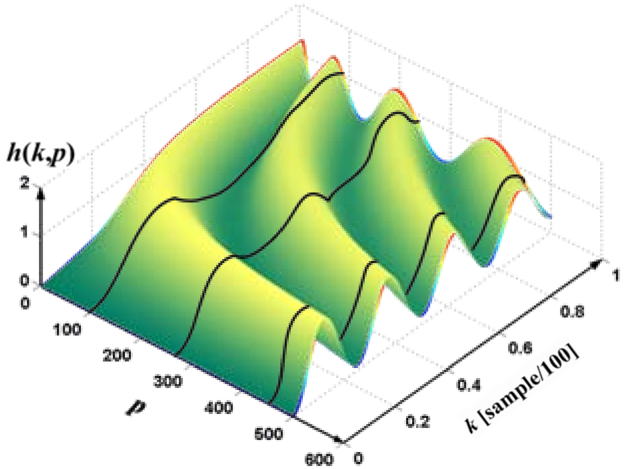
Asymptotically optimal orthonormal basis functions for LPV systems identification

Paul M.J. Van den Hof
Roland Tóth
Peter Heuberger

Delft Center for Systems and Control
Delft University of Technology



Workshop on Systems and Control Theory in honor of Jozsef Bokor on his 60th birthday, Budapest, 9 september 2008

Delft Center for Systems and Control

Delft University of Technology

Contents of the presentation

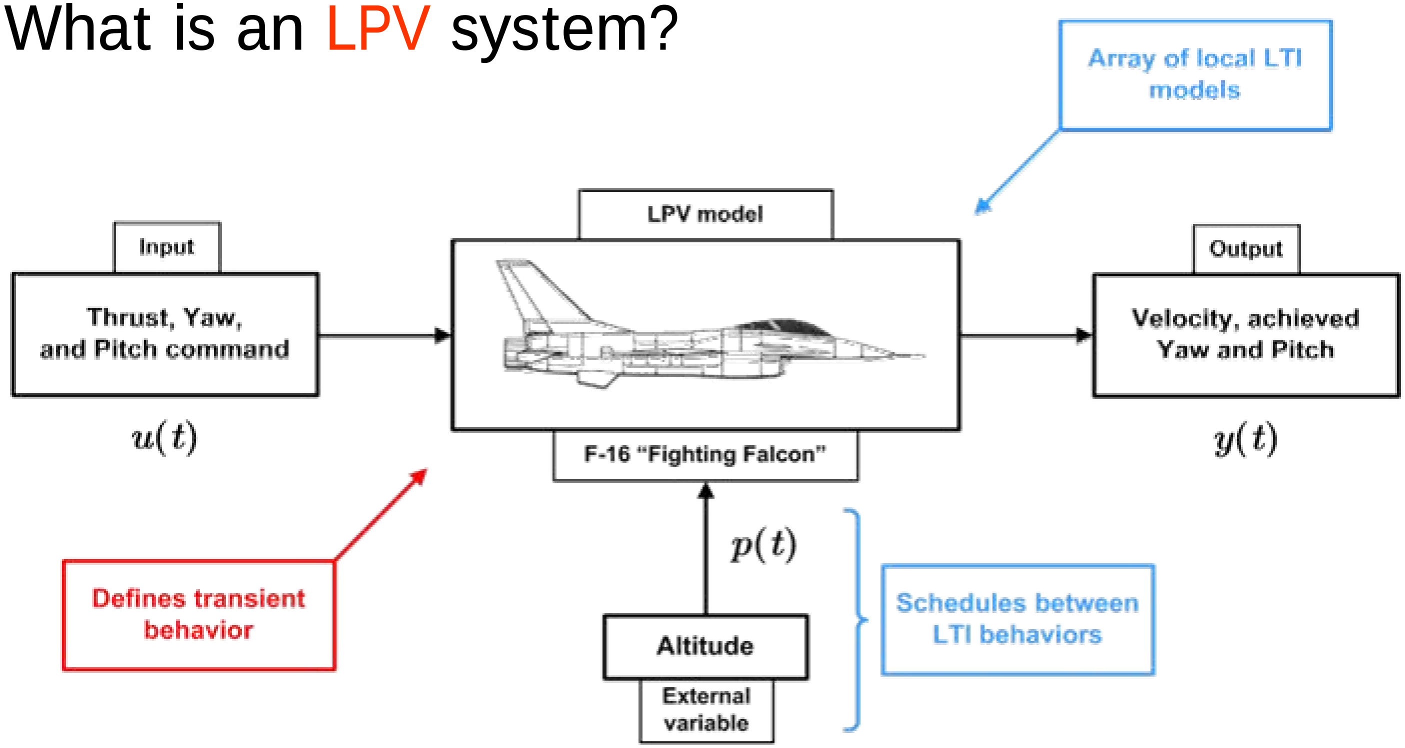
- Questions related to LPV identification
- Model structures for LPV systems
- An orthonormal basis function approach
- Example

