System Identification in Dynamic Networks

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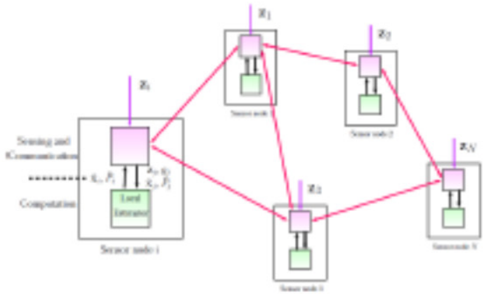
Where innovation starts

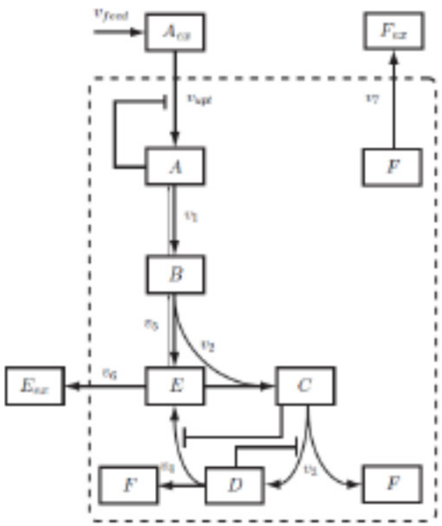
Introduction – dynamic networks

Distributed Control



source: Simonetto 2012

Biological Systems

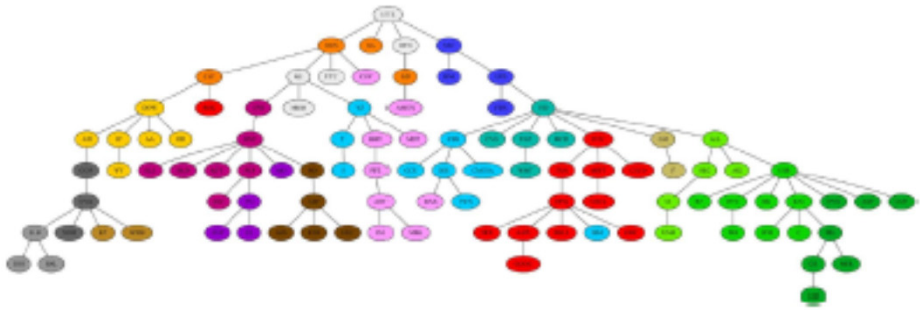


source: Hillen 2012

Power Systems

source: Pierre et al. 2012

Financial Systems

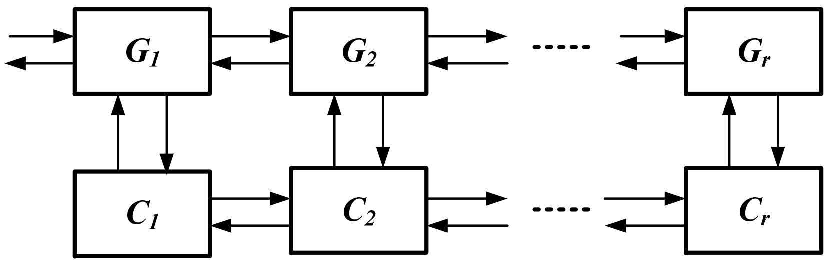


source: Materassi et al. 2010

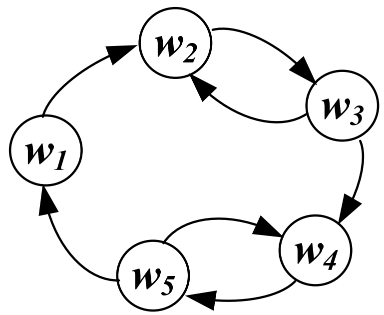
Introduction – dynamic networks

Dynamical systems in emerging fields have a more complex structure:

distributed control system (1d-cascade)



dynamic network



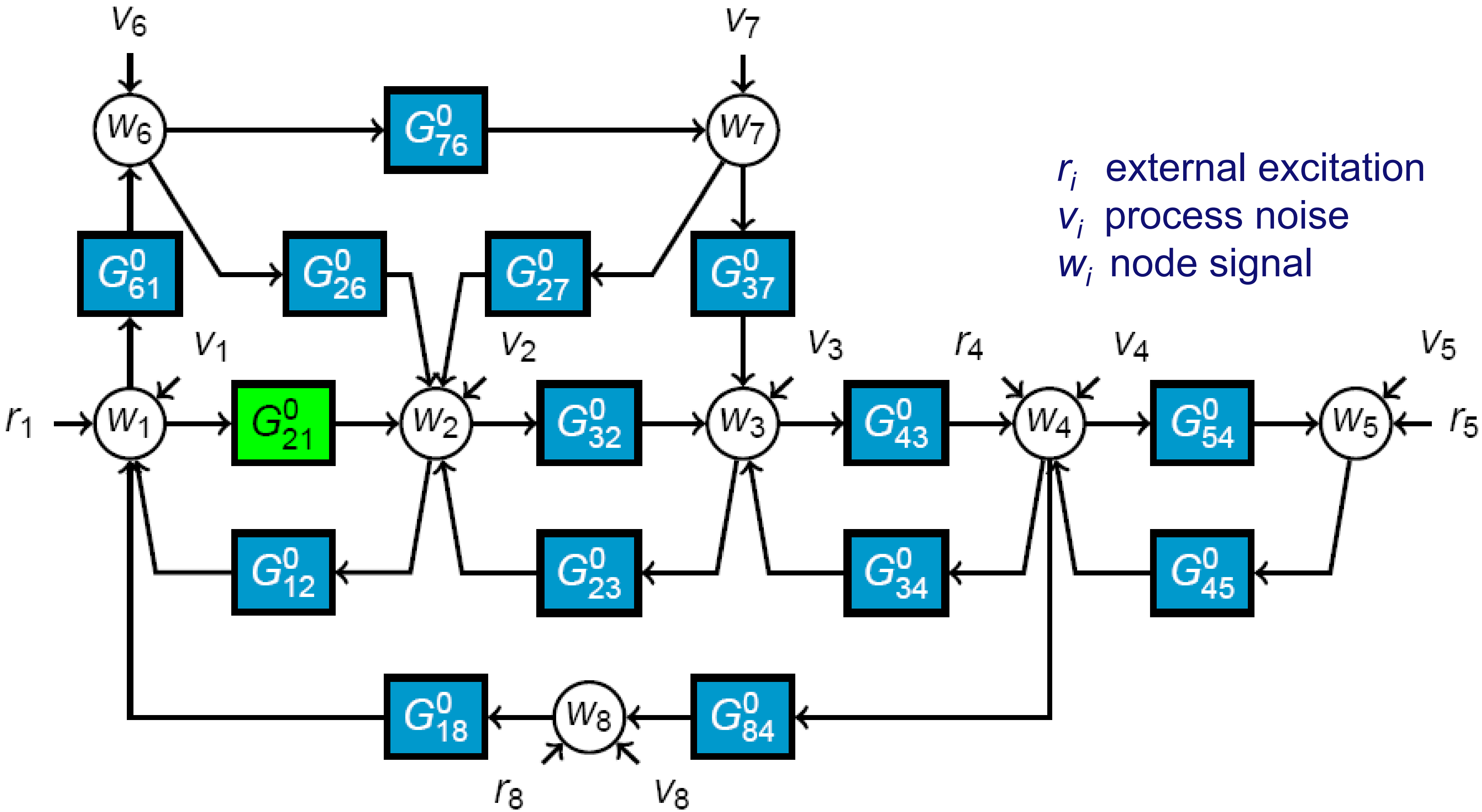
(distributed systems, multi-agent systems, biological networks, smart grids,…..)

For on-line monitoring / control / diagnosis it is attractive to be able to ***identify***

- (changing) dynamics of particular modules
- (changing) interconnection structure

What are relevant identification questions that appear?

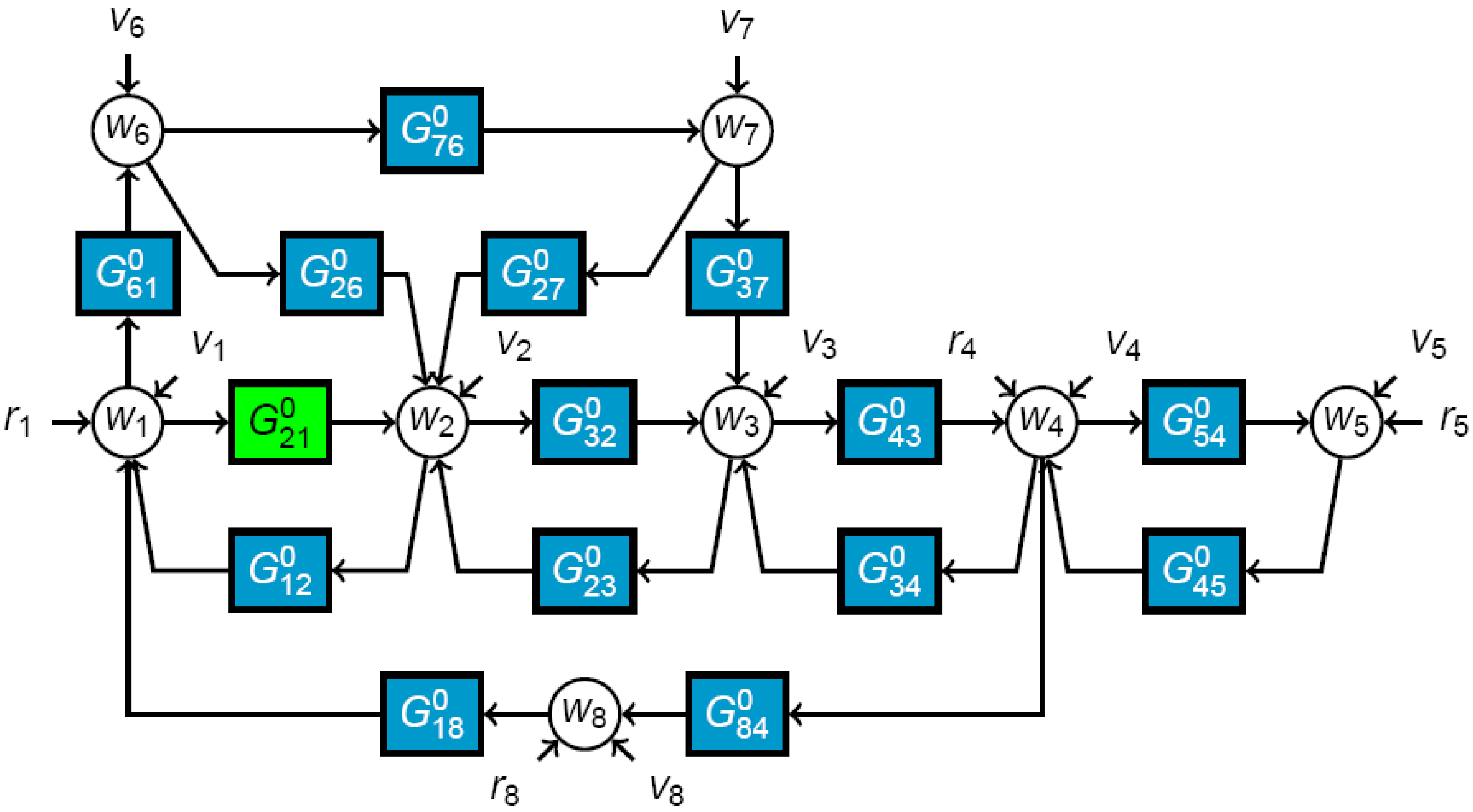
Introduction



r_i external excitation
v_i process noise
w_i node signal

Some modules may be known (e.g. controllers)

