TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Identification in interconnected systems – modeling, structural aspects and MATLAB toolbox

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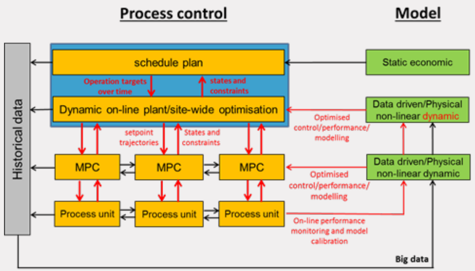
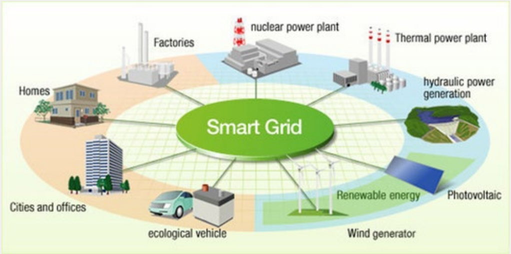
Introduction – dynamic networks

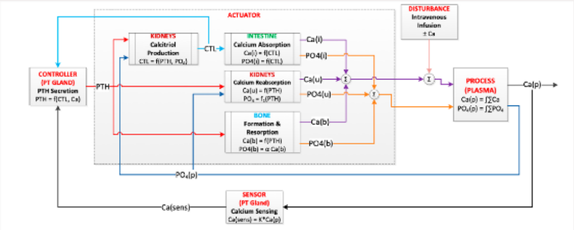
Decentralized process control



Smart power grid

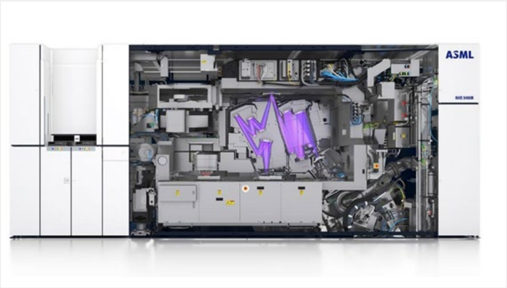


Physiological models

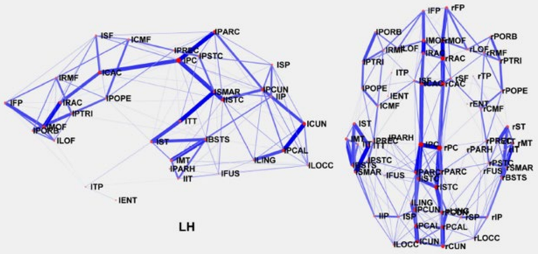


Christie, Achenie and Ogunnaike (2014)

Complex machines

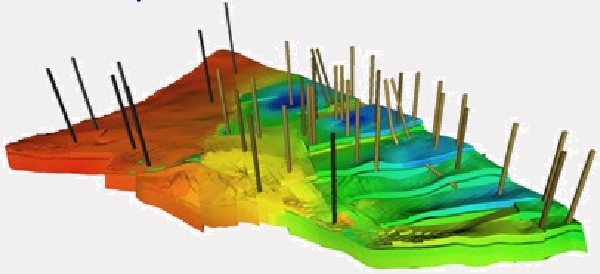


Brain network



P. Hagmann et al. (2008)

Hydrocarbon reservoirs



Mansoori (2014)

Introduction

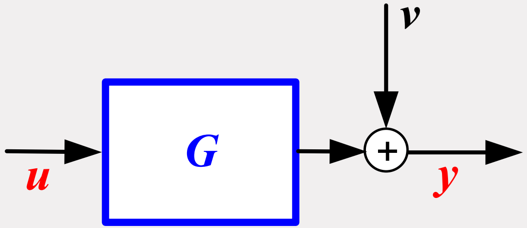
Overall trend:

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control, optimization, diagnostics
- Data is “everywhere”, AI/machine learning tools
- Model-based operations require accurate/relevant models
- → **Learning models/actions from data** (including physical insights when available)

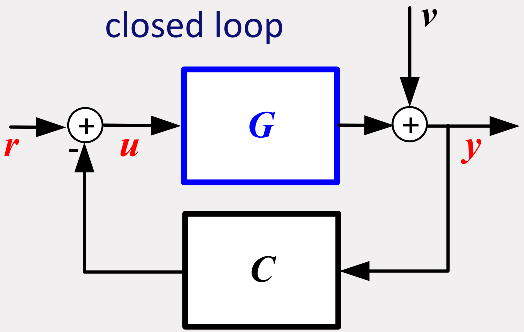
Introduction

The classical (multivariable) data-driven modeling problems[1] :

open loop



closed loop



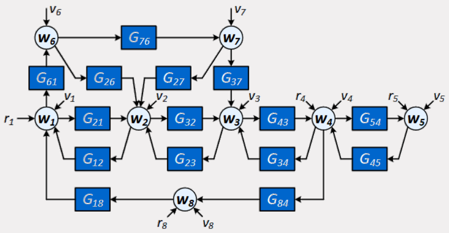
Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

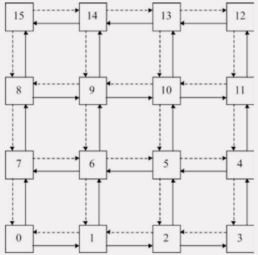
In interconnected systems (networks) the **structure / topology** becomes important to include

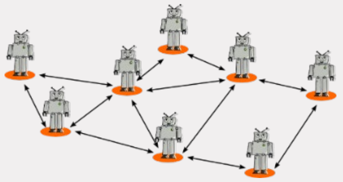
[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

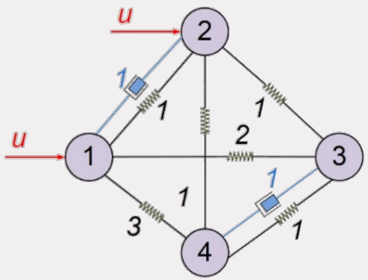
Network models

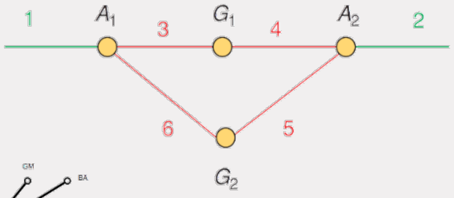
- dynamic elements with cause-effect
- handling feedback loops (cycles)
- centered around measured signals
- allow disturbances and probing signals

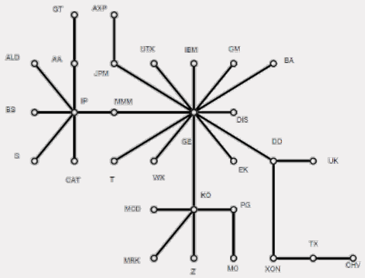


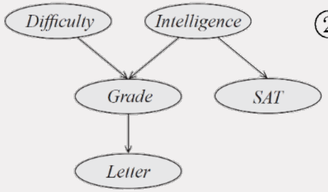


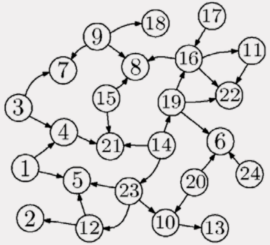


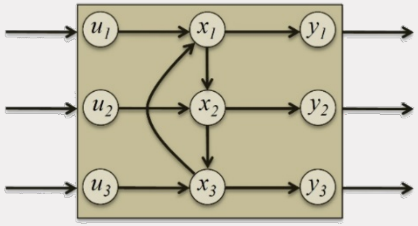


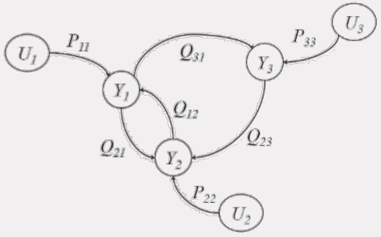






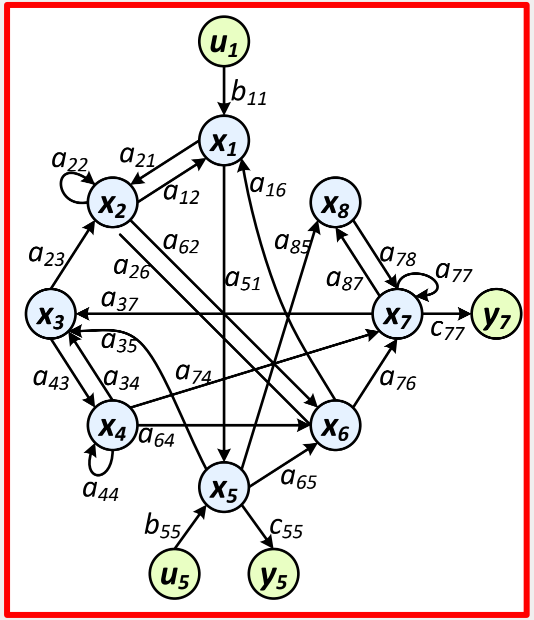






D. Materassi and M.V. Salapaka (2012)
www.momo.cs.okayama-u.ac.jp
J.C. Willems (2007)
E.A. Carara and F.G. Moraes (2008)
P.M.J. Van den Hof et al (2013)
X.Cheng (2019)
R.N. Mantegna (1999)
D. Koller and N. Friedman (2009)
P.E. Paré et al (2013)
E. Yeung et al (2010)

Network models



State space representation

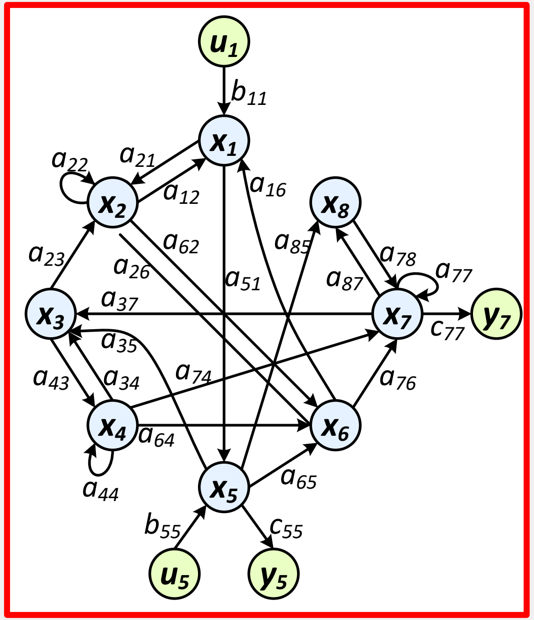
$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k) + Du(k)
\end{aligned}
$$

- States as **nodes** in a (directed) graph
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in links
- Ultimate break-down of system structure
- Actuation (u) and sensing (y) reflected by separate links

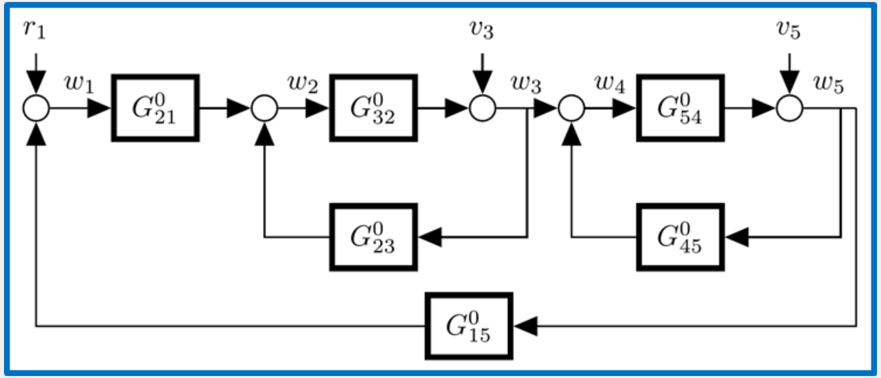
For data-driven modeling problems:

- Lump unmeasured states in dynamic **modules**

Network models



State space representation [1]

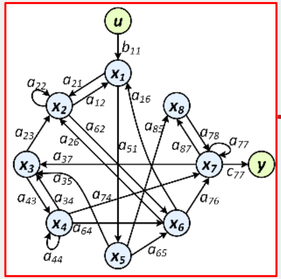


Module representation [2]

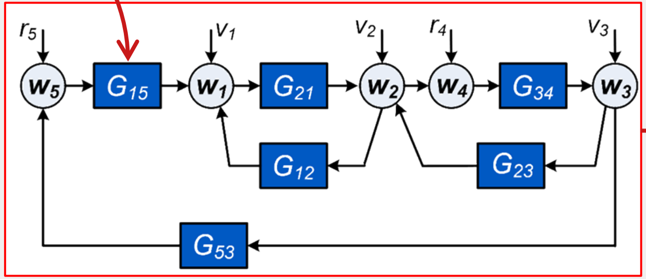
[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…

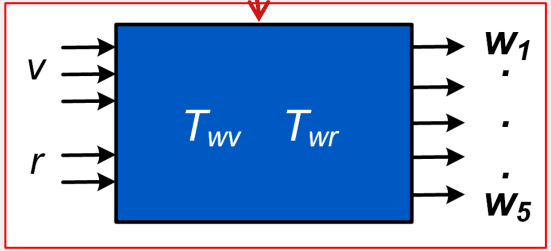
[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,…

