

Identification in interconnected systems – modeling, structural aspects and MATLAB toolbox

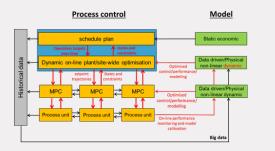
Paul Van den Hof

University of Stuttgart 14 November 2023 www.sysdynet.eu www.pvandenhof.nl p.m.j.vandenhof@tue.nl



Introduction – dynamic networks

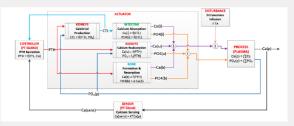
Decentralized process control



Smart power grid



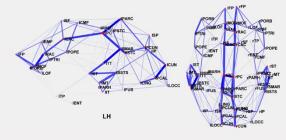
Physiological models



Christie, Achenie and Ogunnaike (2014)

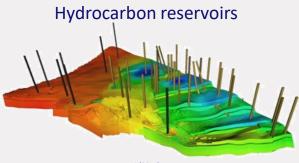
Complex machines





Brain network

P. Hagmann et al. (2008)



Mansoori (2014)



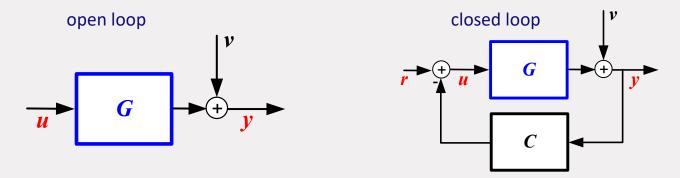
Introduction

Overall trend:

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control, optimization, diagnostics
- Data is "everywhere", AI/machine learning tools
- Model-based operations require accurate/relevant models
- → Learning models/actions from data (including physical insights when available)

Introduction

The classical (multivariable) data-driven modeling problems^[1]:



Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

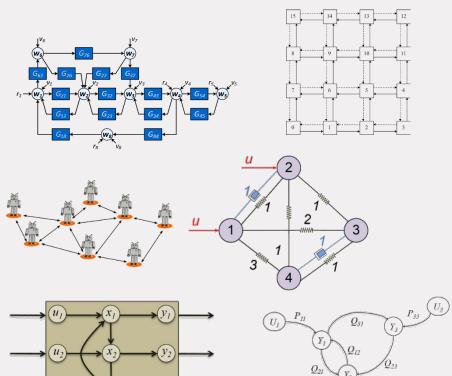
In interconnected systems (networks) the **structure / topology** becomes important to include

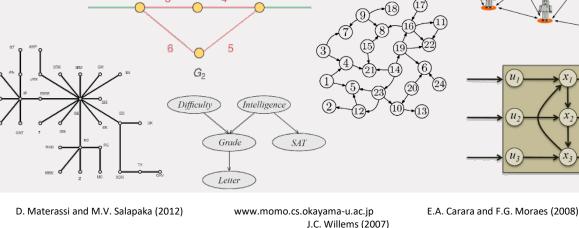
Network models

- dynamic elements with cause-effect
- handling feedback loops (cycles)
- centered around measured signals

G

allow disturbances and probing signals





D. Koller and N. Friedman (2009)

P.M.J. Va X.Cheng (2019)

P.E. Paré et al (2013)

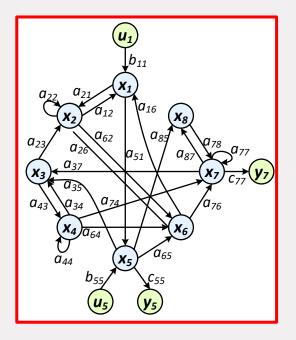
P.M.J. Van den Hof et al (2013)) E. Yeung et al (2010)



R.N. Mantegna (1999)

5

Network models



State space representation

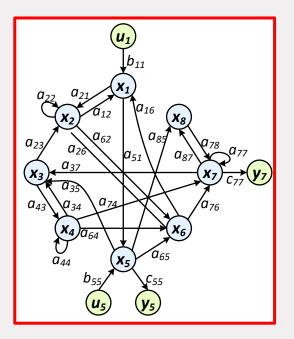
 $egin{array}{rcl} x(k+1)&=&Ax(k)+Bu(k)\ y(k)&=&Cx(k)+Du(k) \end{array}$

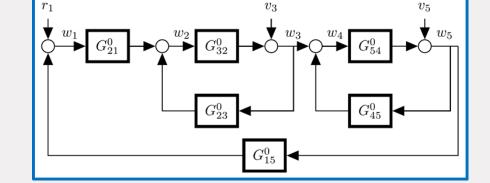
- States as **nodes** in a (directed) graph
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in links
- Ultimate break-down of system structure
- Actuation (u) and sensing (y) reflected by separate links

For data-driven modeling problems:

Lump unmeasured states in dynamic modules

Network models





Module representation ^[2]

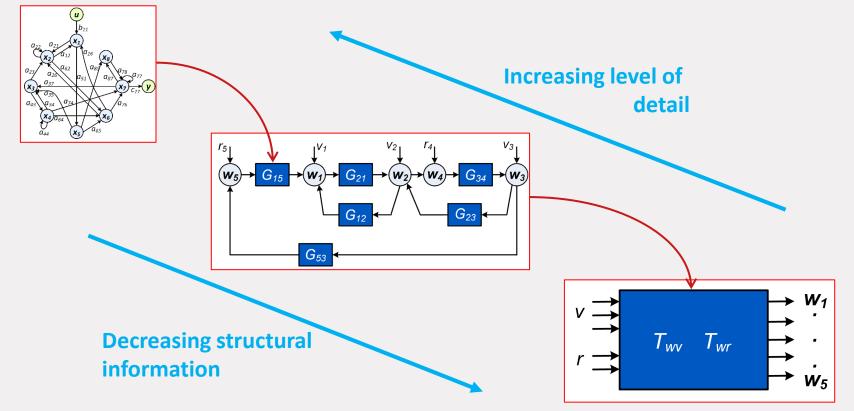
State space representation ^[1]

[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,...