Introduction – relevant identification questions

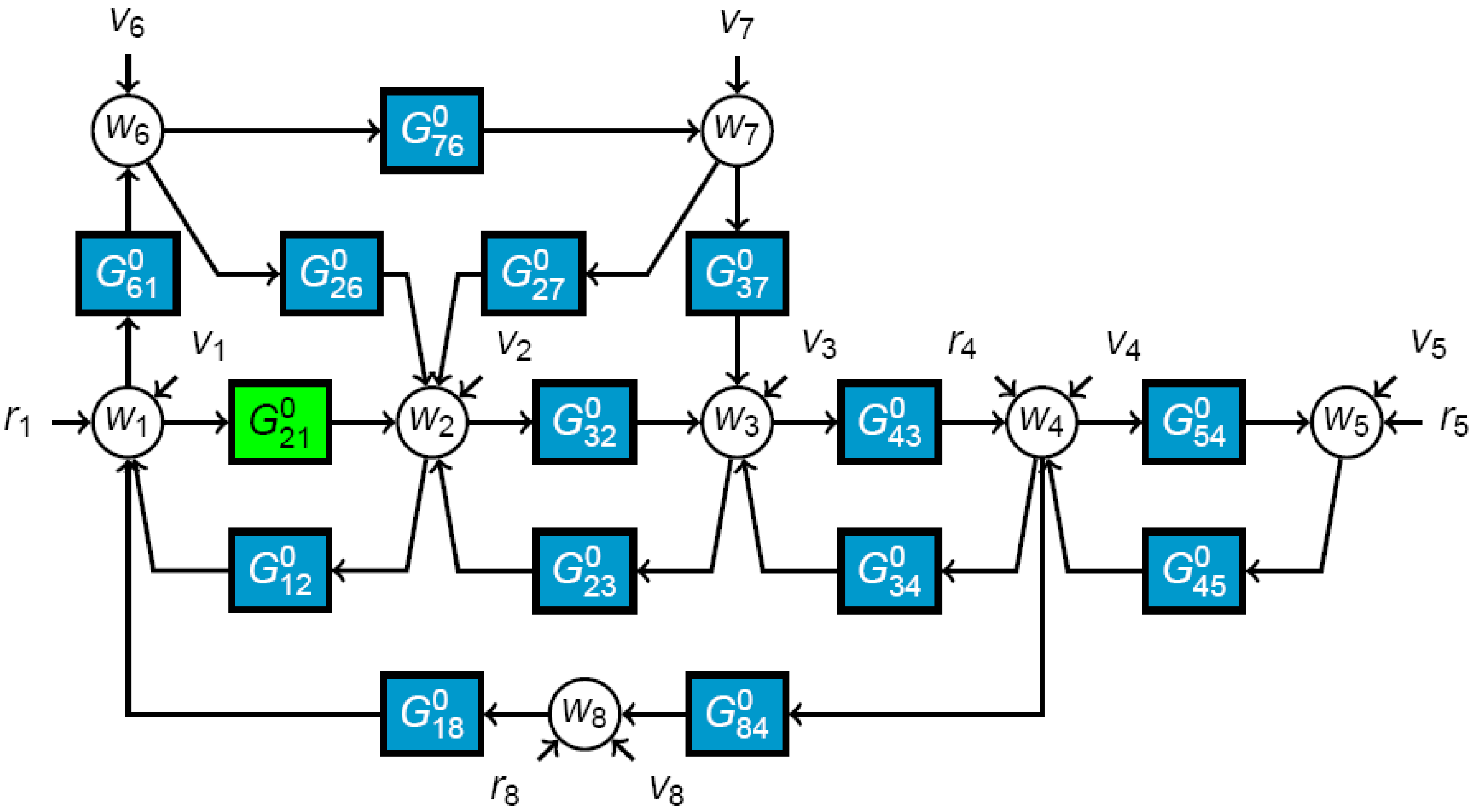


How to perform "local" identification (i.e. estimating only a single module)?

Where to put sensors and actuators for optimal accuracy?

How to utilize known structure/topology and known modules?

Introduction – relevant identification questions



Can we identify the topology?

Can we deal with sensor noise?

Do we need directions of arrows?

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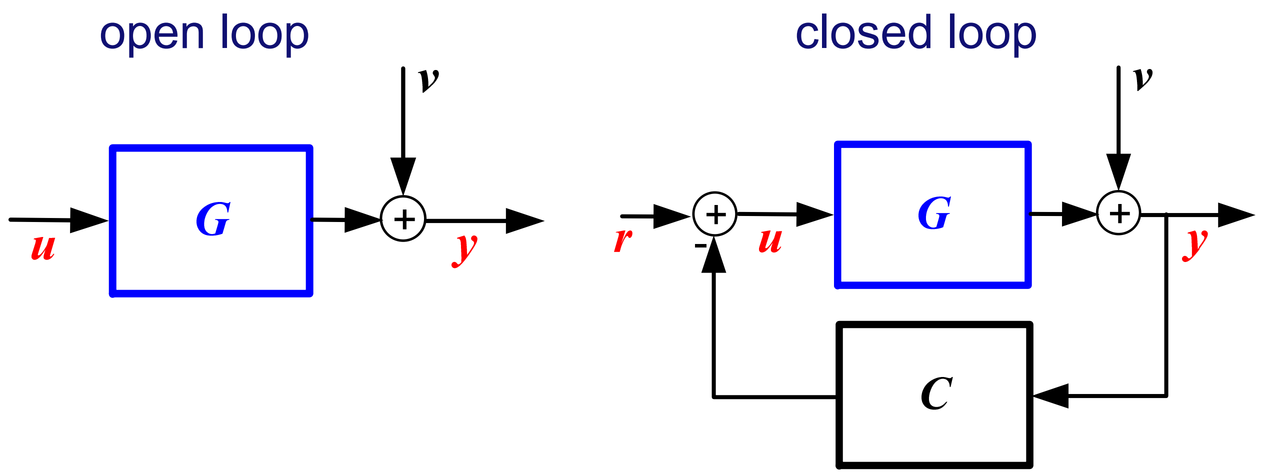
Introduction - identification

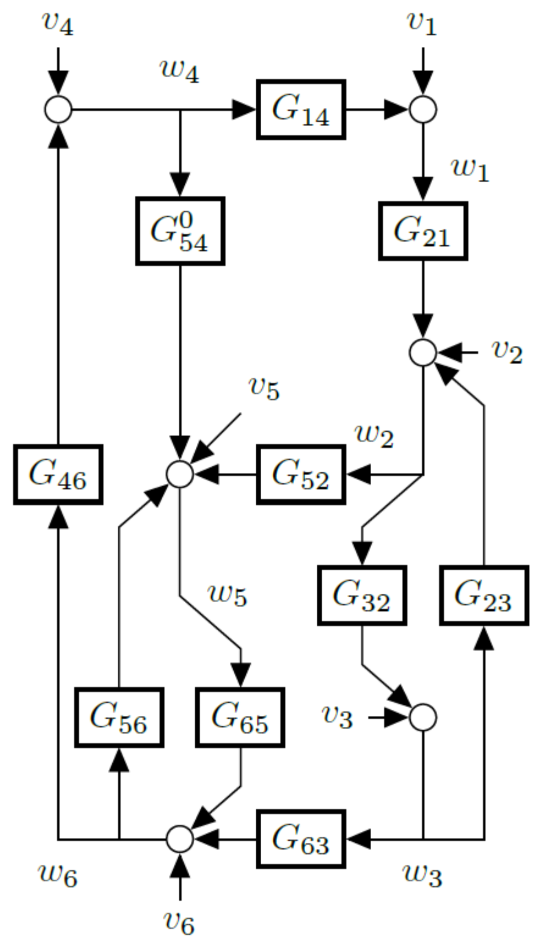
The classical identification problems:



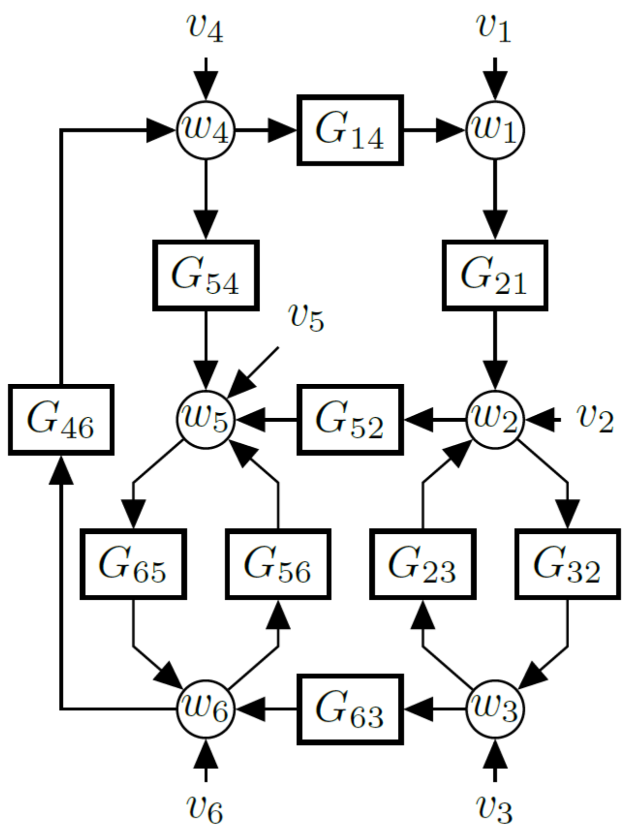
Identify a plant model $\hat{G}$ on the basis of measured signals *u*, *y* (and possibly *r*)

- **We have to move from fixed and known configuration to deal with and exploit *structure* in the problem.**

Network Diagrams



Represented as



Labels of internal variables placed inside summations

Introduction

Current literature

Numerical fast algorithms for **spatially distributed systems** with
identical modules (Fraanje, Verhaegen, Werner), or
non-identical ones (Torres, van Wingerden, Verhaegen, Sarwar, Salapaka, Haber)

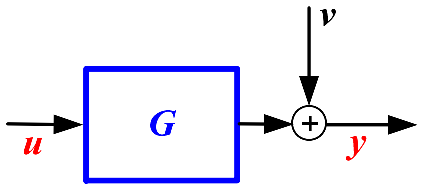
Contributions to **topology detection**: Chiuso, Materassi, Innocenti, Salapaka, Yuan, Stan, Warnick, Goncalves, Sanandaji, Vincent, Wakin, further exploring and utilizing the concept of Granger causality.

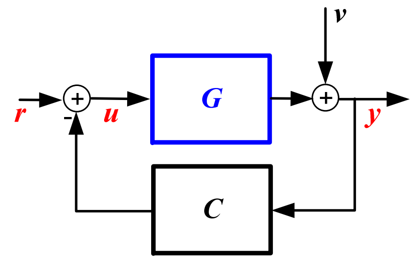
Here: focus on **prediction error methods** and concepts for identification in generally structured (linear) dynamic networks

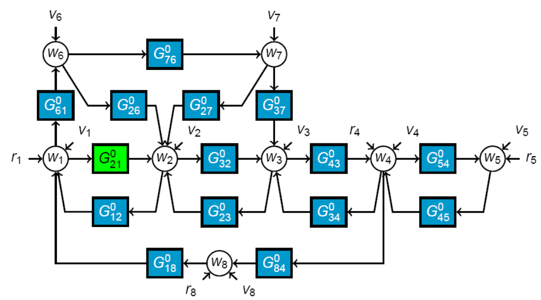
Contents

Towards dynamic network identification

- The basic (prediction error) tools: direct and 2s
- Dynamic network setup
- Single module identification - consistency
  - full MISO models
  - predictor input (sensor) selection
- Sensor noise – the errors-in-variables problem
- Discussion / Wrap-up







Methods for closed-loop identification

1. Direct method

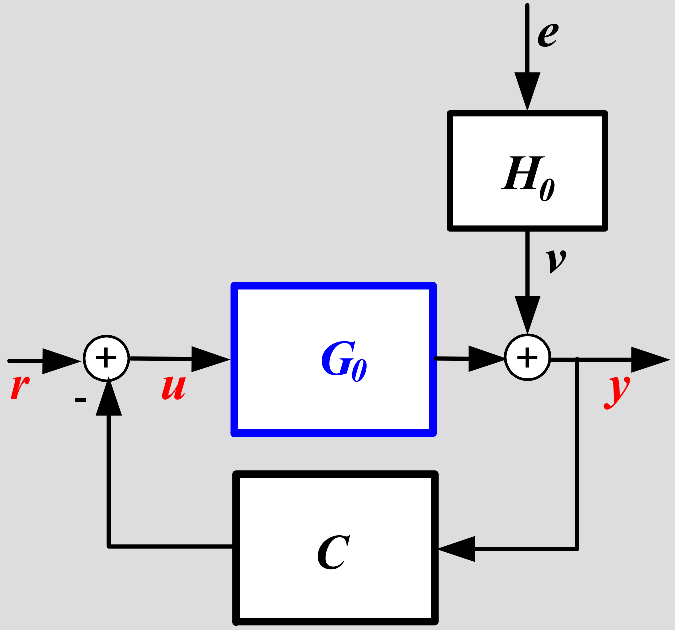
Relying on full-order noise modelling

$$\varepsilon(t,\theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)]$$

prediction error $\varepsilon(t,\theta)$ to become a white noise signal $e(t)$ in the optimum.