Dynamic network models - zooming



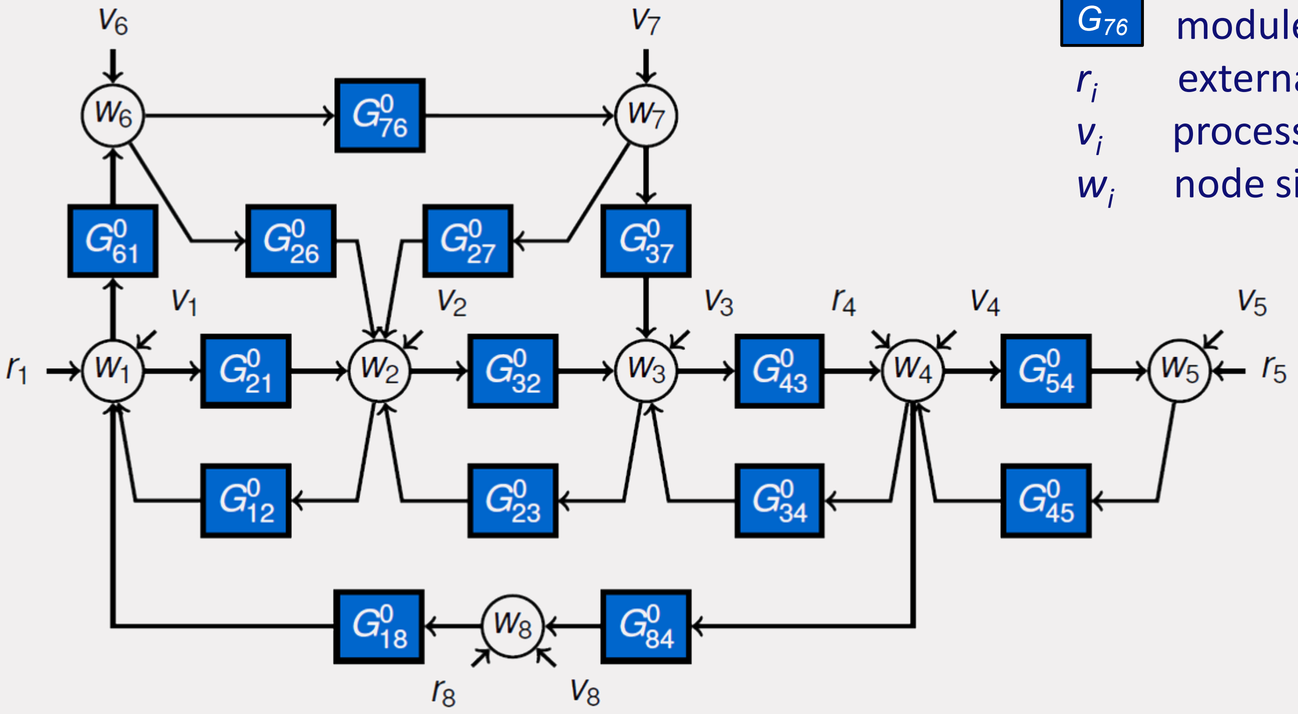
Increasing level of detail





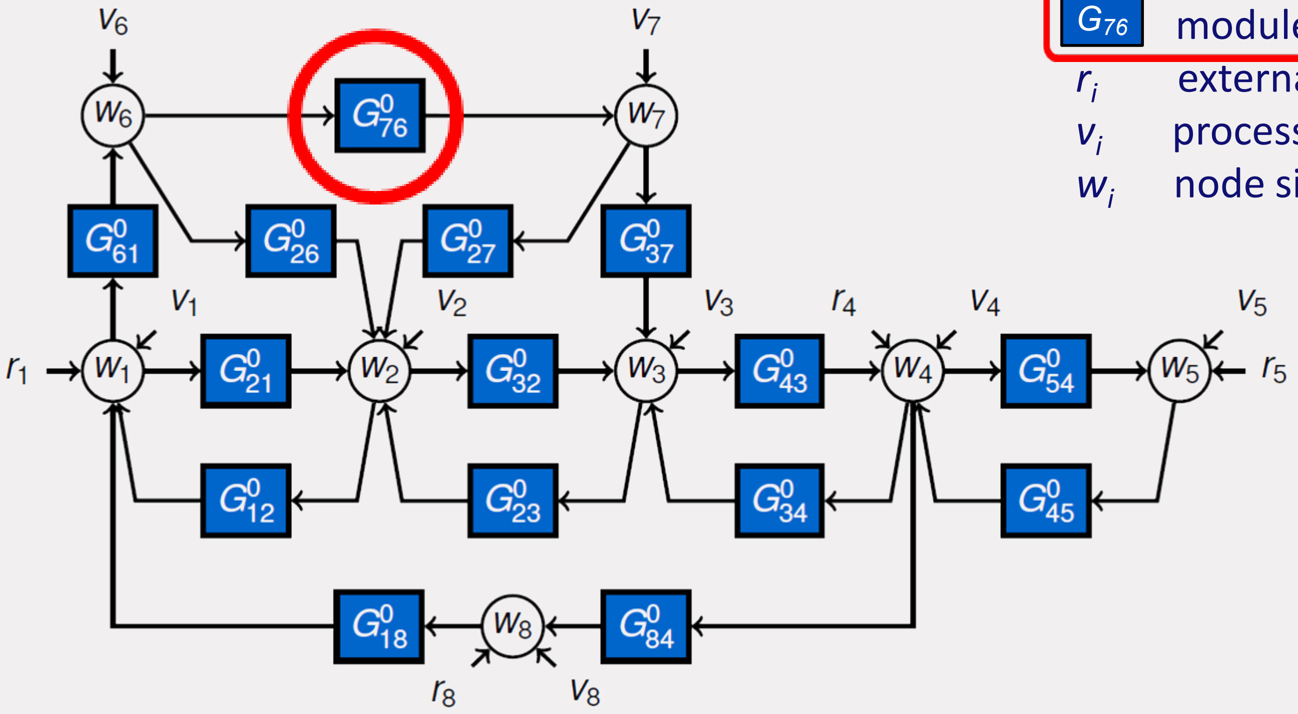
Decreasing structural information

Dynamic network setup



G_{76} module
r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup



G_{76} module
r_i external excitation
v_i process noise
w_i node signal

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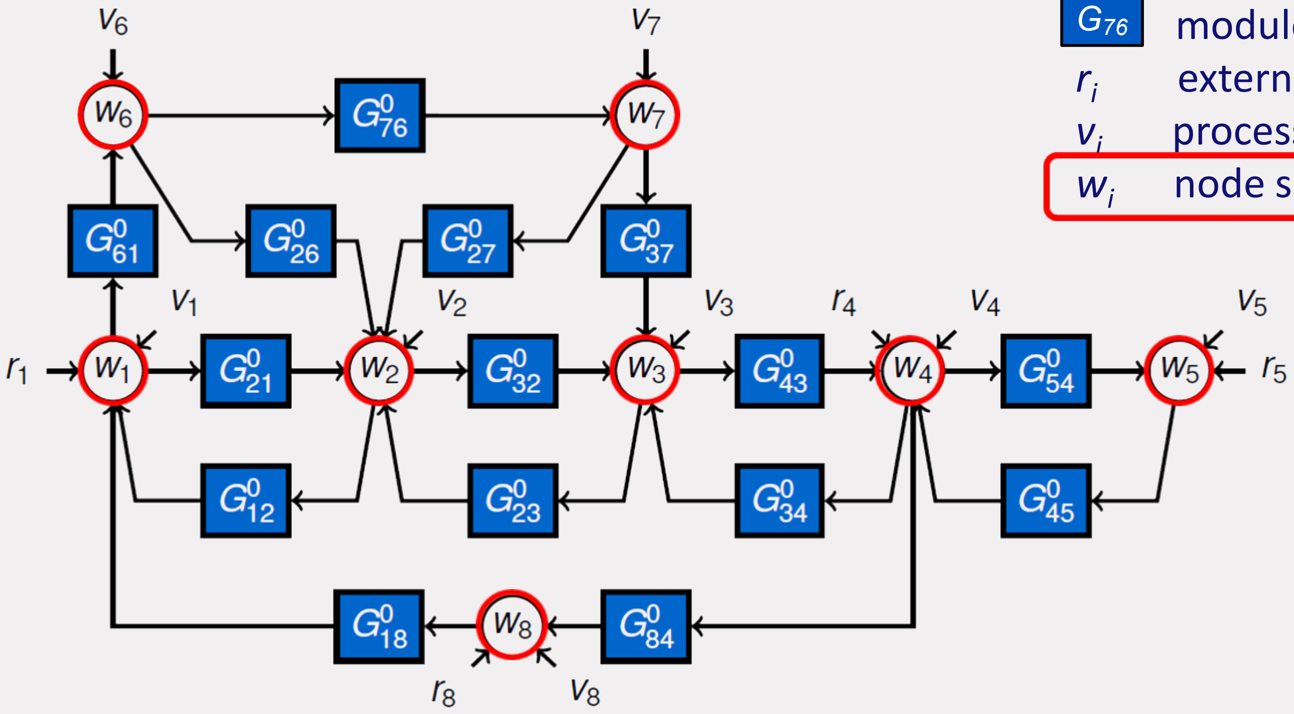
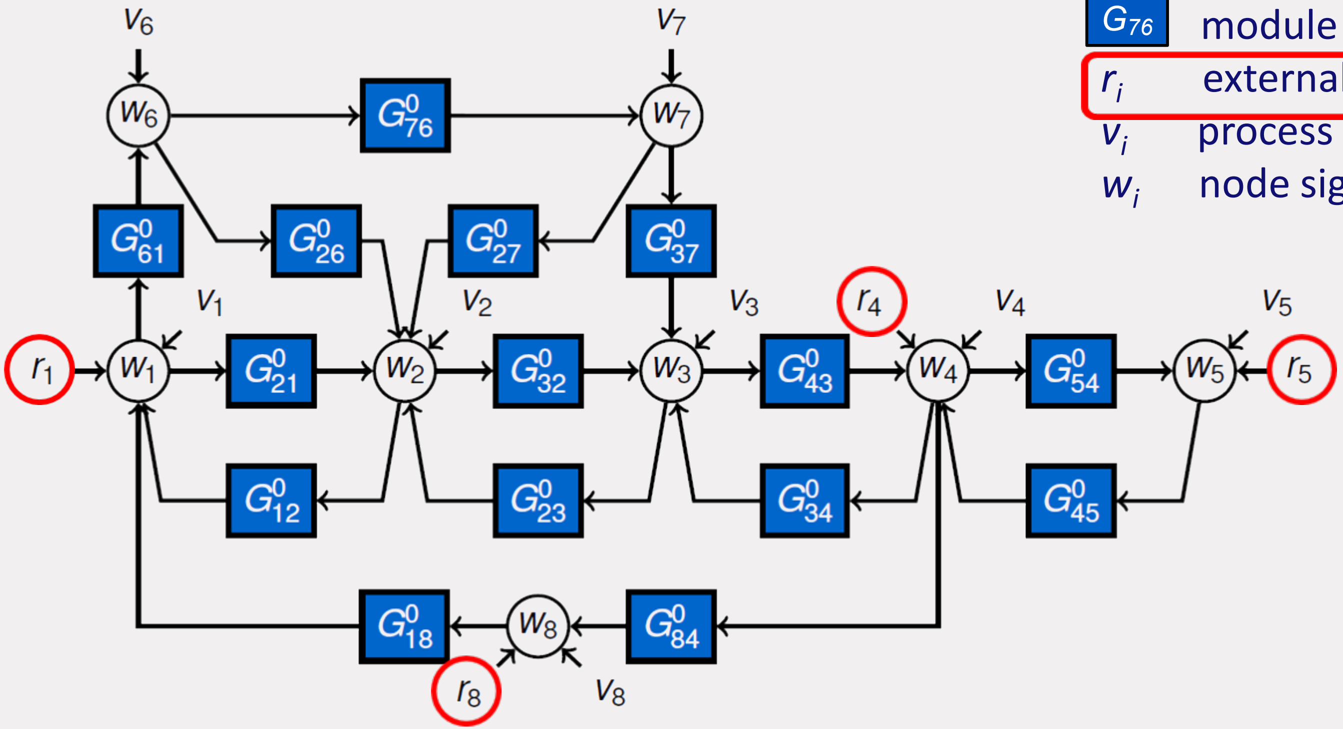
Dynamic network setup



G_{76}	module
r_i	external excitation
v_i	process noise
w_i	node signal

Dynamic network setup



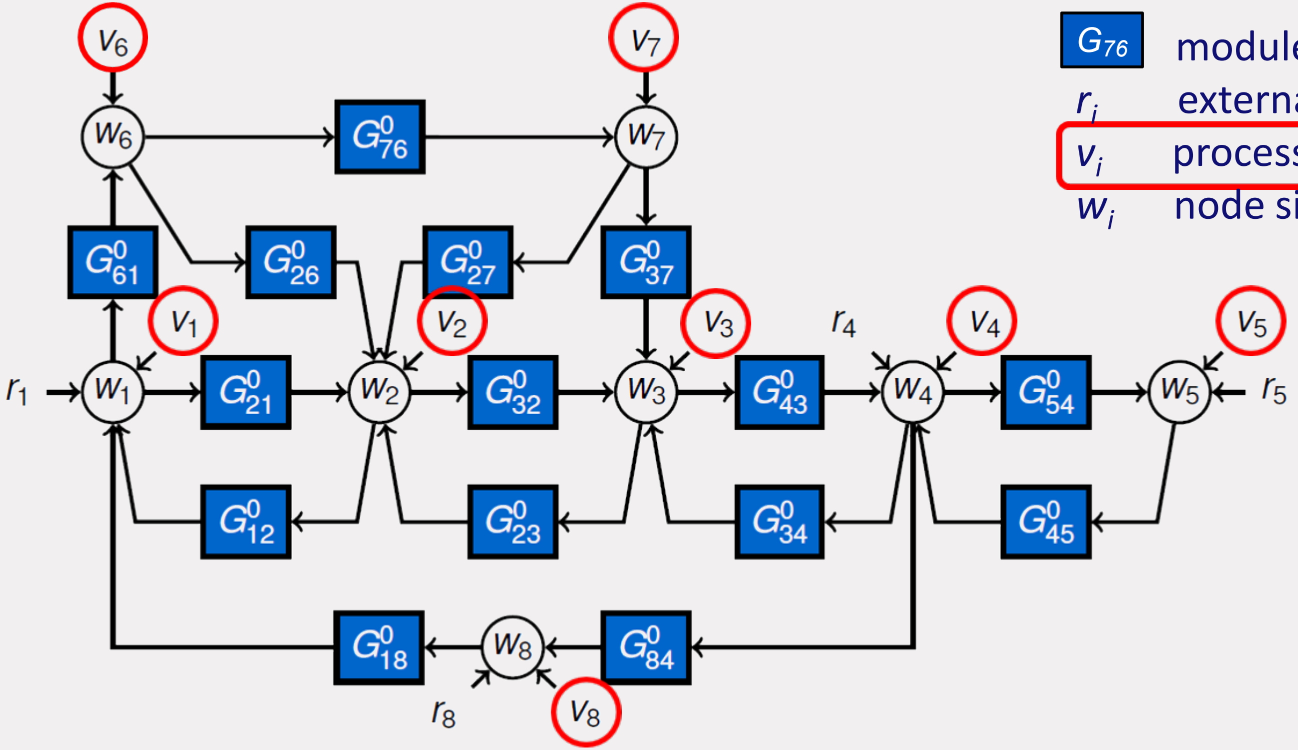
G_{76} module

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



G_{76} module

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup

Collecting all equations:

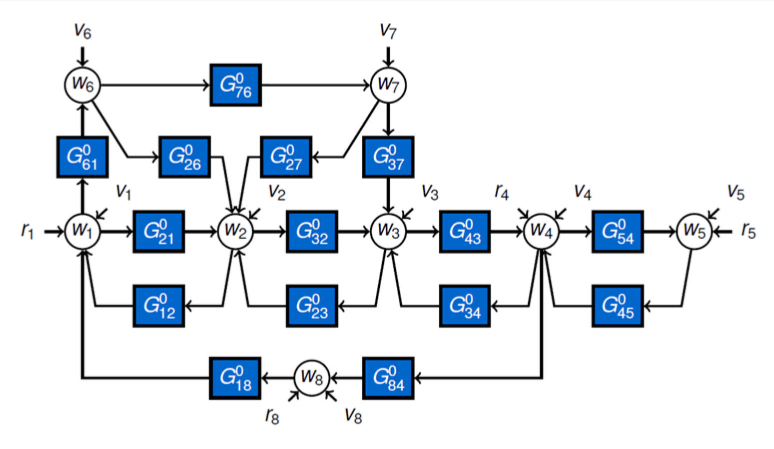
$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + \underbrace{R^0(q)r(t)}_{u(t)} + v(t); \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r and e are called external signals.