Control and identification of nonlinear systems - challenges

- Expand the scope beyond the current LTI framework
- Handling systems in different operating regimes
- Attractive structure: LPV
  for systems with “regime”-dependent (linear) dynamics
- Tools for controller synthesis
- Several algorithms for LPV model identification, but
- Not yet solved in a structured way

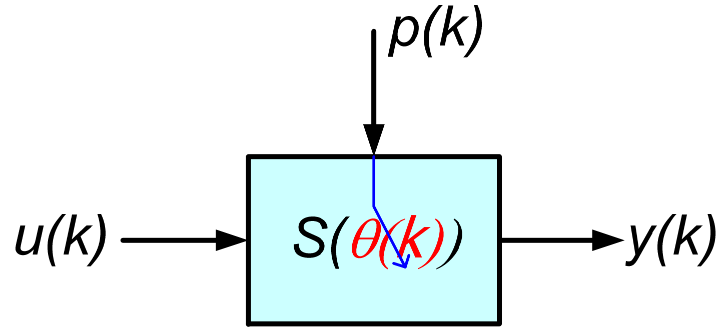
LPV systems

- What is an LPV system?



[Lockheed Martin]

LPV Models



- LPV framework (SISO), $p(k) : \mathbb{Z} \to \mathbb{P}$
  - State-space model representation (SS)

$$\begin{aligned} x(k+1) &= A(p(k))\,x(k) + B(p(k))\,u(k) \\ y(k) &= C(p(k))\,x(k) + D(p(k))\,u(k) \end{aligned}$$

  - I/O model representation, $n_a \geq n_b > 0$

$$y(k) = -\sum_{l=1}^{n_a} a_l(p(k))\,y(k-l) + \sum_{l=0}^{n_b} b_l(p(k))\,u(k-l)$$

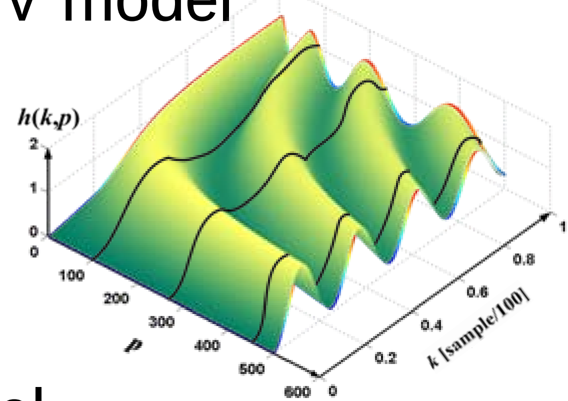
Usually use is restricted to static (nonlinear) maps *p(k)* → *θ(k)*

Issues in LPV identification

Approaches to the identification problem:

Local:

- Identify local linear models (for fixed scheduling $p(k) = c_i$)
- Use global data to interpolate into an LPV model



Global:

- Determine a global LPV model structure
- Use global data to estimate an LPV model

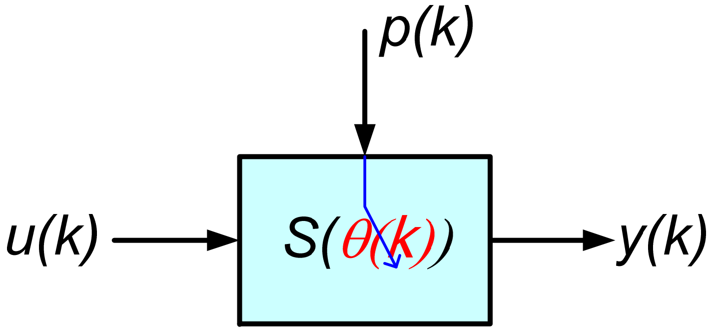
Both PE and subspace methods can be followed

Issues in LPV identification (cont'd)

What are appropriate model structures?
how can they be defined?
what are criteria to select them?

