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Data-driven modeling in linear dynamic networks – Identifiability

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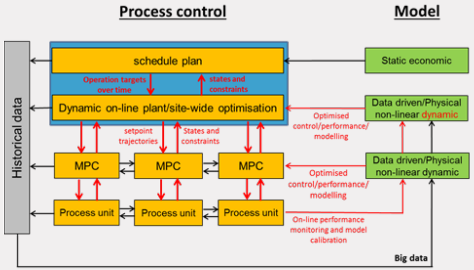
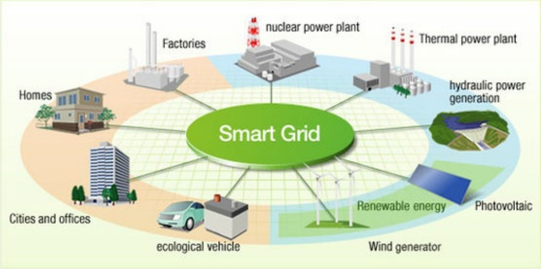
European Research Council

Introduction – dynamic networks

Decentralized process control



Smart power grid

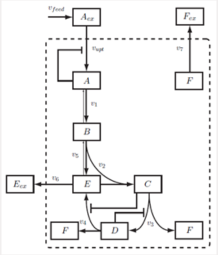


Pierre et al. (2012)

Autonomous driving

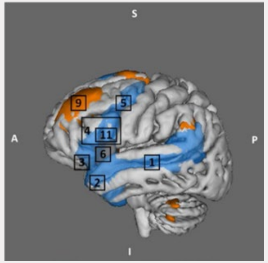
www.envidia.com

Metabolic network

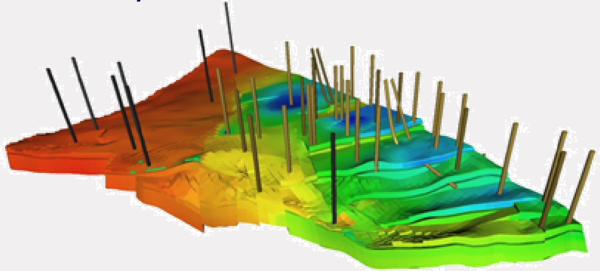


Hillen (2012)