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

Dynamic network setup



Measured time series:

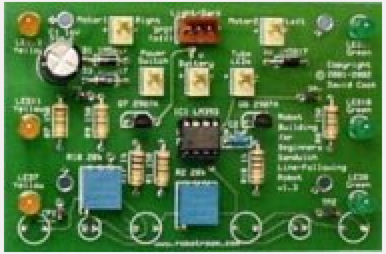
$$\{w_i(t)\}_{i=1,\cdots L};\ \ \{r_j(t)\}_{j=1,\cdots K}$$

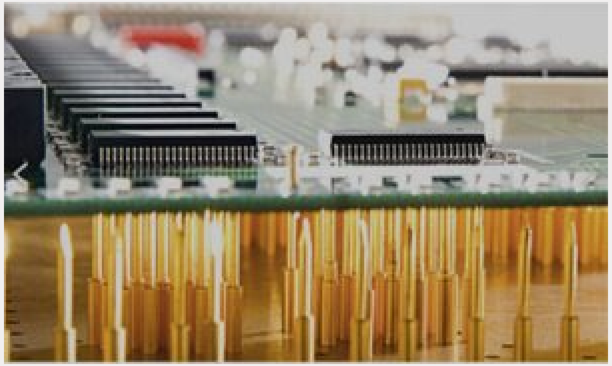
Many challenging data-driven modeling and diagnostics challenges appear

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Diagnostics and fault detection
- Distributed identification

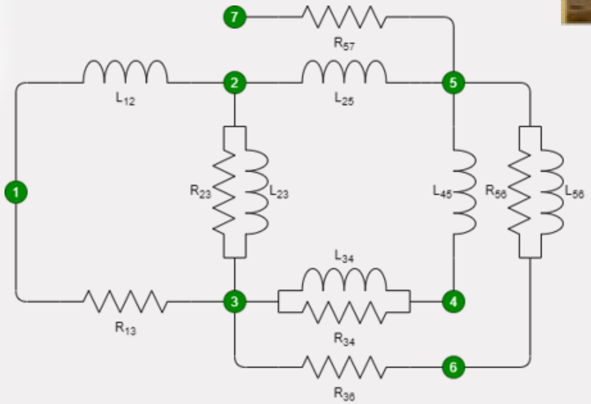
- User prior knowledge of modules
- Scalable algorithms

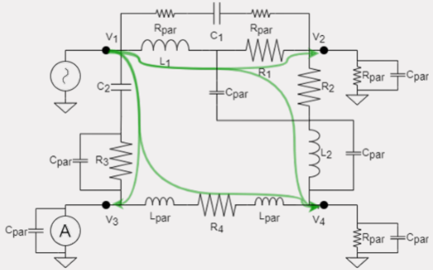
Application: Printed Circuit Board (PCB) Testing





Source: Altium

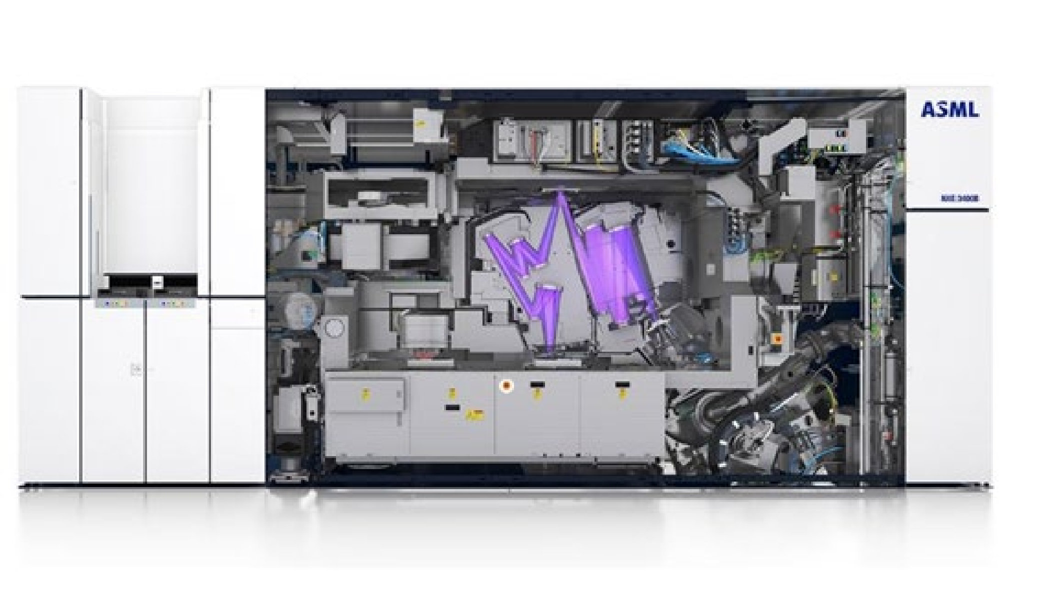




Detection of

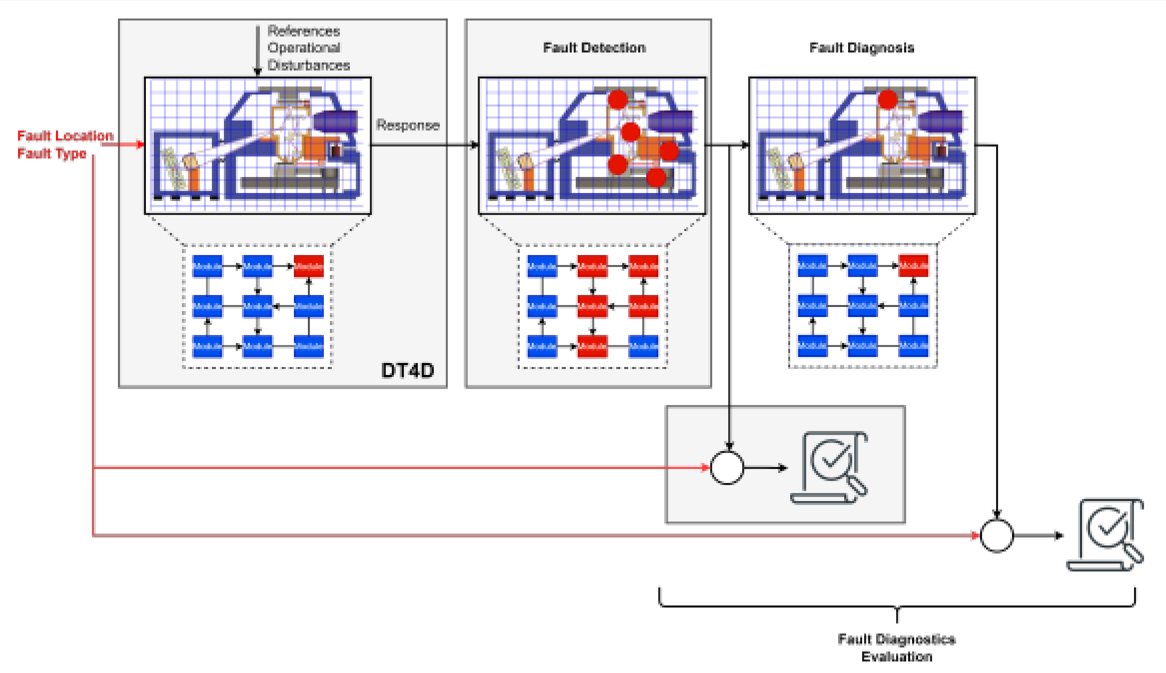
- component failures
- parasitic effects

Application: diagnostics in lithography waferscanners



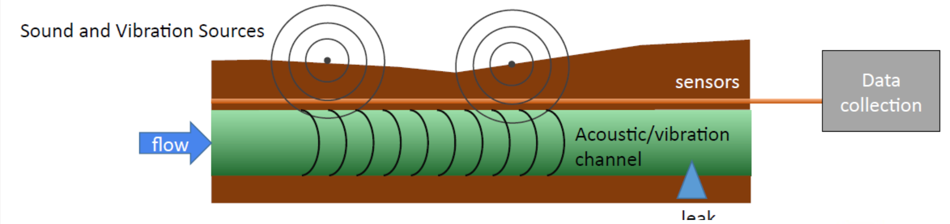
- 200M€ machine
- Highly complex machine dynamics
- Many interconnected subsystems
- Need for very fast recovery from faults
- Tools for automated diagnostics

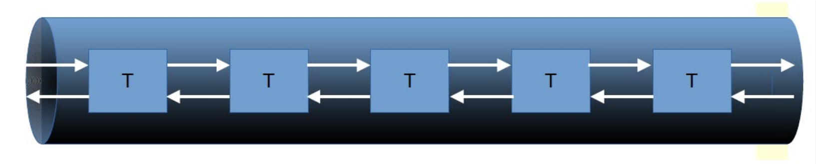
Application: diagnostics in lithography waferscanners

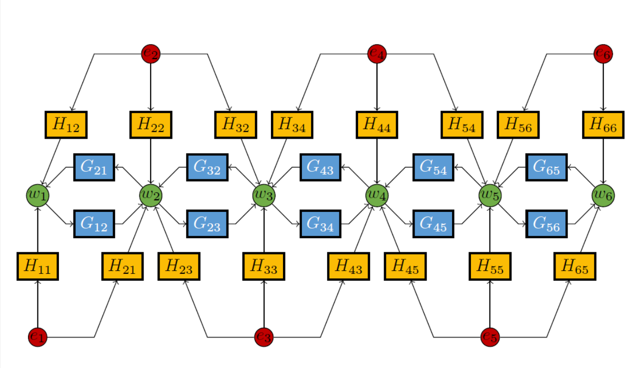


- Digital twin, for detection and diagnosis
- Exploit interconnection structure

Leak detection in gas pipelines with acoustic sensors



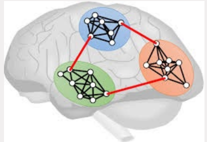




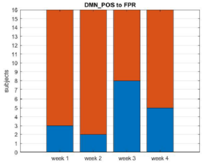
- Use operational data to detect changes in network model dynamics
- Map model changes to physical causes

Slide figures from Arne Dankers, Hifi Engineering Inc., Calgary, Canada (ECC Tutorial session 2021)

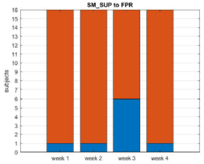
Neurodynamic effect of listening to Mozart music



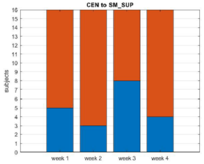
Identifying changes in network connections in the brain, after intensely listening for one week (Sonate K448), based on fMRI data



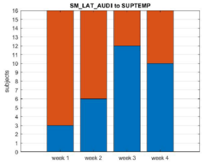
(a) Connection from the posterior default mode network to the fronto-parietal right network.



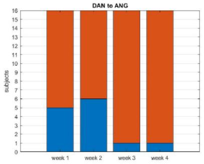
(b) Connection from the sensori-motor superior network to the fronto-parietal right network.



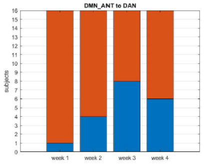
(c) Connection from the central executive network to the sensori-motor superior network.



(d) Connection from the lateral sensori-motor network to the superior temporal gyrus.



(e) Connection from the dorsal attention network to the angular gyrus.



(f) Connection from the anterior default mode network to the dorsal attention network.

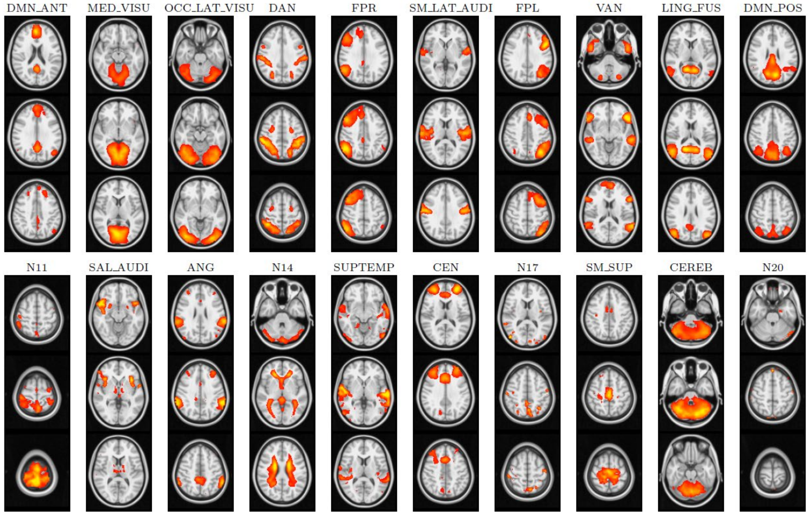
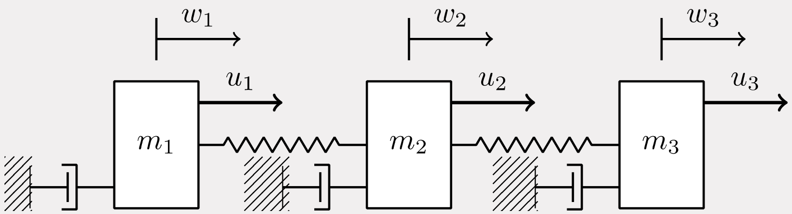


Figure 3: Spatial maps of the 20 active brain networks found through the ICA decomposition. Each image consists of 3 relevant horizontal slices of the brain, where the spatial map is indicated by the red color scale.