And many more questions related to estimation accuracy, experiment design, validation, ...

LPV Model Structures



- State-space model (SS)

$$\begin{aligned} x(k+1) &= A(p(k))\,x(k) + B(p(k))\,u(k) \\ y(k) &= C(p(k))\,x(k) + D(p(k))\,u(k) \end{aligned}$$

- I/O model

$$y(k) = -\sum_{l=1}^{n_a} a_l(p(k))\,y(k-l) + \sum_{l=0}^{n_b} b_l(p(k))\,u(k-l) \qquad n_a \geq n_b > 0$$

Question: are these structures equivalent (as in the LTI case)?

Answer: in general not if you restrict $p \rightarrow \theta$ to be static; dynamic p-depencies are generally required

(Tóth et al., ECC 2007)

Delft Center for Systems and Control

LPV Model Structures

Consequence 1:

- Mapping estimated IO models to SS or vice versa, while retaining a static dependence of the scheduling function introduces (substantial) errors

This points to the need to either

- estimate the LPV model in the same model structure where physical information on the (static) effect of p is available, or
- Include a dynamic map $p \rightarrow \theta$ in the model structure

LPV Model Structures

Consequence 2:

- We need appropriately defined notions of equivalence between LPV systems (and definitions of LPV systems as a start)

Note: transfer function is not available for this purpose
as it is time-varying

Solution:

Through Willems' behavioural framework:

$$S = (\mathbb{T}, \mathbb{P}, \mathbb{W}, \mathcal{B}) \qquad \mathcal{B} \subset \mathbb{P} \times \mathbb{W}$$

leads to well-defined notion of equivalence in terms of $\mathcal{B}$

LPV Model Structures

Generic representation of an LPV system behaviour:

$$\sum_{i=0}^{n} (r_i \diamond p) q^i w = 0 \qquad \text{or} \qquad (R(q) \diamond p) w = 0$$

where $r_i \diamond p$ represents any quotient of homeomorphic functions of p and shifted versions of p; $r_i \in \mathbb{R}^p$

Result:

LPV system equivalence,

canonical forms in SS and IO form, etc., (Tóth 2008)

Taking account of dynamic phenomena in w *and* p

LPV Model Structures

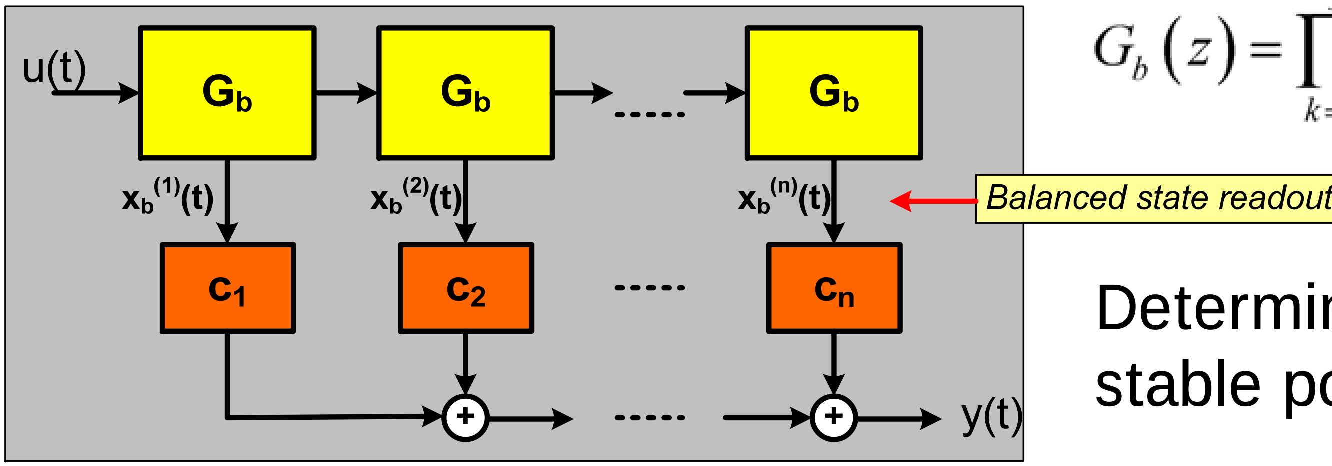
Additional aspects

- In linear PE identification we benefit from linearity-in-the-parameters;
  Can this be maintained?