Brain network

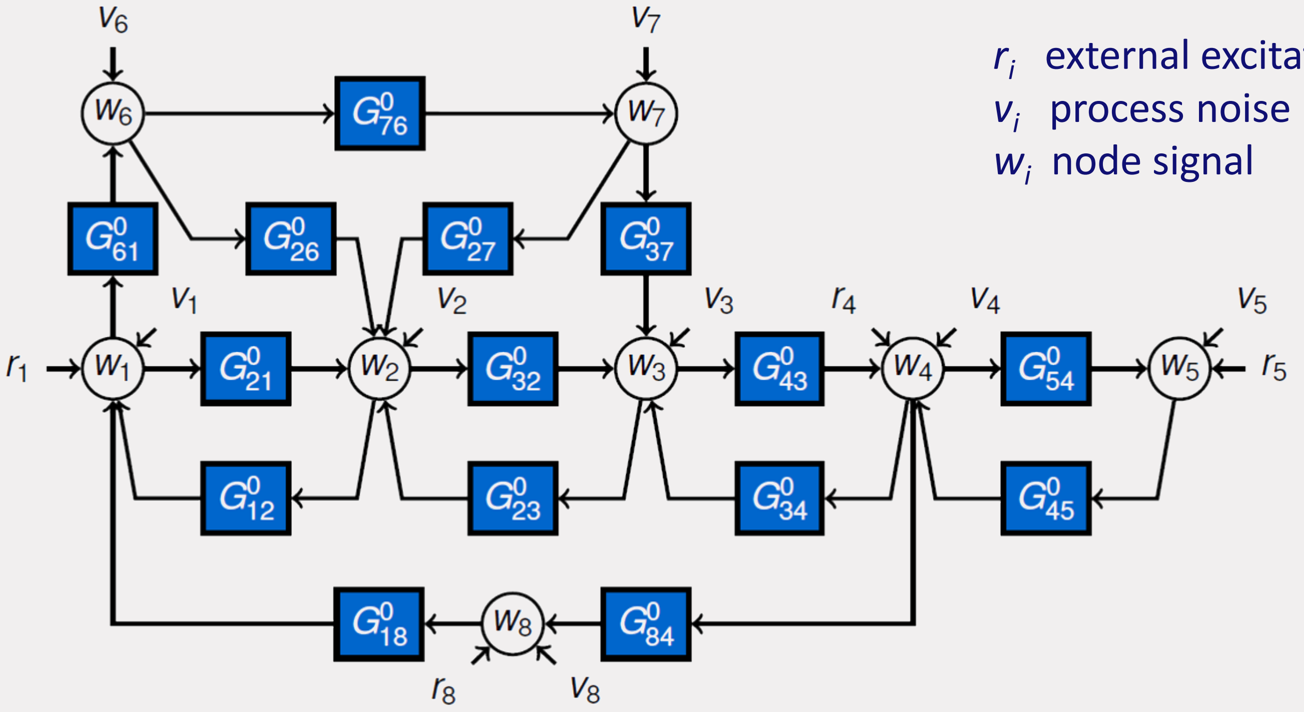


Hydrocarbon reservoirs



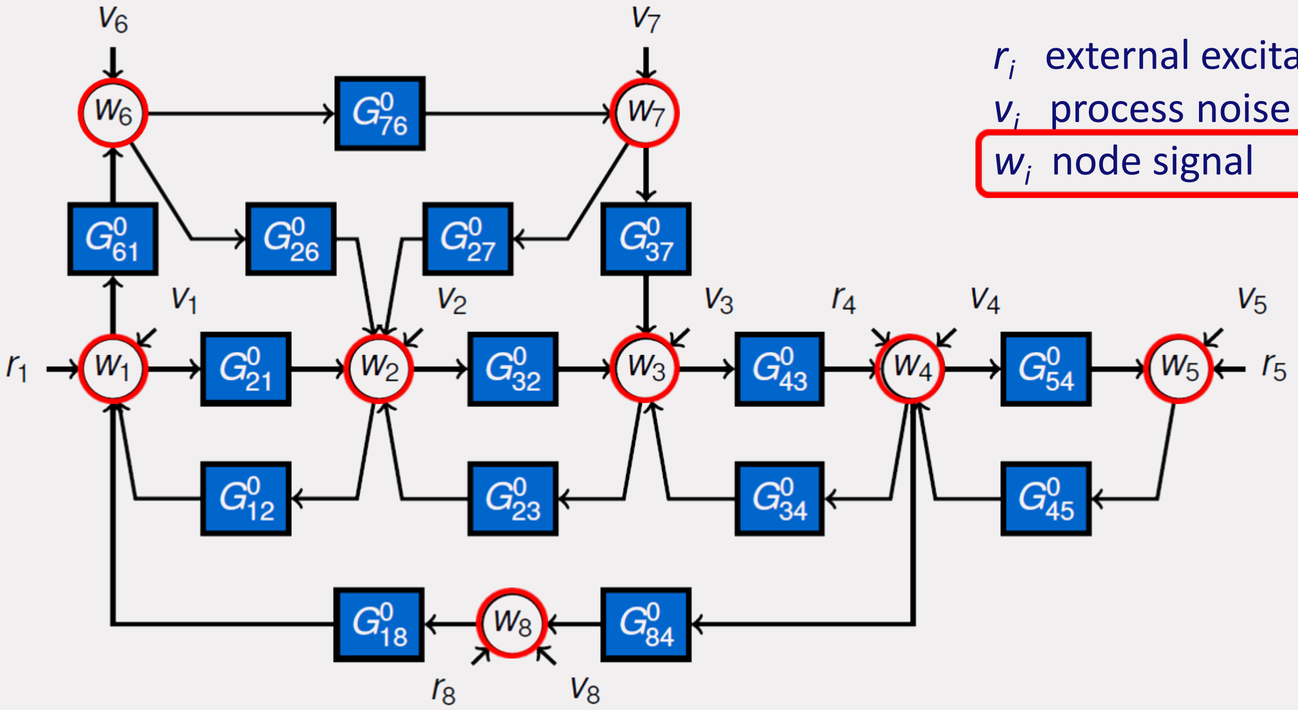
Mansoori (2014)