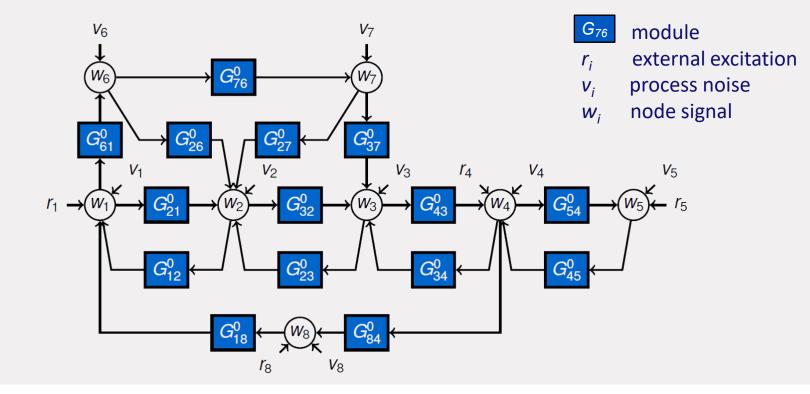
[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,...

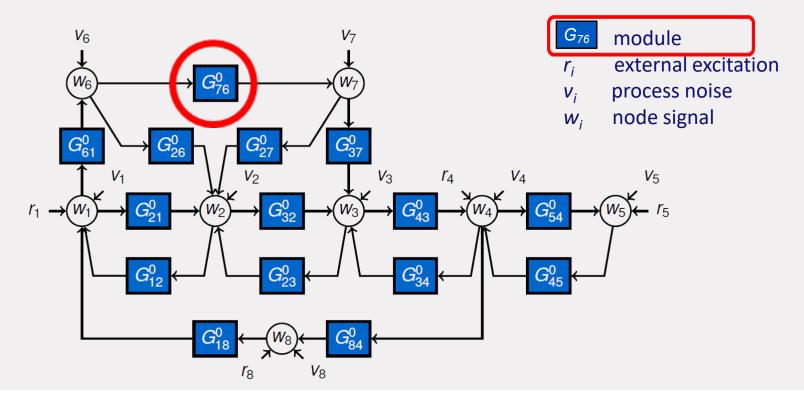


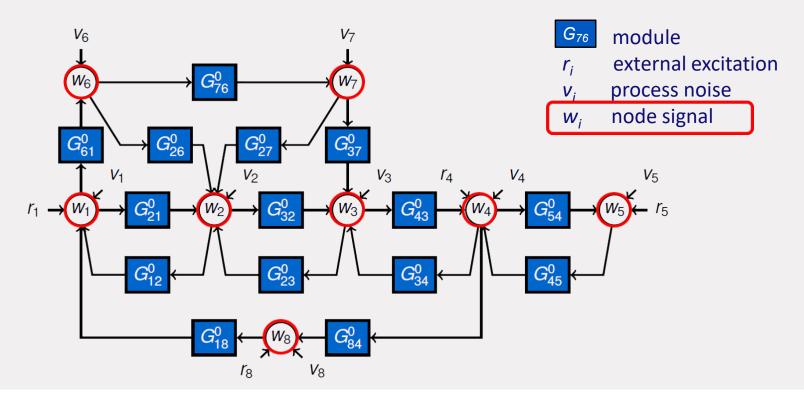
Dynamic network models - zooming

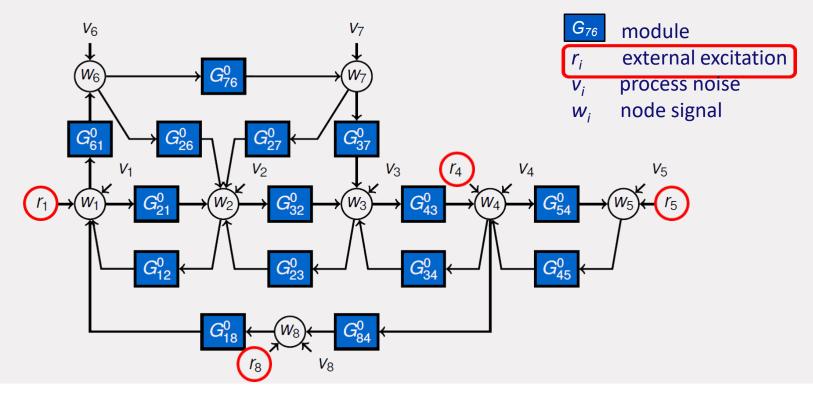


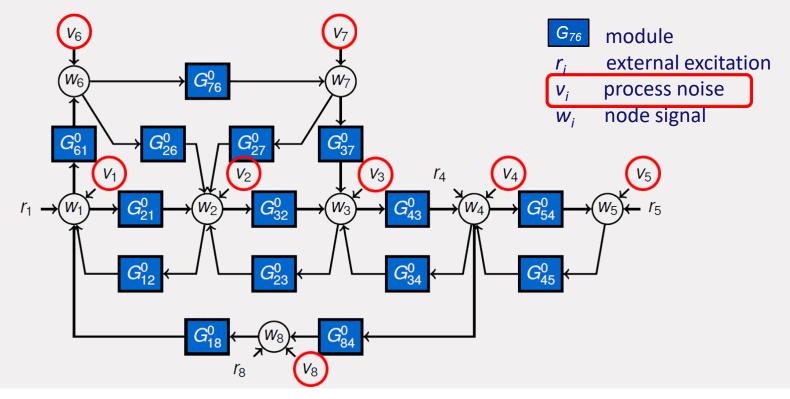












Collecting all equations:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G_{12}^0 & \cdots & G_{1L}^0 \\ G_{21}^0 & 0 & \cdots & G_{2L}^0 \\ \vdots & \ddots & \ddots & \vdots \\ G_{L1}^0 & G_{L2}^0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

$$\underbrace{ w_1 \\ w_2 \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

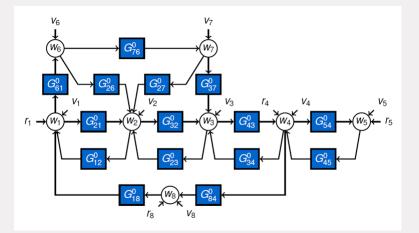
$$\underbrace{ w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

• Typically
$$R^0$$
 is just a (static) selection matrix, indicating which nodes have an excitation signal

• The topology of the network is encoded in the structure (non-zero entries) of G^0 .

 $\dot{u(t)}$

• *r* and *e* are called external signals.