- Interpolating local linear state space models is hard when McMillan degree varies over local models;
  Can we accommodate this?

An orthonormal basis functions approach

For (local) linear models: $G(z) = \sum_{k=1}^{n} c_k F_k(z)$



$$G_b(z) = \prod_{k=1}^{n_f} \frac{1 - z\zeta_k^*}{z - \zeta_k}$$

Determined by a set of stable pole locations

Choice of basis poles determines rate of convergence of the series expansion

An orthonormal basis functions approach

Opportunities for LPV models:

$$y(k) = \sum_{k=1}^{n} c_k(p) F_k(q) u(k)$$

- Scheduling of coefficients c_k retains linearity-in-the parameters
- If basis can be chosen (globally) fixed, interpolation of local models becomes interpolation of $\{c_k(\bar{p}_i)\}_{i=1,\cdots n_l}$

  $\bar{p}_i$ constant scheduling signal belonging to local model i

- No problem with interpolation of models with different McMillan degrees

An orthonormal basis functions approach

Opportunities for LPV models:

$$y(k) = \sum_{k=1}^{n} c_k(p) F_k(q) u(k)$$

- Scheduling of coefficients c_k retains linearity-in-the parameters
- If basis can be chosen (globally) fixed, interpolation of local models becomes interpolation of $\{c_k(\bar{p}_i)\}_{i=1,\cdots n_l}$

Question:

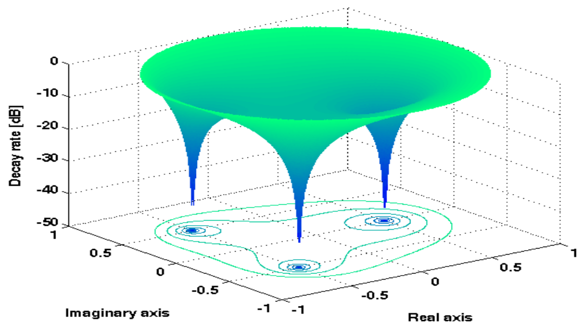
How to choose the global basis functions $F_k(q)$?

An orthonormal basis functions approach

Basis selection:

- Identify a number of local linear models in several in different regimes $\bar{p}_i$
- Plot all identified poles in the complex plane
- Cluster the poles in groups and find optimal cluster centres (basis poles)
- So as to minimize a distance measure that is relevant for the (worst case) length of the resulting series expansions

Optimal basis selection

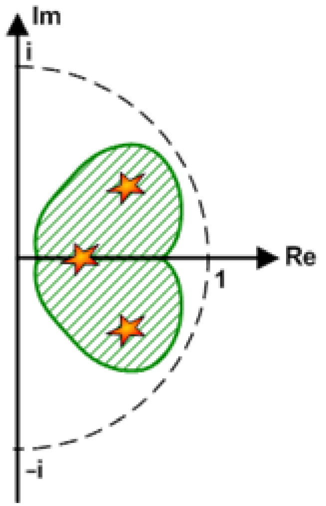


The Kolmogorov n-width theory

- Result (Oliveira e Silva):
  - $G_b(z)$ an inner function
  - Let $\mathbb{K}$ be the set of systems having poles in the region:

$$\left\{ z \in \mathbb{D} \middle| \left| G_b\left(z^{-1}\right) \right| \leq \rho \right\}$$

  - The OBFs, generated by $G_b(z)$ are optimal in the n-width sense

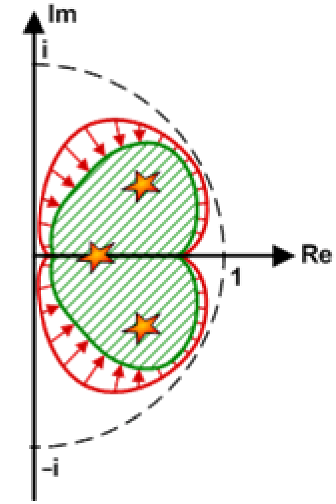


Optimal basis selection

- The inverse Kolmogorov n-width theory
  - Given a region of poles: Ω
  - Try to approximate it as

$$\Omega \approx \Omega(\Xi_n, \rho) = \left\{ z \in \mathbb{D} \middle| \left| G_b\left(z^{-1}\right) \right| \leq \rho \right\}$$

