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# Data-driven model learning in linear dynamic networks

Paul M.J. Van den Hof

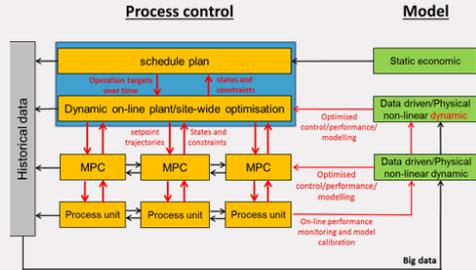
Control Systems Group, Department of Electrical Engineering

EAISI Seminar, 28 January 2020

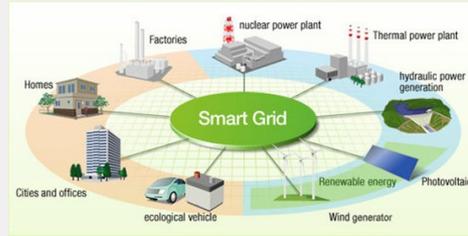
[www.sysdynet.eu](http://www.sysdynet.eu)  
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[p.m.j.vandenhof@tue.nl](mailto:p.m.j.vandenhof@tue.nl)

# Introduction – dynamic networks

## Decentralized process control



## Smart power grid



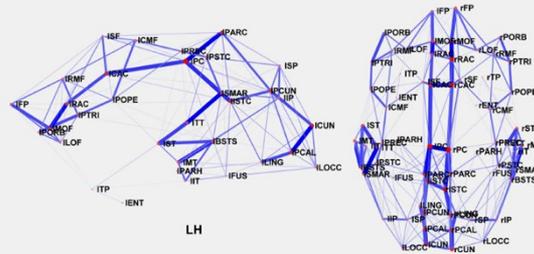
Pierre et al. (2012)

## Autonomous driving



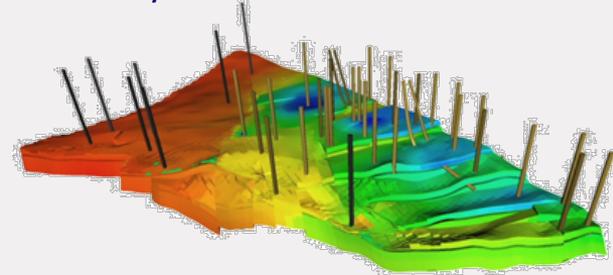
www.nvidia.com

## Brain network



P. Hagmann et al. (2008)

## Hydrocarbon reservoirs



Mansoori (2014)

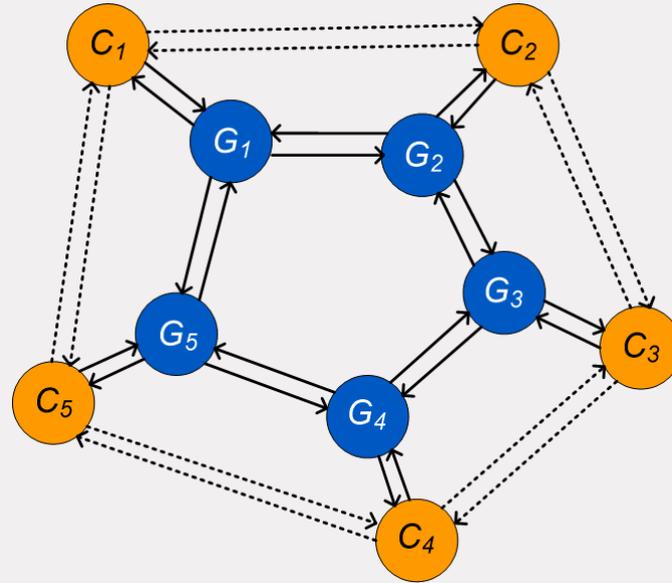
# Introduction

## Overall characterization from a system's perspective:

- (Large-scale) interconnected systems
- Each subsystem is **dynamic** (dynamic relations between time series)  
a subsystem's output is dependent on past inputs
- Optimal operation of the systems through  
distributed / multi-agent type monitoring, control and optimization problems
- Data is “everywhere”, big data era
- Model-based operations require accurate/relevant models
- → **Learning models from data** (including physical insights when available)

# Introduction

Distributed / multi-agent control:

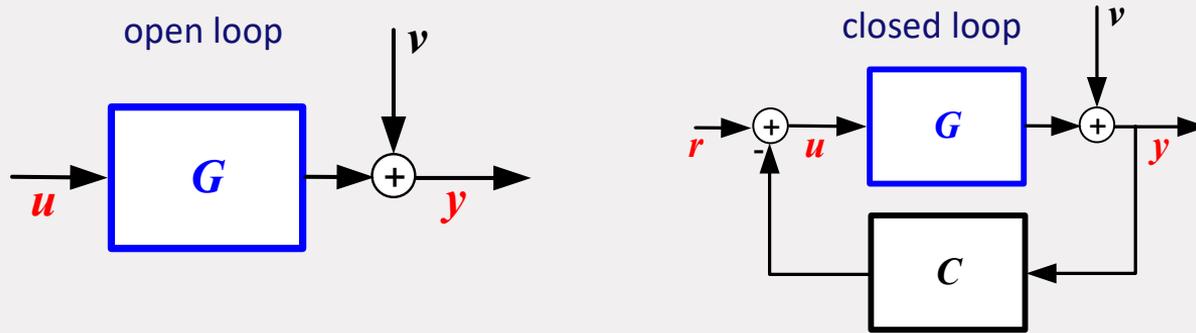


With both physical and communication links between systems  $G_i$  and controllers  $C_i$

How to address data-driven modelling problems in such a setting?