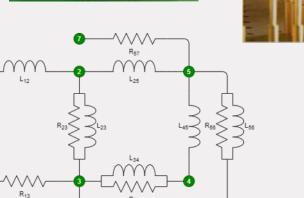
Measured time series: $\{w_i(t)\}_{i=1,\dots L}; \{r_j(t)\}_{j=1,\dots K}$ Many challenging data-driven modeling and diagnostics challenges appear

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Diagnostics and fault detection
- Distributed identification
- User prior knowledge of modules
- Scalable algorithms

Application: Printed Circuit Board (PCB) Testing

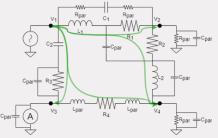








Source: Altium

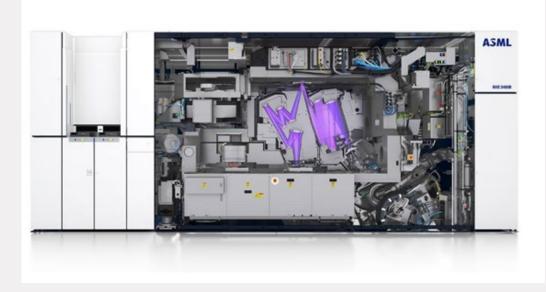


Detection of

- component failures
- parasitic effects

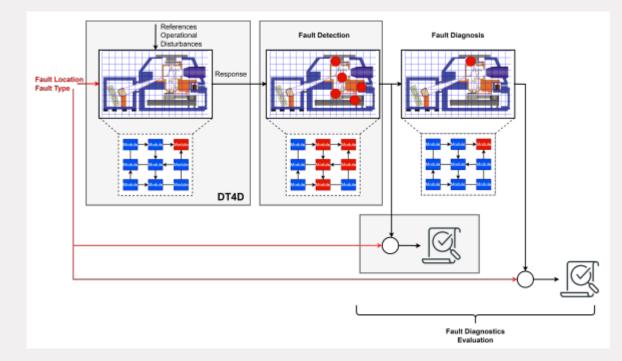


Application: diagnostics in lithography waferscanners



- 200M€ machine
- Highly complex
 machine dynamics
- Many interconnected subsystems
- Need for very fast recovery from faults
- Tools for automated diagnostics

Application: diagnostics in lithography waferscanners

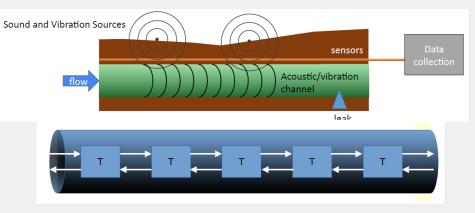


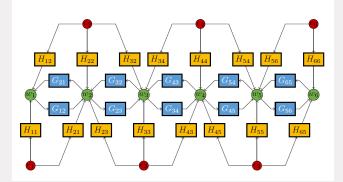
- Digital twin, for detection and diagnosis
- Exploit interconnection structure



Leak detection in gas pipelines with acoustic sensors







- Use operational data to detect changes in network model dynamics
- Map model changes to physical causes



[1] R. van Esch, S. Shi, A. Bernas, S. Zinger, A. Aldenkamp, PVdH, Computers in Biology and Medicine, Vol. 127, December 2020

Neurodynamic effect of listening to Mozart music

Identifying changes in network connections in the brain, after intensely listening for one week (Sonate K448), based on fMRI data



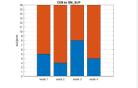
OMN POS to EE

(d) Connection from the lateral sensori-motor network to the superior temporal gyrus.

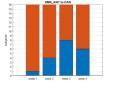
(b) Connection from the sensori-motor superior network to the fronto-parietal right network.



(e) Connection from the dorsal attention network to the angular gyrus.



(c) Connection from the central executive network to the sensori-motor superior network.



(f) Connection from the anterior default mode network to the dorsal attention network.

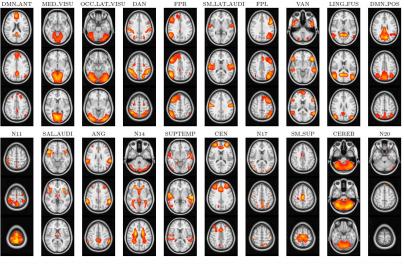
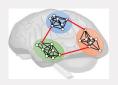


Figure 3: Spatial maps of the 20 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.



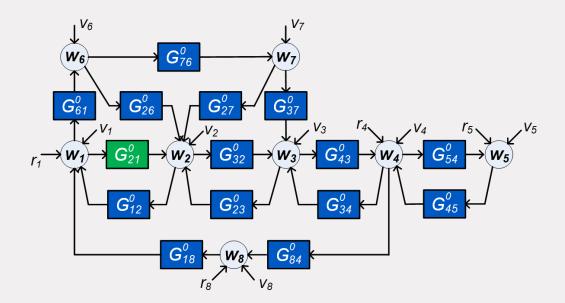


DMN POS

VAN



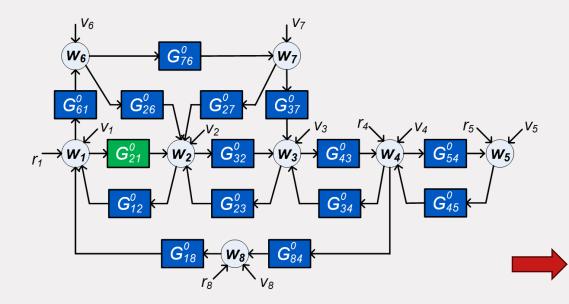
known topology



For a network with **known topology**:

- Identify G⁰₂₁ on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?





Different types of methods:

Indirect methods:

• Rely on mappings $r \rightarrow w$ and on sufficient excitation signals r

Direct methods:

• Rely on mappings $w \to w$ and use excitation from both r and v signals



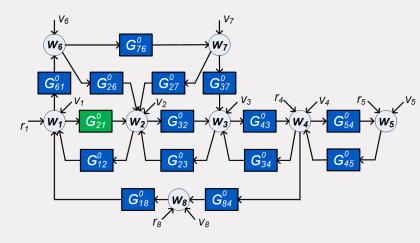
Local direct method:

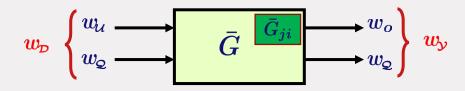
(consistency and minimum variance properties)

Select a subnetwork:

- Predicted outputs: $w_{\mathcal{Y}}$
- Predictor inputs: $w_{\!\mathcal{D}}$

such that prediction error minimization leads to an accurate estimate of G_{21}^0