Dynamic network model – module framework



r_i external excitation
v_i process noise
w_i node signal

Dynamic network model – module framework



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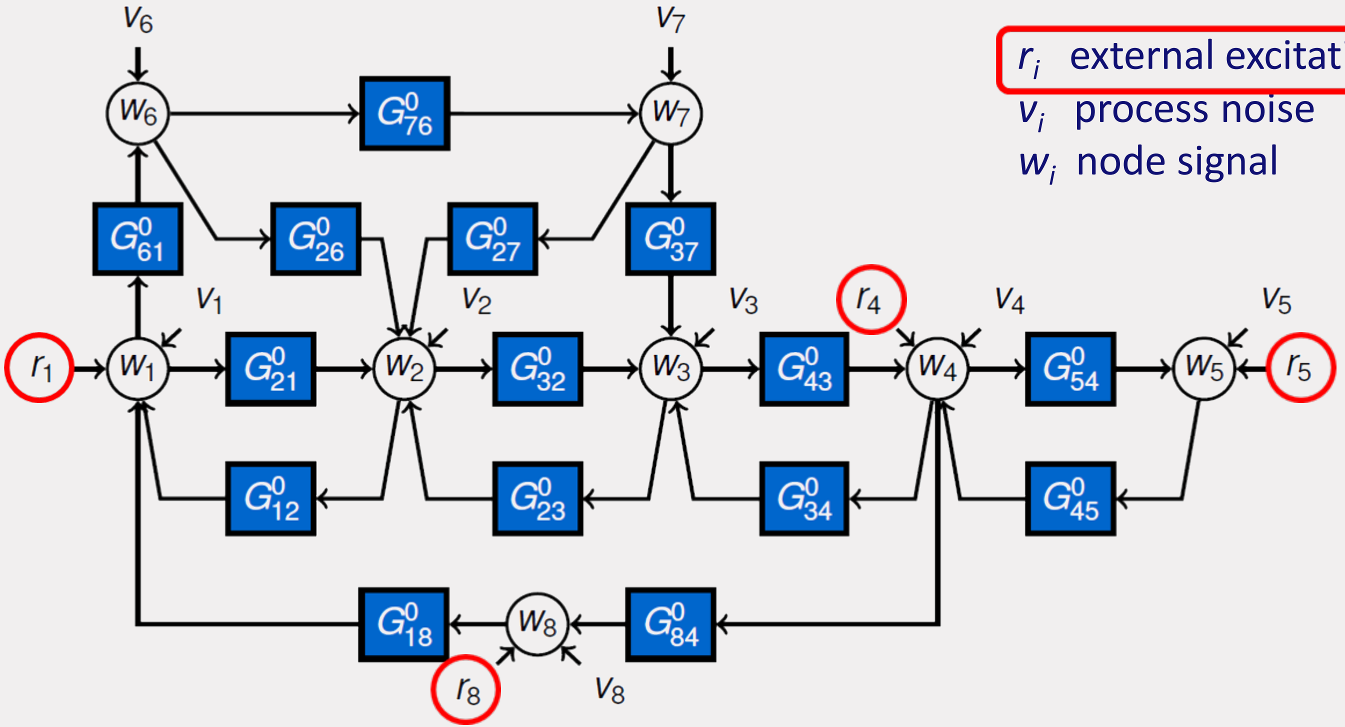

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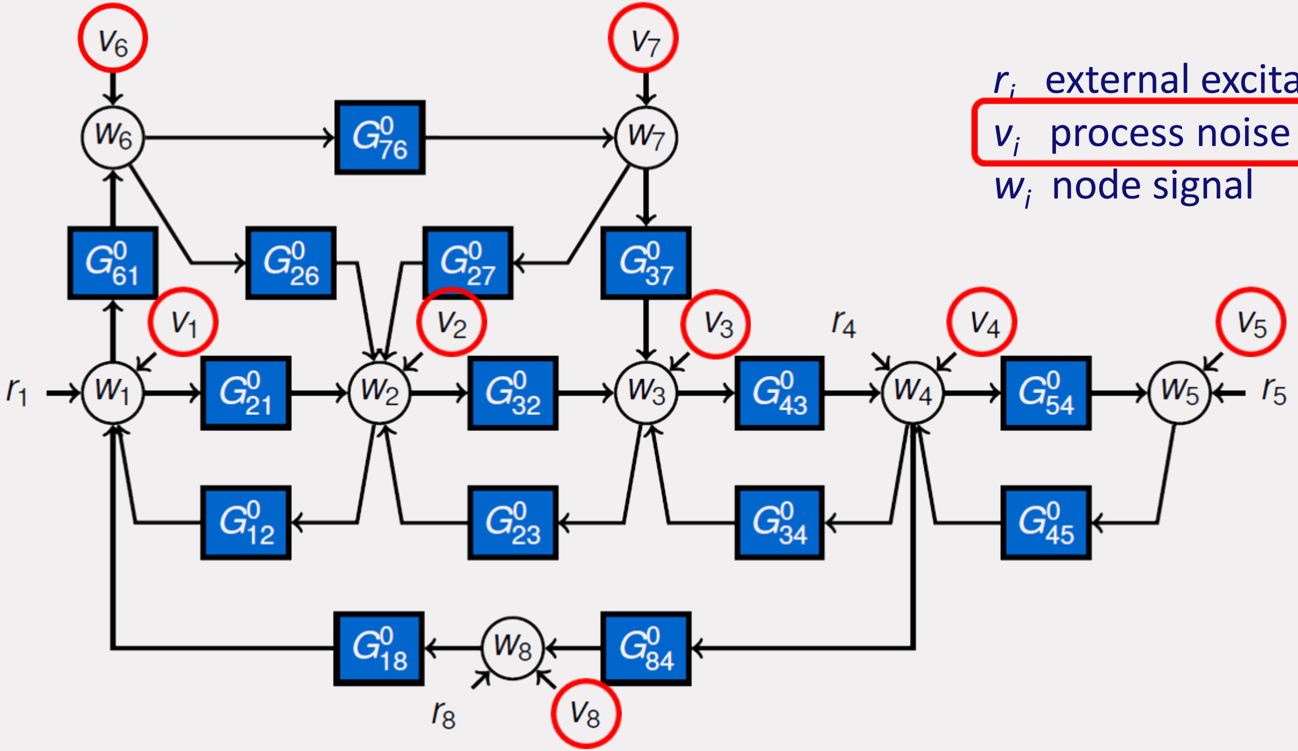
Dynamic network model – module framework