[1] R. van Esch, S. Shi, A. Bernas, S. Zinger, A. Aldenkamp, PVdH, *Computers in Biology and Medicine*, *Vol. 127*, December 2020

Alternative models

In connecting physical systems, there is often no predetermined direction of information [1]



Example: resistor / spring connection in electrical / mechanical system:

Resistor

$$I = \frac{1}{R}(V_1 - V_2)$$

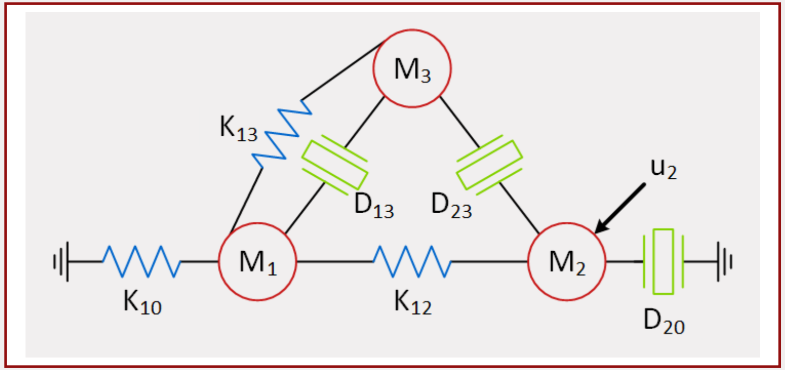
Spring

$$F = K(x_1 - x_2)$$

Difference of node signals drives the interaction: **diffusive coupling**

Alternative models

Diffusively coupled networks:



$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & D_{20} & \\ & & 0 \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{10} & & \\ & 0 & \\ & & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$+ \begin{bmatrix} D_{13} & 0 & -D_{13} \\ 0 & D_{23} & -D_{23} \\ -D_{13} & -D_{23} & D_{13}+D_{23} \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} + \begin{bmatrix} K_{12}+K_{13} & -K_{12} & -K_{13} \\ -K_{12} & K_{12} & 0 \\ -K_{13} & 0 & K_{13} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix}$$

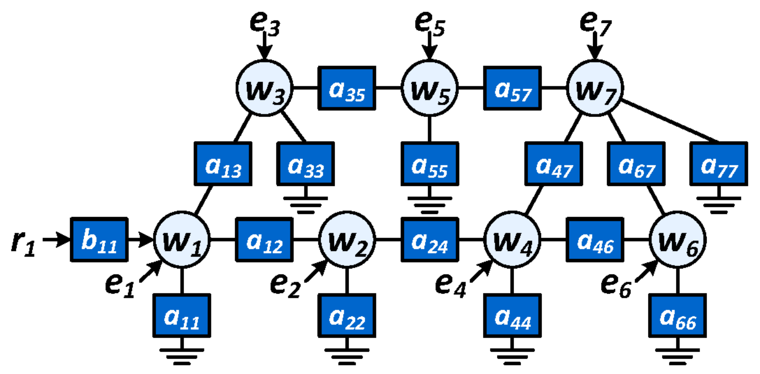
$$M_j \ddot{w}_j(t) + D_{j0}\dot{w}_j(t) + \sum_{k \neq j} D_{jk}(\dot{w}_j(t) - \dot{w}_k(t)) + K_{j0} w_j(t) + \sum_{k \neq j} K_{jk}(w_j(t) - w_k(t)) = u_j(t),$$

$$[\ \underbrace{A(p)}_{diagonal} + \underbrace{B(p)}_{Laplacian}\ ]\, w(t) = u(t) \qquad A(p), B(p) \ \text{polynomial} \qquad p = \frac{d}{dt}$$

Alternative models

Diffusively coupled networks

The related graph is bi-directional:



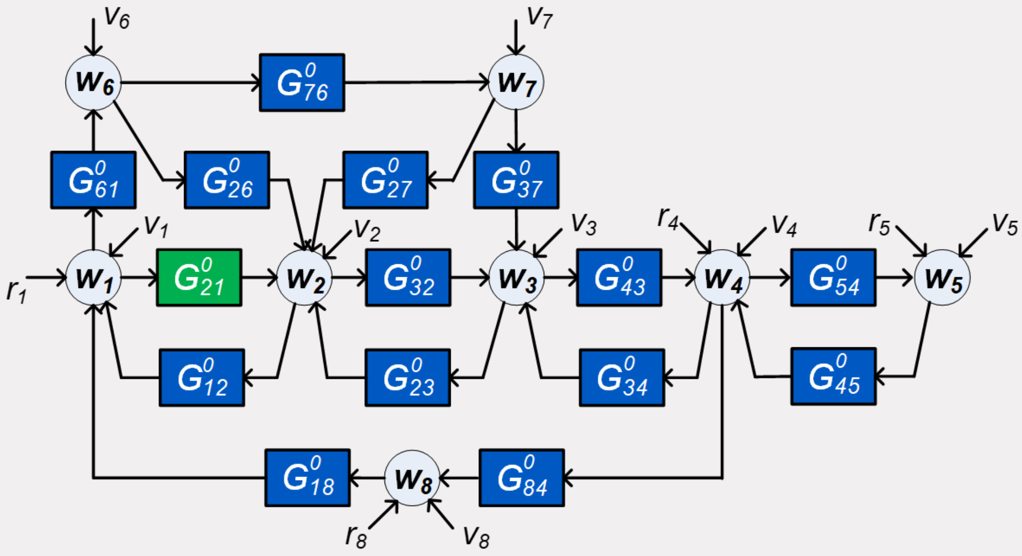
$$[\ \underbrace{Q(p)}_{diagonal} - \underbrace{P(p)}_{hollow\&symmetric}\ ]\, w(t) = u(t)$$

Q, P polynomial

Single module identification

known topology

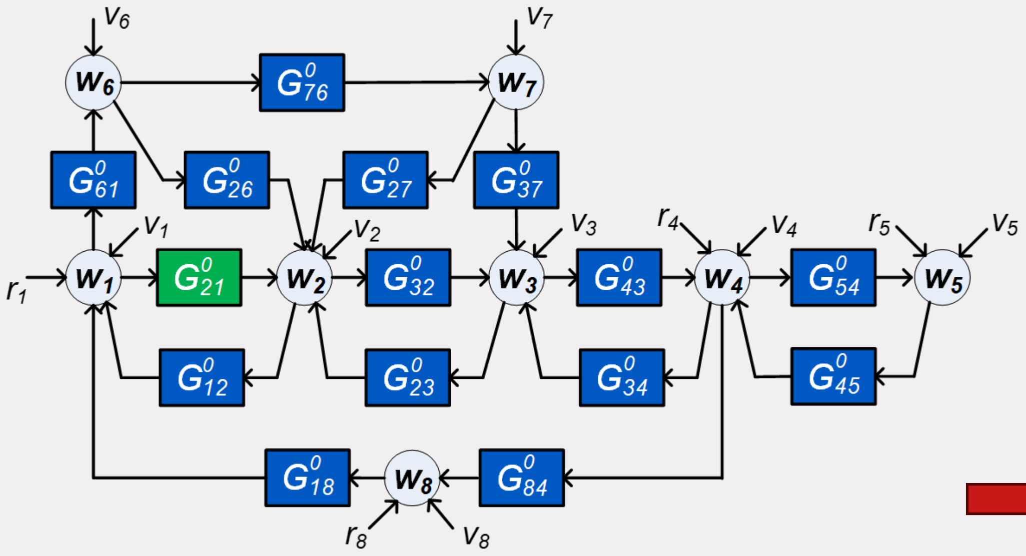
Single module identification



For a network with
known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?

Single module identification



Different types of methods:

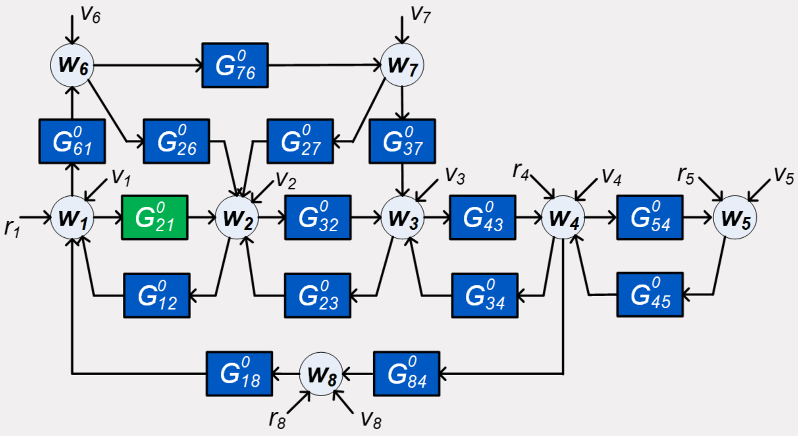
Indirect methods:

- Rely on mappings $\boldsymbol{r} \rightarrow \boldsymbol{w}$ and on sufficient excitation signals $\boldsymbol{r}$

Direct methods:

- Rely on mappings $\boldsymbol{w} \rightarrow \boldsymbol{w}$ and use excitation from both $\boldsymbol{r}$ and $\boldsymbol{v}$ signals

Single module identification



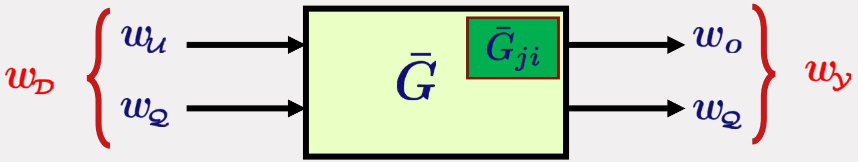
Local direct method:

(consistency and minimum variance properties)

Select a subnetwork:

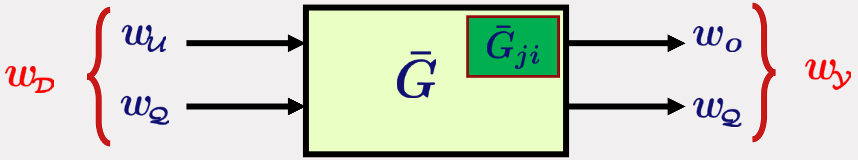
- Predicted outputs: $w_{\mathcal{Y}}$
- Predictor inputs: $w_{\mathcal{D}}$

such that prediction error minimization leads to an accurate estimate of G^0_{21}



Note: same node signals can appear in input and output

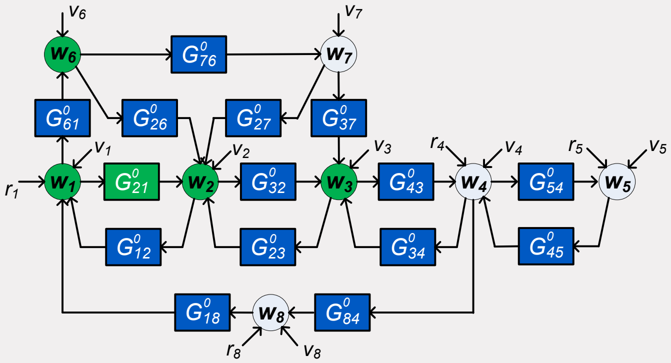
Single module identification

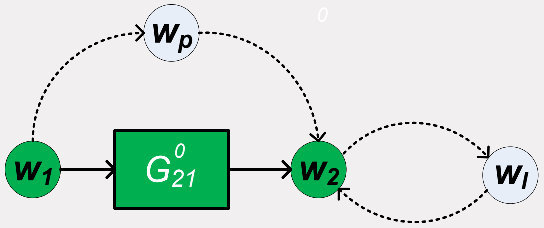


Conditions for arriving at an accurate (consistent) model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$ when removing discarded nodes (immersion)
2. Handling of confounding variables
3. Data-informativity
4. *Technical condition on presence of delays*

Single module identification - module invariance





A sufficient condition for module invariance:

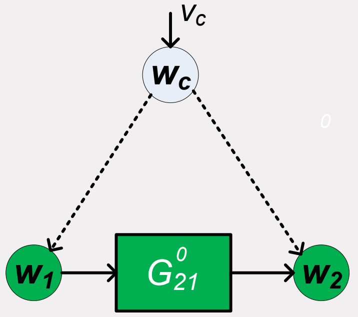
All parallel paths, and loops around the output,
should be "blocked" by a measured node that is present in $w_{\mathcal{D}}$

All other signals can be removed/immersed from the network[2]

Alternative graph-based formulation in terms of disconnecting sets in [3]

[1] Dankers et al., TAC 2016
[3] Shi et al., Automatica 2022
[2] Generalizations available in Linder&Enqvist (2017), Weerts et al, (2020)

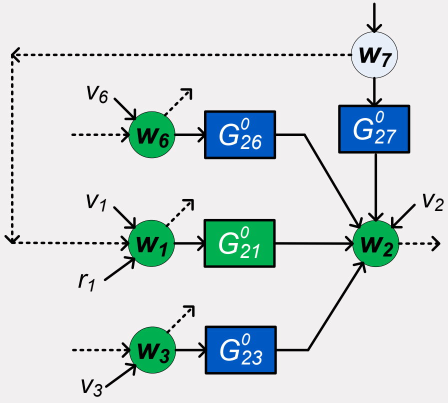
Single module identification - confounding variables



Confounding variable [1][2]**:**

Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.

In networks they can appear in two different ways:



Direct:

- If disturbances on inputs and outputs are correlated.