Note: same node signals can appear in input and output

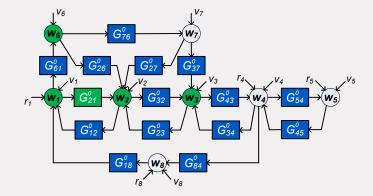


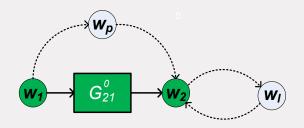
$$w_{\mathcal{D}} \left\{ \begin{array}{c} w_{\mathcal{U}} & \longrightarrow & \bar{G} \\ w_{\mathcal{Q}} & \longrightarrow & \bar{G} \end{array} \right\} \left[\begin{array}{c} \bar{G}_{ji} & \longrightarrow & w_o \\ & & & & & \\ & & & & & \\ \end{array} \right\} \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & & \\ & & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & & \\ & & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[\begin{array}{c} w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[w_{\mathcal{D}} & & \\ \end{array} \right] \left[\begin{array}[w_{\mathcal{D}} & & & \\ \end{array} \right] \left[w_{\mathcal{D}} & & \\ \end{array} \\[w_{\mathcal{D}} & & \\ \end{array} \left[w_{\mathcal{D}}$$

Conditions for arriving at an accurate (consistent) model:

- 1. Module invariance: $\bar{G}_{ji} = G_{ji}^0$ when removing discarded nodes (immersion)
- 2. Handling of confounding variables
- 3. Data-informativity
- 4. Technical condition on presence of delays

Single module identification - module invariance





A sufficient condition for module invariance:

All parallel paths, and loops around the output, should be "blocked" by a measured node that is present in w_D

All other signals can be removed/immersed from the network^[2]

Alternative graph-based formulation in terms of disconnecting sets in [3]

[1] Dankers et al., TAC 2016[3] Shi et al., Automatica 2022

[2] Generalizations available in Linder&Enqvist (2017), Weerts et al, (2020)



Single module identification - confounding variables

Confounding variable ^{[1][2]}:

Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.

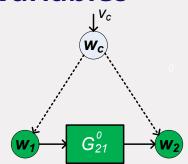
In networks they can appear in two different ways:

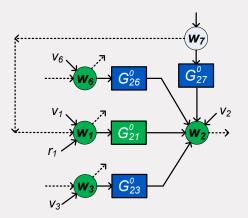
Direct:

• If disturbances on inputs and outputs are correlated.

Indirect:

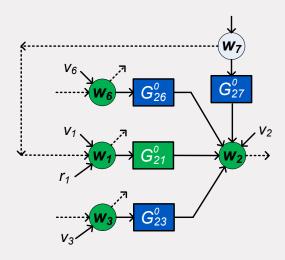
• If non-measured in-neighbors of an output affect signals in the inputs.





Confounding variables

• **Direct** confounding variables



e.g., v_1 is correlated with v_2

In identification we know how to handle correlated disturbances: we model them!

Solution:

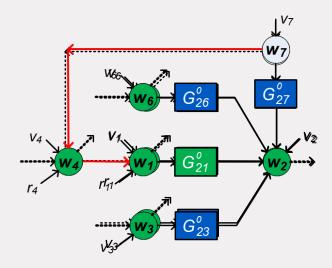
Include w_1 as output and use a multivariate noise model

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{ oldsymbol{w_1}, w_2 \}$$



Confounding variables

• Indirect confounding variable:



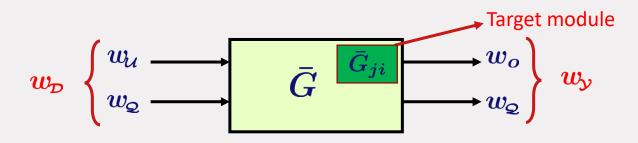
Non-measurable w_7 is a confounding variable

Two possible solutions:

- 1. Include $w_4 \longrightarrow$ add predictor input $w_{\mathcal{D}} = \{w_1, w_3, w_4, w_6\} \quad w_{\mathcal{Y}} = \{w_2\}$ 2. Predict w_1 too \longrightarrow add predictor output $w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$
- There are degrees of freedom in choosing the predictor model

Direct method

General setup:



Different algorithms for arriving at predictor models:

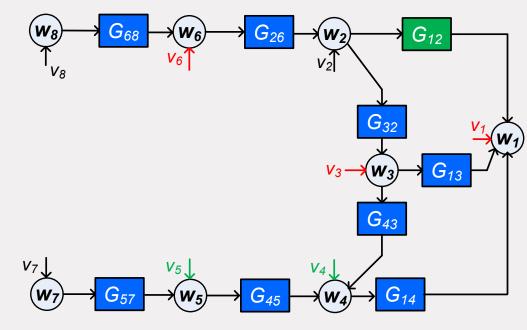
- Full input case:
- Minimum measurement case:
- User selection case :

include all in-neighbors of w_y maximize number of outputs dedicated choice based on measurable nodes



Different strategies – direct method

- Full input case
- Minimum measurements case
- User selection case



Network with v_1 correlated with v_3 and v_6 . v_4 correlated with v_5 .

Full input case

We include all in-neighbors of the predicted outputs as predictor inputs

Maximum use of information in signals

 $w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3,4\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1\}$

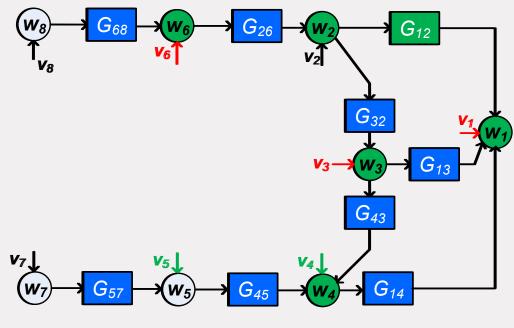
Handling direct confounding variable:

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3,4\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1,3\}$$

Handling indirect confounding variable:

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3,4,6\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1,3\}$$

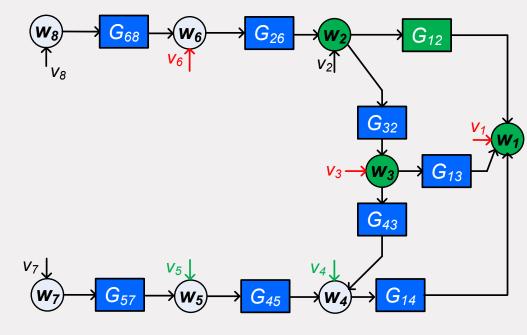
Direct identification $w_{\!\scriptscriptstyle \mathcal{D}} o w_{\!\scriptscriptstyle \mathcal{Y}}$



Minimum measurements case

- Select signals to satisfy the parallel path and loop condition
- Handle all confounding variables by including signals in output