Using only signals u and y, discarding r

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N}\sum_{t=1}^{N}\varepsilon(t,\theta)^2$$



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

Methods for closed-loop identification

1. Direct method

Consistency result [Ljung, 1987]

$$\{G(\hat{\theta}_N), H(\hat{\theta}_N)\} \rightarrow \{G_0, H_0\} \text{ w.p.1}, N \rightarrow \infty$$

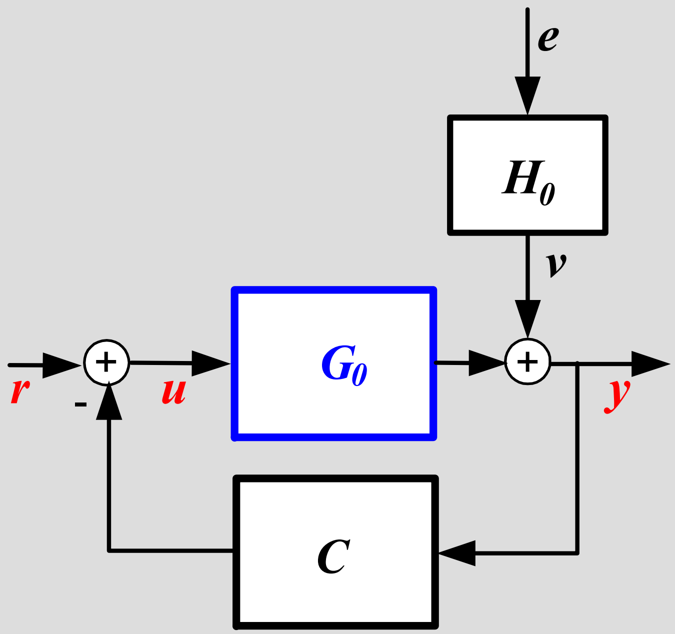
if

- **full order noise model** $(\mathcal{S} \in \mathcal{M})$
- **delay in every loop**
- **sufficient excitation, i.e.**

$$\Phi_z(\omega) > 0 \quad \forall\omega \qquad z = \begin{bmatrix} y \\ u \end{bmatrix}$$

with spectral density

$$\Phi_z(\omega) = \mathcal{F}\{\bar{E}[z(t)z(t-\tau)]\}$$



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

Methods for closed-loop identification

2. Two-stage/projection/IV method

- Relying on measured external excitation
- Decoupling estimation of G_0 and H_0

$$\varepsilon(t,\theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

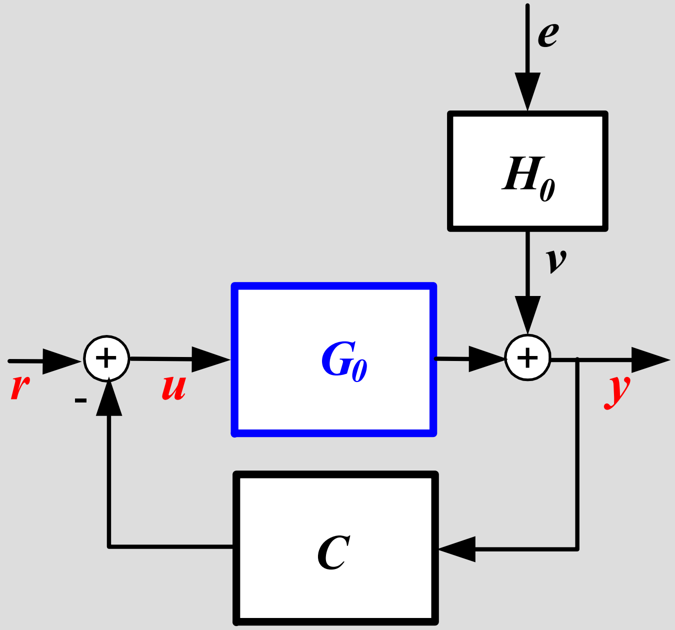
with u^r the signal u projected onto r

such that

$$u = u^r + u^v$$

with u^r and u^v uncorrelated.

Similar least squares criterion.



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

Methods for closed-loop identification

2. Two-stage/projection/IV method

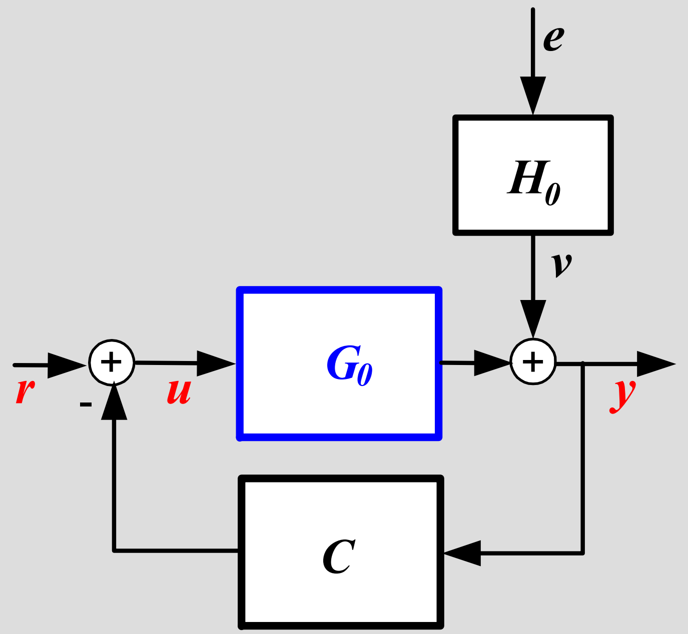
Consistency result [Van den Hof & Schrama, 1993]

$$G(\hat{\theta}_N) \rightarrow G_0 \ \text{w.p.1}, N \rightarrow \infty$$

if

- **full order plant model ($G_0 \in \mathcal{G}$)**
- **no conditions on loop delays**
- **sufficient excitation condition:**

$$\Phi_{u^r}(\omega) > 0 \quad \forall\omega$$



Plant representation

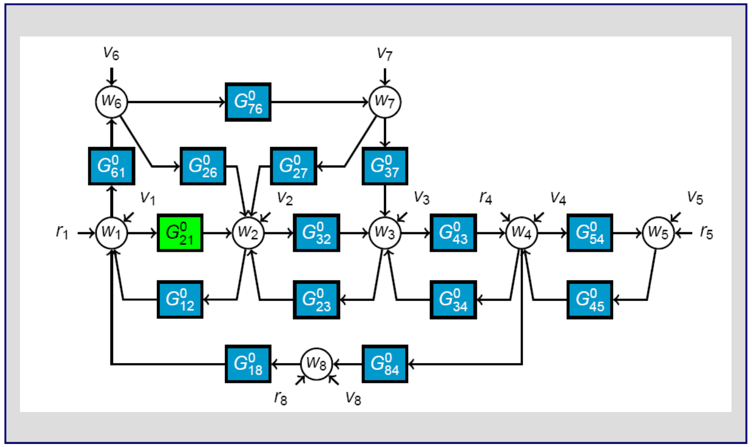
$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

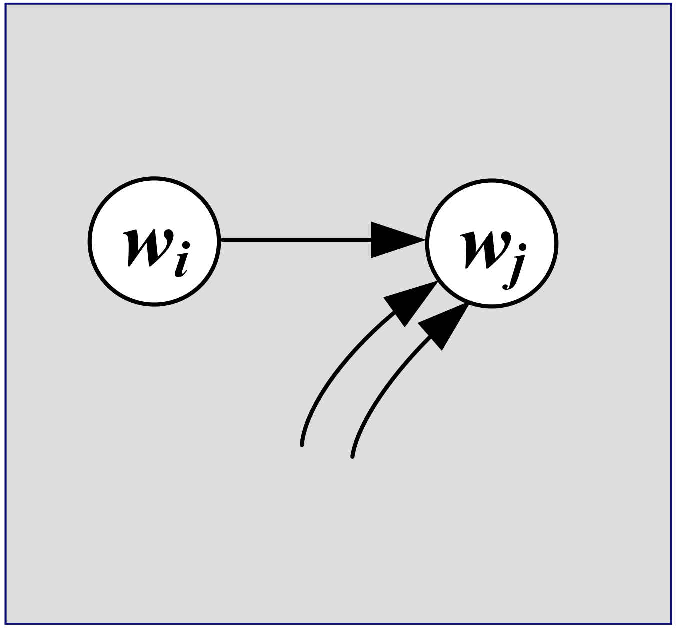
Question

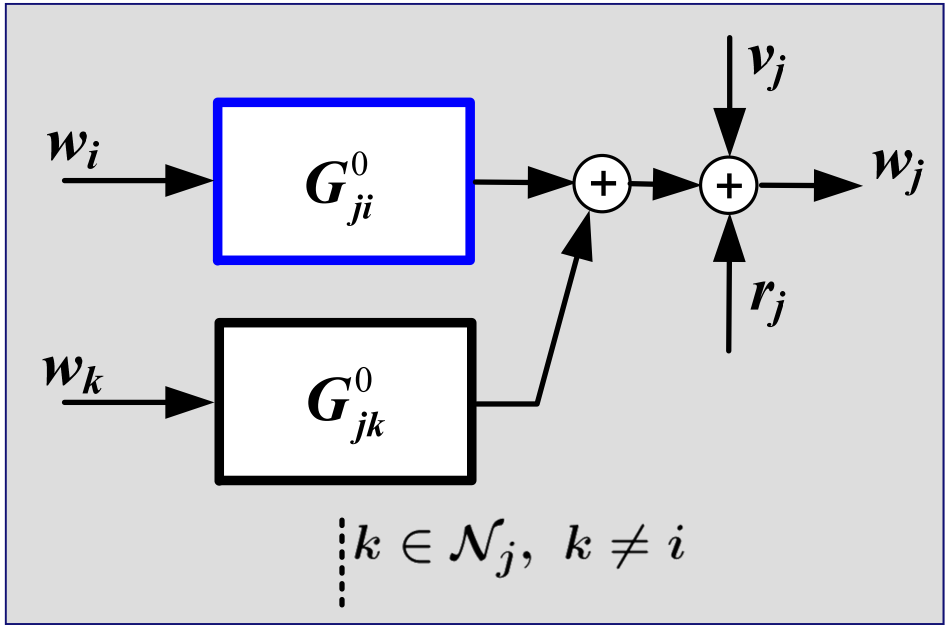
- **Can we utilize these tools for identification of transfer functions in a (complex) dynamic network ?**



Network Setup

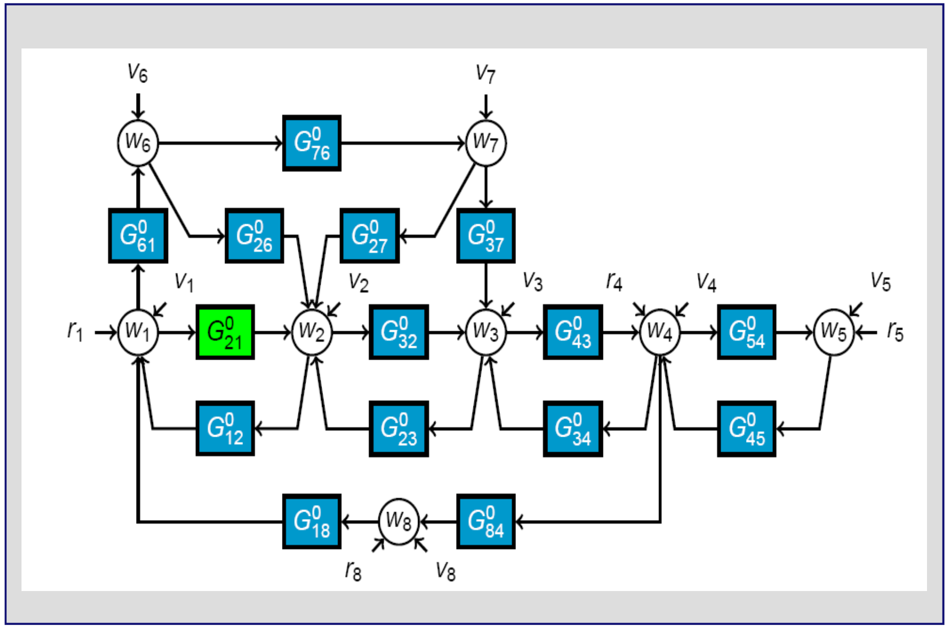
Formalizing one link (transfer between w_i and w_j)





- On each node a disturbance v_j and a reference r_j might be present
- Reference signals are uncorrelated to noise signals
- $\mathcal{N}_j$: set of nodes that has a direct causal link with node j, of which $\mathcal{K}_j$ are known transfers and $\mathcal{U}_j$ unknown.