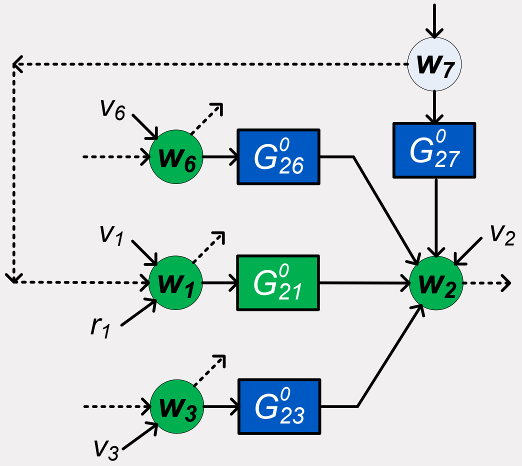
Indirect:

- If non-measured in-neighbors of an output affect signals in the inputs.

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Confounding variables

- **Direct** confounding variables



e.g., v_1 is correlated with v_2

In identification we know how to handle correlated disturbances: we model them!

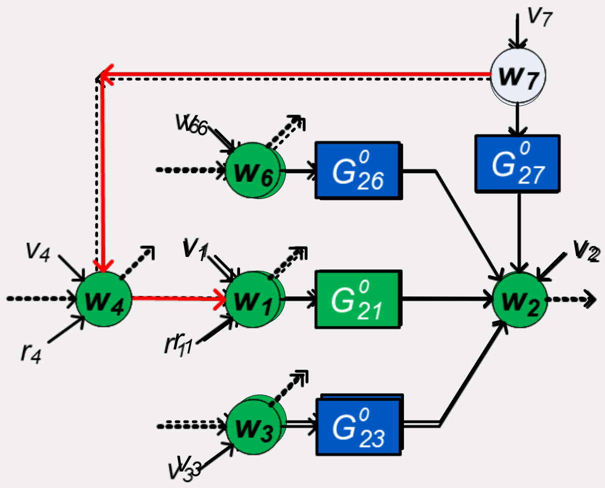
Solution:

Include w_1 as output and use a multivariate noise model

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

Confounding variables

- **Indirect** confounding variable:



Non-measurable w_7 is a confounding variable

Two possible solutions:

1. Include w_4 ➡ add predictor input

$$w_{\mathcal{D}} = \{w_1, w_3, w_4, w_6\} \quad w_{\mathcal{Y}} = \{w_2\}$$

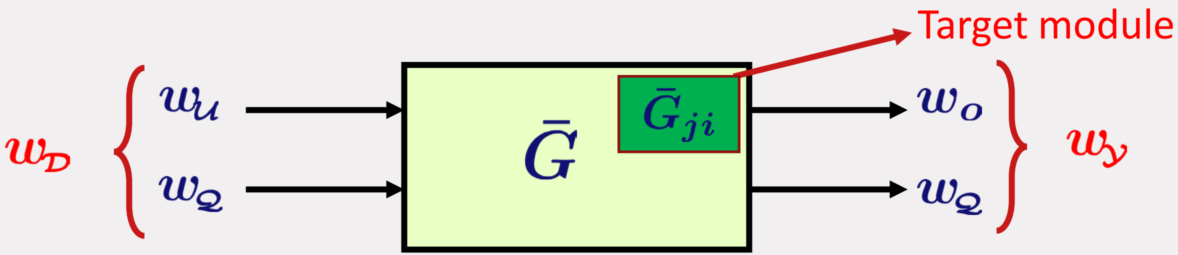
2. Predict w_1 too ➡ add predictor output

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

- There are degrees of freedom in choosing the predictor model

Direct method

General setup:

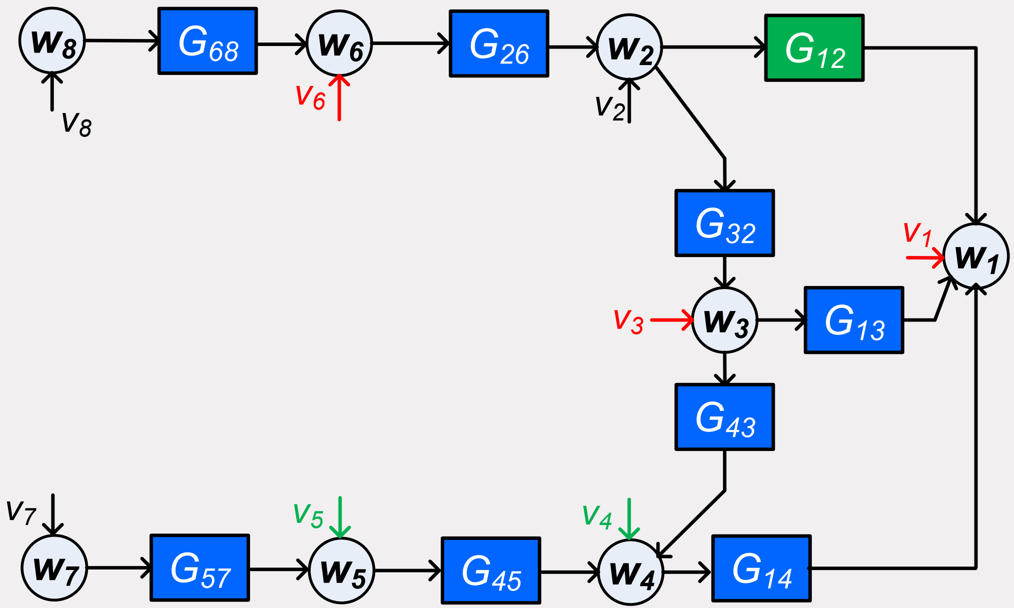


Different algorithms for arriving at predictor models:

- Full input case: include all in-neighbors of $w_{\mathcal{Y}}$
- Minimum measurement case: maximize number of outputs
- User selection case : dedicated choice based on measurable nodes

Different strategies – direct method

- Full input case
- Minimum measurements case
- User selection case



Network with v_1 correlated with v_3 and v_6.
v_4 correlated with v_5.

Full input case

We include all in-neighbors of the predicted outputs as predictor inputs

Maximum use of information in signals

$$w_{\mathcal{D}} = \{2,3,4\} \quad w_{\mathcal{Y}} = \{1\}$$

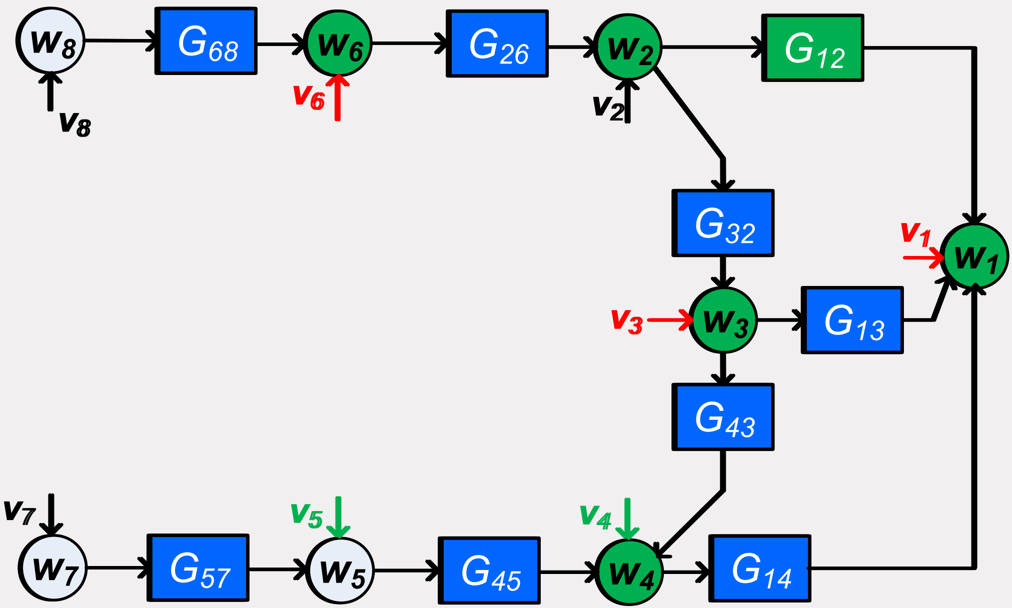
Handling direct confounding variable:

$$w_{\mathcal{D}} = \{2,3,4\} \quad w_{\mathcal{Y}} = \{1,3\}$$

Handling indirect confounding variable:

$$w_{\mathcal{D}} = \{2,3,4,6\} \quad w_{\mathcal{Y}} = \{1,3\}$$

Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$



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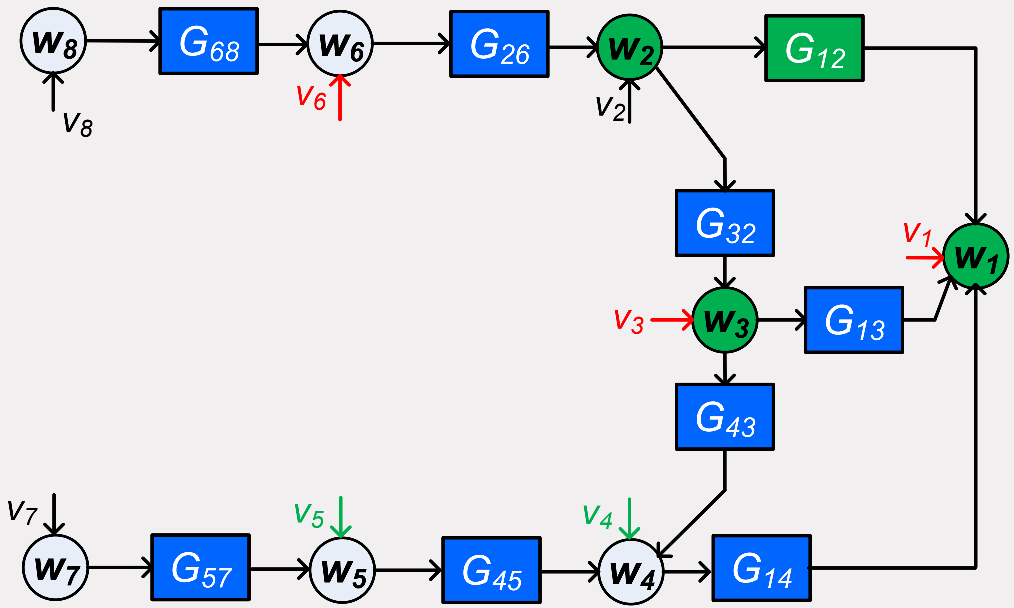
Minimum measurements case

- Select signals to satisfy the parallel path and loop condition
- Handle all confounding variables by including signals in output

$$w_{\mathcal{D}} = \{2, 3\} \quad w_{\mathcal{Y}} = \{1, 2, 3\}$$

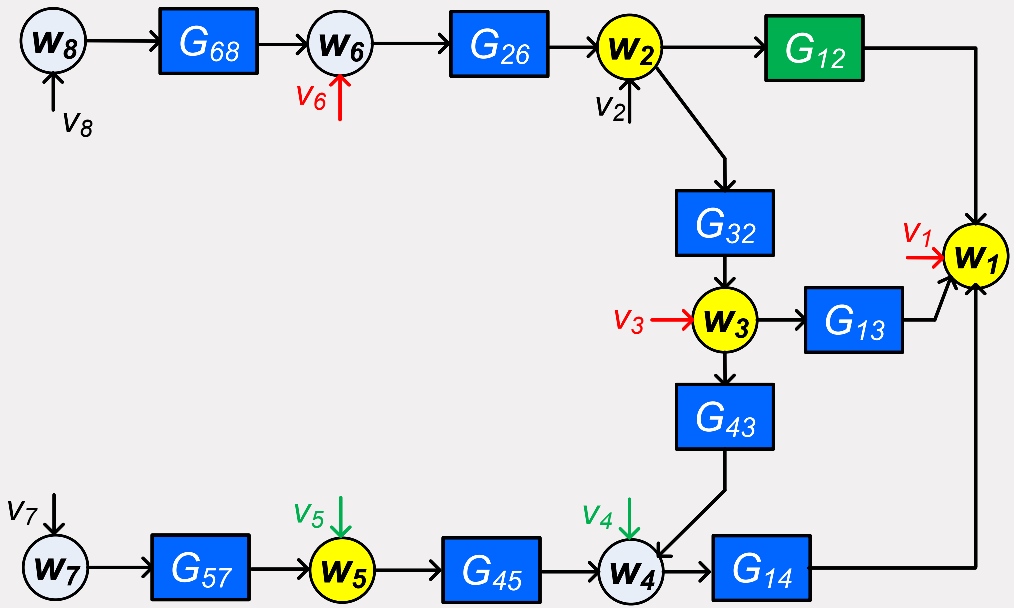


Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$

User selection case

- The user does not have access to all node signals
- Four node signals can be measured
- Parallel path and loop condition is satisfied
- Start with:

$$w_{\mathcal{D}} = \{2, 3\} \quad w_{\mathcal{Y}} = \{1\}$$



User selection case

$$w_{\mathcal{D}} = \{2,3\} \quad w_{\mathcal{Y}} = \{1\}$$

Handling direct confounding variable:

$$w_{\mathcal{D}} = \{2,3\} \quad w_{\mathcal{Y}} = \{1,3\}$$

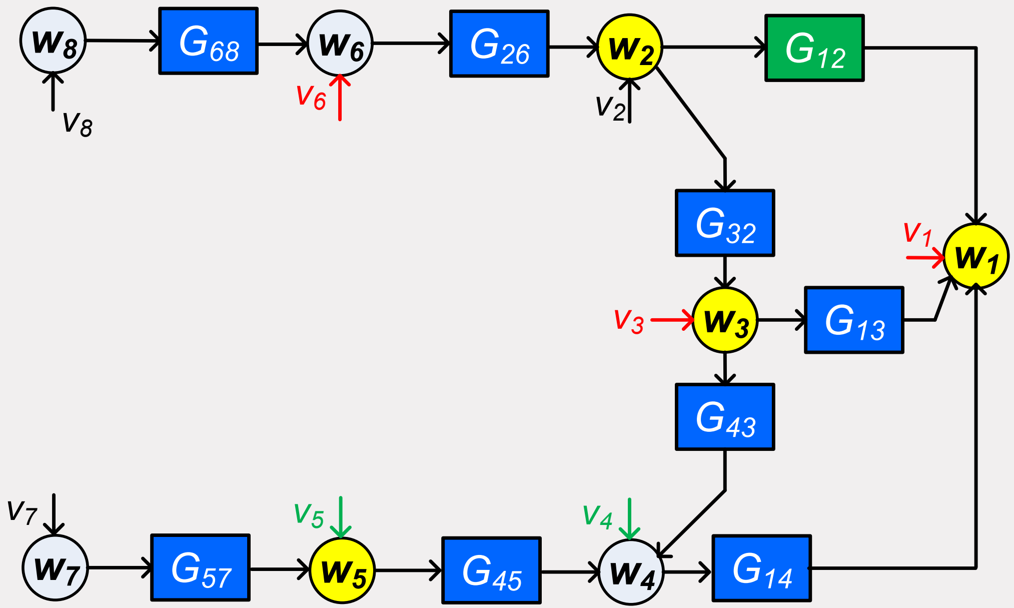
Indirect confounding variables:

(v_4, v_5):

$$w_{\mathcal{D}} = \{2,3,5\} \quad w_{\mathcal{Y}} = \{1,3,5\}$$

v_6:

$$w_{\mathcal{D}} = \{2,3,5\} \quad w_{\mathcal{Y}} = \{1,2,3,5\}$$



Direct identification $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$

Different strategies for same network and target module

Same network with different identification setups that lead to **consistent estimate of the target module** with **Maximum likelihood properties** based on the strategy used.