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1,2,3\}$$



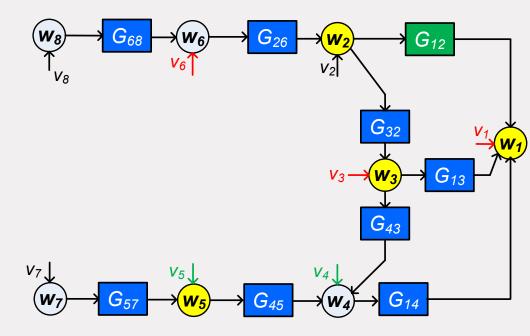
Direct identification $w_{\!\scriptscriptstyle \mathcal{D}} o w_{\!\scriptscriptstyle \mathcal{Y}}$



User selection case

- The user does not have access to all node signals
- Four node signals can be measured
- Parallel path and loop condition is satisfied
- Start with:

 $w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1\}$





User selection case

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1\}$$

Handling direct confounding variable:

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1,3\}$$

Indirect confounding variables:

 W_8

 V_8

G₆₈

 W_6

G₂₆

 V_2

 V_3

Direct identification $w_{\!\scriptscriptstyle \mathcal{D}} o w_{\!\scriptscriptstyle \mathcal{Y}}$



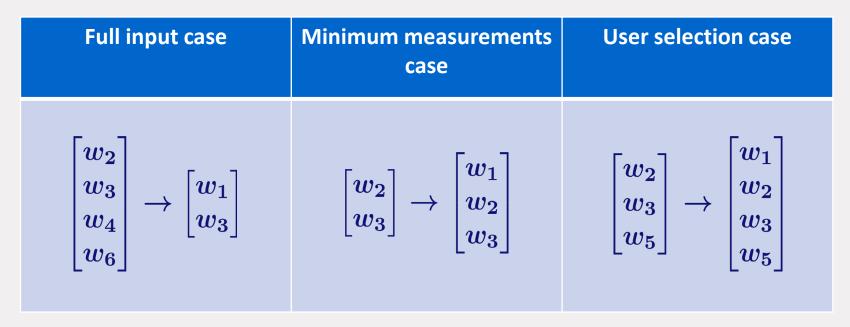
G₁₂

 G_{13}

G₃₂

Different strategies for same network and target module

Same network with different identification setups that lead to **consistent estimate of the target module** with **Maximum likelihood properties** based on the strategy used.



Data informativity conditions still need to be added.

Single module identification

Serious degrees of freedom in selecting the predictor model to satisfy the first two conditions:

- 1. Module invariance PPL test
- 2. Handling confounding variables

While presuming that data-informativity can always be satisfied by adding sufficient # of r-signals.



Single module identification – data-informativity

When focusing on estimating only the row with target module $ar{G}_{ii}$

 $w_j(t) = \bar{G}_{j*}(q,\theta) w_{\mathcal{D}}(t) + \bar{H}_{j*}(q,\theta) \xi_{\mathcal{V}}(t) + \bar{J}_{j*}(q,\theta) u_{\mathcal{K}}(t) + \bar{S}_{j*} u_{\mathcal{P}}(t)$

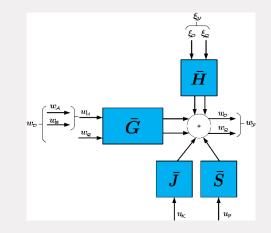
Typical data-informativity condition for estimating \bar{G}_{ii} :

 $\kappa^{[j]}$ persistently exciting

 $\Phi_{\kappa^{[j]}}(\omega)>0~~ ext{for almost all}~\omega$

$$\kappa^{[j]}(t) := egin{bmatrix} w_{\mathcal{D}_j}(t) \ \xi_{\mathcal{Y}_j}(t) \ u_{\mathcal{K}_j}(t) \end{bmatrix}$$

inputs corresponding to parametrized terms in the predictor model

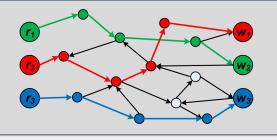


Rank-based condition can generically be satisfied based on a graph-based condition

Data informativity (path-based condition)

A signal y(t) = F(q)x(t) with x persistently exciting, is persistently exciting iff F has **full row rank**.

This condition can be verified in a generic sense, by considering the **generic rank** of $F^{[1],[2]}$



$$b_{\mathcal{R}\to\mathcal{W}}=3$$

linking to the maximum number of vertex disjoint paths between inputs and outputs

 $\kappa^{[j]}$ persistently exciting holds **generically** if there are $dim(\kappa^{[j]})$ vertex disjoint paths between external signals $\{u, e\}$ and $\kappa^{[j]} =$

 $= \begin{bmatrix} w_{\mathcal{D}_j} \\ \xi_j \\ w_j \end{bmatrix}$

Equivalently:

 $dim(w_{\mathcal{D}_j})$ vertex disjoint paths between $\{u,e\}ackslash\{\xi_{\mathcal{V}_j},u_{\mathcal{K}_j}\}$ and $w_{\mathcal{D}_j}$

 [1] Van der Woude, 1991
 [3] VdH et al., CDC 2020.

 [2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.

Data informativity (path-based condition)

Equivalently: $dim(w_{\mathcal{D}_j})$ vertex disjoint paths between $\{u, e\} \setminus \{\xi_{\mathcal{V}_j}, u_{\mathcal{K}_j}\}$ and $w_{\mathcal{D}_j}$

All input nodes need excitation, and white noise terms that have a link to w_y are excluded.

The more inputs, the more external signals required

The more outputs, less noise signals can be used



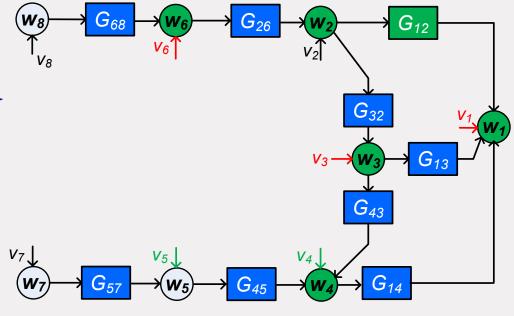
Full input case

Predictor model:

$$w_{\!\scriptscriptstyle \mathcal{D}} = \{2,3,4,6\} \;\; w_{\!\scriptscriptstyle \mathcal{Y}} = \{1,3\}$$

Excluded disturbances: v_1, v_3, v_6

 w_2 is excited by v_2 w_3 is not excited w_4 is excited by v_4 w_6 is excited by v_8