# Introduction

The classical (multivariable) identification problems<sup>[1]</sup> :



Estimate a plant model  $\hat{G}$  on the basis of measured time series signals  $u(t)$ ,  $y(t)$ , and possibly  $r(t)$ ,  $t=0, \dots, N$ , focusing on *continuous LTI dynamics*.

$G$  represents a (linear) differential equation (of a particular finite order)

We have to move from a simple and fixed configuration to deal with **structure** in the problem.

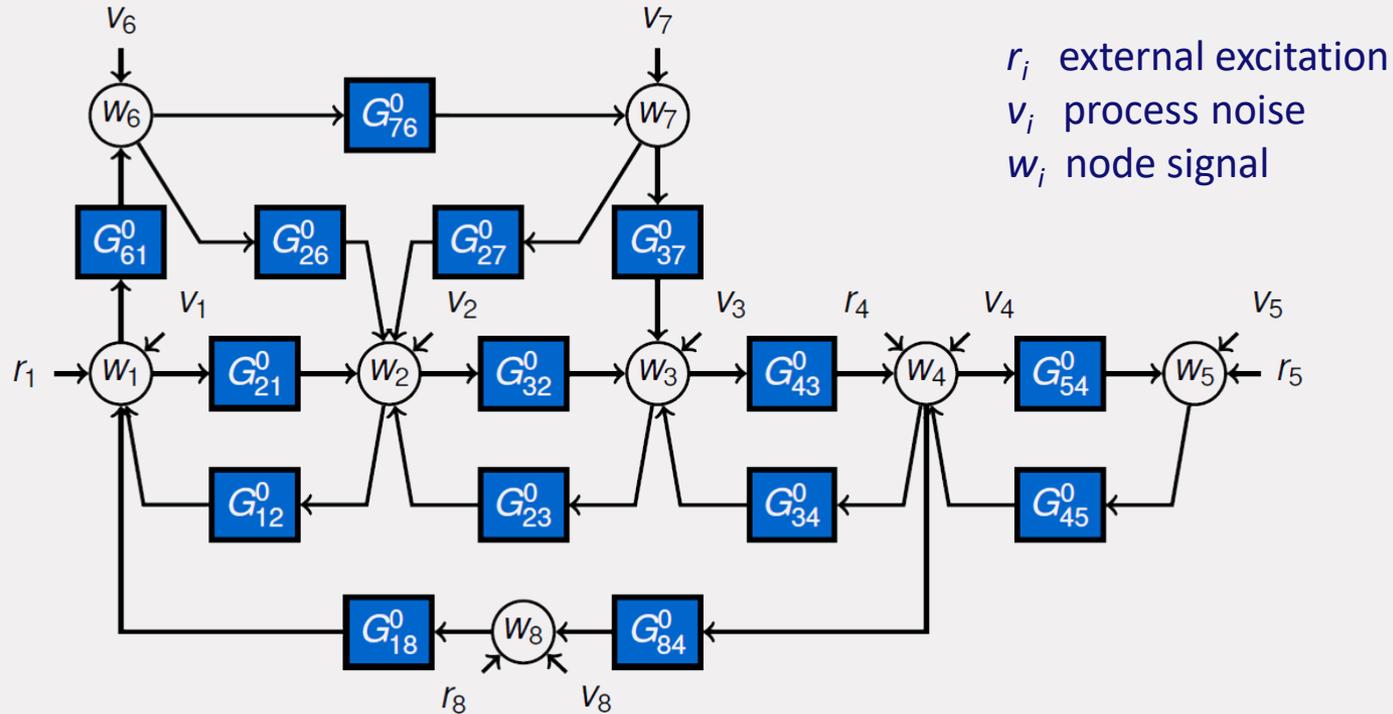
<sup>[1]</sup> Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

# Contents

- Introduction and motivation
- How to model a dynamic network?
- Relevant learning problems
- One example: generic identifiability