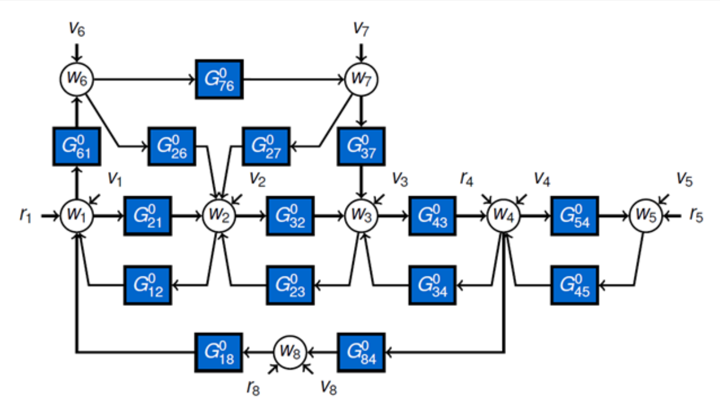


r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



Assumptions:

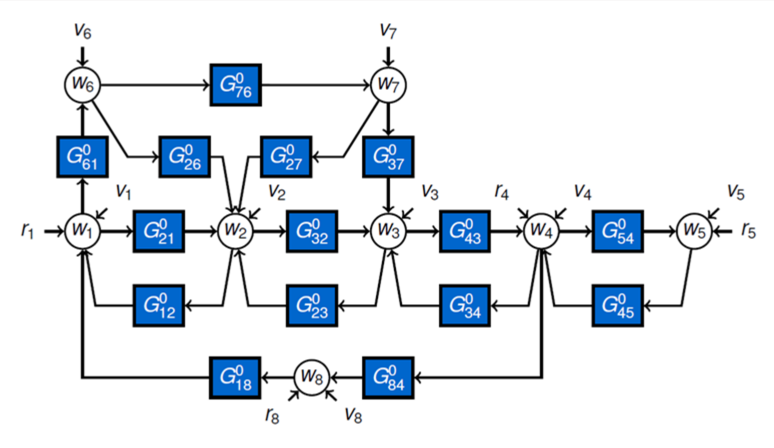
- Total of L nodes
- Network is well-posed and stable
- Modules are dynamic, may be unstable
- Disturbances are stationary stochastic and can be correlated

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t)$$

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
P.M.J. Van den Hof, A.G. Dankers, P.S.C. Heuberger and X. Bombois. Automatica, 2013.