Network Setup



Assumptions:

- **Total of L nodes**
- **Network is well-posed**
  $I - G^0$ causally invertible
- **Stable (all signals bounded)**
- **All $w_m, m = 1, \cdots L,$ measured, as well as all present r_m**
- **Modules may be unstable**

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

Network Setup

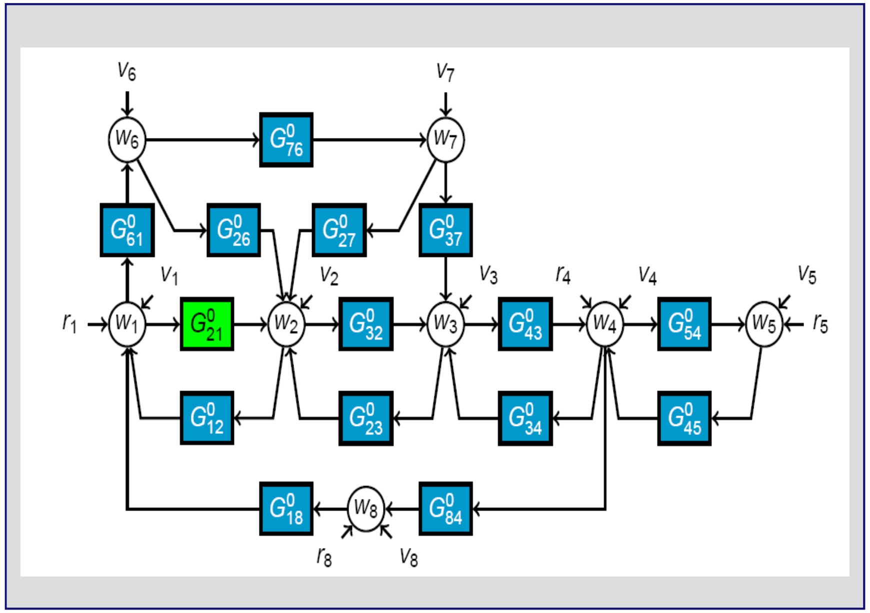
Options for identifying a module:

- **Identify the full MIMO system:**

$$w = (I - G^0)^{-1}[r + v]$$

from measured r and w.

Global approach with "standard" tools



- **Identify a local (set of) module(s)**
  from a (sub)set of measured r_k and w_ℓ

Local approach with "new" tools and structural conditions

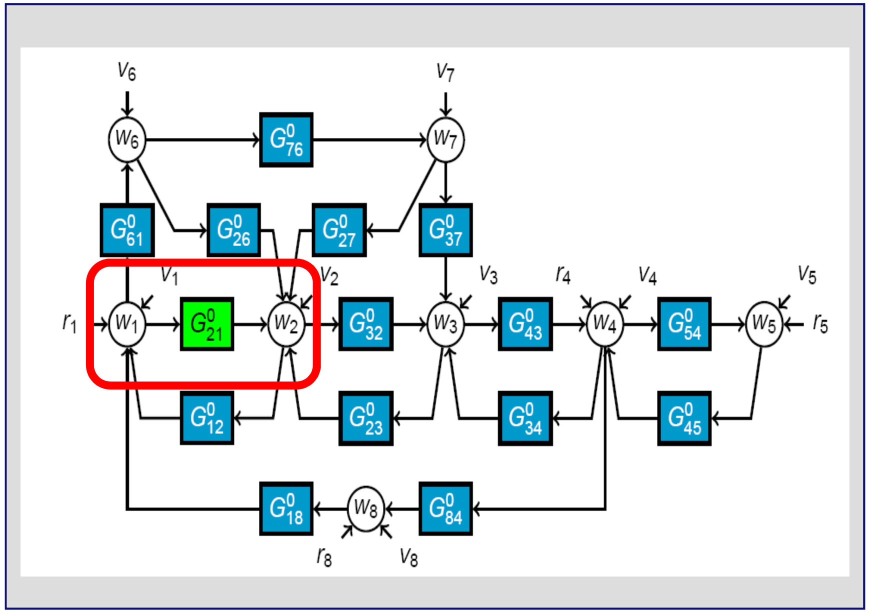
Network Setup

How to identify a module:

Suppose we are interested in $\boldsymbol{G^0_{21}}$

Can it be identified from measured input $\boldsymbol{w_1}$ and output $\boldsymbol{w_2}$?

Typically bias will occur due to "neglecting" the rest of the network



- Non-modelled disturbances on $\boldsymbol{w_2}$ can create problems
- The observed transfer between $\boldsymbol{w_1}$ and $\boldsymbol{w_2}$ is not necessarily equal to $\boldsymbol{G^0_{21}}$

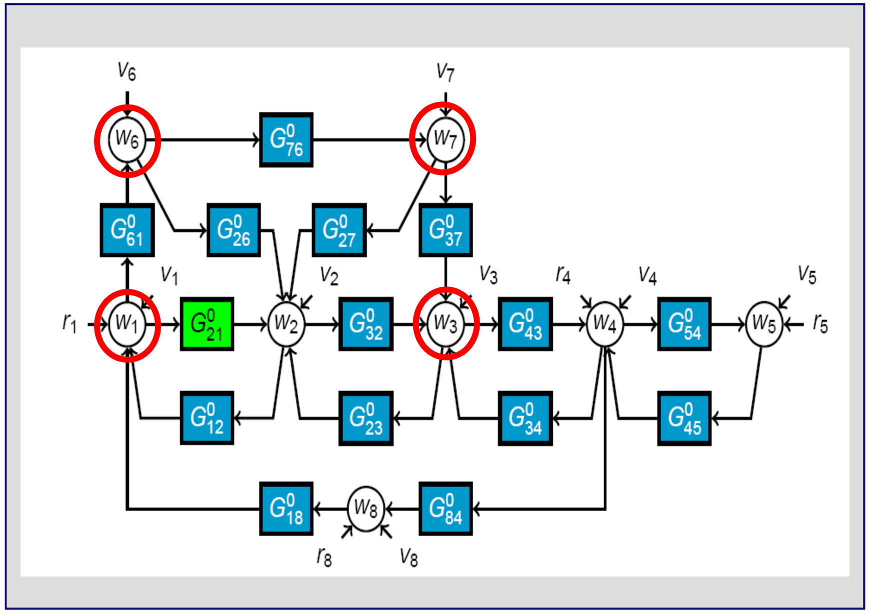
Network Setup

How to identify a module:

Two approaches for finding G^0_{21}

- **Full MISO approach:**
  Include all node signals that directly map into w_2 in an input vector, and identify a MISO model

- **Predictor input selection:**
  Formulate conditions for checking the sufficiency of set of nodes to include as inputs in a MISO model



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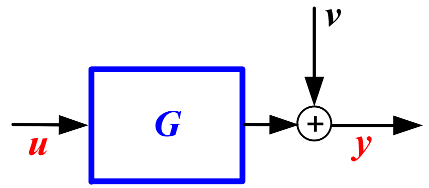
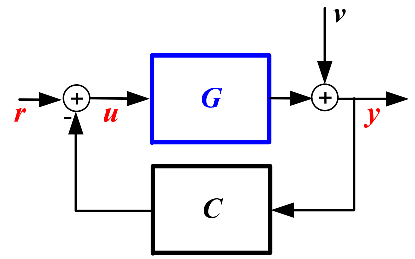
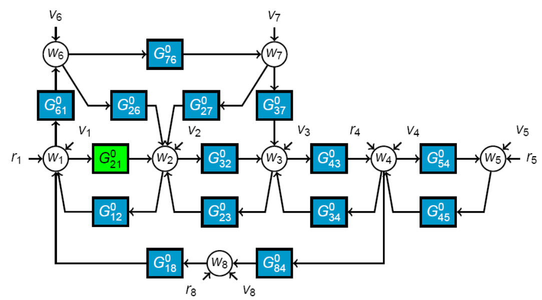






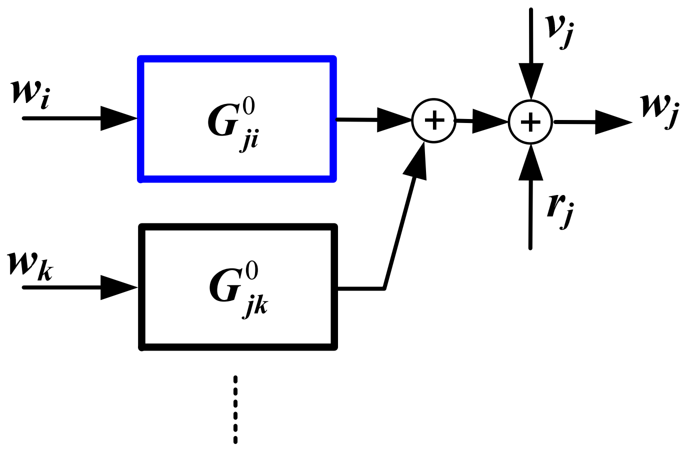
Full MISO models – Direct method

Module of interest: G_{ji}^0

Separate the remaining modules: G_{jk}^0

into **known** transfers: $G_{jk}^0,\ k \in \mathcal{K}_j$

and **unknown** transfers: $G_{jk}^0,\ k \in \mathcal{U}_j^i$



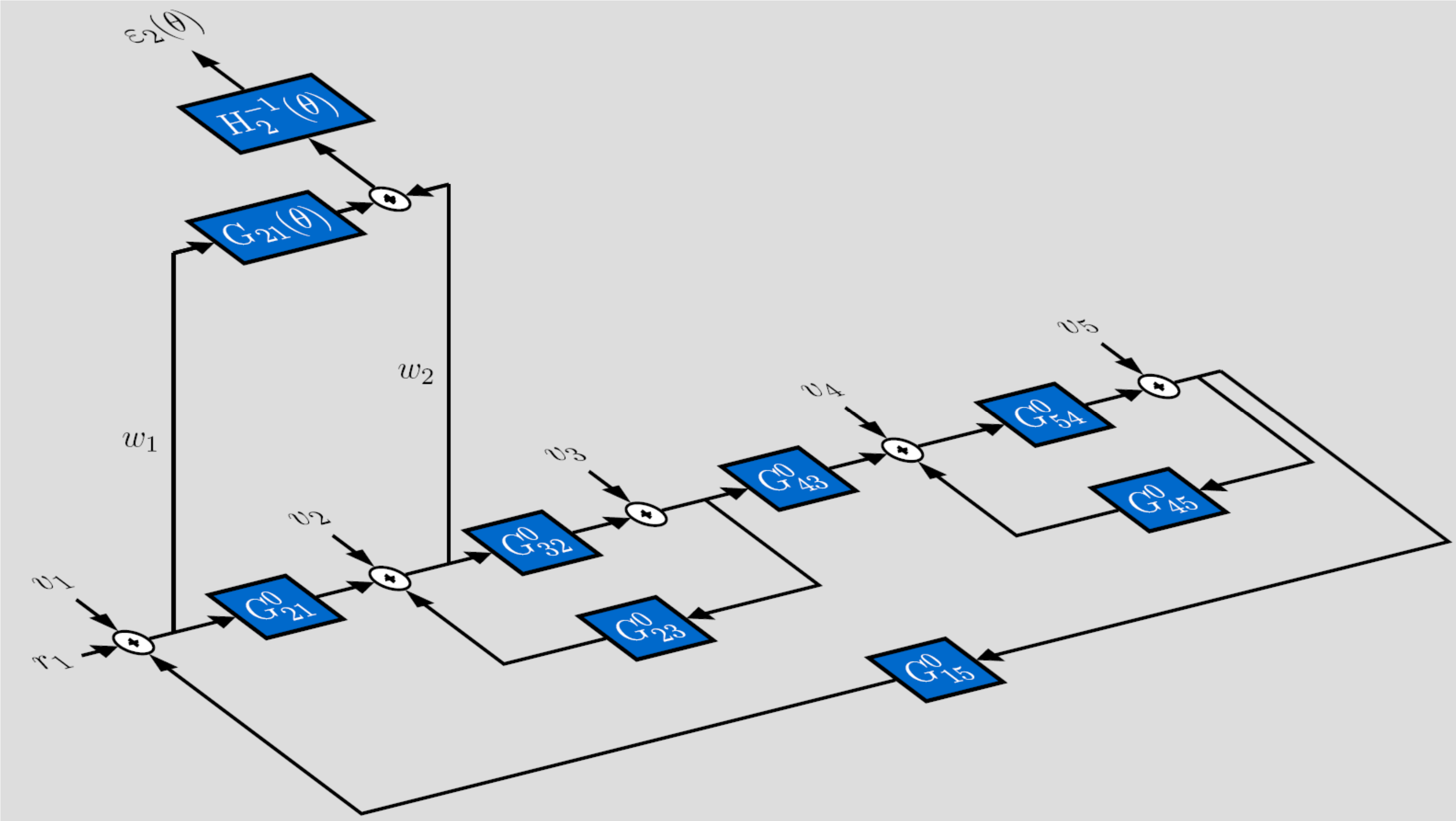
A MISO approach:

$$\varepsilon(t,\theta) = H_j(\theta)^{-1}[\underbrace{w_j - r_j - \sum_{k\in\mathcal{K}_j} G_{jk}^0 w_k}_{\tilde{w}_j \text{ known}} - G_{ji}(\theta)w_i - \sum_{k\in\mathcal{U}_j^i} G_{jk}(\theta)w_k]$$