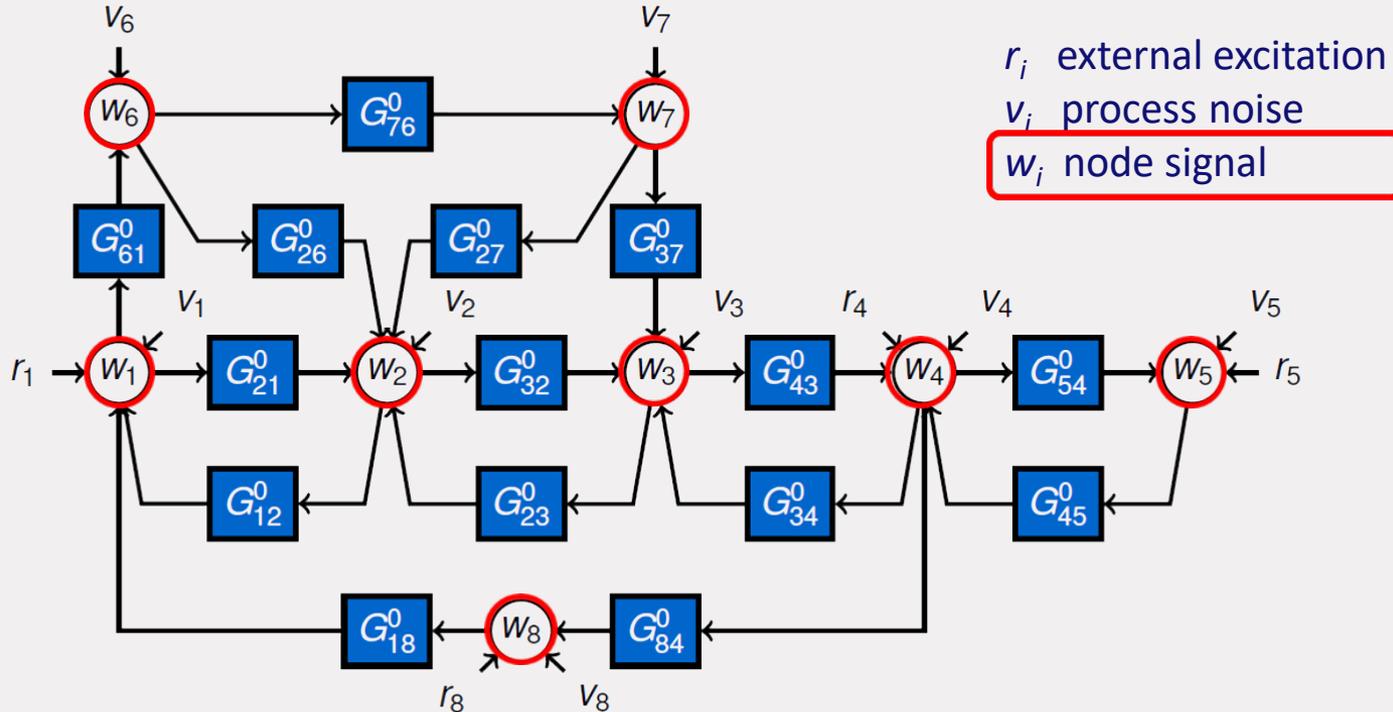
# Dynamic network setup

Directed graph with subsystems on the links, and signals in the nodes



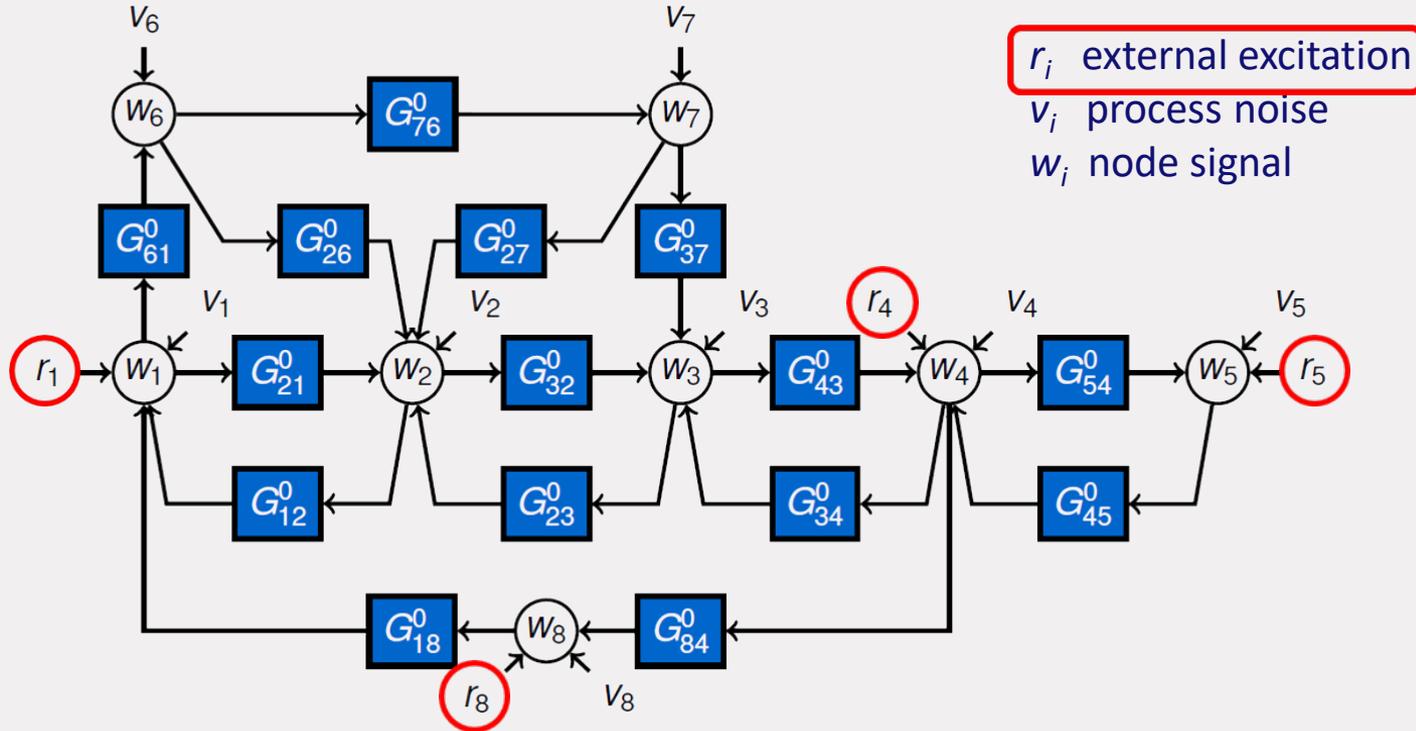
# Dynamic network setup

Directed graph with subsystems on the links, and signals in the nodes



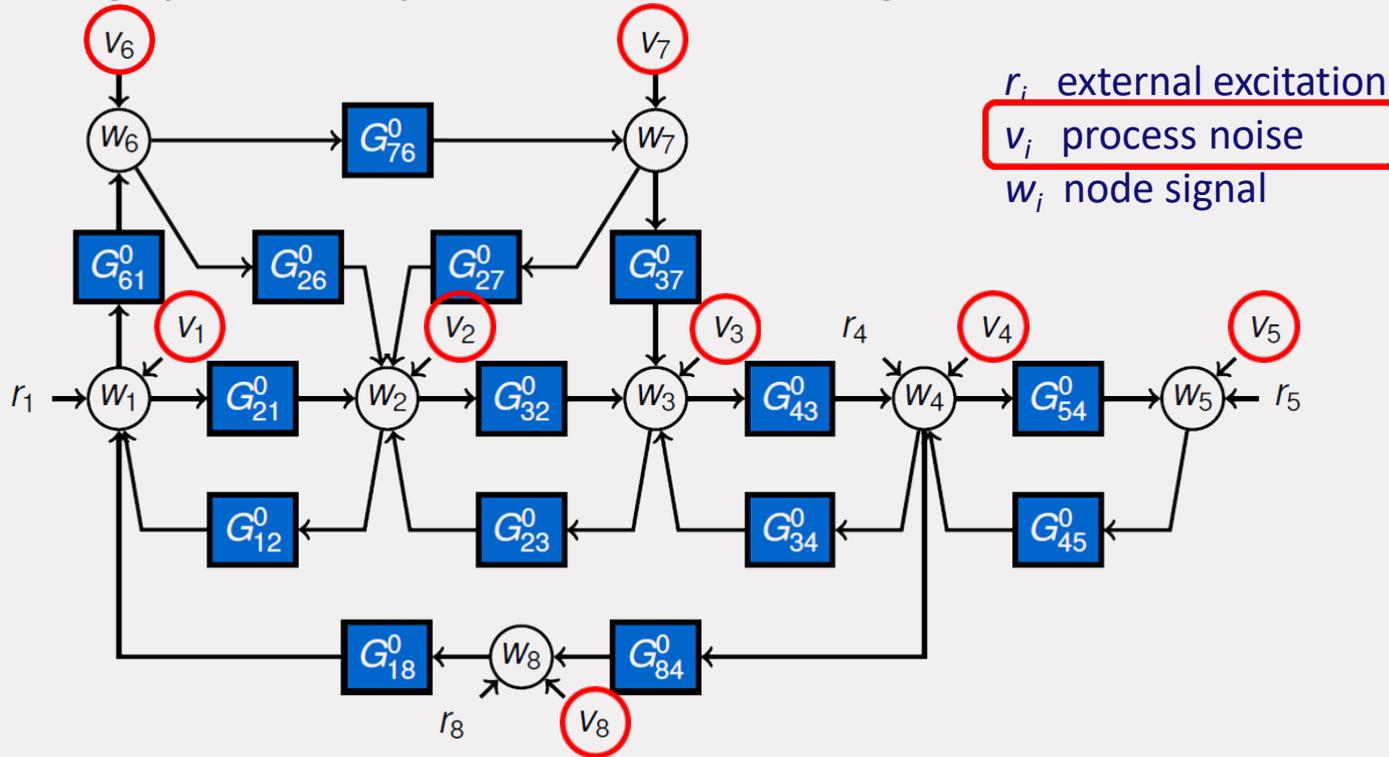
# Dynamic network setup

Directed graph with subsystems on the links, and signals in the nodes

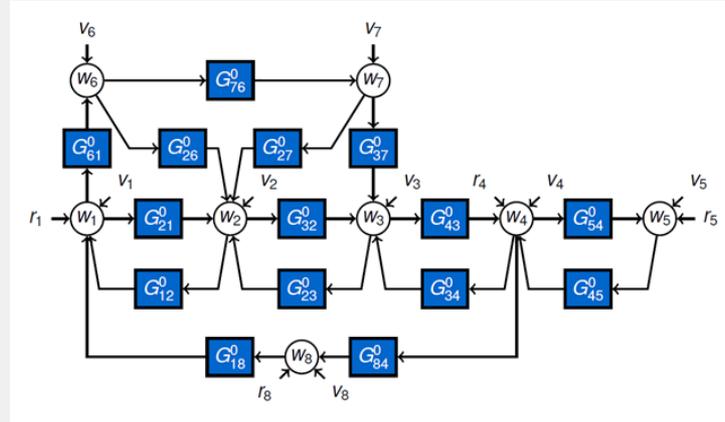