Dynamic network model – module framework



Measured time series signals:

$\{w_i\}_{i=1,\cdots L}; \ \{r_j\}_{j=1,\cdots K}$

Many new identification questions can be formulated:

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Scalable algorithms

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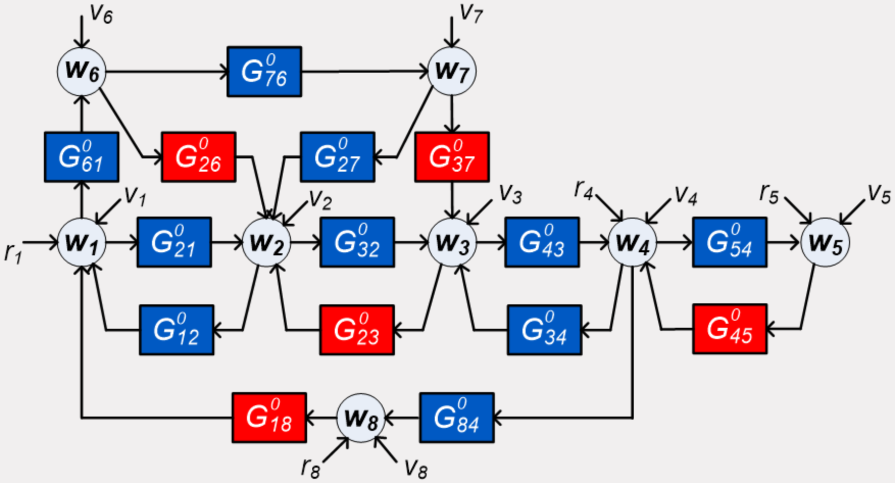
Network Identifiability

Network identifiability



blue = unknown
red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals w_i, r_i ?

Where to add excitation signals r such that we can?

Starting assumption: all signals w_i, r_i that are present can be measured.

Network identifiability

Network: $$w = Gw + Rr + He$$

$$w = (I - G^{-1}[Rr + He]$$

Denote: $w = T \begin{bmatrix} r \\ e \end{bmatrix}$

Where T can typically be identified from data (under some conditions)

Consider a **network model set**: $\mathcal{M} = \{M(\theta) = (G(\theta), R(\theta), H(\theta)), \theta \in \Theta\}$

Network identifiability of $\mathcal{M}$ is defined by a unique mapping: $T \rightarrow M$

Is there a single model in the model set that matches a ``measured'' T ?

Generic identifiability holds if this is true for *almost all* models in $\mathcal{M}$

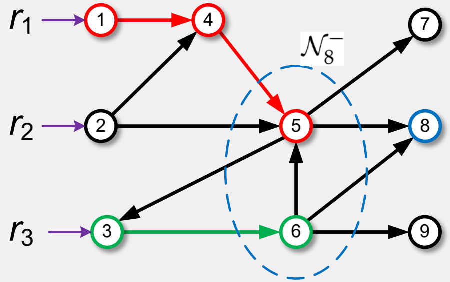
Network identifiability

Theorem – generic identifiability

For each node signal w_j, let $\mathcal{P}_j$ be the set of in-neighbours of w_j that map to w_j through a parametrized module.
Then, under fairly general conditions,

$\mathcal{M}$ is generically network identifiable if and only if for all j :

- There are $|\mathcal{P}_j|$ vertex disjoint paths from external excitation signals $(r, e)^1$ to the node signals in $\mathcal{P}_j$

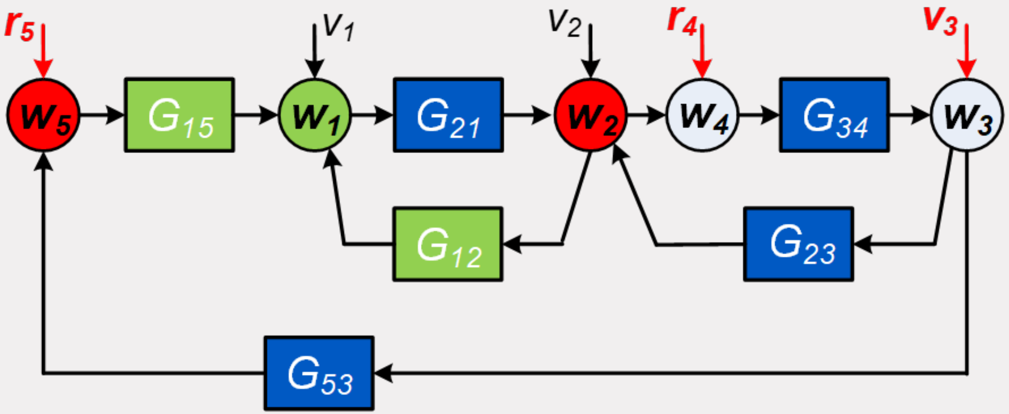


[1] that appear nonparametrized in w_j

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC-2019
[3] Weerts et al, SYSID2015; Weerts et al., Automatica, March 2018; Weerts et al CDC 2018.