An additional excitation signal is required on $oldsymbol{w_3}$

Single module identification

$$w_{\mathcal{D}} \left\{ \begin{array}{c} w_{\mathcal{U}} & \longrightarrow & \bar{G} \\ w_{\mathcal{Q}} & \longrightarrow & \bar{G} \end{array} \right\} \begin{array}{c} \bar{G}_{ji} & \longrightarrow & w_{o} \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ \end{array} \right\} \left\{ \begin{array}{c} w_{\mathcal{U}} & \longrightarrow & w_{o} \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ \end{array} \right\} \left\{ \begin{array}{c} w_{\mathcal{U}} & \longrightarrow & w_{o} \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ \end{array} \right\} \left\{ \begin{array}{c} w_{\mathcal{U}} & \longrightarrow & w_{o} \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & &$$

Conditions for arriving at an accurate model:

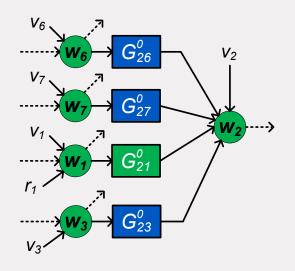
- 1. Module invariance: $\bar{G}_{ji} = G_{ji}^0$
- 2. Handling of confounding variables
- 3. Data-informativity
- 4. Technical conditions on presence of delays

Path-based conditions on the network graph



Single module identification

Typical solution:



- MISO (sometimes MIMO) estimation problem
- to be solved by your favorite estimation algorithm

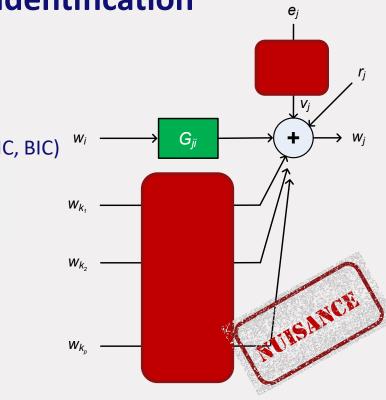


Machine learning in local module identification

- MISO identification with all modules parameterized
- Brings in two major problems :
 - Large number of parameters to estimate
 - Model order selection step for each module (CV, AIC, BIC)
- For 5 modules, combinations = 244,140,625

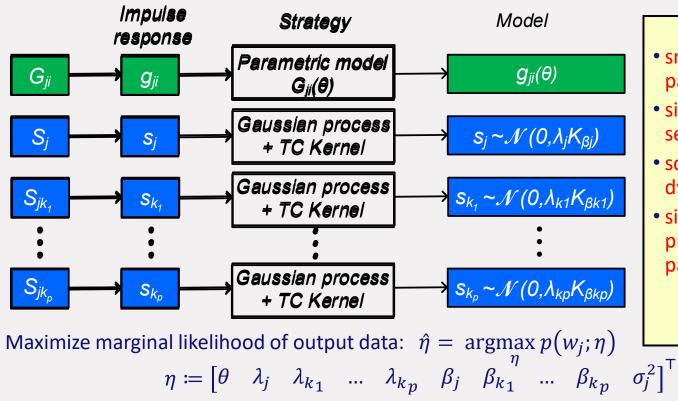


• We need only the target module. No NUISANCE!





Machine learning in local module identification

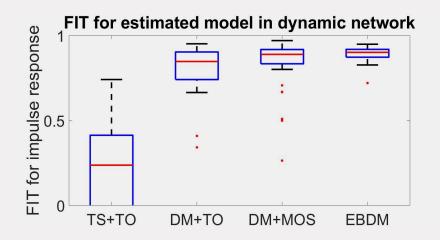


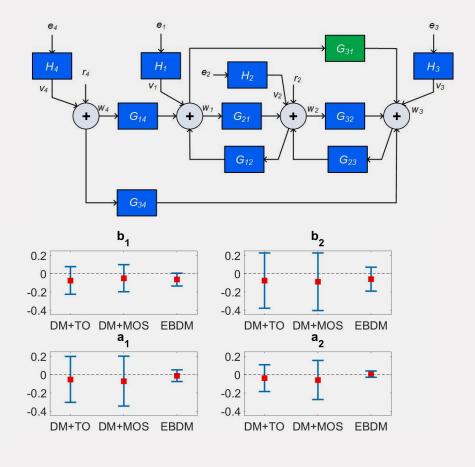
- smaller no. of parameters
- simpler model order selection step
- scalable to large dynamic networks
- simpler optimization problems to estimate parameters

[1] Everitt et al., *Automatica* 2017.[2] K.R. Ramaswamy et al., *Automatica*, 2021.

Numerical simulation

- Identify G₃₁ given data
- ▶ 50 independent MC simulation
- Data = 500





Summary single module identification

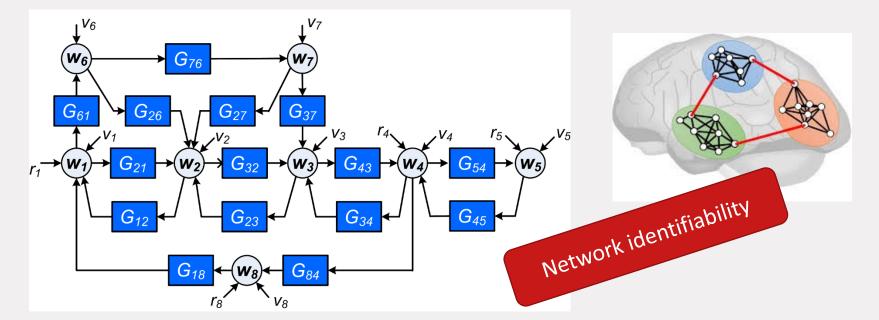
- Path-based conditions that the predictor model should satisfy
- Different algorithms for synthesizing predictor model, with freedom in sensor / actuator placement
- Methods for **consistent** and **minimum variance** module estimation, and effective (scalable) algorithms
- Algorithms can be preceded by (nonparametric) local **topology estimation**^[1]



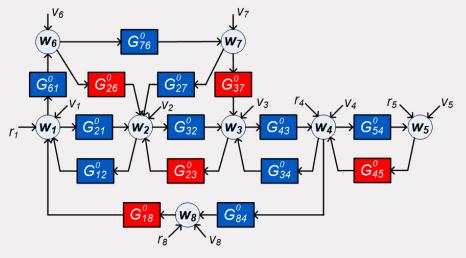


Generic network identifiability

Full network identification



Under which conditions can we estimate the topology and/or dynamics of the full network?



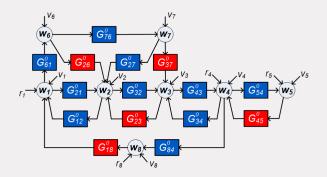
blue = unknown red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals *w*, *r* ?