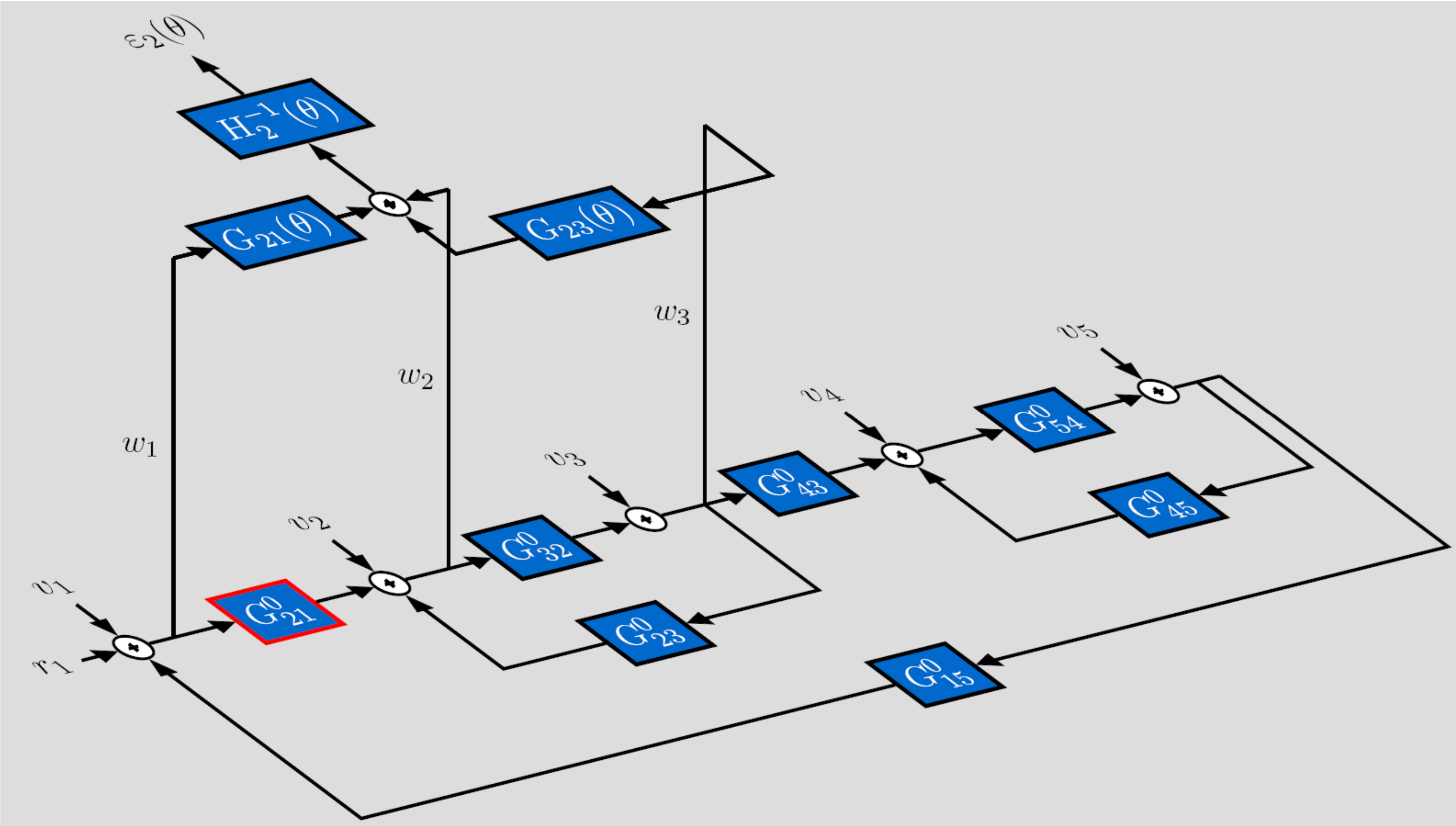
$G_{jk}^0,\ k \in \mathcal{U}_j$

Simultaneous identification of transfers $G_{jk}^0,\ k \in \mathcal{U}_j^i$ and a noise model for v_j

Network Identification – Direct method



Network Identification – Direct method



Network Identification – Direct method

Result direct method

The plant models $G_{jk}(\theta),\ k \in \mathcal{U}_j$ are consistently estimated if:

- All parametrized plant and noise models are correctly parametrized,
$$G_{jk}(\theta),\ k \in \mathcal{U}_j;\ H_j(\theta)\ \ (\mathcal{S} \in \mathcal{M})$$
- Every loop in the network that runs through node j has at least one delay (no algebraic loop)
- $\Phi_z(\omega) > 0\ \ \forall\omega$, for $z := vec\{w_j, \{w_k\}_{k\in\mathcal{U}_j}\}$
(excitation condition)
- Noise source v_j is uncorrelated with all other noise terms in the network

[P.M.J. Van den Hof, A. Dankers, P.S.C. Heuberger and X. Bombois. *Automatica*, October 2013]

Network Identification – Two-stage method

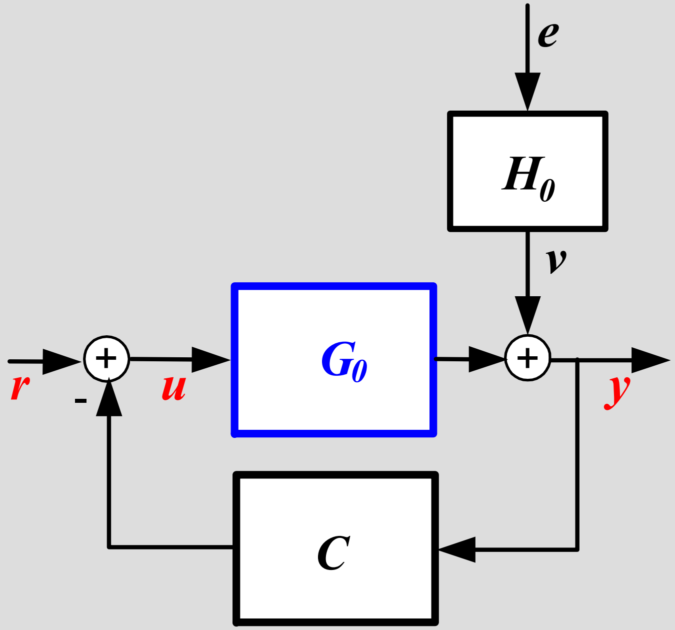
Recall the two-stage/projection/IV approach:

Project $\boldsymbol{u}$ onto an external signal $\boldsymbol{r}$
that is uncorrelated to $\boldsymbol{v}$

$$\varepsilon(t,\theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

$$u = u^r + u^v$$

with $\boldsymbol{u^r}$ and $\boldsymbol{u^v}$ uncorrelated.



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

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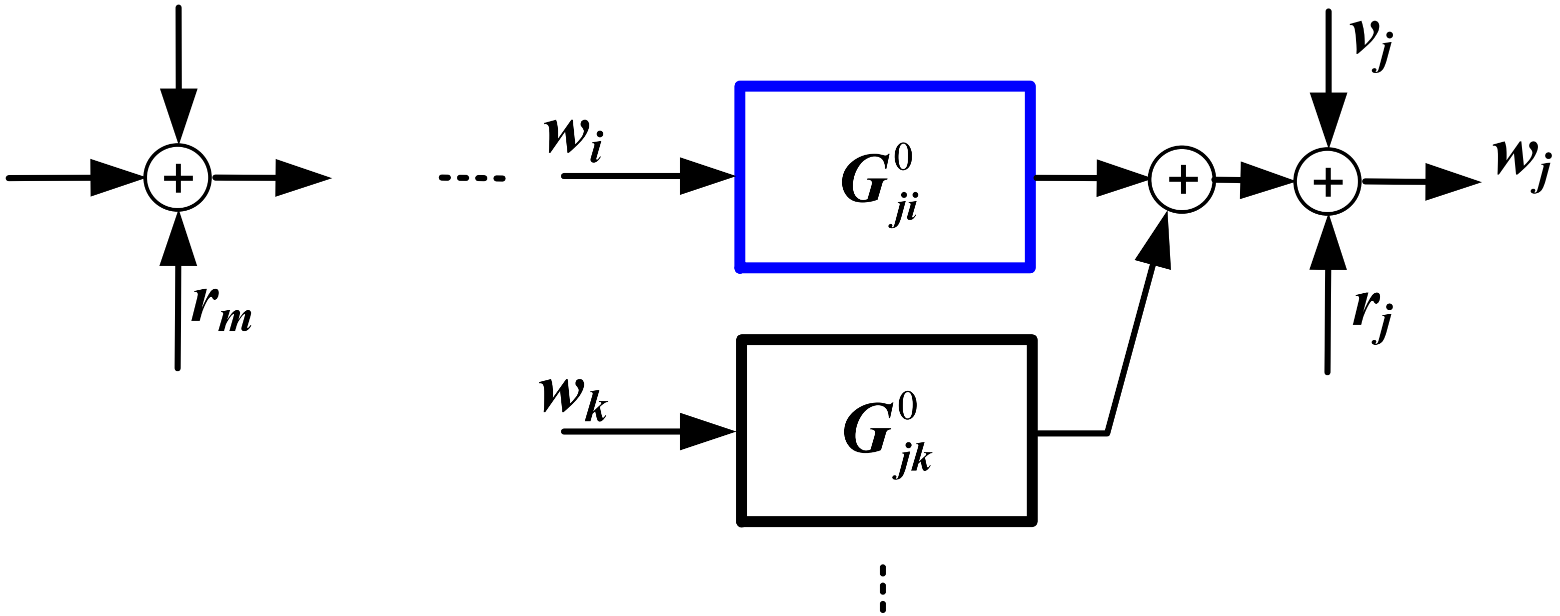
Network Identification – Two-stage method

Main approach:

- Look for an external reference signal that has a connection with w_i
- And that does not act as an unmodelled disturbance on w_j

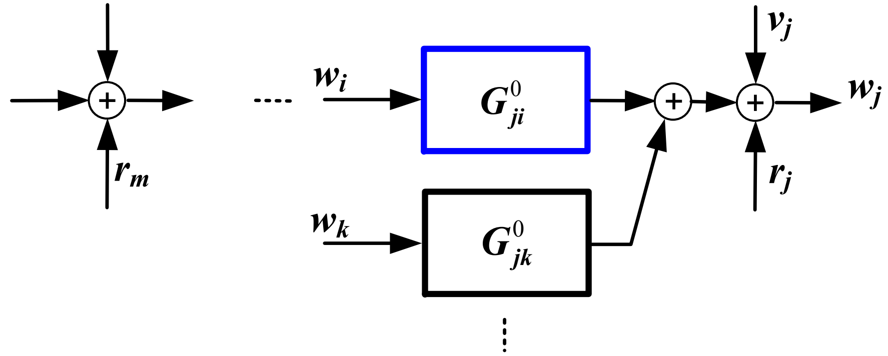


Network Identification – Two-stage method

Algorithm:

- Determine whether there exists an r_m such that $w_i^{r_m}$ is sufficiently exciting
- Construct:

$$\tilde{w}_j = \underbrace{w_j - r_j - \sum_{k \in \mathcal{K}_j} G_{jk}^0 w_k}_{\text{known terms}}$$



- Identify G_{ji}^0 through PE identification with prediction error

$$\varepsilon(t,\theta) = H_j(\rho)^{-1}[\tilde{w}_j - \sum_{k \in U_{is}} G_{jk}(\theta) w_k^{r_m}]$$

where all inputs $k \in \mathcal{U}_{is}$ are considered that are correlated to r_m

- This extends to multiple signals r_m

Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated if:

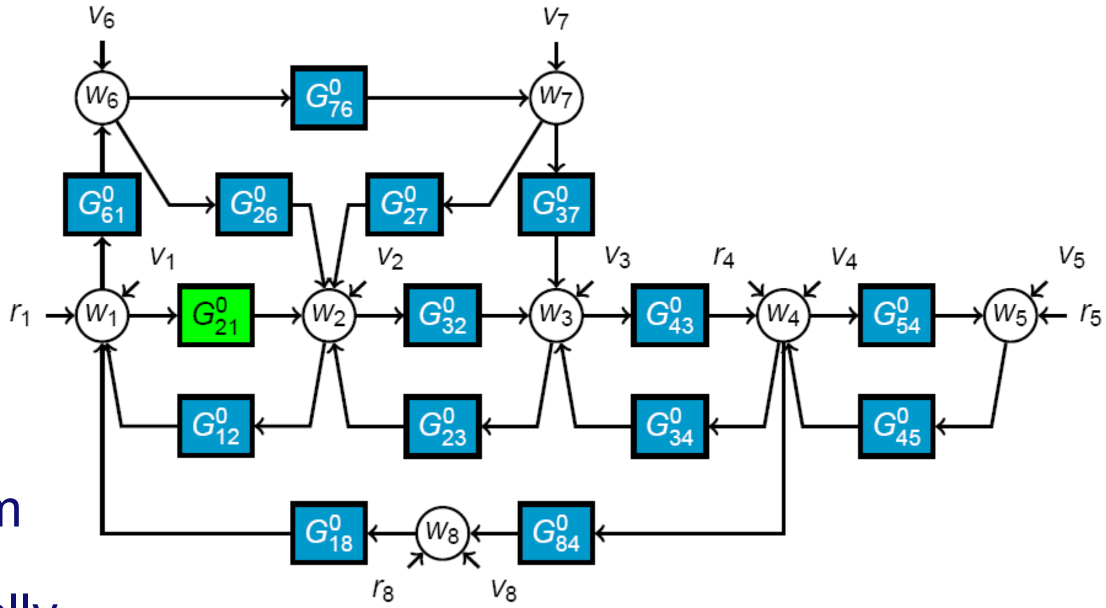
- The plant models $G_{jk}(\theta)$ are correctly parametrized $k \in \mathcal{U}_{is}$
- The vector of (projected) input signals is sufficiently exciting
- Excitation signals are uncorrelated to noise disturbances

[P.M.J. Van den Hof, A. Dankers, P.S.C. Heuberger and X. Bombois. *Automatica*, October 2013]

Network Identification – Two-stage method

Example

- External signal r_1
- Input nodes to w_2 that are correlated with r_1 : w_1, w_6, w_7, w_3
- So 4 input, 1 output problem
- Projected inputs will generally not be sufficiently exciting (we need 4 independent sources)



- Include r_4, r_5 and r_8 as external signals
- Input nodes remain the same

Network Identification – Two-stage method

Observations:

- Consistent identification of single transfers is possible, dependent on network topology and reference excitation
- Full noise models are not necessary
- No conditions on uncorrelated noise sources, nor on absence of algebraic loops
- Excitation conditions on (projected) input signals can be limiting
- Network topology conditions on r_m can simply be checked by tools from graph theory

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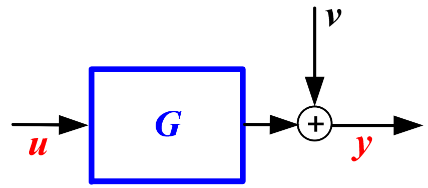
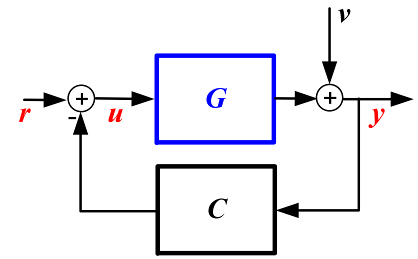
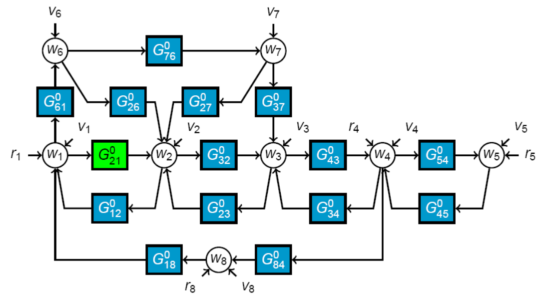






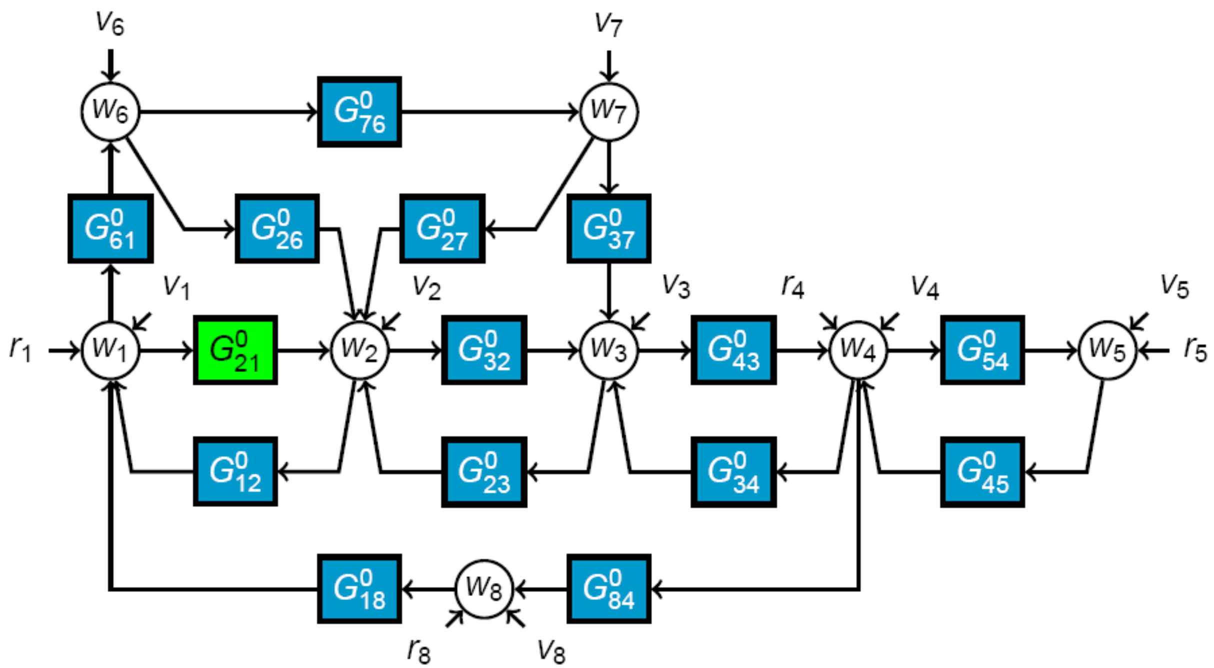
Predictor input selection

- So far: predictor input choice not very flexible
- What if some signals are hard (expensive) to measure?
- What if we would like to have flexibility in placing sensors?
- Can we formulate (more relaxed) conditions on nodes to be measured, for allowing a consistent module estimate?

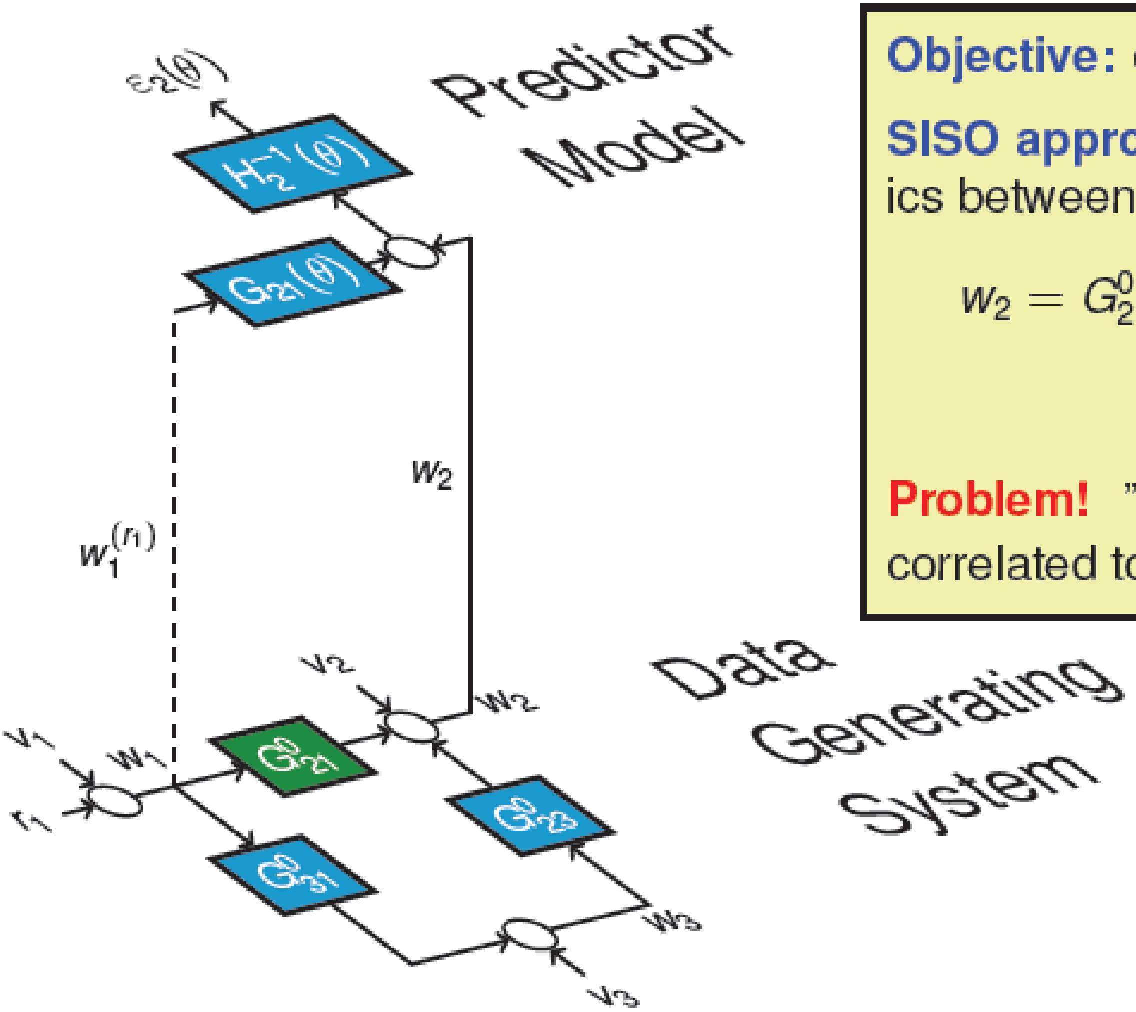
Predictor input selection

There are two basic mechanisms that "deteriorate" the transfer $\boldsymbol{G^0_{ji}}$ when observed through the input/output signals $\boldsymbol{w_i}$ and $\boldsymbol{w_j}$

1. **Parallel paths**
2. **Loops around $\boldsymbol{w_j}$**



First mechanism: parallel paths



Objective: consistently estimate G^0_{21}.