Example 5-node network

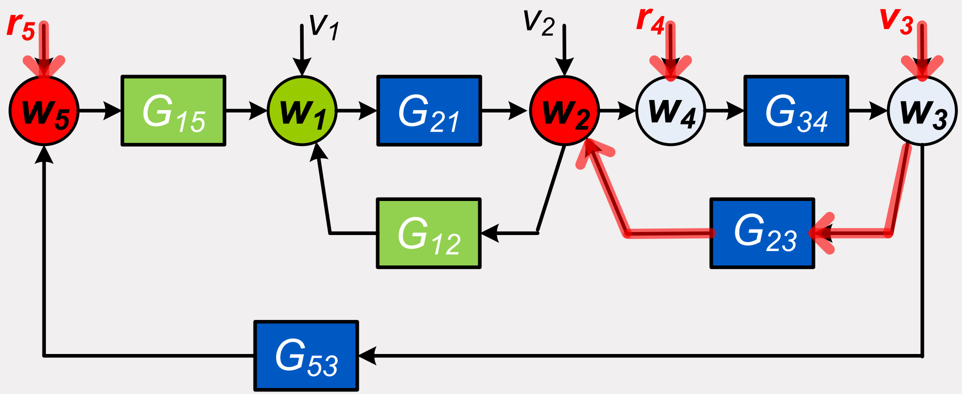
Verifying the rank condition for w_1 :



$j = 1$: Evaluate the number of vertex disjoint paths $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

Example 5-node network

Verifying the rank condition for w_1:



$$\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \longrightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$$

$\boxed{\text{2 vertex-disjoint paths} \rightarrow \text{full row rank 2}}$ ✓

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017 [3] Weerts et al., CDC 2018

Generic identifiability

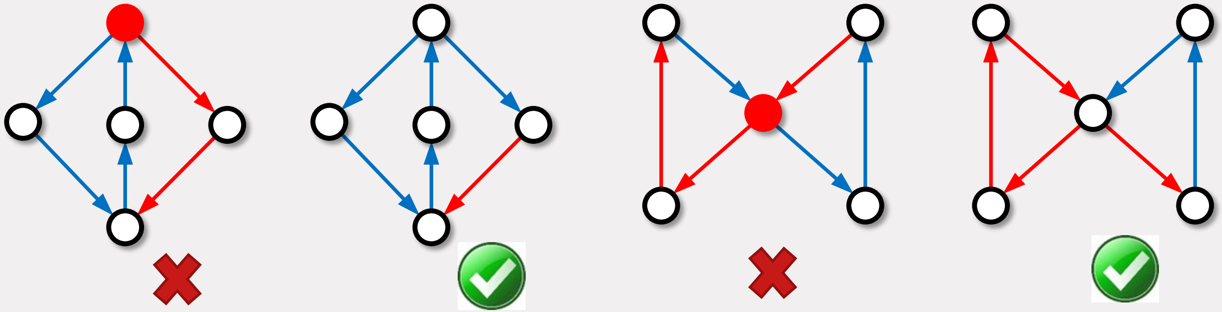
Result provides an analysis tool, but is less suited for the question:

Given a parametrized network model set: where to add external excitation signals so as to achieve generic network identifiability?

Graph-based synthesis solution for full network

Decompose network in disjoint pseudo-trees:

- Connected directed graphs, where nodes have maximum indegree 1
- Edges are disjoint and all out-neighbours of a node are in the same pseudo-tree



- Any network can be decomposed into a set of disjoint pseudo-trees

Graph-based synthesis solution for full network

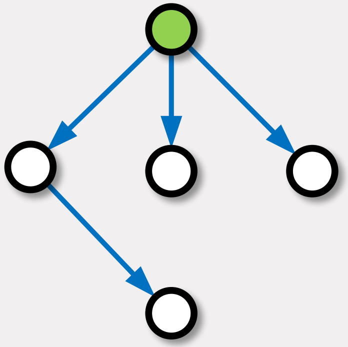
Result[1]

A network is generically identifiable if

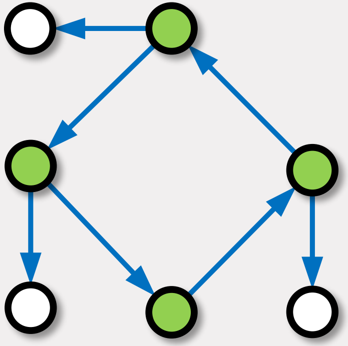
- It can be decomposed in K disjoint pseudo-trees, and
- There are K independent external signals entering at a **root** of each pseudo-tree

Two typical pseudo-trees:

Tree with root in green

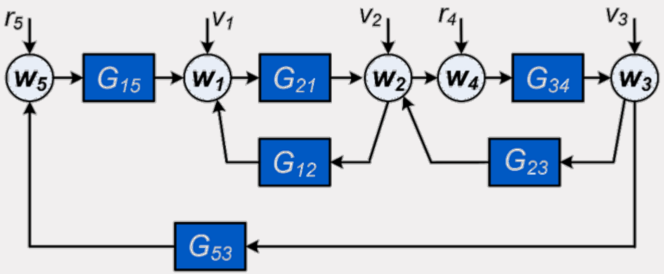


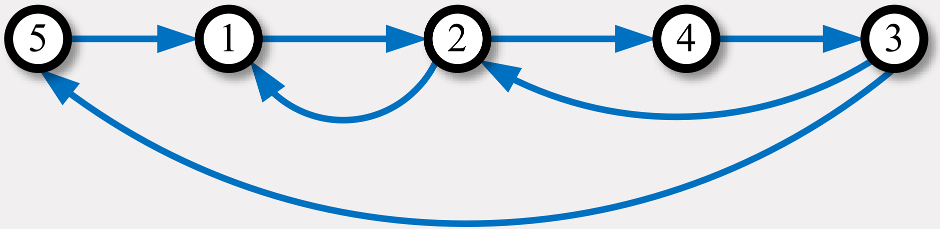
Cycle with outgoing trees;
Any node in cycle is root



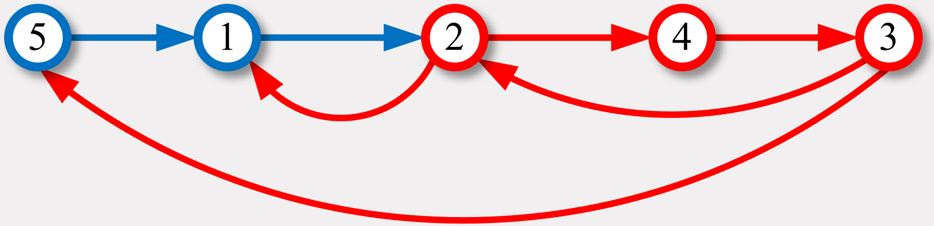
[1] X. Cheng, S. Shi and PVdH, CDC 2019.

Where to allocate external excitations for network identifiability?

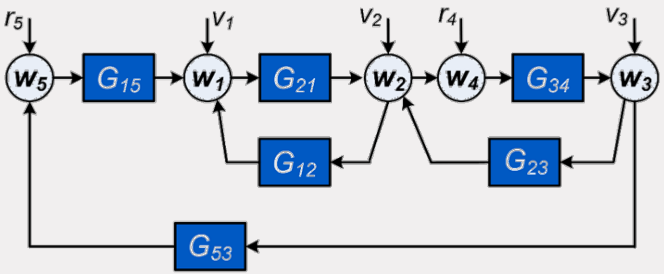


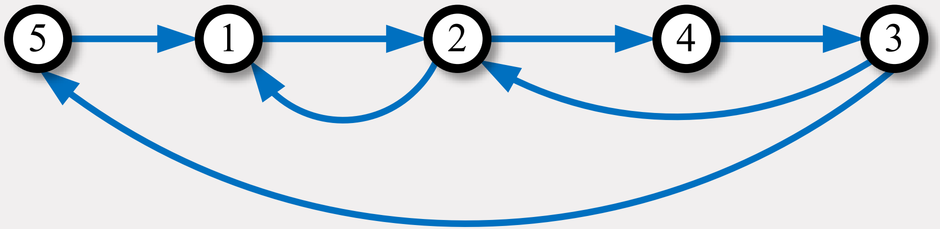


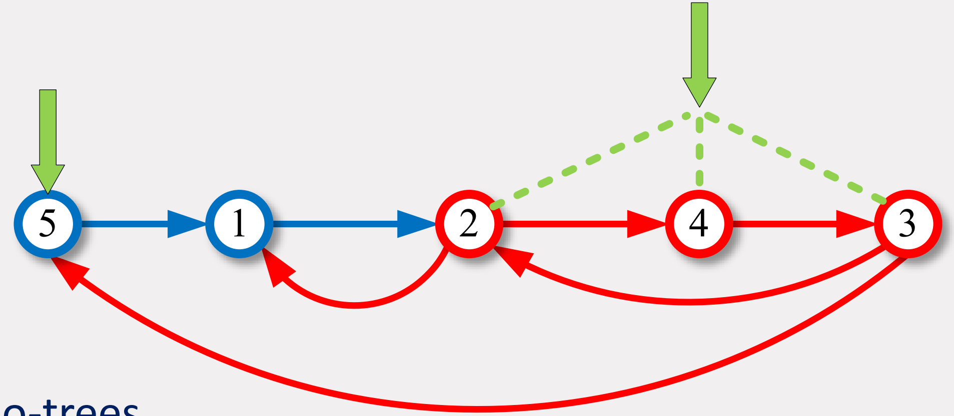
Two disjoint pseudo-trees



Where to allocate external excitations for network identifiability?