Full input case	Minimum measurements case	User selection case
$\begin{bmatrix} w_2 \\ w_3 \\ w_4 \\ w_6 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_3 \end{bmatrix}$	$\begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$	$\begin{bmatrix} w_2 \\ w_3 \\ w_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_5 \end{bmatrix}$

Data informativity conditions still need to be added.

Single module identification

Serious degrees of freedom in selecting the predictor model to satisfy the first two conditions:

1. Module invariance – PPL test
2. Handling confounding variables

While presuming that data-informativity can always be satisfied by adding sufficient # of r-signals.

Single module identification – data-informativity

When focusing on estimating only the row with target module $\bar{G}_{ji}$

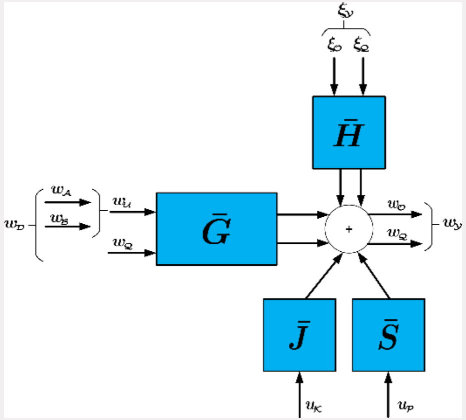
$$w_j(t) = \bar{G}_{j*}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}_{j*}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}_{j*}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}_{j*}u_{\mathcal{P}}(t)$$

Typical data-informativity condition for estimating $\bar{G}_{ji}$:

$\kappa^{[j]}$ persistently exciting $\boxed{\Phi_{\kappa^{[j]}}(\omega) > 0 \text{ for almost all } \omega}$

$$\kappa^{[j]}(t) := \begin{bmatrix} w_{\mathcal{D}_j}(t) \\ \xi_{\mathcal{Y}_j}(t) \\ u_{\mathcal{K}_j}(t) \end{bmatrix}$$

inputs corresponding to parametrized terms in the predictor model



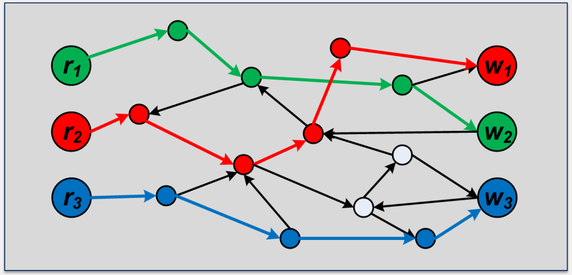
Rank-based condition can generically be satisfied based on a graph-based condition

[1] VdH et al, under preparation for submission, 2023.

Data informativity (path-based condition)

A signal $y(t) = F(q)x(t)$ with x persistently exciting, is persistently exciting iff F has **full row rank**.

This condition can be verified in a generic sense, by considering the **generic rank** of F [1],[2]



$b_{\mathcal{R}\to\mathcal{W}} = 3$

linking to the maximum number of **vertex disjoint paths** between inputs and outputs

$\kappa^{[j]}$ persistently exciting holds **generically** if there are $dim(\kappa^{[j]})$ **vertex disjoint paths** between external signals $\{u, e\}$ and $\kappa^{[j]} = \begin{bmatrix} w_{\mathcal{D}_j} \\ \xi_{\mathcal{Y}_j} \\ u_{\mathcal{K}_j} \end{bmatrix}$

Equivalently:

$dim(w_{\mathcal{D}_j})$ vertex disjoint paths between $\{u, e\}\backslash\{\xi_{\mathcal{Y}_j}, u_{\mathcal{K}_j}\}$ and $w_{\mathcal{D}_j}$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.
[3] VdH et al., CDC 2020.

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Data informativity (path-based condition)

Equivalently:

$dim(w_{\mathcal{D}_j})$ vertex disjoint paths between $\{u,e\}\backslash\{\xi_{\mathcal{Y}_j}, u_{\mathcal{K}_j}\}$ and $w_{\mathcal{D}_j}$

All input nodes need excitation, and
white noise terms that have a link to $w_{\mathcal{Y}}$ are excluded.

The **more inputs**, the more external signals required

The **more outputs**, less noise signals can be used

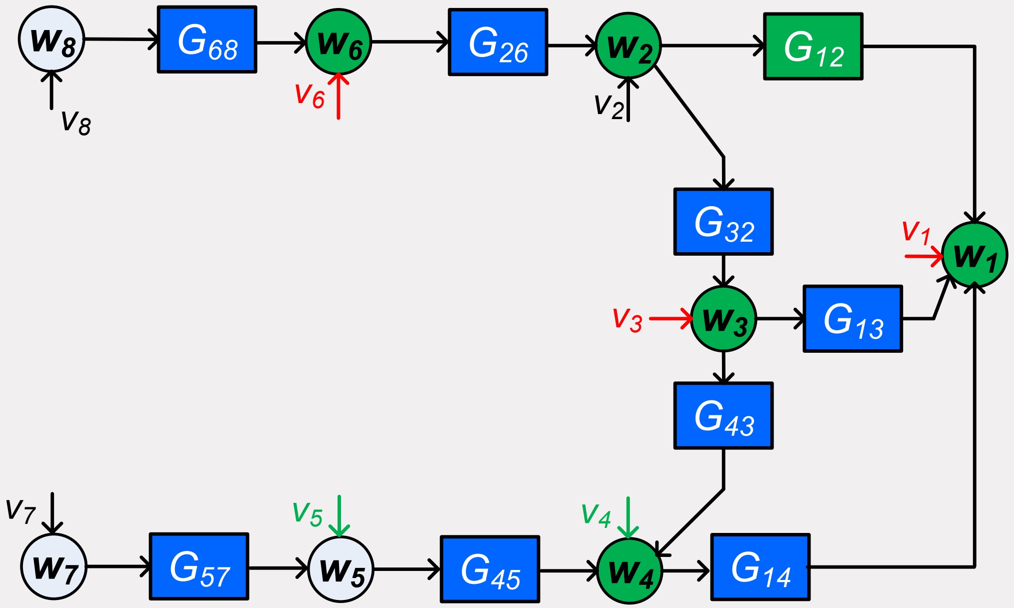
Full input case

Predictor model:

$$\mathcal{w}_{\mathcal{D}} = \{2, 3, 4, 6\} \quad \mathcal{w}_{\mathcal{Y}} = \{1, 3\}$$

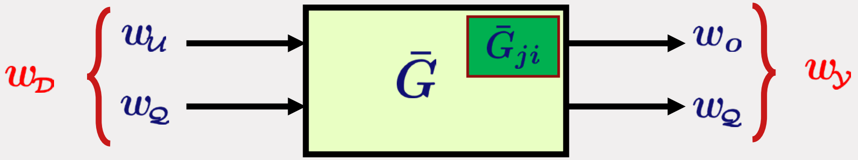
Excluded disturbances: v_1, v_3, v_6

w_2 is excited by v_2
w_3 is not excited
w_4 is excited by v_4
w_6 is excited by v_8



An additional excitation signal is required on w_3

Single module identification



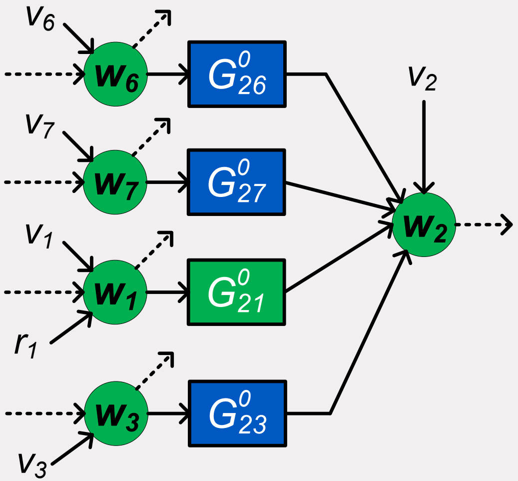
Conditions for arriving at an accurate model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$
2. Handling of confounding variables
3. Data-informativity
4. *Technical conditions on presence of delays*

Path-based conditions on the network graph

Single module identification

Typical solution:



- MISO (sometimes MIMO) estimation problem
- to be solved by your favorite estimation algorithm

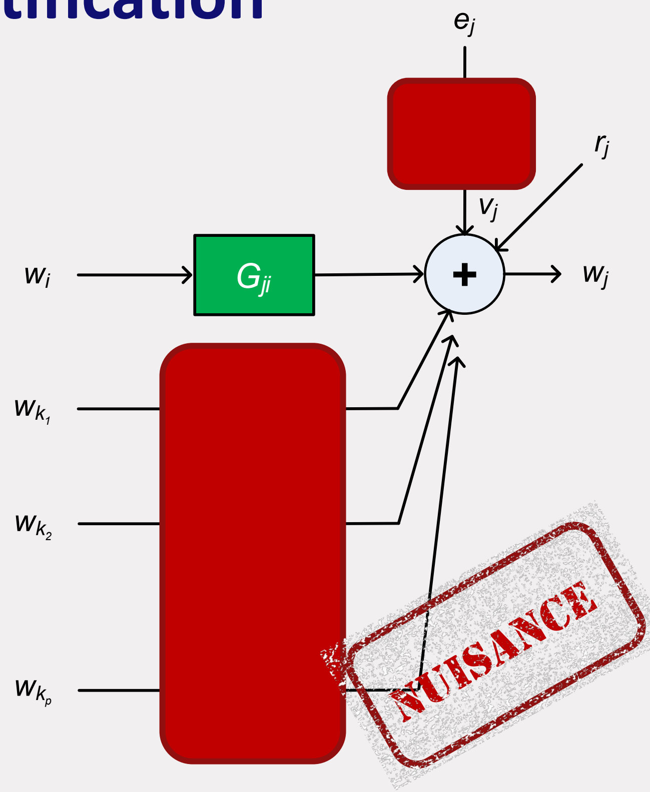
Machine learning in local module identification

- MISO identification with all modules parameterized
- Brings in two major problems :
  - Large number of parameters to estimate
  - Model order selection step for each module (CV, AIC, BIC)

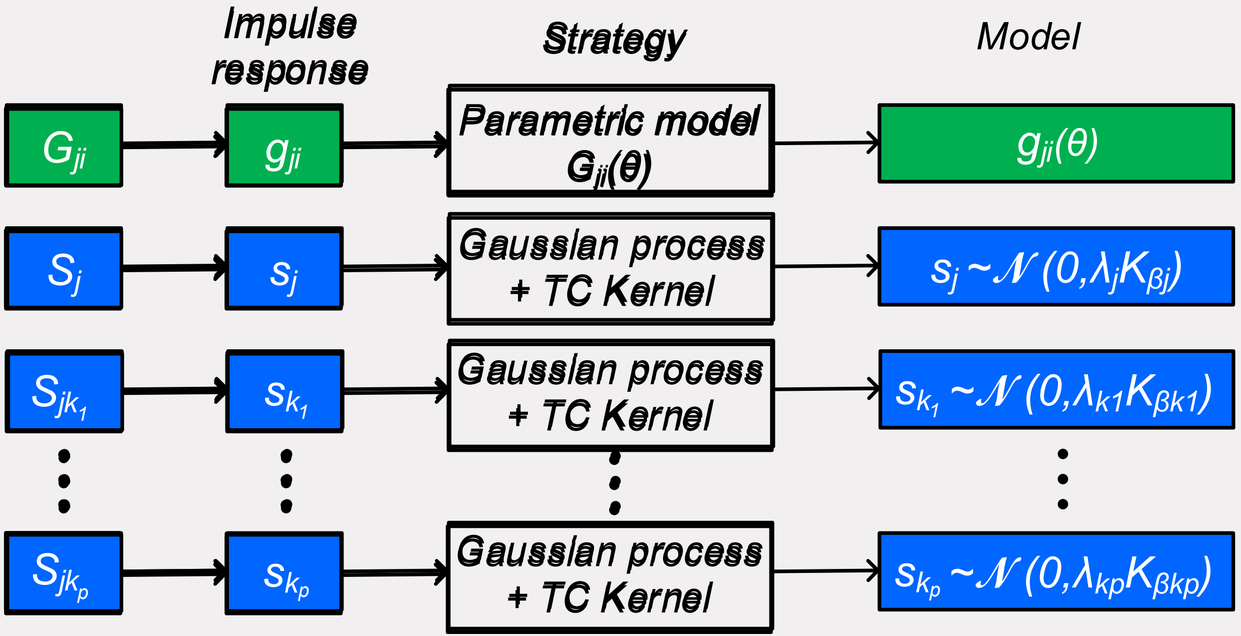
- For 5 modules, combinations = 244,140,625

Increases variance
Computationally challenging

- We need only the target module. No NUISANCE!