The identifiability problem:

The network model:



$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

can be transformed with any rational P(q):

 $\boldsymbol{P(q)}\boldsymbol{w(t)} = \boldsymbol{P(q)}\{\boldsymbol{G(q)}\boldsymbol{w(t)} + \boldsymbol{R(q)}\boldsymbol{r(t)} + \boldsymbol{H(q)}\boldsymbol{e(t)}\}$

to an **equivalent model**:

 $w(t) = ilde{G}(q)w(t) + ilde{R}(q)r(t) + ilde{H}(q)e(t)$

Nonuniqueness, unless there are structural constraints on G, R, H.



Consider a network model set:

 $\mathcal{M} = \{(G(heta), R(heta), H(heta))\}_{ heta \in \Theta}$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

Generic identifiability of \mathcal{M} :

- There do not exist distinct equivalent models (generating the same data)
- for almost all models in the set.



Definition Network identifiability^[1]

For a network model set \mathcal{M} , consider a model $M(heta_0) \in \mathcal{M}$ and the implication

$$\begin{cases} T_{wr}(q,\theta_0) = T_{wr}(q,\theta_1) \\ \Phi_{\bar{v}}(\omega,\theta_0) = \Phi_{\bar{v}}(\omega,\theta_1) \end{cases} \\ \end{cases} \Longrightarrow \left\{ \begin{array}{l} M(\theta_0) = M(\theta_1), \\ \text{for all } M(\theta_1) \in \mathcal{M} \end{array} \right.$$

Then ${\cal M}$ is

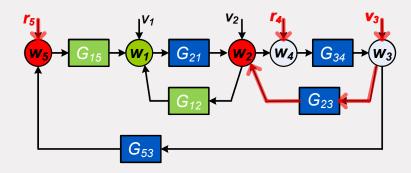
54

- globally identifiable from (w,r) at $M(heta_0)$ if the implication holds for $M(heta_0)$;
- globally identifiable from (w,r) if it holds for all $M(heta_0)\in\mathcal{M}$;
- generically identifiable^[2] from (w,r) if it holds for almost all $M(heta_0)\in\mathcal{M}$;

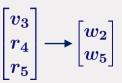


Example 5-node network

Conditions for identifiability **—** rank conditions on transfer function



Full row rank of

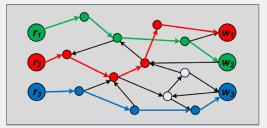


For the **generic case**, the rank can be calculated by a graph-based condition^{[1],[2]}:

Generic rank = number of vertex-disjoint paths

2 vertex-disjoint paths \rightarrow full row rank 2

The rank condition has to be checked for all nodes.



[1] Van der Woude, 1991
 [2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019

Synthesis solution for network identifiability

Allocating external signals for generic identifiability:

Cover the graph of the network model set by a set of disjoint pseudo-trees
 Pseudo-trees:

Tree with root in green

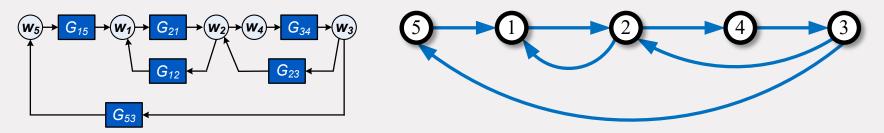
Cycle with outgoing trees; Any node in cycle is root

Edges are disjoint and all out-neighbours of a node are in the same pseudo-tree

2. Assign an independent external signal (r or e) at a root of each pseudo-tree.

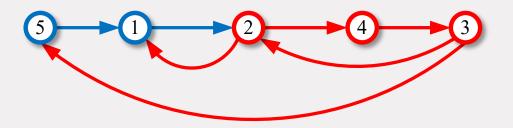
This guarantees generic identifiability of the model set.

Where to allocate external excitations for network identifiability?



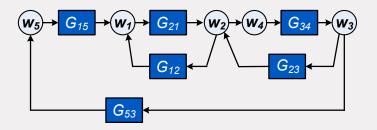
All indicated modules are parametrized

Two disjoint pseudo-trees

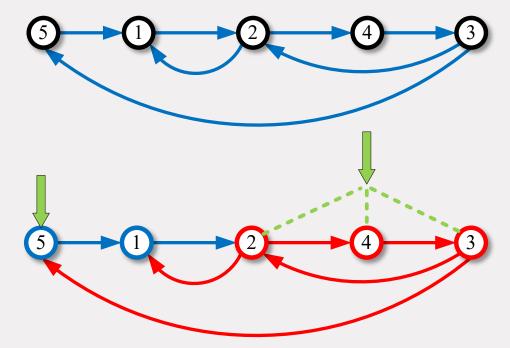




Where to allocate external excitations for network identifiability?



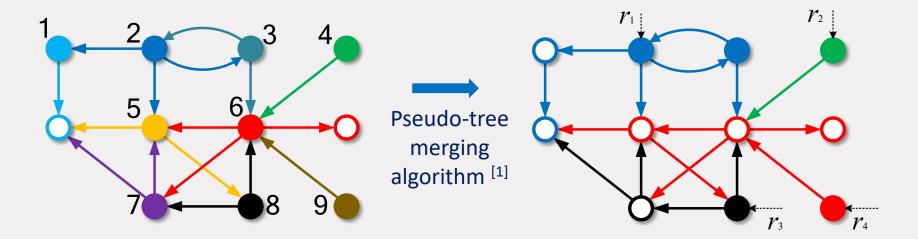
Two independent excitations guarantee generic network identifiability





[1] X. Cheng, S. Shi and PVdH, CDC 2019.

Where to allocate external excitations for network identifiability?



- Nodes are signals w and external signals (r, e) that are input to a parametrized link
- Known (nonparametrized) links do not need to be covered



[1] X. Cheng, S. Shi and PVdH, TAC, February 2022.

Summary identifiability of full network

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Topology of parametrized modules
- Graphic-based tool for synthesizing allocation of external signals

Extensions:

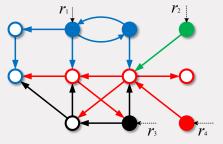
• Situations where not all node signals are measured ^[1]



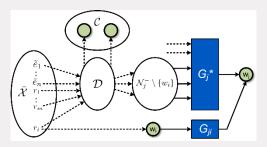
[1] Bazanella, CDC 2019.

Related topics...

• Excitation allocation for full network identifiability^[1]

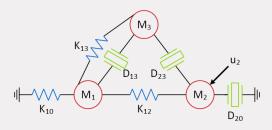


• Subnetwork identifiability^[3]

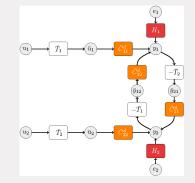


Cheng et al., IEEE-TAC, February 2022.
 Shi et al., IEEE-TAC, January 2023.