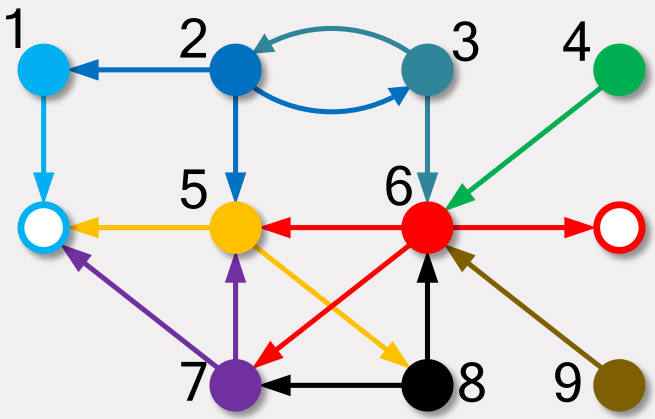


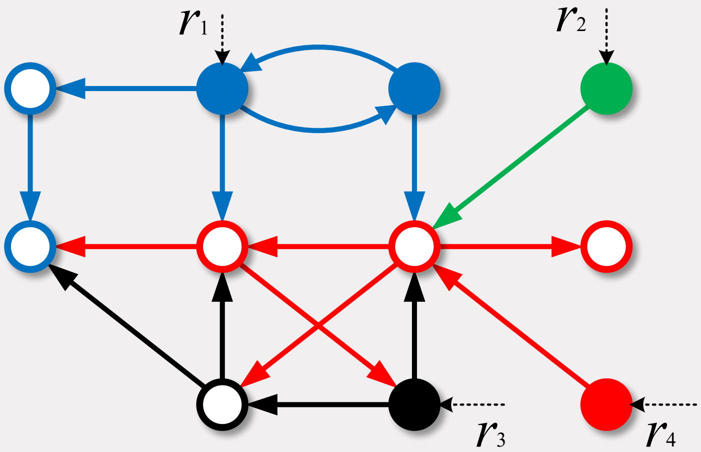


Two independent excitations guarantee network identifiability

Algorithm available for merging pseudo-trees.

Where to allocate external excitations for network identifiability?





- Nodes are signals w and external signals (r, e) when they are input to parametrized links
- Result extends to the presence of known (nonparametrized links): they can be excluded from the covering

Summary identifiability

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Prior knowledge on modules

- Graphic-based tool for synthesizing allocation of external excitation signals

So far:

- All node signals assumed to be measured
- Fully applicable to the situation $p < L$ (i.e. reduced-rank noise)
- Extensions towards identifiability of a single module [1],[2],[3]

[1] Hendrickx, Gevers & Bazanella, CDC 2017, IEEE-TAC 2019
[2] Weerts et al., CDC 2018
[3] Shi et al., IFAC 2020 submitted

Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Identifiability of linear dynamic networks. *Automatica*, 89, pp. 247-258, March 2018.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Single module identifiability in linear dynamic networks. Proc. 57th IEEE CDC 2018, ArXiv 1803.02586.
- X. Cheng, S. Shi and P.M.J. Van den Hof (2019). Allocation of excitation signals for generic identifiability of linear dynamic networks. Proc. 2019 CDC. ArXiv 1910.04525.
- K.R. Ramaswamy, G. Bottegal and P.M.J. Van den Hof (2018). Local module identification in dynamic networks using regularized kernel-based methods. Proc. 57th IEEE CDC 2018.
- P.M.J. Van den Hof, K.R. Ramaswamy, A.G. Dankers and G. Bottegal (2019). Local module identification in dynamic networks with correlated noise: the full input case. Proc. 2019 CDC. ArXiv 1809.07502.
- K.R. Ramaswamy and P.M.J. Van den Hof (2019). A local direct method for module identification in dynamic networks with correlated noise. Proc. CDC 2019. ArXiv:1908.00976

Papers available at www.pvandenhof.nl

The end