Model-based control and optimization in reservoir engineering

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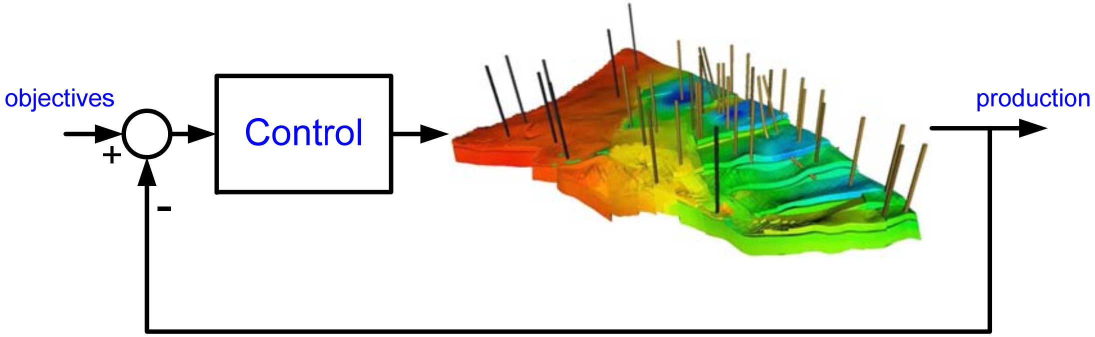

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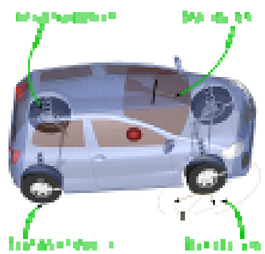
Contents

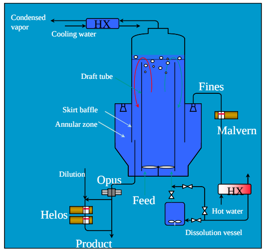
- Introduction
- Estimating states and parameters - identification
- Identifiability
- Controllability and observability
- Discussion

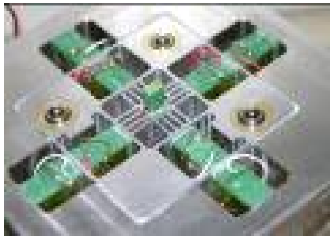
Systems and Control

- Successes of advanced control are widespread -
  from aerospace to vehicles, robots, and chemical plants







- Effective use of dynamic models (and their limitations) is of central importance

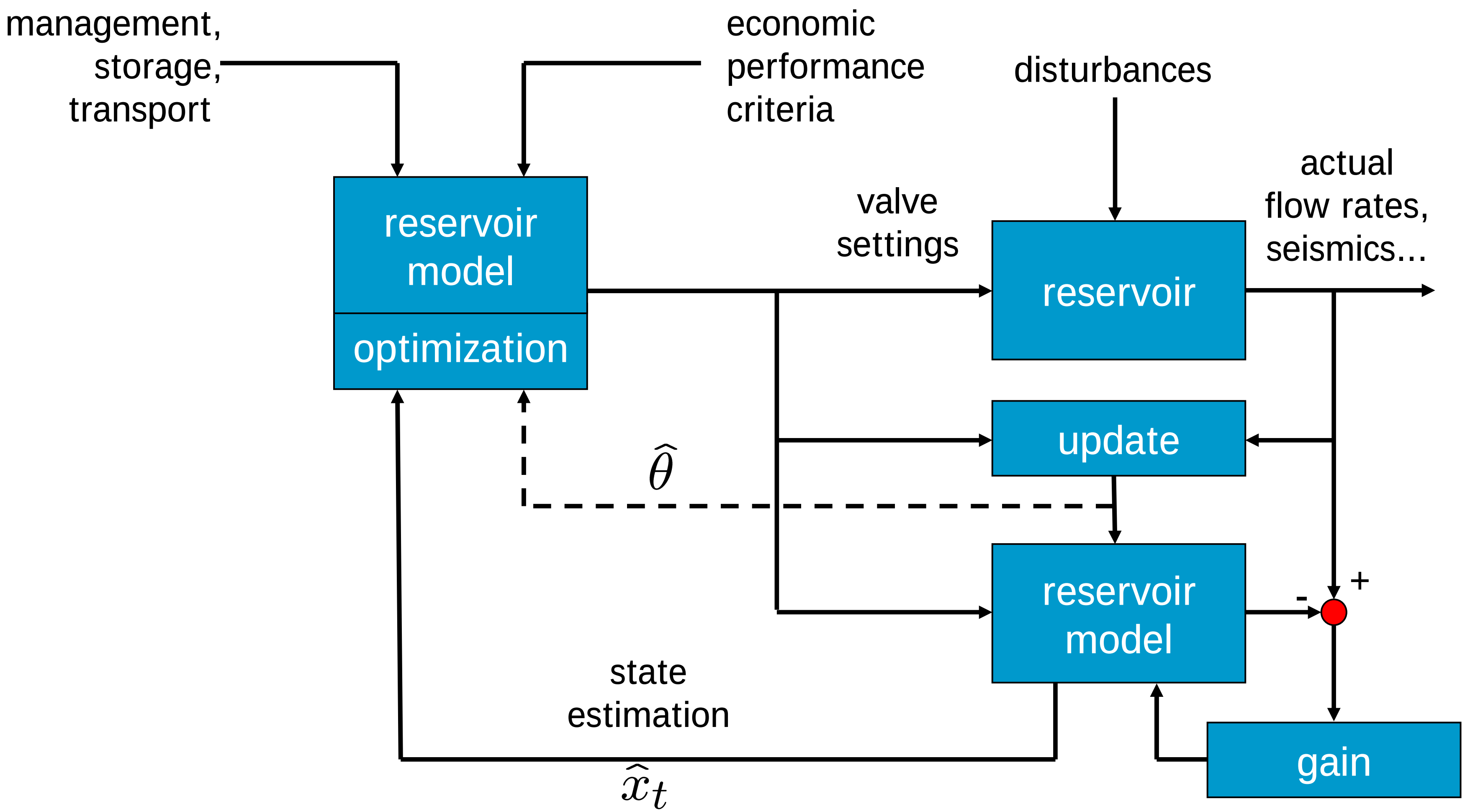
The reservoir problem:

- Challenging and attractive!
- Poorly known models
- Highly nonlinear behaviour
- One-shot (batch) type of process
- High levels of uncertainty in information
- Large scale (manipulated/measured variables and more)
- High computational load
- Slow / low sampling rates
- Options for learning/adaptation

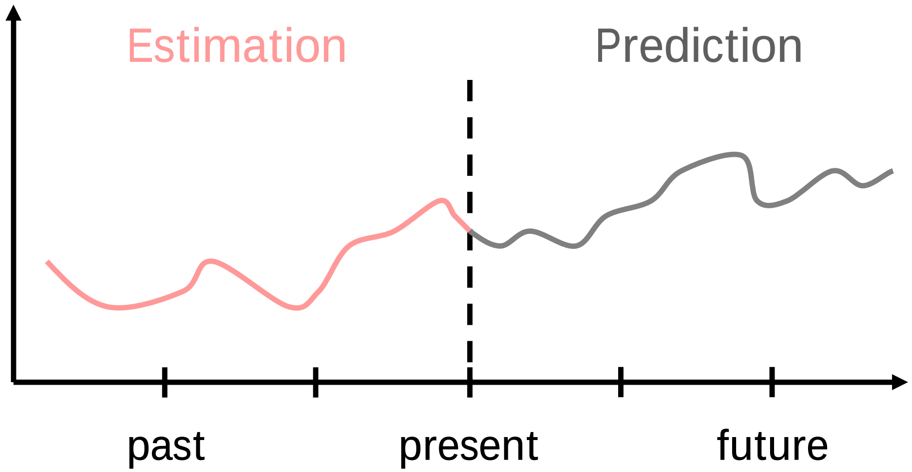
Here we will focus on issues around model construction and estimation / data assimilation / history matching

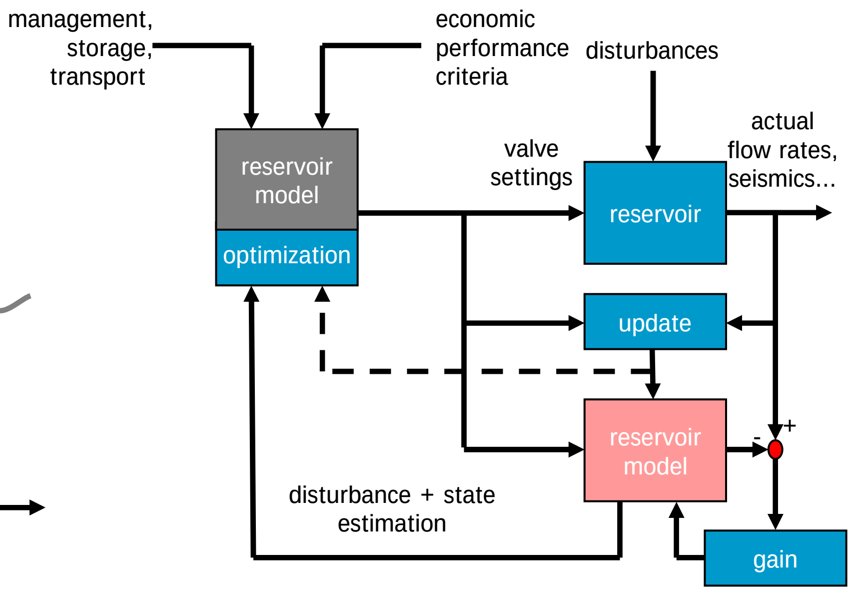
with reference to tools from systems and control theory

Estimating states and parameters



Two roles of reservoir models





- Reservoir model used for two distinct tasks: state estimation and prediction.
- Distinct role of
  parameters: essential model properties
  states: initial conditions for predictions