• Diffusively coupled networks^[2]



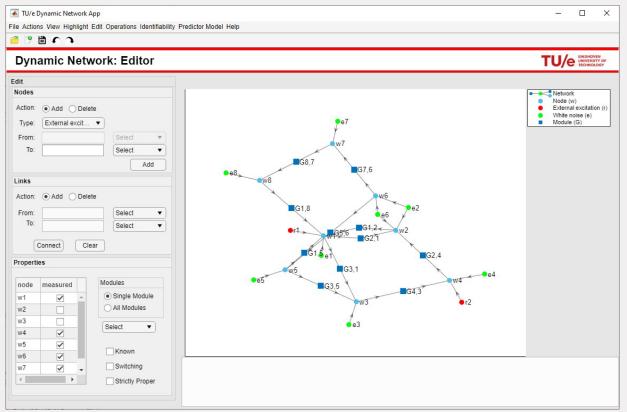
• Distributed controller identification^[4]



[2] Kivits et al., IEEE- TAC, June 2023.[4] Steentjes, PhD thesis, June 2022.

TU/e

Algorithms implemented in SYSDYNET App and Toolbox



Structural analysis and operations on dynamic networks

- Edit and manipulate
- Assign properties to nodes and modules
- Immersion of nodes, PPL test
- Generic identifiability analysis and synthesis
- Predictor model construction for single module ID

to be complemented with

- estimation algorithms for single module and full network ID;
- topology estimation

TU/e

Beta-version to be downloaded from <u>www.sysdynet.net</u>

ERC SYSDYNET Team: data-driven modeling in dynamic networks

Research team:





Arne Dankers





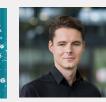
Harm Weerts



Shengling Shi



Karthik Ramaswamy





Tom Steentjes



Lizan Kivits



Stefanie Fonken



Xiaodong Cheng Giulio Bottegal Mircea Lazar



Tiis Donkers



Jobert Ludlage



Wim Liebregts Ilja v. Oort



+ MSc students

Co-authors, contributors and discussion partners:

Donatello Materassi, Manfred Deistler, Michel Gevers, Jonas Linder, Sean Warnick, Alessandro Chiuso, Håkan Hjalmarsson, Miguel Galrinho, Martin Enqvist, Johan Schoukens, Xavier Bombois, Peter Heuberger, Péter Csurcsia Minneapolis, Vienna, Louvain-la-Neuve, Linköping, KTH Stockholm, Padova, Brussels, Salt Lake City, Lyon.





63

Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
- A. Dankers, P.M.J. Van den Hof, X. Bombois and P.S.C. Heuberger (2015). Errors-in-variables identification in dynamic networks - consistency results for an instrumental variable approach. *Automatica*, Vol. 62, pp. 39-50, 2015.
- A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois (2016). Identification of dynamic models in complex networks with predictior error methods - predictor input selection. *IEEE Trans. Autom. Contr.*, *61 (4)*, pp. 937-952, 2016.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Identifiability of linear dynamic networks. *Automatica*, 89, pp. 247-258, March 2018.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Prediction error identification of linear dynamic networks with rank-reduced noise. *Automatica*, *98*, pp. 256-268, December 2018.
- H.H.M. Weerts, J. Linder, M. Enqvist and P.M.J. Van den Hof (2019). Abstractions of linear dynamic networks for input selection in local module identification. *Automatica, Vol. 117*, July 2020.
- R.J.C. van Esch, S. Shi, A. Bernas, S. Zinger, A.P. Aldenkamp and P.M.J. Van den Hof (2020). A Bayesian method for inference of effective connectivity in brain networks for detecting the Mozart effect. *Computers in Biology and Medicine, Vol. 127*, paper 104055, December 2020.
- K.R. Ramaswamy, G. Bottegal and P.M.J. Van den Hof (2020). Learning linear models in a dynamic network using regularized kernel-based methods. *Automatica, Vol. 129*, Article 109591, July 2021.
- P.M.J. Van den Hof and K.R. Ramasmwamy (2021). Learning local modules in dynamic networks. *Proc. of Machine Learning Res.*, Vol. 144, pp. 176-188.

- K.R. Ramaswamy and P.M.J. Van den Hof (2021). A local direct method for module identification in dynamic networks with correlated noise. *IEEE Trans. Automatic Control, Vol.* 66, no. 11, pp. 3237-3252, November 2021.
- X. Cheng, S. Shi and P.M.J. Van den Hof (2022). Allocation of excitation signals for generic identifiability of linear dynamic networks. *IEEE Trans. Automatic Control*, *Vol.* 67, no. 2, pp. 692-705, February 2022.
- S. Shi, X. Cheng and P.M.J. Van den Hof (2022). Generic identifiability of subnetworks in a linear dynamic network: the full measurement case. *Automatica, Vol. 117* (110093), March 2022.
- S.J.M. Fonken, K.R. Ramaswamy and P.M.J. Van den Hof (2022). A scalable multi-step least squares method for network identification with unknown disturbance topology. *Automatica, Vol. 141* (110295), July 2022.
- K.R. Ramaswamy, P.Z. Csurcsia, J. Schoukens and P.M.J. Van den Hof (2022). A frequency domain approach for local module identification in dynamic networks. *Automatica, Vol. 142* (110370), August 2022.
- S. Shi, X. Cheng and P.M.J. Van den Hof (2023). Single module identifiability in linear dynamic networks with partial excitation and measurement. *IEEE Trans. Automatic Control, Vol. 68(1),* pp. 285-300, January 2023.
- X.Bombois, K. Colin, P.M.J. Van den Hof and H. Hjalmarsson (2023). On the informativity of direct identification experiments in dynamical networks. *Automatica, Vol. 148* (110742), February 2023.
- E.M.M. Kivits and P.M.J. Van den Hof (2023). Identification of diffusively coupled linear networks through structured polynomial models. *IEEE Trans. Automatic Control, Vol.* 68(6), pp. 3513-3528, June 2023.
- T.R.V. Steentjes, M. Lazar and P.M.J. Van den Hof. On a canonical distributed controller in the behavioral framework. *Syst.* &*Control Lett.*, Vol. 179 (105581), September 2023.
- X. Cheng, S. Shi, I. Lestas and P.M.J. Van den Hof. A necessary condition for network identifiability with partial excitation and measurement. *IEEE Trans. Automatic Control, Vol. 68 (11)*, pp.6820-6827, November 2023.

Papers available at www.sysdynet.eu



The end