# Dynamic network setup

Directed graph with subsystems on the links, and signals in the nodes



# Dynamic network setup



## Assumptions:

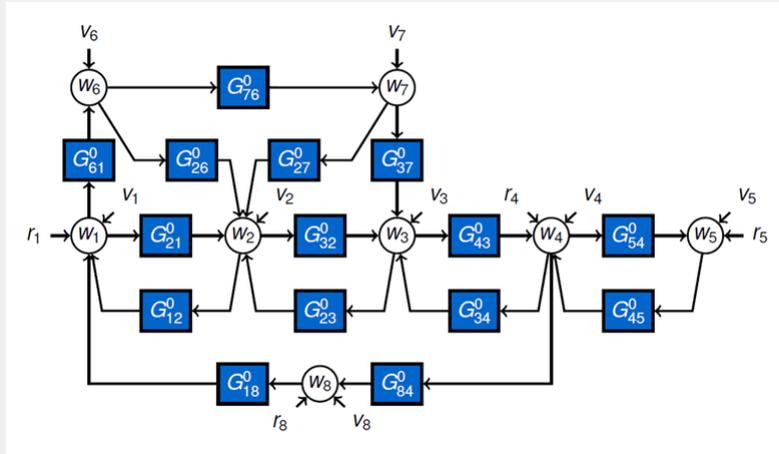
- Total of  $L$  nodes
- Network is well-posed and stable
- Modules are dynamic, may be unstable
- Disturbances are stationary stochastic and can be correlated

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \cdots & G^0_{2L} \\ \vdots & \cdots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

