) more reliable and can be validated
- But are limited in their long-term predictions

- Locally identified black-box linear models, could be replaced by
  - limited complexity (identifiable, nonlinear) physical models
  - simple nonlinear black box extensions (LPV)

- Model uncertainties need to be quantified, and incorporated in the (long term) optimization strategy [Van Essen et al, SPE J, 2009]

Summary

- The development / handling of appropriate models is a key issue in closed-loop reservoir management.
- In comparison with process technology, limiting features include the nonlinear and batch-type of the process, together with highly uncertain process knowledge
- Large-scale reservoir models are not identifiable from production data, nor can they be validated
- A proper balance between data-induced and priors-induced modelling should be achieved when estimating models, focussing at the control-relevant dynamics
- Model uncertainty should be specified and incorporated in the optimization at all levels; a separation of time scales is one of the options to support this

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