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Sensor Configuration Problem: Application to a Membrane Separation Unit

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Introduction

- The effectiveness of state and parameter estimation algorithms depends on which variables are measured.
- The *degree of observability* measures can be utilized for comparing different output channels.
- These measures are defined from the spectral properties of *observability gramian*,
  - Minimum eigenvalue
  - Determinant
  - Trace
  - Condition number
- For nonlinear systems, gramian is calculated by;
  - Linearization based techniques;
  - Lie algebra (geometric) based techniques;
  - Data-driven techniques.

Problem Statement: For large-scale models with differential-algebraic equations (DAEs), configure the sensor allocation such that the system is observable and (locally) identifiable.

Methodology

- We use data-based techniques to construct the *empirical observability gramian*, by using the input-output data.
- By perturbing the initial states or parameters, the correlation between the output measurements is used for the gramian.

$$\hat{W}_o^i(t_0,t_f) := \sum_{l=1}^{r}\sum_{m=1}^{s}\frac{1}{r}\frac{1}{s}\frac{1}{c_m^2}\int_{t_0}^{t_f} T_l \Psi^{lm}(t,i) T_l^T\,dt, \quad (1)$$

$$T^{r,n_x} = \{T_1,\dots,T_r;\ T_l \in \mathbb{R}^{n_x\times n_x},\ T_l^T T_l = I,\ l=1,\dots,r\},$$

$$M = \{c_1,\dots,c_s;\ c_s \in \mathbb{R},\ c_m > 0,\ m=1,\dots,s\},$$

$$\Psi_{jk}^{lm}(t,i)(t) = (y_i^{jlm}(t) - \bar{y}_i(t))^T (y_i^{klm}(t) - \bar{y}_i(t)),$$

- $\bar{y}_i(t)(\bar{x}_0)$ is the unperturbed output (initial condition),
- $y_i^{jlm}(t)$ is the output from the perturbed initial condition $x^{jlm}(0) = c_m T_l e_j + \bar{x}_0$,
- Both initial conditions are driven with the same input signal.

Observation:

- In literature $\bar{y}$ is taken as the steady state value.
- For unstable systems, we use the unperturbed trajectory.
- For smooth dynamics, empirical observability gramian converges to the true observability gramian.

Case Study

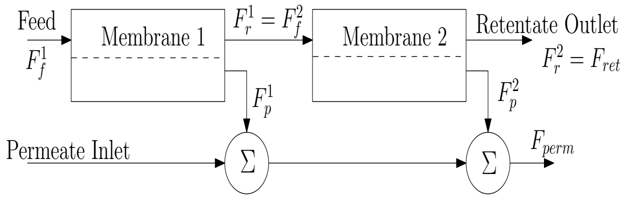

Figure 1: Two membrane system, the dynamics are taken from [1].

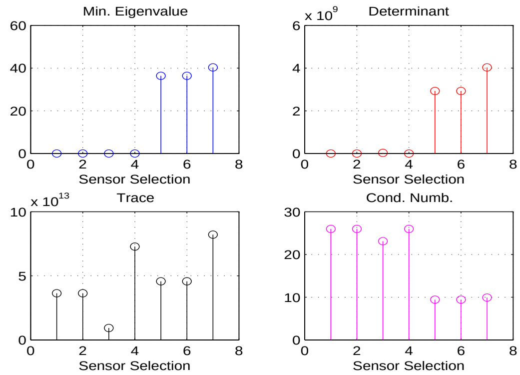

Figure 2: DoO measures for different sensor selections measuring F_r^1, F_r^2, F_{perm}.

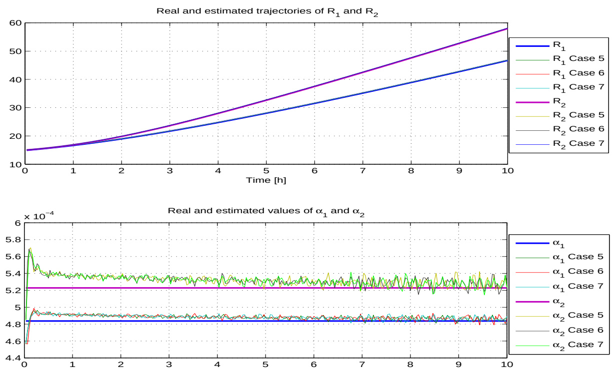

Figure 3: State and parameter estimation results.

Conclusion: For complex, large-scale, DAE systems, detailed rigorous models can be efficiently utilized for sensor selection problem via empirical observability gramian.

References

[1] Guadix, Antonio, et al. "Optimal design and operation of continuous ultrafiltration plants." Journal of membrane science 235(1) (2004): 131-138.