$$q^{-1}w(t) = w(t - 1)$$

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t)$$

# Model learning problems

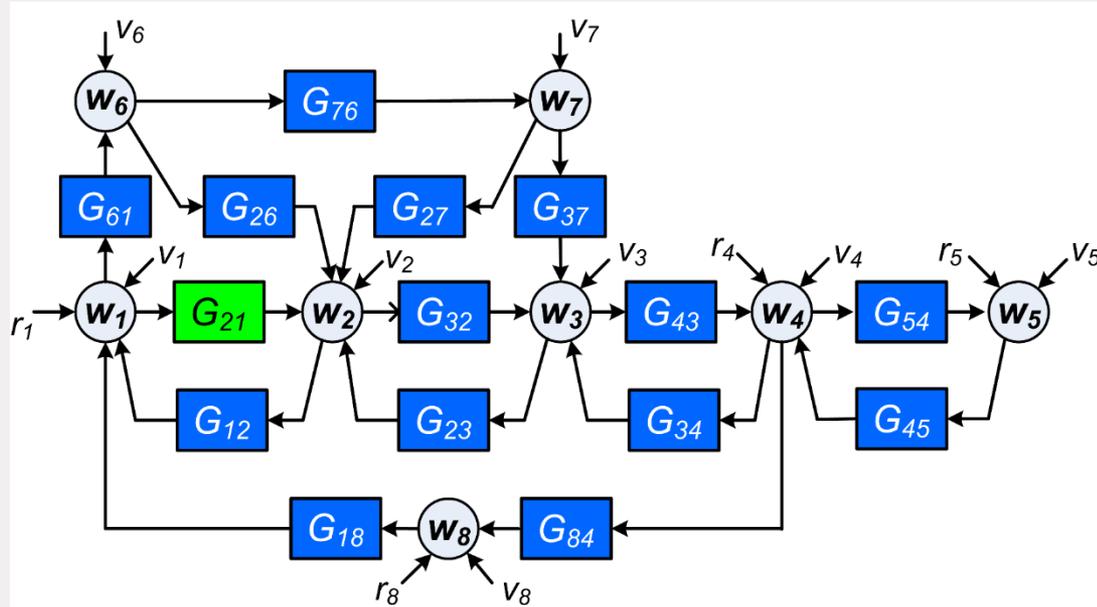


Many new data-driven modeling / learning questions can be formulated:

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Sensor and excitation selection
- Fault detection
- Experiment design
- User prior knowledge of modules
- Scalable algorithms