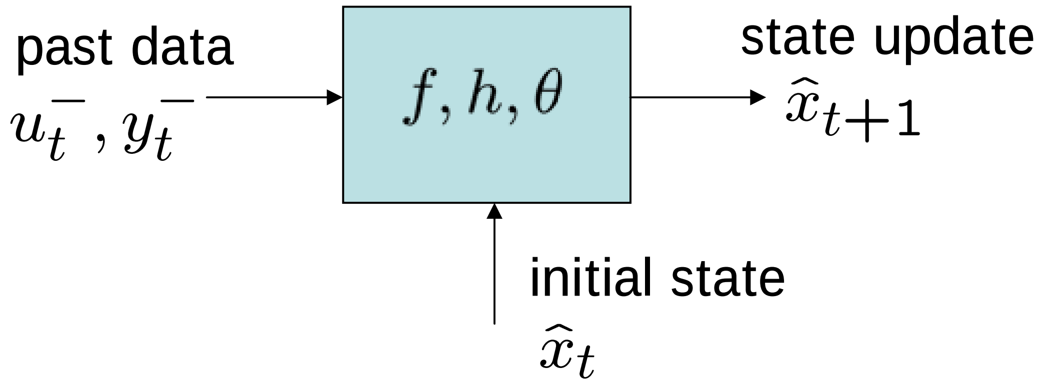
Parameter and state estimation in data assimilation

$$x_{t+1} = f(x_t, \theta, u_t)$$
$$y_t = h(x_t, \theta, u_t)$$

x_t : saturations, pressures

θ : e.g. permeabilities

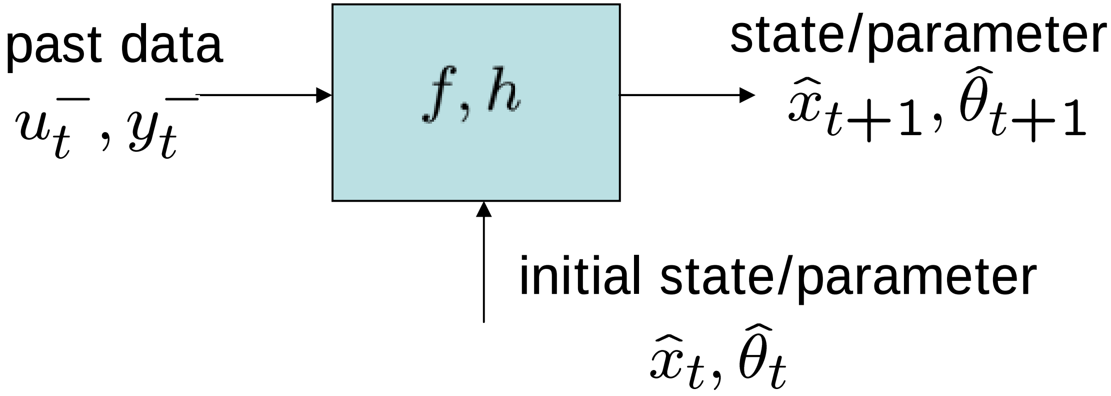
Model-based state estimation:



Options: Ensemble Kalman Filter (EnKF) (Evensen, 2006)

Parameter and state estimation in data assimilation

If parameters are unknown, they can be estimated by incorporating them into the state vector:



Can everything that you do not know be estimated?

With respect to large-scale parameter vector:

- Singular parameter-update matrix
  (data not sufficiently informative)
- Parameters are updated only in directions where data contains information (in the best case)

Result and reliability is crucially dependent on initial (prior) model

Matching the history may add/contribute little to the priors

Problem of identifiability

With respect to large-scale state vector:

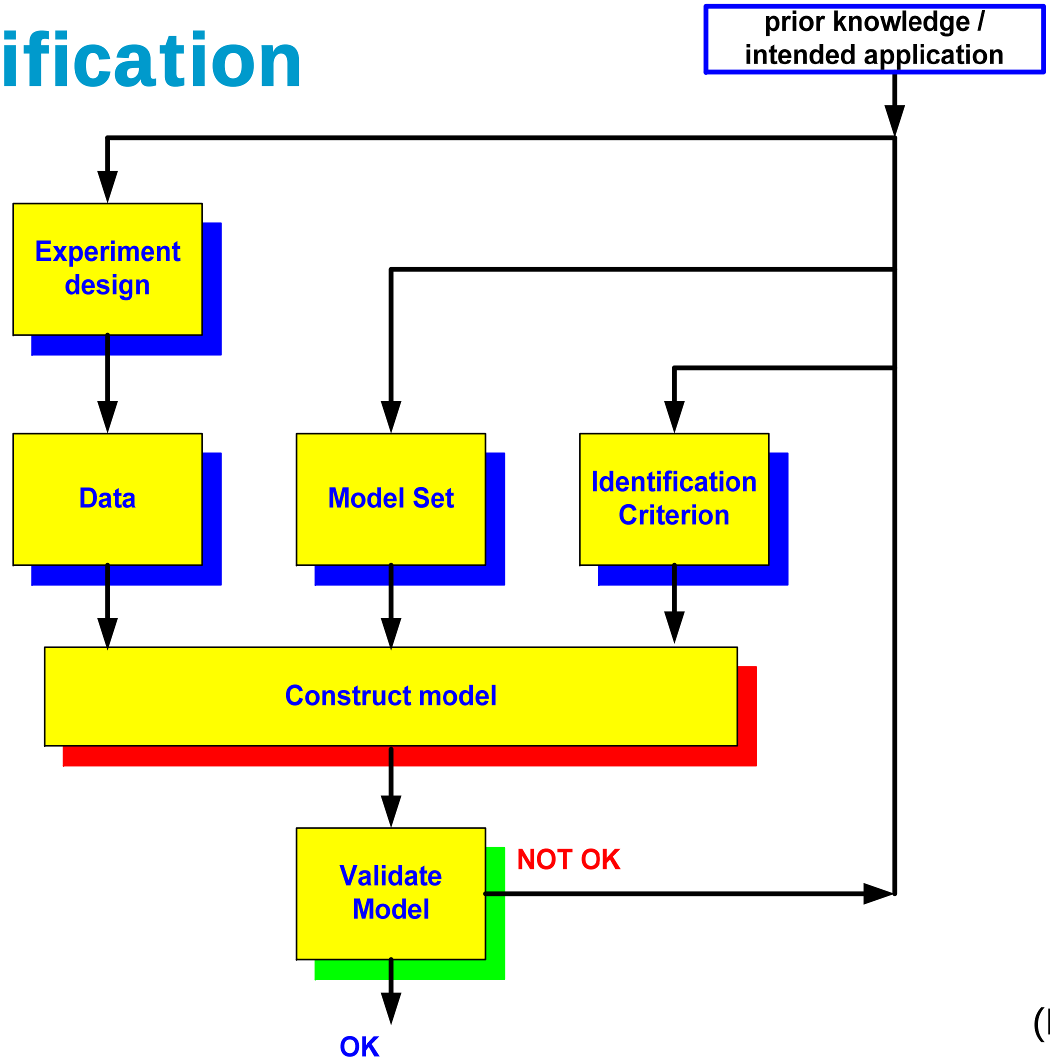
- Similar mechanisms
- States are updated only in directions where data contains information

Only that part of the state space that can be appropriately observed and controlled is relevant for the optimization

Reservoir models typically live in low-dimensional spaces

Problems of controllability and observability

Parameter estimation and system identification

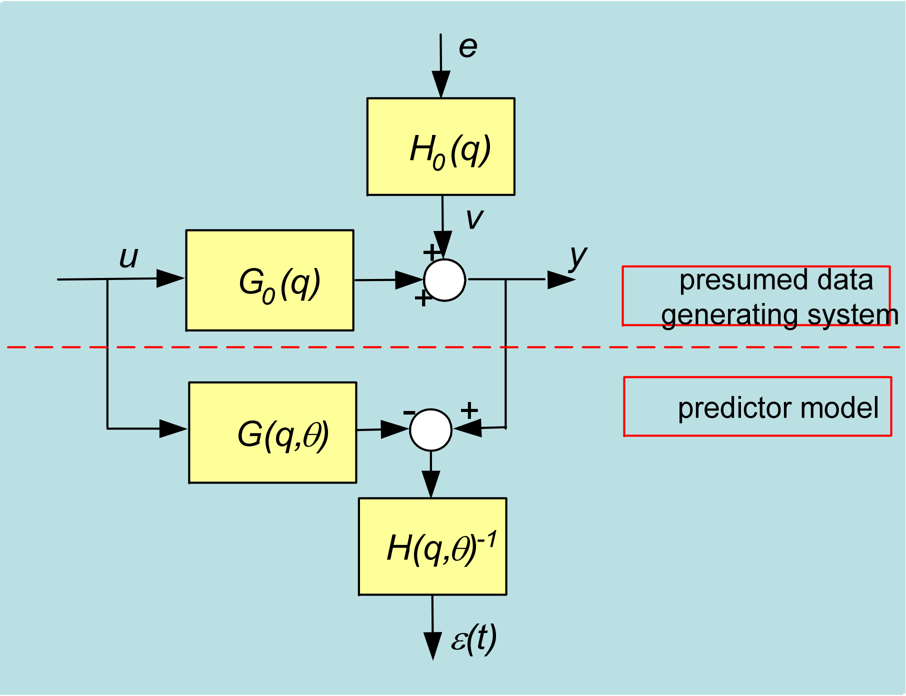


(Ljung, 1987)