Machine learning in local module identification



- smaller no. of parameters
- simpler model order selection step
- scalable to large dynamic networks
- simpler optimization problems to estimate parameters

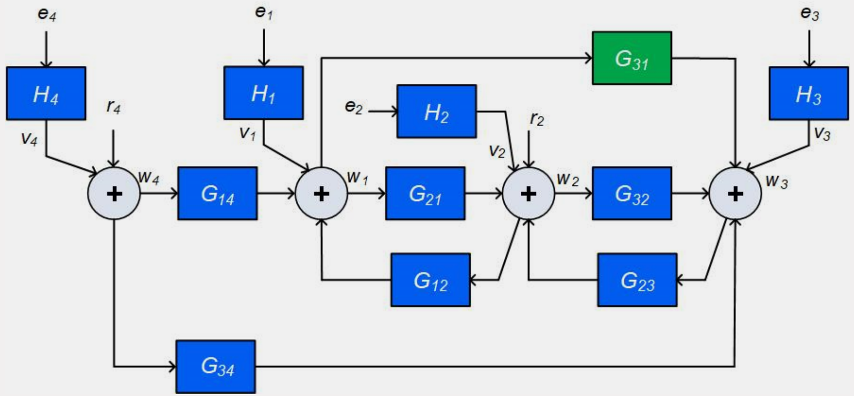
Maximize marginal likelihood of output data: $\hat{\eta} = \underset{\eta}{\operatorname{argmax}}\, p(w_j; \eta)$

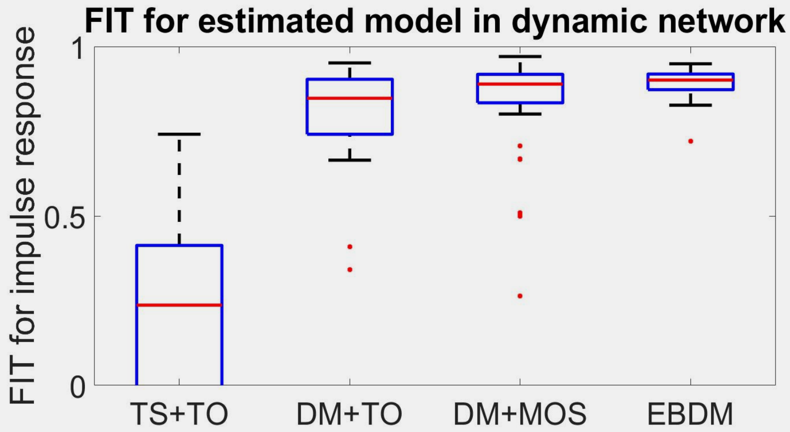
$$\eta := [\theta \quad \lambda_j \quad \lambda_{k_1} \quad \dots \quad \lambda_{k_p} \quad \beta_j \quad \beta_{k_1} \quad \dots \quad \beta_{k_p} \quad \sigma_j^2]^\top$$

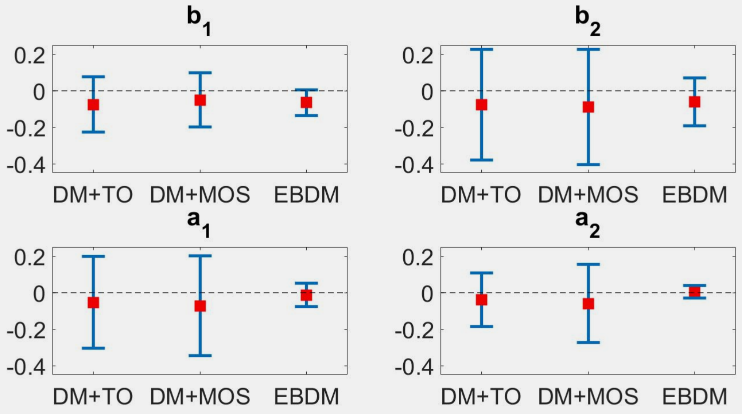
[1] Everitt et al., *Automatica* 2017.
[2] K.R. Ramaswamy et al., *Automatica*, 2021.

Numerical simulation

- Identify G_{31} given data
- 50 independent MC simulation
- Data = 500







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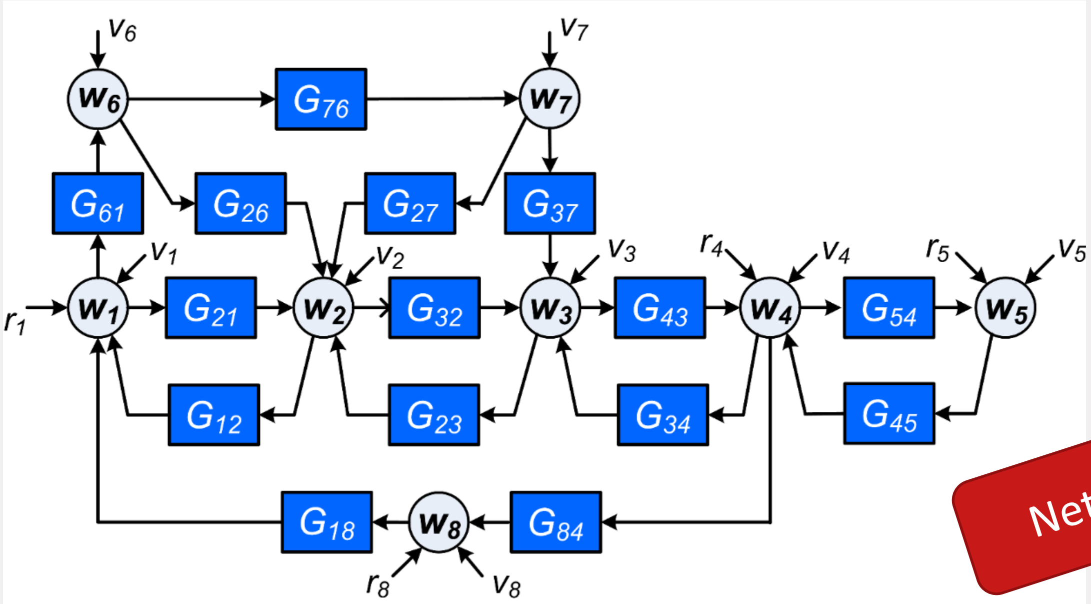
Summary single module identification

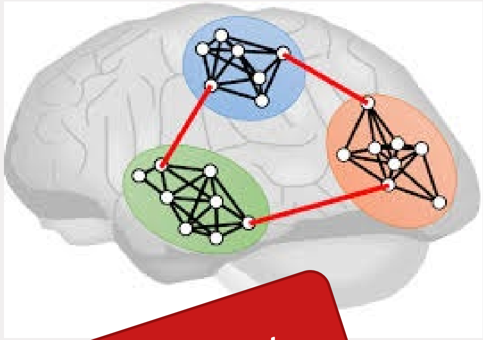
- **Path-based conditions** that the predictor model should satisfy
- Different algorithms for synthesizing predictor model, with freedom in sensor / actuator placement
- Methods for **consistent** and **minimum variance** module estimation, and effective (scalable) algorithms
- Algorithms can be preceded by (nonparametric) local **topology estimation**[1]

[1] Rajagopal, Ramaswamy and VdH, *CDC* 2021.

Generic network identifiability

Full network identification

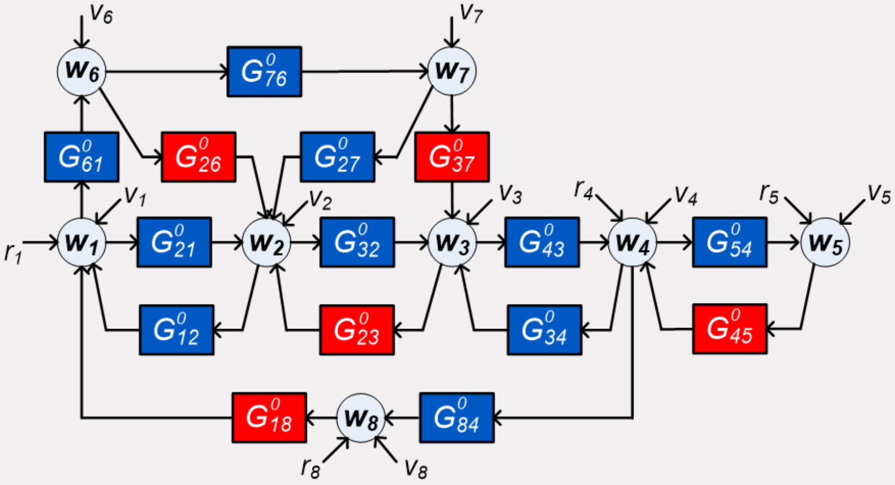




Network identifiability

Under which conditions can we estimate the topology and/or dynamics of the full network?

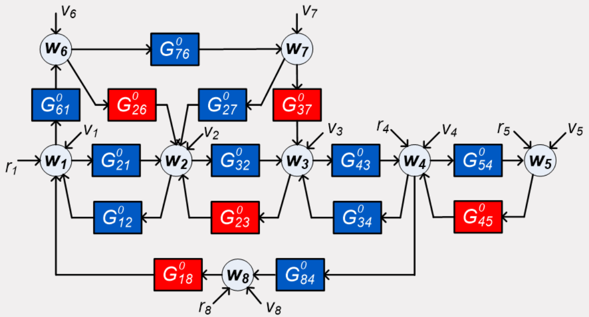
Network identifiability



blue = unknown
red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals w, r ?

Network identifiability



The identifiability problem:

The network **model**:

$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

can be transformed with any rational $P(q)$:

$$P(q)w(t) = P(q)\{G(q)w(t)+R(q)r(t)+H(q)e(t)\}$$

to an **equivalent model**:

$$w(t) = \tilde{G}(q)w(t) + \tilde{R}(q)r(t) + \tilde{H}(q)e(t)$$

→ **Nonuniqueness,** unless there are structural constraints on G, R, H.

[1] Weerts, Linder et al., Automatica, 2019.
[2] Bottegal et al., SYSID 2017

Network identifiability

Consider a **network model set**:

$$\mathcal{M} = \{(G(\theta), R(\theta), H(\theta))\}_{\theta\in\Theta}$$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

Generic identifiability of $\mathcal{M}$:

- There do not exist distinct equivalent models (generating the same data)
- for almost all models in the set.

[1] Weerts et al., SYSID2015; Weerts et al., Automatica, March 2018;
[2] Bazanella, CDC2017; Hendrickx et al., IEEE-TAC, 2019.

Network identifiability

Definition Network identifiability[1]

For a network model set $\mathcal{M}$, consider a model $M(\theta_0) \in \mathcal{M}$ and the implication

$$\left.\begin{array}{c} T_{wr}(q,\theta_0) = T_{wr}(q,\theta_1) \\ \Phi_{\bar{v}}(\omega,\theta_0) = \Phi_{\bar{v}}(\omega,\theta_1) \end{array}\right\} \Longrightarrow \left\{ \; M(\theta_0) = M(\theta_1), \right.$$

for all $M(\theta_1) \in \mathcal{M}$

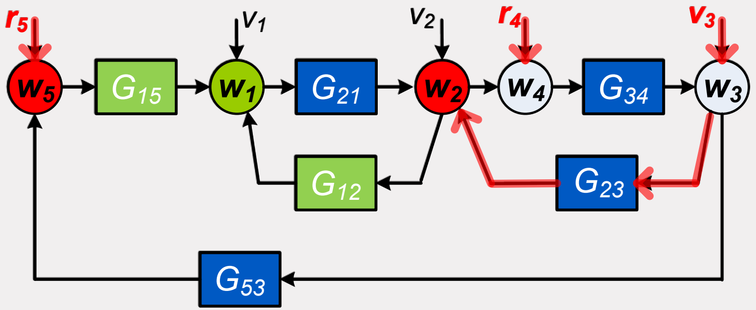
Then $\mathcal{M}$ is

- globally identifiable from (w,r) at $M(\theta_0)$ if the implication holds for $M(\theta_0)$;
- globally identifiable from (w,r) if it holds for all $M(\theta_0) \in \mathcal{M}$;
- generically identifiable[2] from (w,r) if it holds for almost all $M(\theta_0) \in \mathcal{M}$;

[1] Weerts et al., Automatica, March 2018;
[2] Hendrickx et al., IEEE-TAC, 2019.

Example 5-node network

Conditions for identifiability → rank conditions on transfer function



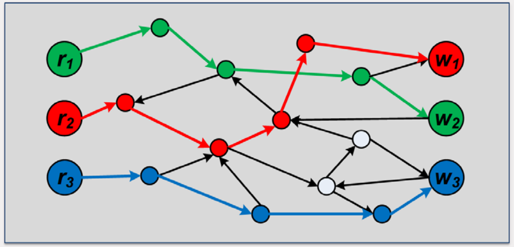
Full row rank of

$$\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \longrightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$$

For the **generic case**, the rank can be calculated by a graph-based condition[1],[2] :

Generic rank = number of vertex-disjoint paths

2 vertex-disjoint paths → full row rank 2



The rank condition has to be checked for all nodes.

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019

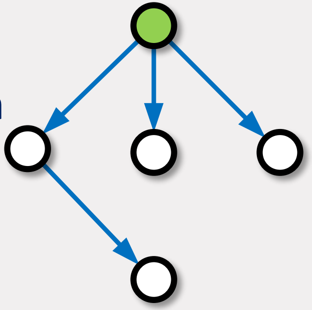
Synthesis solution for network identifiability

Allocating external signals for generic identifiability:

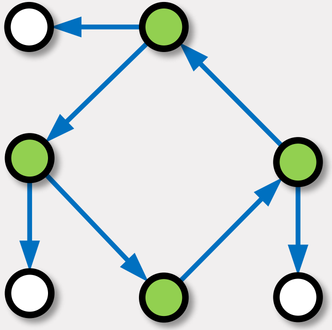
1. Cover the graph of the network model set by a set of **disjoint pseudo-trees**

   Pseudo-trees:

   Tree with root in green



   Cycle with outgoing trees;
   Any node in cycle is root



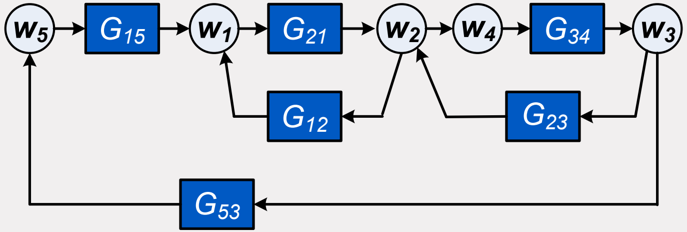
   Edges are **disjoint** and all out-neighbours of a node are in the same pseudo-tree

2. Assign an independent external signal (r or e) at a root of each pseudo-tree.

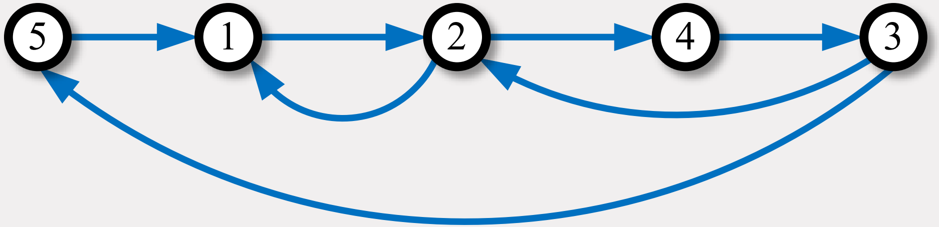
   This guarantees generic identifiability of the model set.