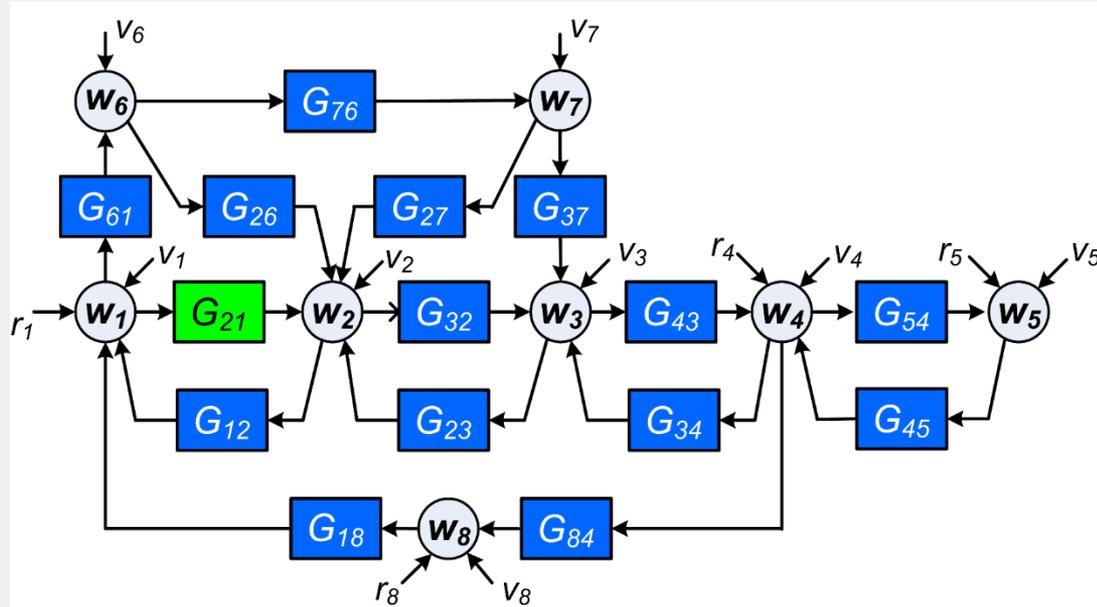


# Model learning problems



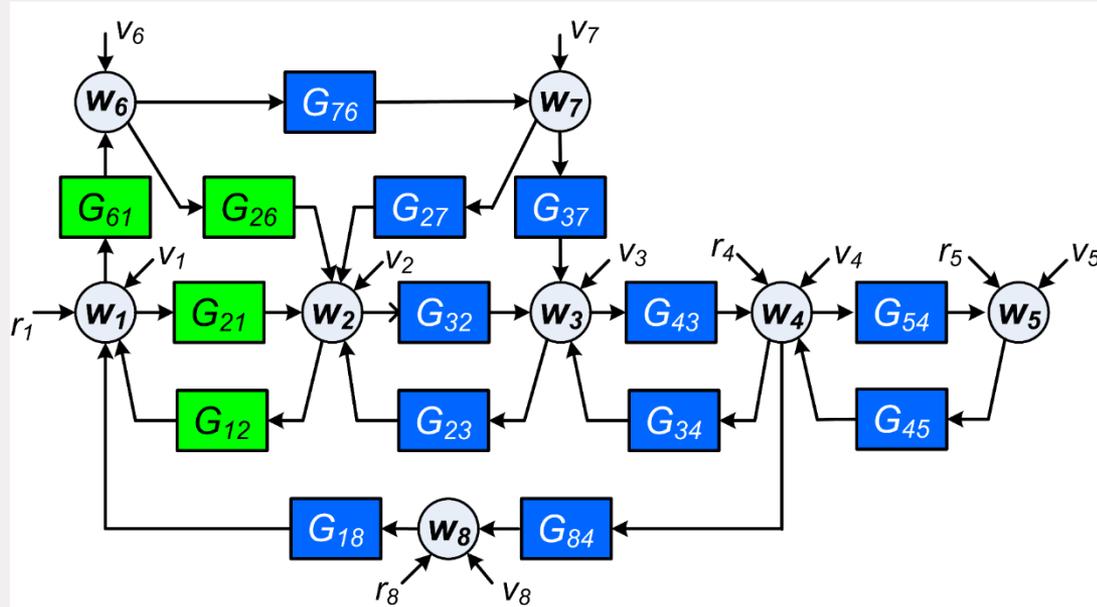
How/when can we learn a local module from data  
(with known/unknown network topology)? Which signals to measure?

# Model learning problems



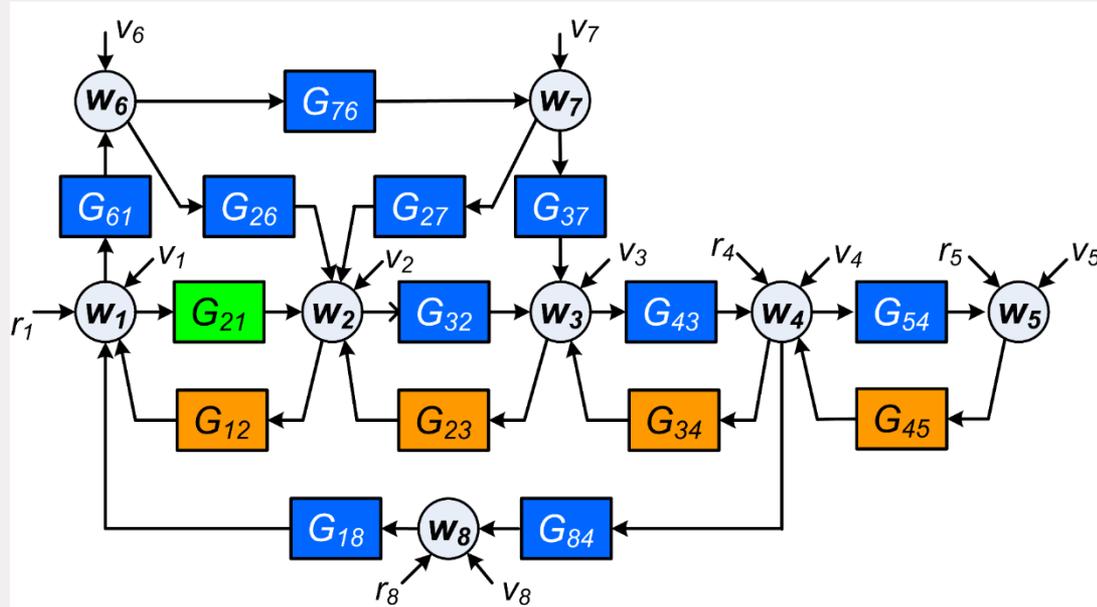
Where to optimally locate sensors and actuators?