Parameter estimation in identification

Parameter estimation by applying LS/ML criterion to (linearized) model prediction errors

e.g. θ are parameters that describe permeabilities

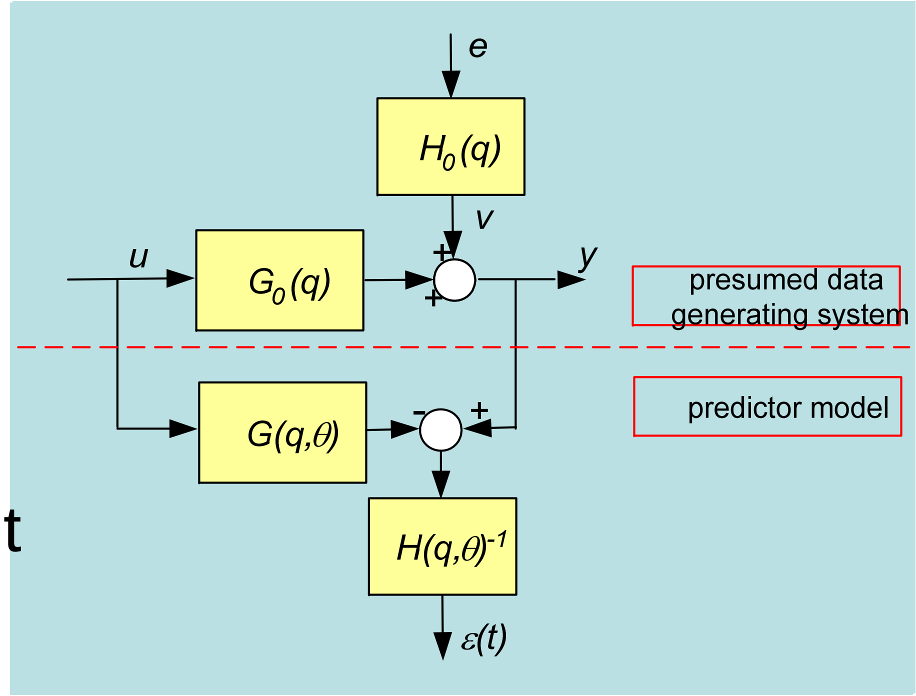


$$\min_{\theta \in \Theta} \sum_{t=1}^{N} \varepsilon^T(t,\theta)\varepsilon(t,\theta) \rightarrow \hat{\theta}$$

Parameter estimation in identification

Particular issues/tools:

- Experiment design
(u has to excite dynamics)
- Best model fit is not the goal
- Validation should prevent overfit of parameters
- Mature theory and tools for linear models



Structural identifiability

Starting from (linearized) state space form:

$$\begin{aligned} x_{t+1} &= A(\theta)x_t + B(\theta)u_t \\ y_t &= C(\theta)x_t \end{aligned}$$

the model dynamics is represented in its i/o transfer function form:

$$G(q,\theta) = C(\theta)[qI - A(\theta)]^{-1}B(\theta)$$

with q the shift operator: $qx_t = x_{t+1}$

Principle problem of physical model structures

Different $\theta' s$ might lead to the same dynamic models $G(\theta)$

This points to a *lack of structural identifiability*

There does not exist experimental data that can solve this!

Solutions:

- Apply regularization (additional penalty term on criterion) to enforce a unique solution (does not guarantee a *sensible* solution for $\widehat{\theta}$)
- Find (identifiable) parametrization of reduced dimension

Structural identifiability (cont'd)

A model structure is locally (i/o) identifiable at $\hat{\theta}$ if for any two parameters θ_1, θ_2 in the neighbourhood of $\hat{\theta}$ it holds that

$$\{G(q,\theta_1) = G(q,\theta_2)\} \Longrightarrow \theta_1 = \theta_2$$

At a particular point $\hat{\theta}$ the identifiable subspace of Θ can be computed! This leads to a map

$$\rho = T\theta \quad \text{with} \quad dim(\rho) << dim(\theta)$$

Van Doren et al., Proc. IFAC World Congress, 2008, to appear.

Tool:

Analyse (svd) the matrix $\mathcal{I}_r := \left(\frac{\partial \overrightarrow{S}_r(\theta)}{\partial \theta}\right)\left(\frac{\partial \overrightarrow{S}_r(\theta)}{\partial \theta}\right)^T \Bigg|_{\theta=\hat{\theta}}$

with $\overrightarrow{S}_r(\theta) := [\ \overrightarrow{M}(1,\theta)\ \ \overrightarrow{M}(2,\theta)\ \ \ldots\ \ \overrightarrow{M}(r,\theta)\ ]$

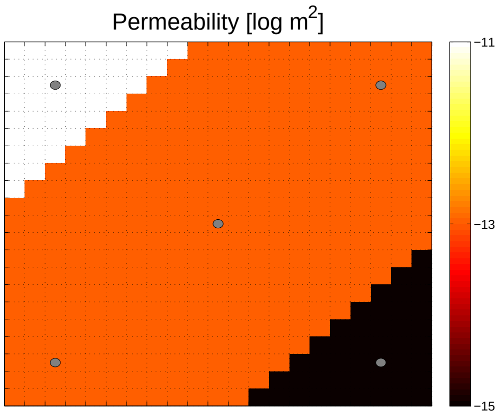
and Markov parameters $\overrightarrow{M}(k,\theta) = CA^{k-1}B$