$$\rho = \text{decay rate of error}$$

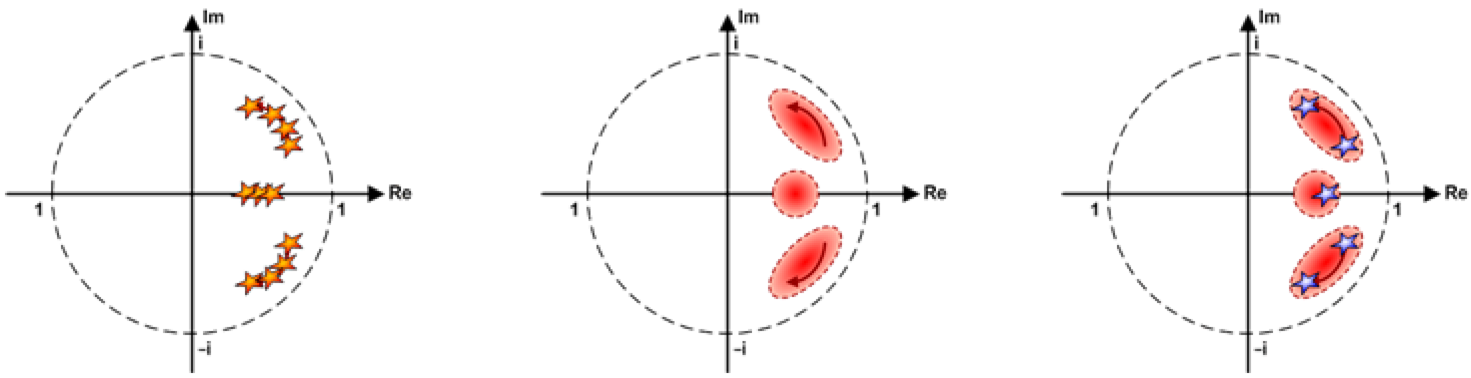


  - The n optimal OBF poles obtained through (Kolmogorov measure minimization):

$$\min_{\Xi_n \subset \mathbb{D}} \rho = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} \left| G_b\left(z^{-1}\right) \right| = \min_{\Xi_n \subset \mathbb{D}} \max_{z \in \Omega} \left| \prod_{k=1}^{n} \frac{z - \zeta_k}{1 - z\zeta_k^*} \right|$$

A Fuzzy Clustering approach

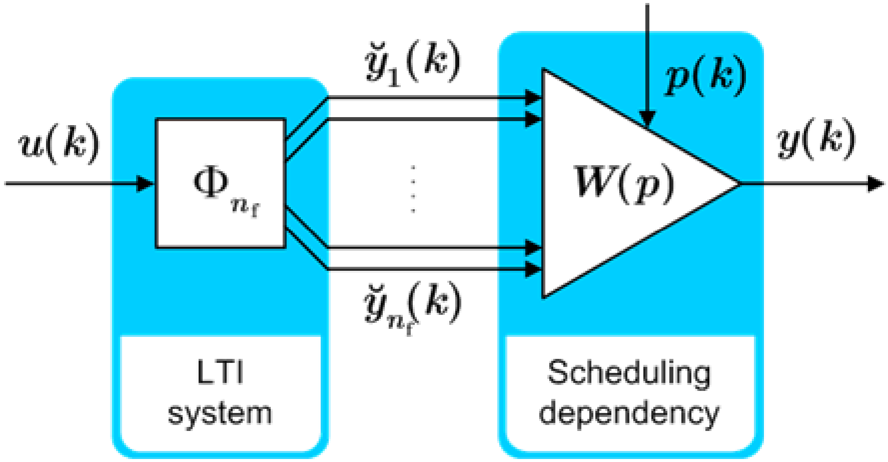
- Optimization problem in practice
  - Prior knowledge & local pole observations Ω
  - Clustering of $\mathcal{Z}$ to obtain $\mathcal{Z} = \{z_1, \ldots, z_N\}$
  - Application of Kolmogorov n-width



Delft Center for Systems and Control

An orthonormal basis functions approach

The following global model structure results:



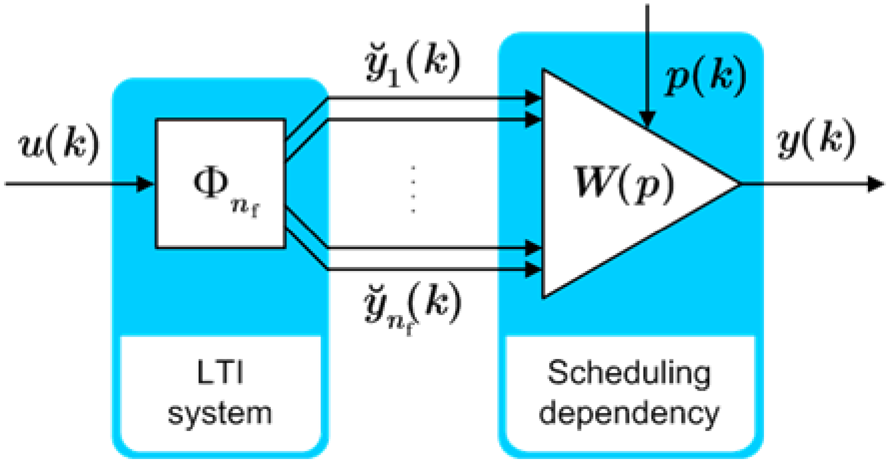
Wiener LPV model

Static p-depence is linearly parametrized (e.g. polynomial, splines)

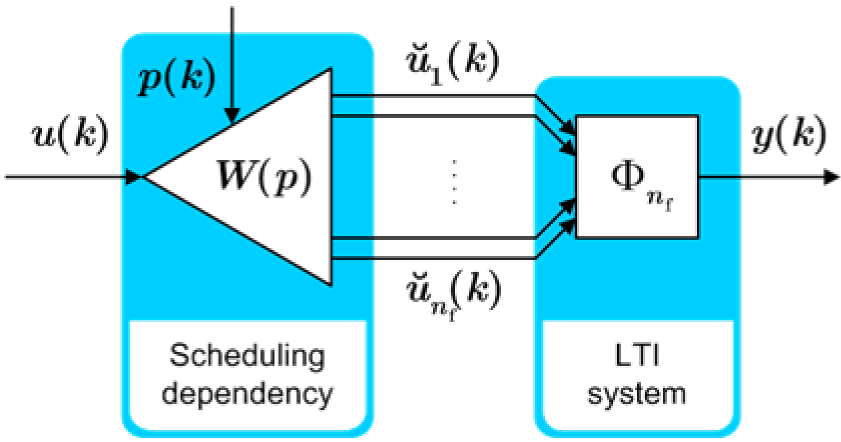
Estimation through linear regression (OE-form)

An orthonormal basis functions approach

Different alternatives:



Wiener LPV model



Hammerstein LPV model

Different results due to the finite expansion, and the static p-dependence

Example

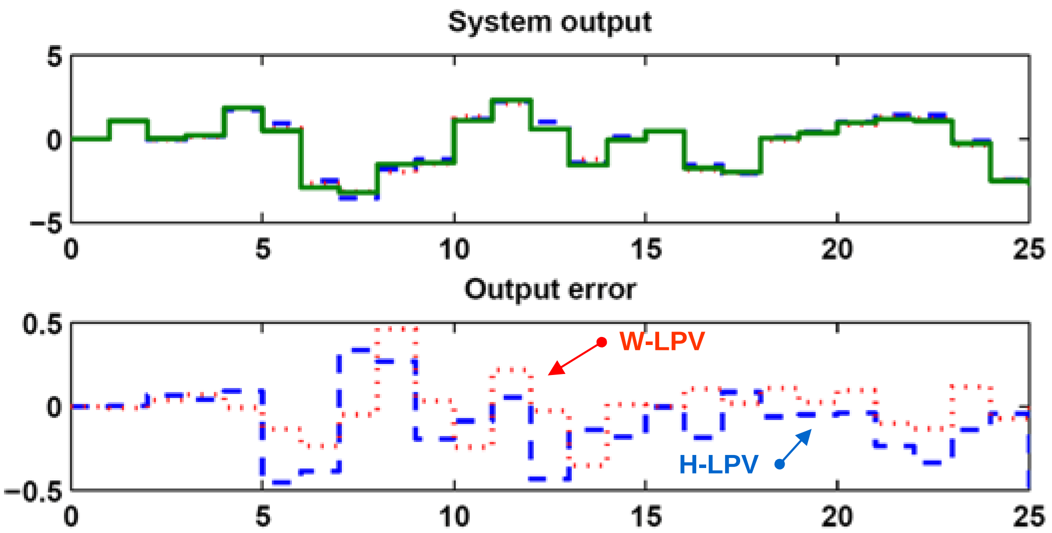
- Example (LPV-I/O identification)
  - LPV system $\mathcal{S}$ with I/O representation:

$$a_0(p(k))\,y(k) = b_1(p(k))\,u(k-1) - \sum_{l=1}^{5} a_l(p(k))\,y(k-l)$$

$$\begin{aligned}
a_0(p) &= 0.58 - 0.1p, & a_1(p) &= -\frac{511}{860} - \frac{48}{215}p^2 + 0.3(\cos(p) - \sin(p)),\\
a_2(p) &= \frac{61}{110} - 0.2\sin(p), & a_3(p) &= -\frac{23}{85} + 0.2\sin(p),\\
a_4(p) &= \frac{12}{125} - 0.1\sin(p), & a_5(p) &= -0.003,\ b_1(p) = \cos(p).
\end{aligned}$$

with $\mathbb{P} = [0.6, 0.8]$. Identify it with W-LPV and H-LPV OBF models!

OBFs based LPV identification



Global identification of $\mathcal{S}$

model	SNR	MSE	BTF	VAF
W-LPV	15 dB	0.0572	83.69%	97.34%
H-LPV	15 dB	0.0973	78.72%	95.48%

7 OBFs

Data: $p \in \mathcal{U}(0.6, 0.8)$, $u \in \mathcal{U}(-1, 1)$

500 samples long

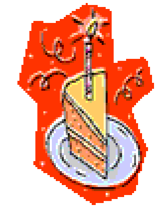
Noise: $v_e \in \mathcal{N}(0, 0.5)$

output additive

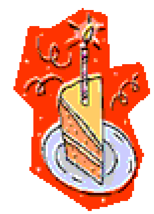
Delft Center for Systems and Control

Conclusions

- LPV models are an effective engineering tool for dealing with nonlinear systems.
- LPV model structures for identification are studied and basic structures and phenomena have been clarified.
- OBF's provide a powerful tool for parametrizing relevant classes of LPV systems
- There is work to be done on completing the picture of a general framework for LPV identification.



Peter



Roland

Paul

Jozsef:

A big toast and Congratulations with your 60th birthday

Further reading

- R. Tóth (2008). *Modeling and Identification of Linear Parameter-Varying Systems – An Orthonormal Basis Function Approach.* Dr. Dissertation, Delft University of Technology, December 2008.
- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2008). Asymptotically optimal orthonormal basis functions for LPV system identification. Provisionally accepted for publication in *Automatica.*
- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2008). Flexible model structures for LPV identification with static scheduling dependency. To appear in *Proc. 47th IEEE Conf. Decision and Control*, Cancun, Mexico, 9-11 December 2008.
- R. Tóth, F. Felici, P.S.C. Heuberger and P.M.J. Van den Hof (2008). Crucial aspects of zero-order hold LPV state-space system discretization. In: M.J. Chung, P. Misra and H. Shim (Eds.), *Proc. 17th IFAC World Congress*, 6-11 July 2008, Seoul, Korea, pp. 3246-3251.
- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2007). LPV system identification with globally fixed orthonormal basis functions. *Proc. 46th IEEE Conf. Decision and Control*, 12-14 December 2007, New Orleans, LA.
- R. Tóth, F. Felici, P.S.C. Heuberger and P.M.J. Van den Hof (2007). Discrete time LPV I/O and state space representations, differences of behavior and pitfalls of interpolation. *Proc. European Control Conference 2007, ECC '07*, 2-5 July 2007, Kos, Greece, pp. 5418-5425.
- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2006). Orthonormal basis selection for LPV system identification, the fuzzy-Kolmogorov c-max approach. *Proc. 45th IEEE Conf. Decision and Control*, 13-15 December 2006, San Diego, CA, 2006, pp. 2529-2534.
- R. Tóth, P.S.C. Heuberger and P.M.J. Van den Hof (2006). Optimal pole selection for LPV system identification with OBFs, a clustering approach. *Proc. 14th IFAC Symposium on System Identification*, 27-29 March 2006, Newcastle, Australia, pp. 356-361.