# Model learning problems



Same questions for a subnetwork

# Model learning problems



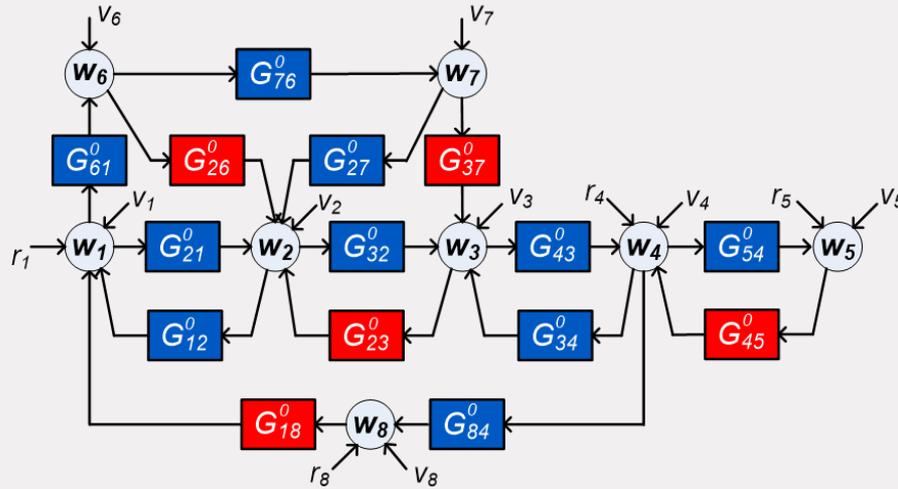
How can we benefit from known (orange) modules?



# Contents

- Introduction and motivation
- How to model a dynamic network?
- Relevant learning problems
- **One example: generic identifiability**

# Network identifiability



blue = unknown  
red = known

**Question:** Can different dynamic networks be *distinguished* from each other from measured signals  $w_i, r_i$ ?

A parametrized network model set  $\mathcal{M}$  is **identifiable** if there is a unique mapping  $T \rightarrow \mathcal{M}$  where  $T$  are the transfer function objects that can be uniquely identified from the data.

# Network identifiability

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Possible correlation between disturbance signals
- Prior knowledge on modules (fixed / parametrized)

# Network identifiability

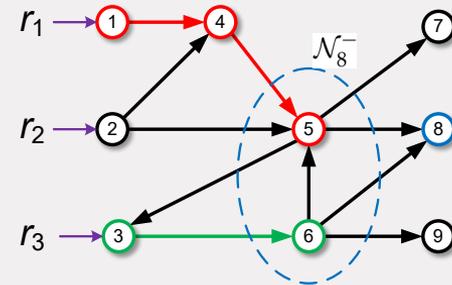
## Theorem – generic identifiability

For each node signal  $w_j$ , let  $\mathcal{P}_j$  be the set of in-neighbours of  $w_j$  that map to  $w_j$  through a parametrized module.

Then, under fairly general conditions,

$\mathcal{M}$  is generically **network identifiable** if and only if for all  $j$  :

- There are  $|\mathcal{P}_j|$  vertex disjoint paths from external excitation signals  $(r, e)$ <sup>1</sup> to the node signals in  $\mathcal{P}_j$



<sup>1</sup> that appear nonparametrized in  $w_j$

[1] Van der Woude, 1991

[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC-2019

[3] Weerts et al, SYSID2015; Weerts et al., Automatica, March 2018; Weerts et al CDC 2018.

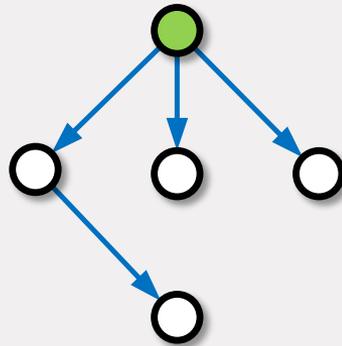
# Graph-based synthesis solution for full network

## Decompose network in **disjoint pseudo-trees**:

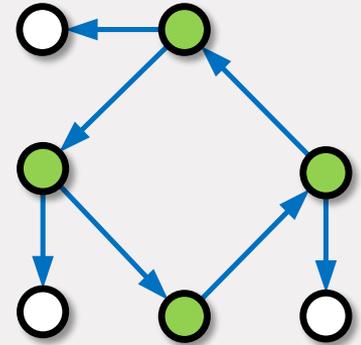
- Connected directed graphs, where nodes have maximum indegree 1
- Edges are disjoint and all out-neighbours of a node are in the same pseudo-tree