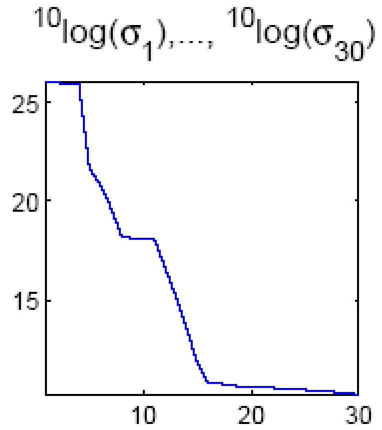
$$\mathcal{I}_r = [U_1\ U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [V_1\ V_2]^T$$

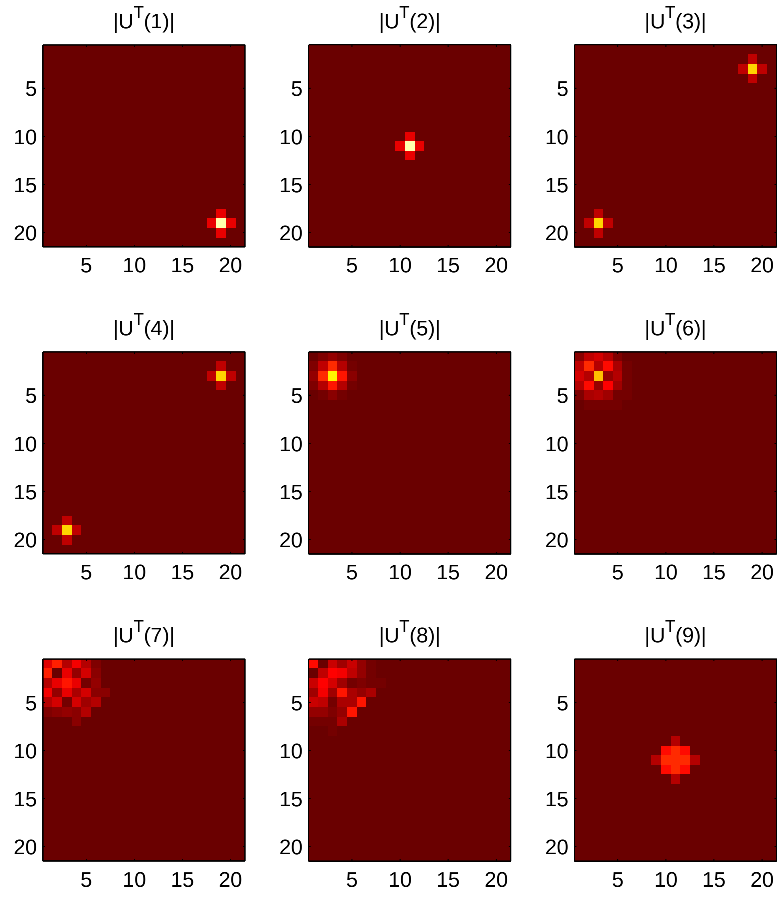
The svd gives the directions in parameter space that have the greatest influence on the i/o dynamics (columns of U_1)

Limitation: only local linearized situation can be handled

Identifiable directions







- Consider a single-phase, 21x21 grid block model with 5 wells.
- Directions in the permeability space that are best identifiable from pressure measurements:

Even if structural identifiability is OK, the input has to excite the dynamics in order to make parameter visible in the data

Analysis extends to nonlinear (two-phase) situation,
by calculating the svd of the Hessian of the cost function

Result:
Insight into information content of data, with respect to parameters to be estimated

Analysis can be applied to geological parametrization

Van Doren et al., ECMOR, 2008

Parameter estimation:

Physical parameters (permeabilities) determine predictive quality but one parameter per grid block leads to excessive over-parametrization (hard to validate)

Controllability and observability

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t \end{aligned} \qquad dim(x_t) = n$$

- Controllability:
  - Can we (dynamically) steer all pressures and saturations by manipulating the inputs?
- Observability:
  - Do all states (dynamically) appear in the observed output?

In the linear(ized) case:

Controllability:

$$rank \underbrace{\begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}}_{\mathcal{C}_n} = n$$

Observability:

$$rank \underbrace{\begin{bmatrix} C^T & A^TC^T & {A^T}^2C^T & \dots & {A^T}^{n-1}C^T \end{bmatrix}}_{\mathcal{O}_n} = n$$

The controllable and observable part of the state space determines that part of the states that affects the i/o mapping of the system

Controllability and observability

Besides a yes/no answer, the notions can be quantified:

Minimum input energy to reach a state x is

$$x^T \mathcal{P}^{-1} x \qquad \mathcal{P} = \mathcal{C}_\infty \mathcal{C}_\infty^T$$