[1] X. Cheng, S. Shi and PVdH, TAC, February 2022.

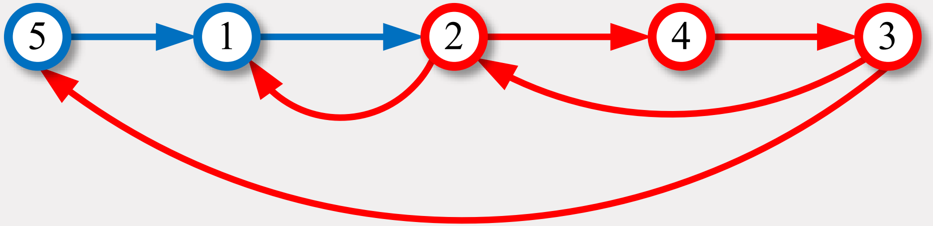
Where to allocate external excitations for network identifiability?



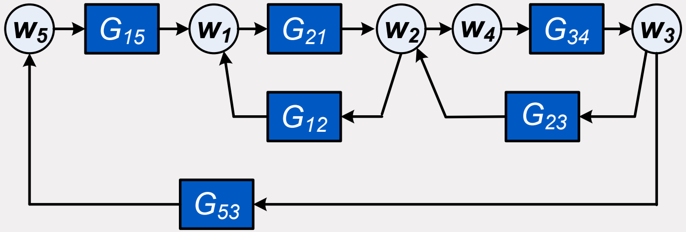
All indicated modules are parametrized

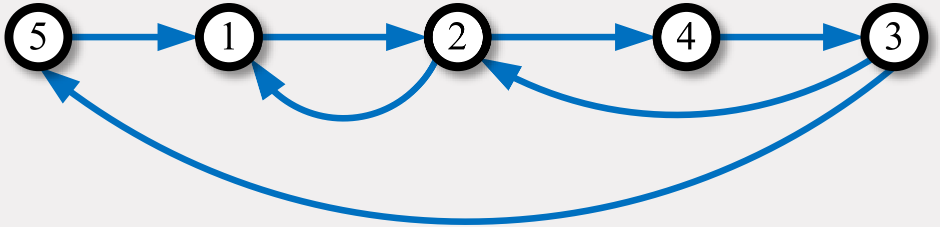


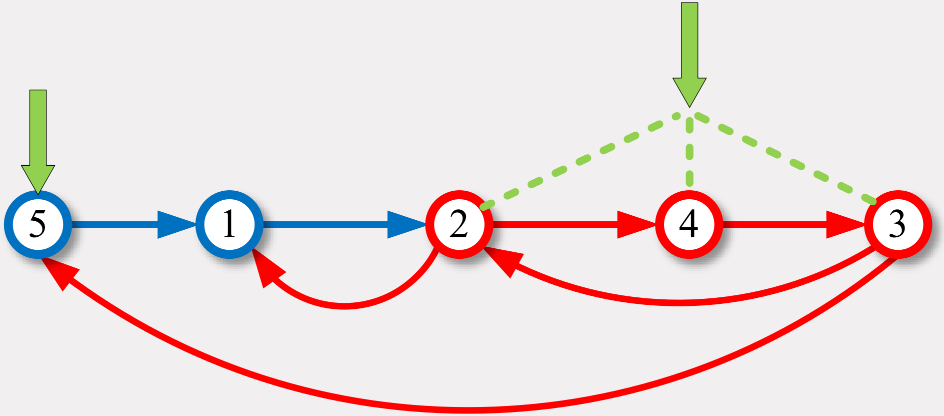
Two disjoint pseudo-trees



Where to allocate external excitations for network identifiability?



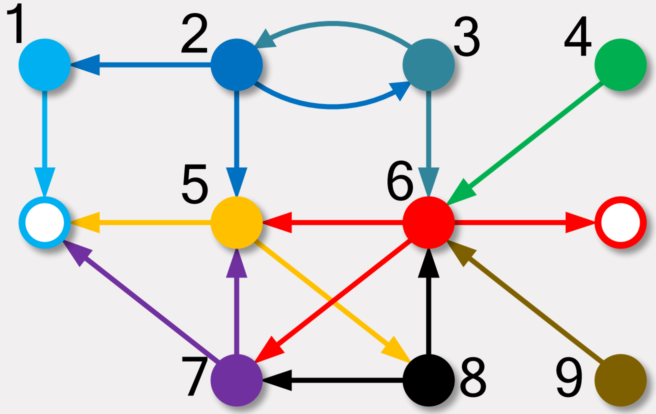


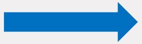


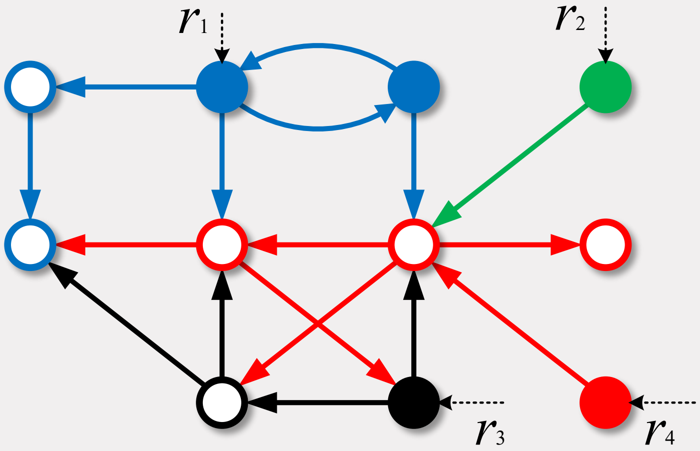
Two independent excitations
guarantee
generic network identifiability

[1] X. Cheng, S. Shi and PVdH, CDC 2019.

Where to allocate external excitations for network identifiability?







- Nodes are signals w and external signals (r, e) that are input to a parametrized link
- Known (nonparametrized) links do not need to be covered

[1] X. Cheng, S. Shi and PVdH, TAC, February 2022.

Summary identifiability of full network

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Topology of parametrized modules

- Graphic-based tool for synthesizing allocation of external signals

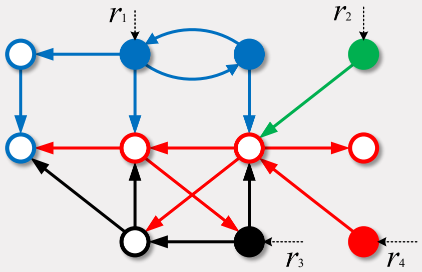
Extensions:

- Situations where not all node signals are measured [1]

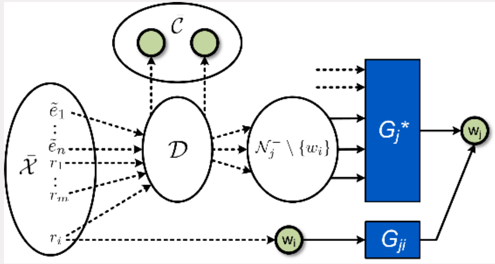
[1] Bazanella, CDC 2019.

Related topics…

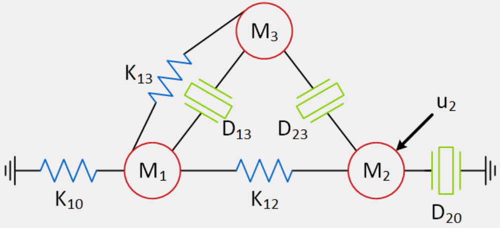
- Excitation allocation for full network identifiability[1]



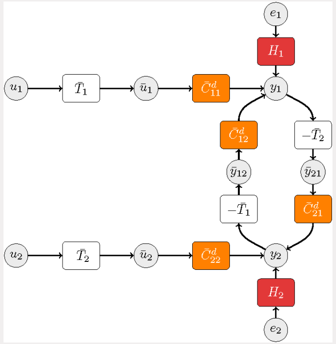
- Subnetwork identifiability[3]



- Diffusively coupled networks [2]

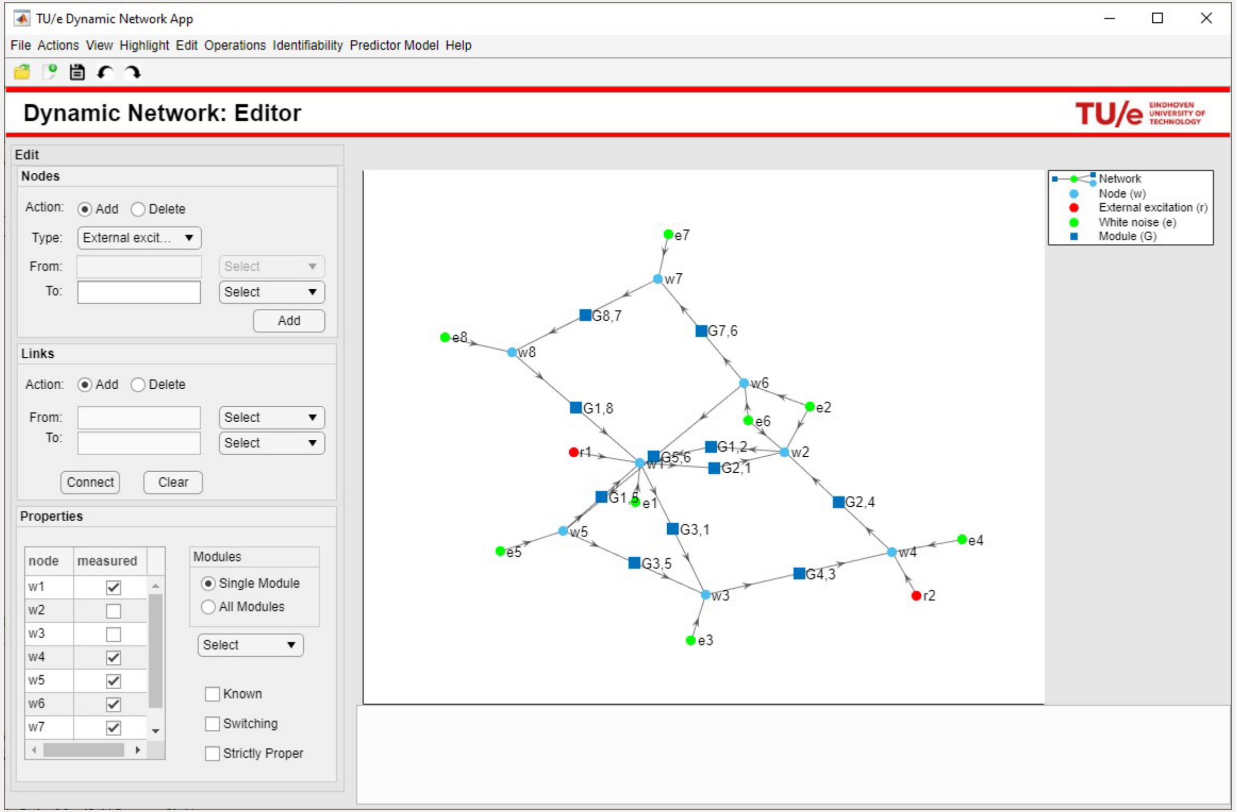


- Distributed controller identification[4]



[1] Cheng et al., IEEE-TAC, February 2022.
[3] Shi et al., IEEE-TAC, January 2023.
[2] Kivits et al., IEEE- TAC, June 2023.
[4] Steentjes, PhD thesis, June 2022.

Algorithms implemented in SYSDYNET App and Toolbox



Structural analysis and operations on dynamic networks

- Edit and manipulate
- Assign properties to nodes and modules
- Immersion of nodes, PPL test
- Generic identifiability analysis and synthesis
- Predictor model construction for single module ID

to be complemented with

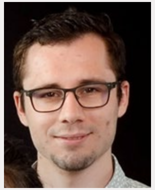
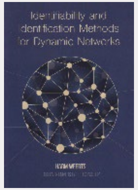
- estimation algorithms for single module and full network ID;
- topology estimation

Beta-version to be downloaded from www.sysdynet.net

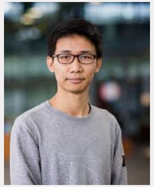
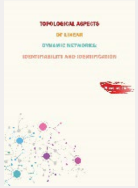
ERC SYSDYNET Team: data-driven modeling in dynamic networks

Research team:

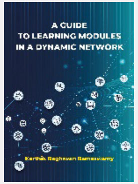

Arne Dankers

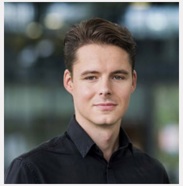
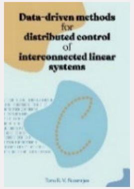
Harm Weerts

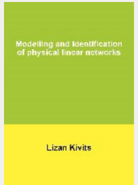
Shengling Shi

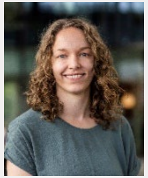
Karthik Ramaswamy

Tom Steentjes

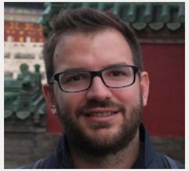



Lizan Kivits



Stefanie Fonken

Xiaodong Cheng



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Minneapolis, Vienna, Louvain-la-Neuve, Linköping, KTH Stockholm, Padova, Brussels, Salt Lake City, Lyon.

European Research Council

Further reading

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Papers available at www.sysdynet.eu

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The end