## Two typical pseudo-trees:

Tree with root in green



Cycle with outgoing trees;  
Any node in cycle is root



# Graph-based synthesis solution for full network

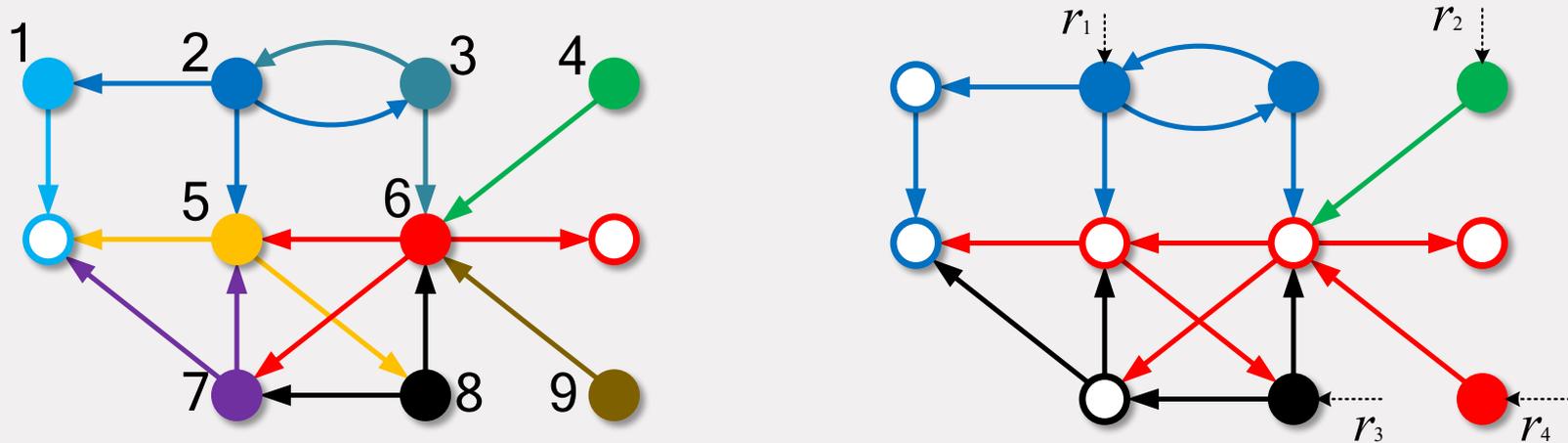
## Result<sup>[1]</sup>

A network is generically identifiable if

- It is decomposed in disjoint pseudo-trees, and
- Each pseudo-tree has an independent excitation at a **root**

[1] X. Cheng, S. Shi and PVdH, CDC 2019.

# Where to allocate external excitations for network identifiability?



- Nodes are signals  $w$  and external signals ( $r, e$ ) when they are input to parametrized links
- Result extends to the presence of known (nonparametrized links): they can be excluded from the covering

# Extensions - Discussion

# Extensions - Discussion

- **Learning algorithms to deal with reduced rank noise** <sup>[1]</sup>
  - number of disturbance terms is larger than number of white sources
  - Optimal identification criterion becomes a **constrained quadratic problem** with ML properties for Gaussian noise
  - Reworked Cramer Rao lower bound
  - Some parameters can be estimated variance free
- **Including sensor noise** <sup>[2]</sup>
  - Errors-in-variables problems can be more easily handled in a network setting

[1] Weerts et al., Automatica, December 2018.

[2] Dankers et al., Automatica, 2015.

# Extensions - Discussion

- **Machine learning tools for estimating large scale models** <sup>[1,2]</sup>
  - Choosing correctly parametrized model sets for all modules is impractical
  - Use of Gaussian process priors for kernel-based estimation of models
- **From centralized to distributed estimation (MISO models)** <sup>[3]</sup>
  - Communication constraints between different agents
  - Recursive (distributed) estimator converges to global optimizer (more slowly)

[1] Everitt et al., Automatica, 2018.

[2] Ramaswamy et al., CDC 2018.

[3] Steentjes et al., IFAC-NECSYS, 2018.

# Discussion

- **Dynamic network identification:**  
intriguing research topic with many open questions
- The (centralized) LTI framework is only just the beginning
- Structural basis for learning algorithms
- Extensions to non-directional graphs (physical networks)
- Including switching / time-varying links

# Acknowledgements

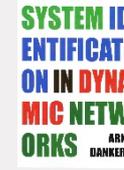


Lizan Kivits, Shengling Shi, Karthik Ramaswamy,  
Tom Steentjes, Mircea Lazar, Jobert Ludlage, Mannes Dreef,  
Tijds Donkers, Giulio Bottegal, Maarten Schoukens, Xiaodong Cheng

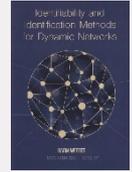
Co-authors, contributors and discussion partners:



Arne Dankers



Harm Weerts



Xavier Bombois  
Peter Heuberger  
Donatello Materassi  
Manfred Deistler  
Michel Gevers  
Jonas Linder  
Sean Warnick  
Alessandro Chiuso  
Hakan Hjalmarsson  
Miguel Galrinho  
Martin Enqvist

# Further reading

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**The end**