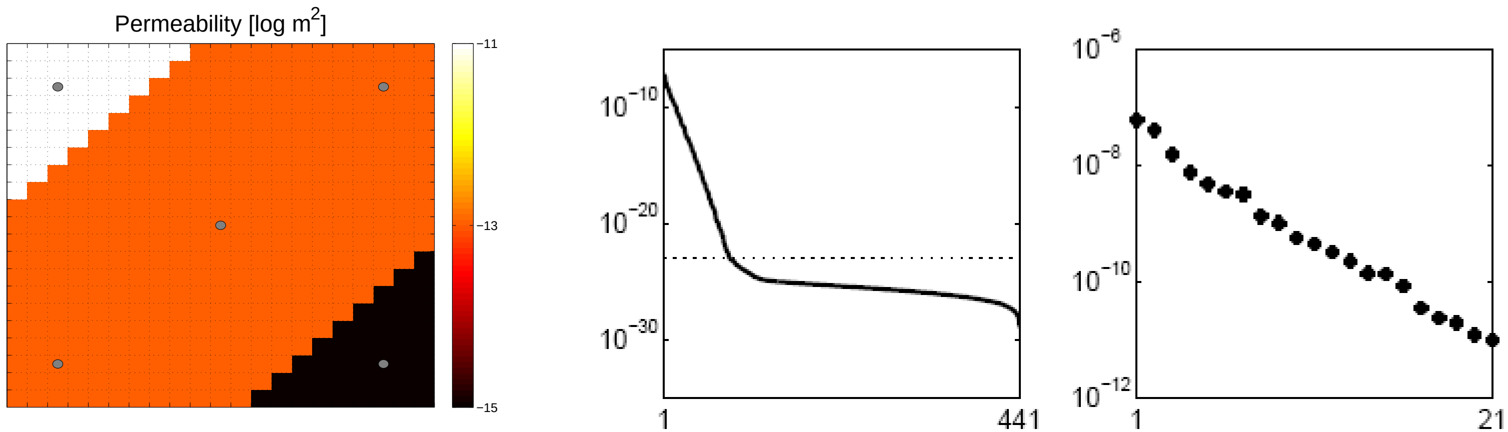
Maximum output energy obtained from state x is

$$x^T \mathcal{Q} x \qquad \mathcal{Q} = \mathcal{O}_\infty^T \mathcal{O}_\infty$$

Small s.v.'s of Grammians $\mathcal{P}, \mathcal{Q}$ refer to state directions that are poorly controllable/observable

Controllability and observability

Eigenvalues of Grammian product $\mathcal{P}\mathcal{Q}$ determine the minimum number of states required to describe the i/o dynamics



Reservoir models live in low-dimensional state space

Zandvliet et al., Comput. Geosciences, submitted, 2008.

Quantifying controllability and observability

- In which area's in the reservoir are the states more controllable and observable than others?
- Notions can be extended to nonlinear situation
- Methodology:
  1. Linearize along current state
  2. Calculate LTV controllability and observability matrices
  3. Visualize dominant directions with SVD (for pressures and saturations separately)

Controllability / observability of pressures

Calculated with LTV controllability and observability matrices

Pressure(k)

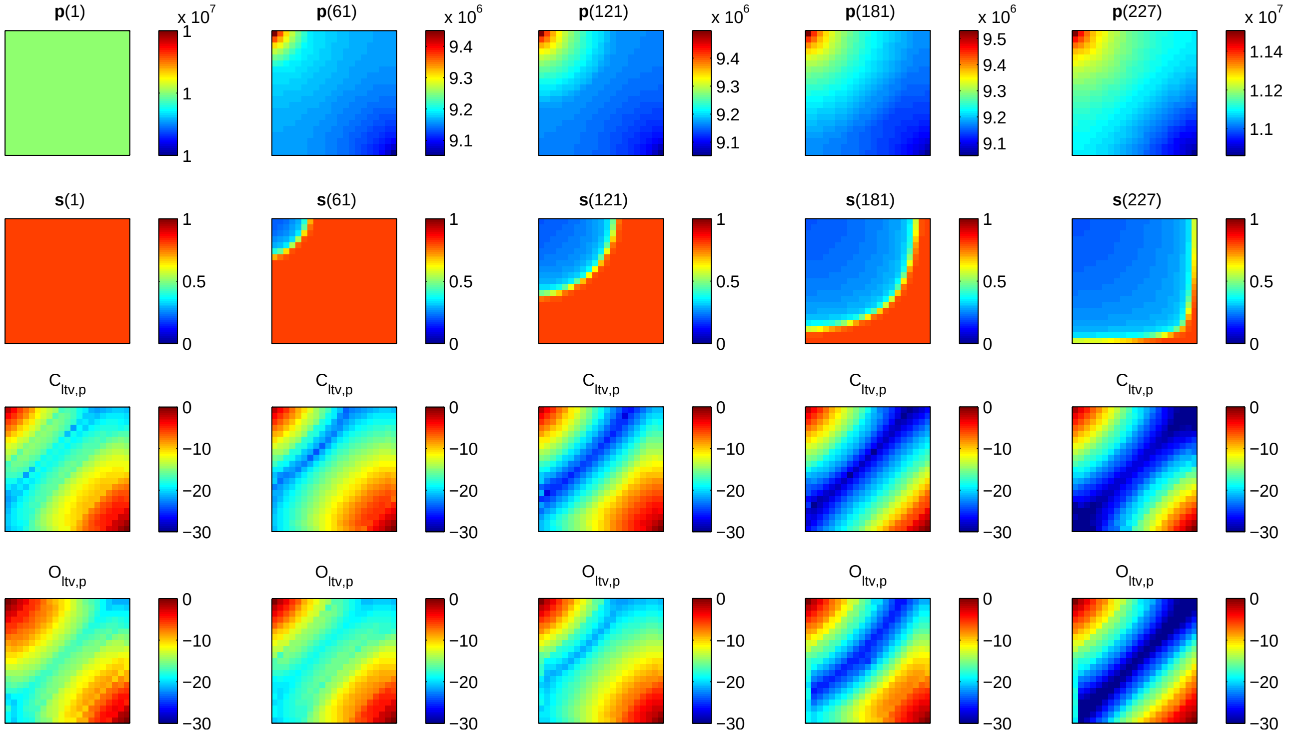
Saturation(k)

Controllable pressures

!!! Logarithmic scale

Observable pressures

!!! Logarithmic scale



time(k) →

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Controllability/observability of saturations

Calculated with empirical Gramians

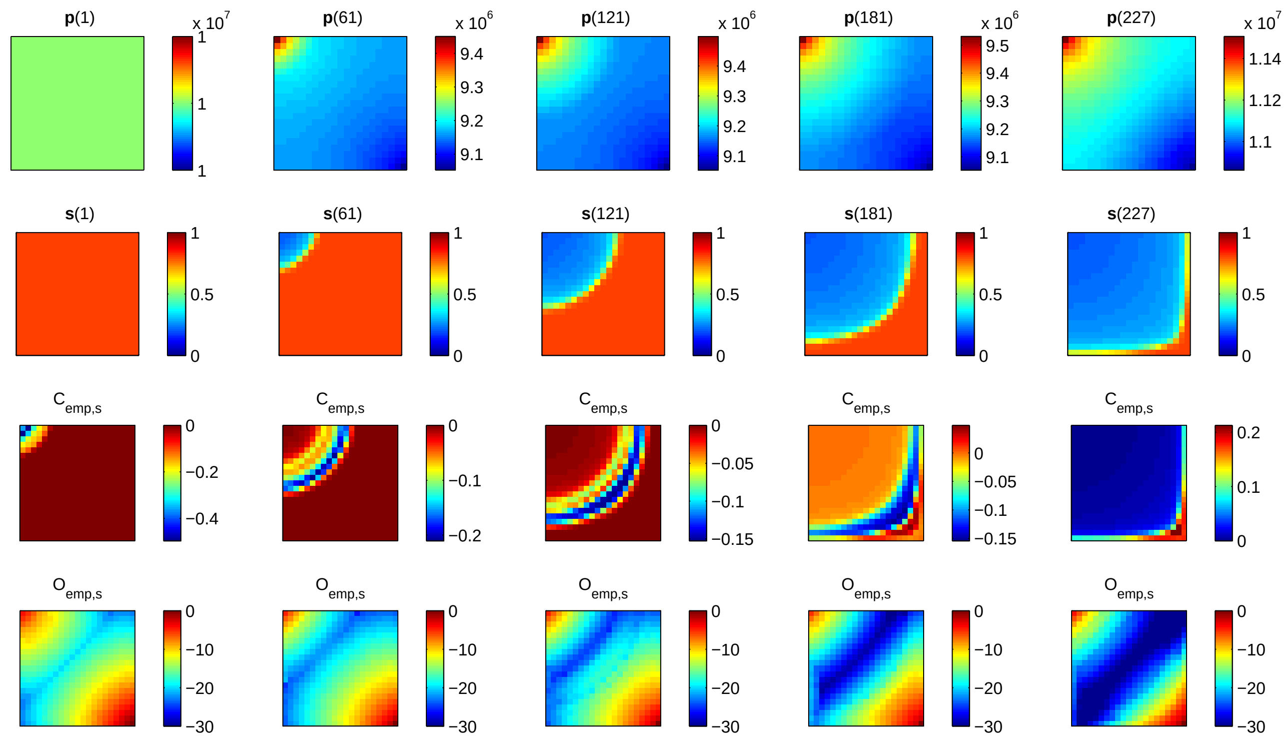
Pressure

Saturation

Controllable saturations

Observable saturations

!!! Logarithmic scale



time(k) →

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Observations

- Most observable phenomena occur in the direct vicinity of the wells
- Saturations are most controllable around the oil-water front
- This would motivate a representation of the state space (e.g. in terms of basis functions), with emphasis on the oil-water front.... Yortsos et al., 2006
- Notions can be instrumental in
  (a) determining the control-relevant model aspects
  (b) optimal well placement

Discussion

- Basic methods and tools have been set, but there remain important and challenging questions
- Complexity reduction of the physical models: limit attention to
  (a) what is known or verifiable by data
  (b) what is relevant for the ultimate optimization….
  (control-relevant modelling)
- Can we increase information content in the data? (learning / dual control)