SISO approach. Try to estimate the dynamics between w_1 and w_2:

$$w_2 = G^0_{21} w_1^{(r_1)} + \underbrace{G^0_{21} w_1^{(v)} + G^0_{23} w_3 + v_2}_{\text{unmodeled term}}$$

Problem! "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

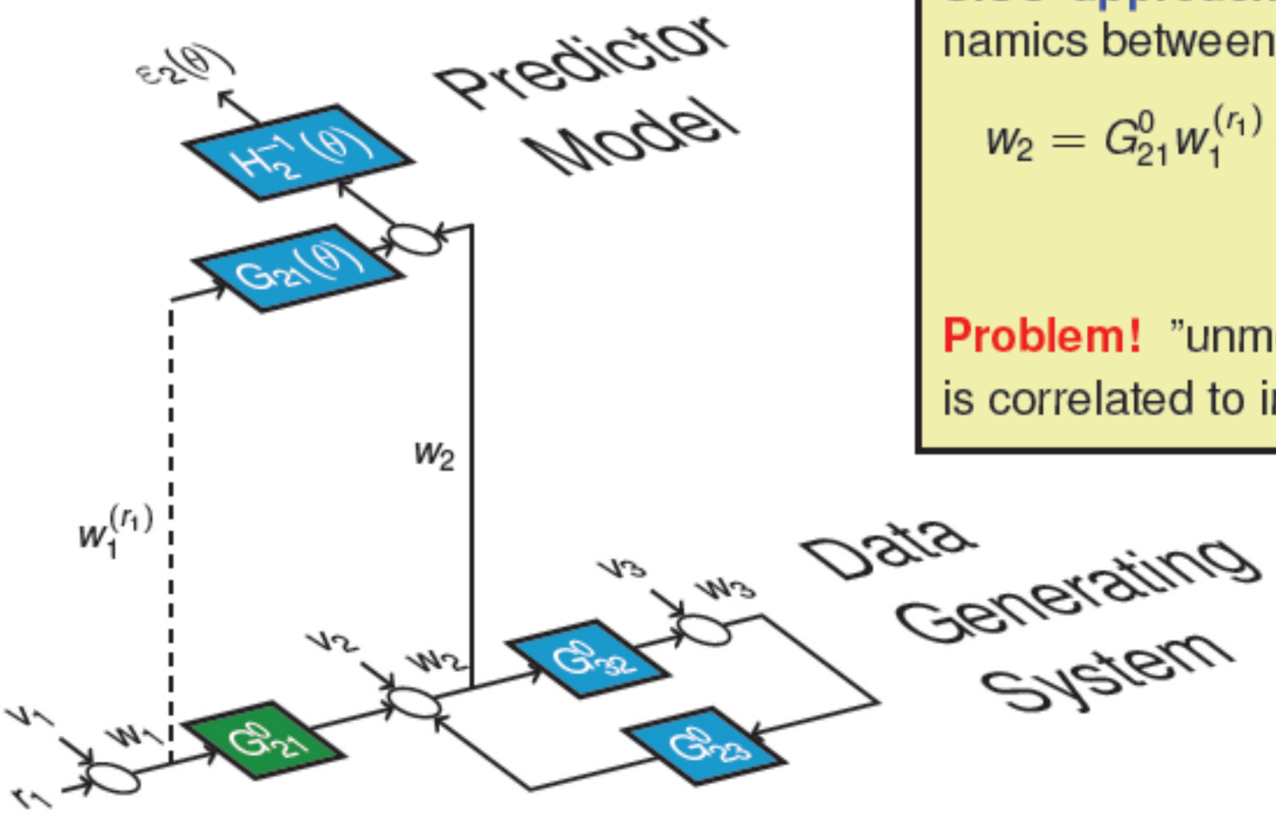
Predictor input selection: condition 1

Objective: obtain an estimate of G_{ji}^0

Consistent estimates of G_{ji}^0 are possible if:

1. w_i is included as predictor input
2. Each path from $w_i \rightarrow w_j$ passes through a node chosen as predictor input

Second mechanism: loops around the output



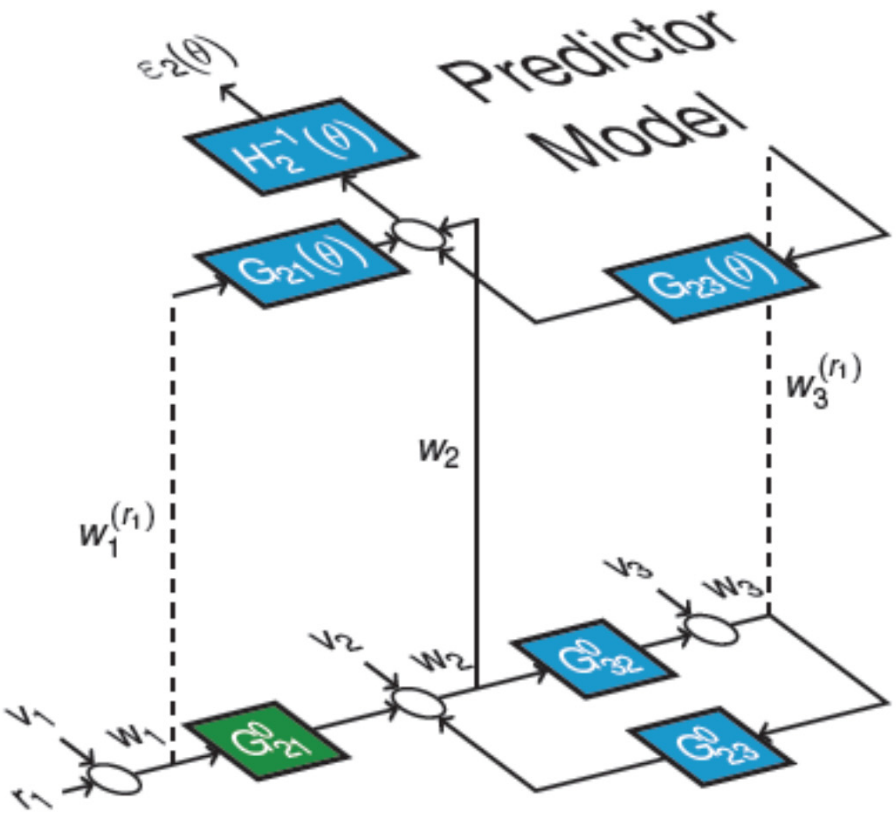
Objective: consistently estimate G_{21}^0.

SISO approach. Try to estimate the dynamics between w_1 and w_2:

$$w_2 = G_{21}^0 w_1^{(r_1)} + \underbrace{G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2}_{\text{unmodeled term}}$$

Problem! "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

Second mechanism: loops around the output



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$$w_2 = G_{21}^0 w_1^{(r_1)} + \underbrace{G_{21}^0 w_1^{(v)} + G_{23}^0 w_3 + v_2}_{\text{unmodeled term}}$$

Problem! "unmodeled term" (noise term) is correlated to input term, $w_1^{(r_1)}$.

Solution: Include $w_3^{(r_1)}$ in the predictor:

$$w_2 = G_{21}^0 w_1^{(r_1)} + G_{23}^0 w_3^{(r_1)} + \underbrace{G_{21}^0 w_1^{(v)} + G_{23}^0 w_3^{(v)} + v_2}_{\text{unmodeled term}}$$

Predictor input selection: condition 1 and 2

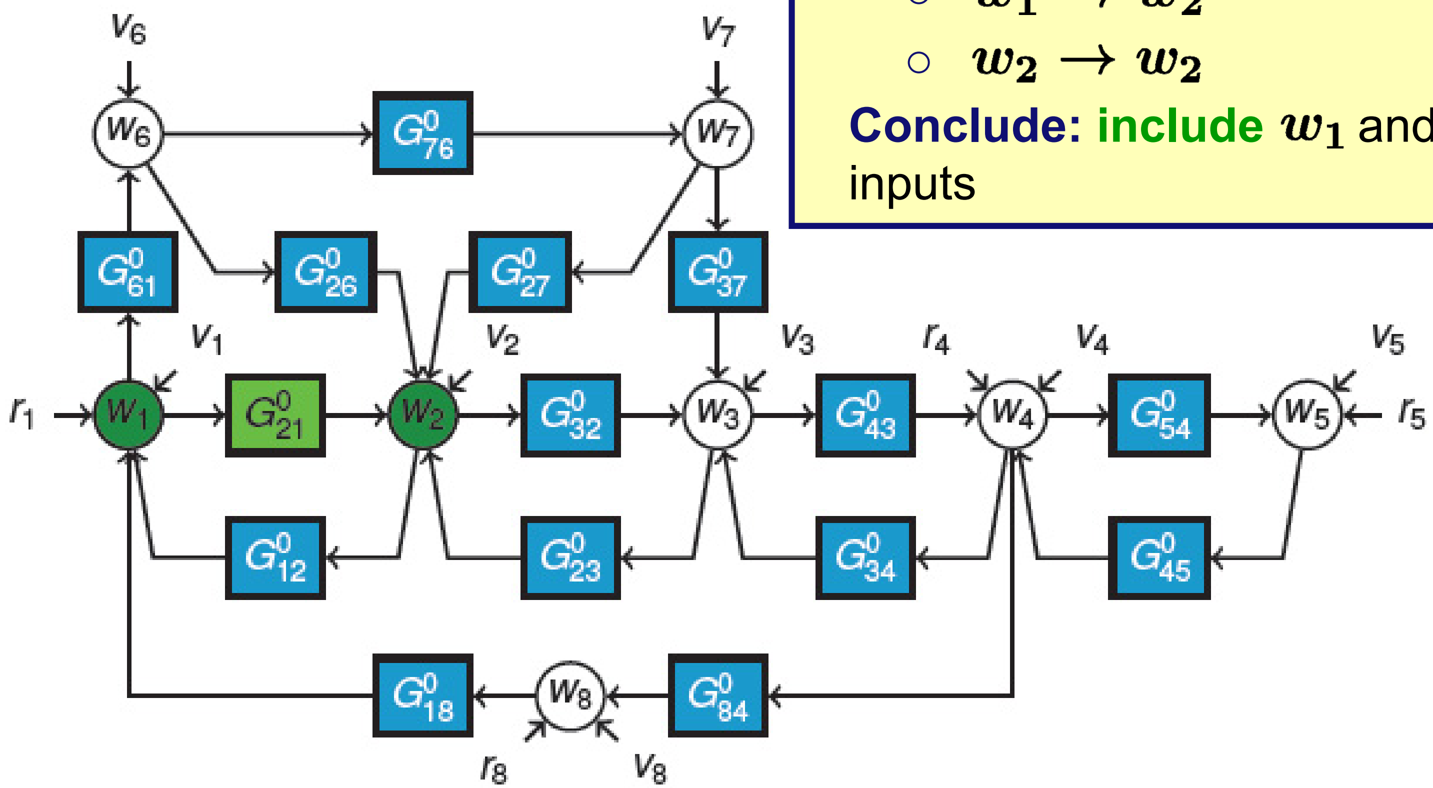
Objective: obtain an estimate of G_{ji}^{0}

Consistent estimates of G_{ji}^{0} are possible if:

1. w_i is included as predictor input
2. Each path from $w_i \to w_j$ passes through a node chosen as predictor input
3. Each loop from $w_j \to w_j$ passes through a node chosen as predictor input

Example with predictor input conditions

Objective: Estimate G_{21}^0.



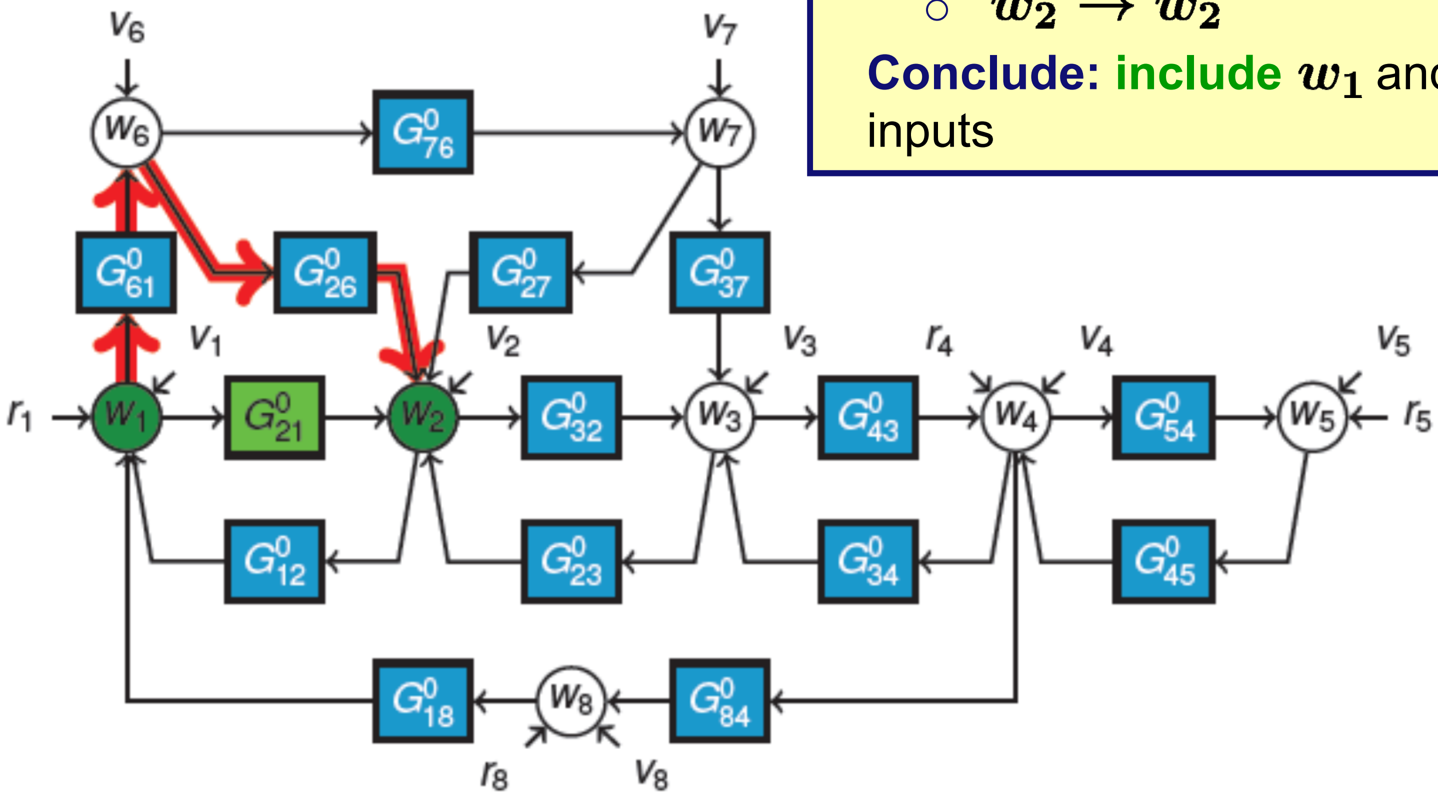
Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1 and **…** as predictor inputs

Example with predictor input conditions

Objective: Estimate G^0_{21}.



Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: **include** w_1 and **…** as predictor inputs

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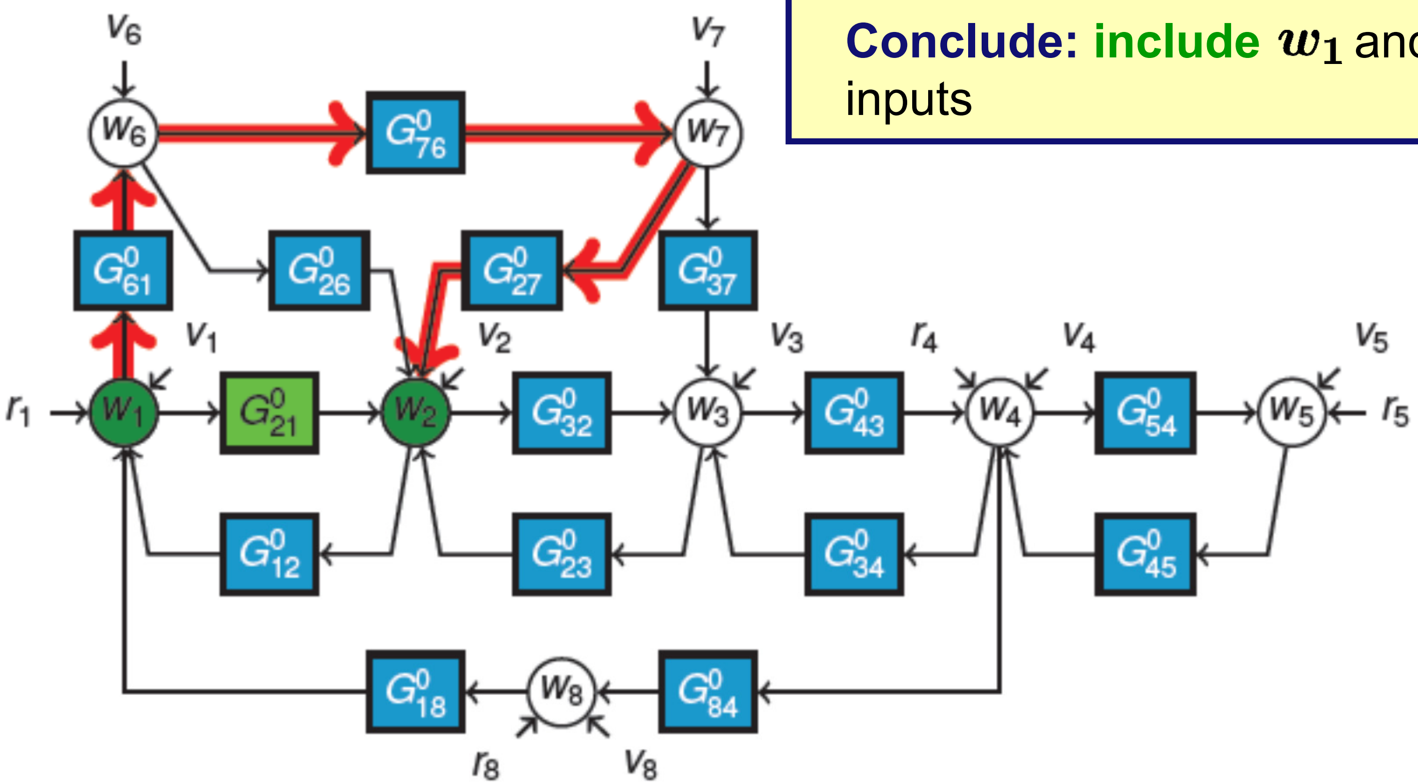
Example with predictor input conditions

Objective: Estimate G_{21}^0.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1 and … as predictor inputs



Example with predictor input conditions

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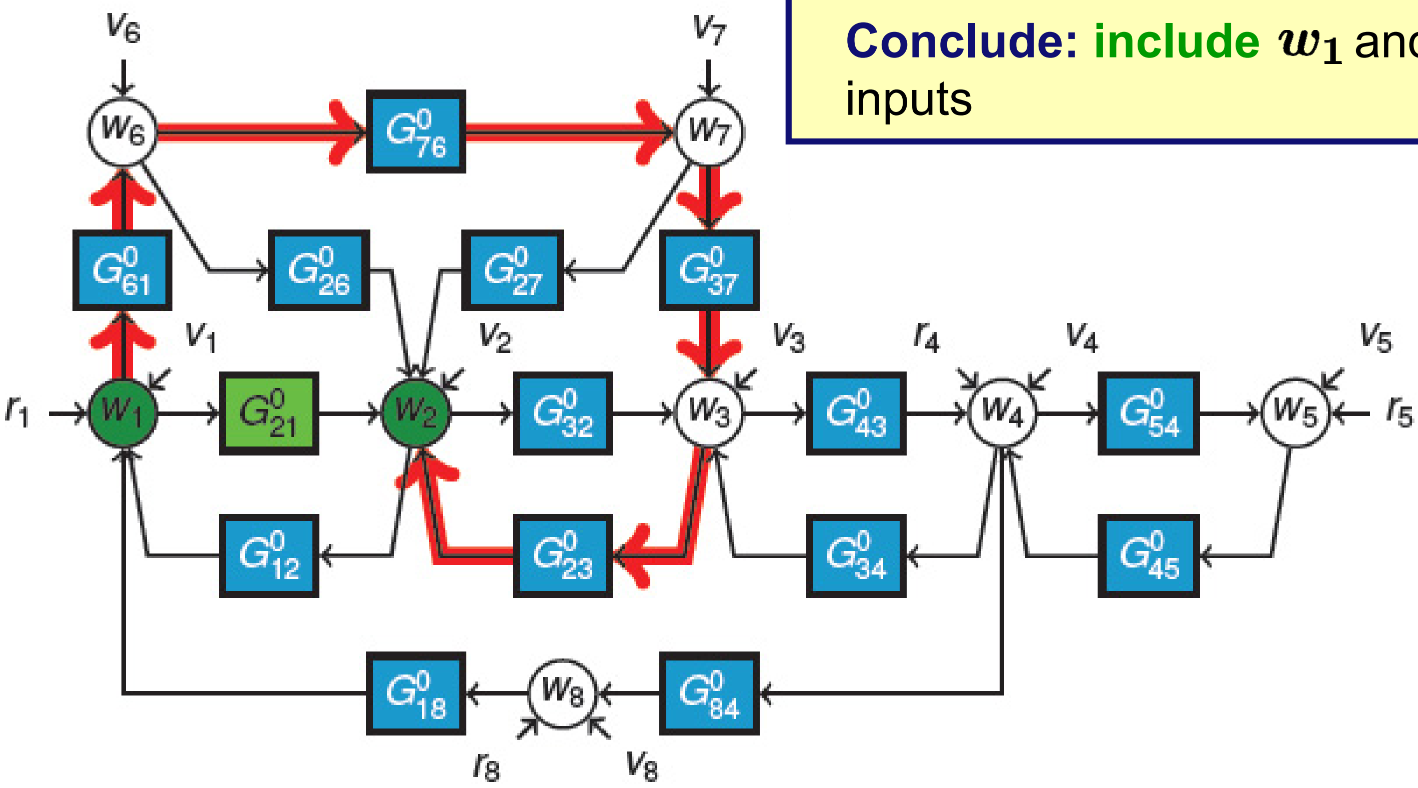


Conditions: Include variable on every path

- $w_1 \to w_2$
- $w_2 \to w_2$

Conclude: **include** w_1 and … as predictor inputs

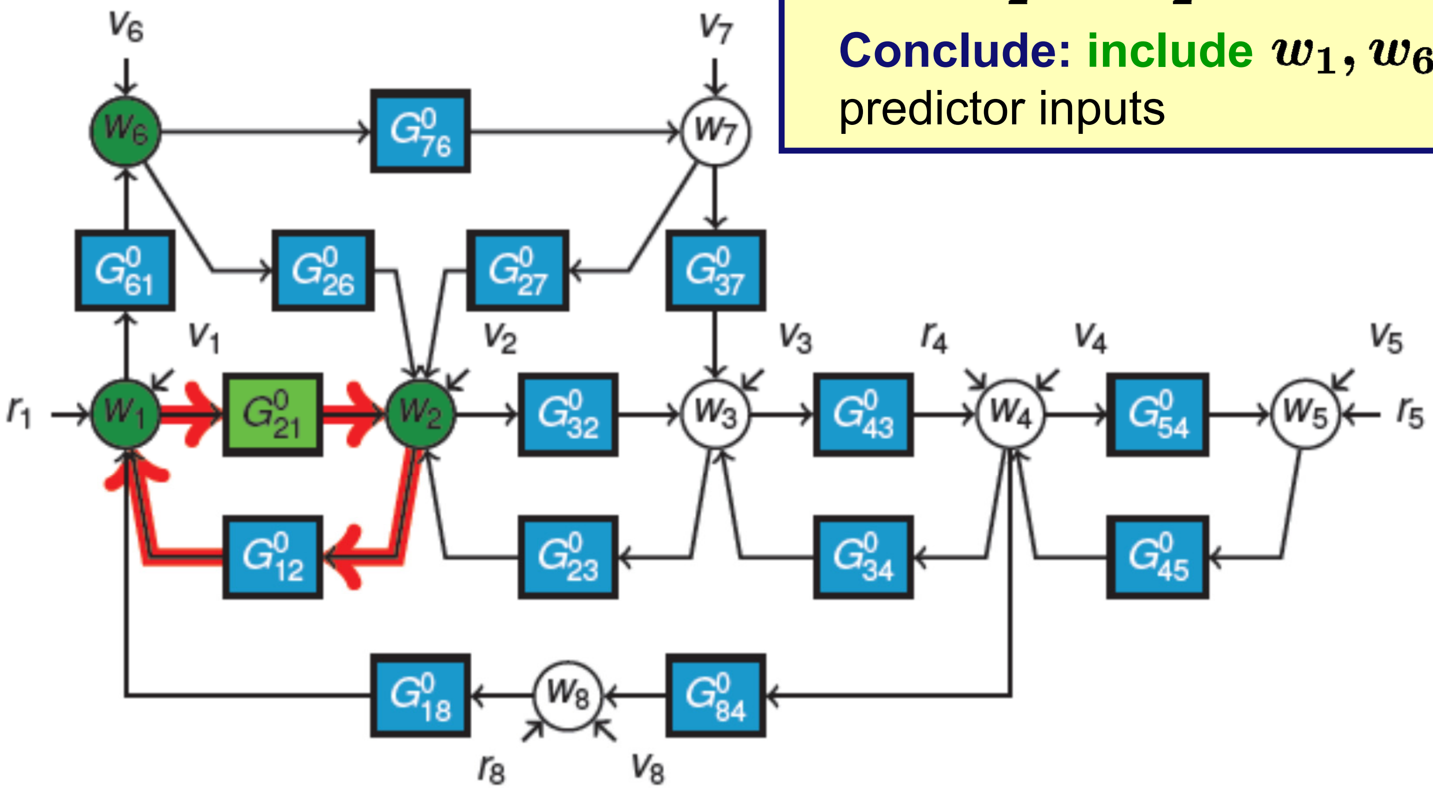
Example with predictor input conditions

Objective: Estimate G_{21}^0.

Conditions: Include variable on every path

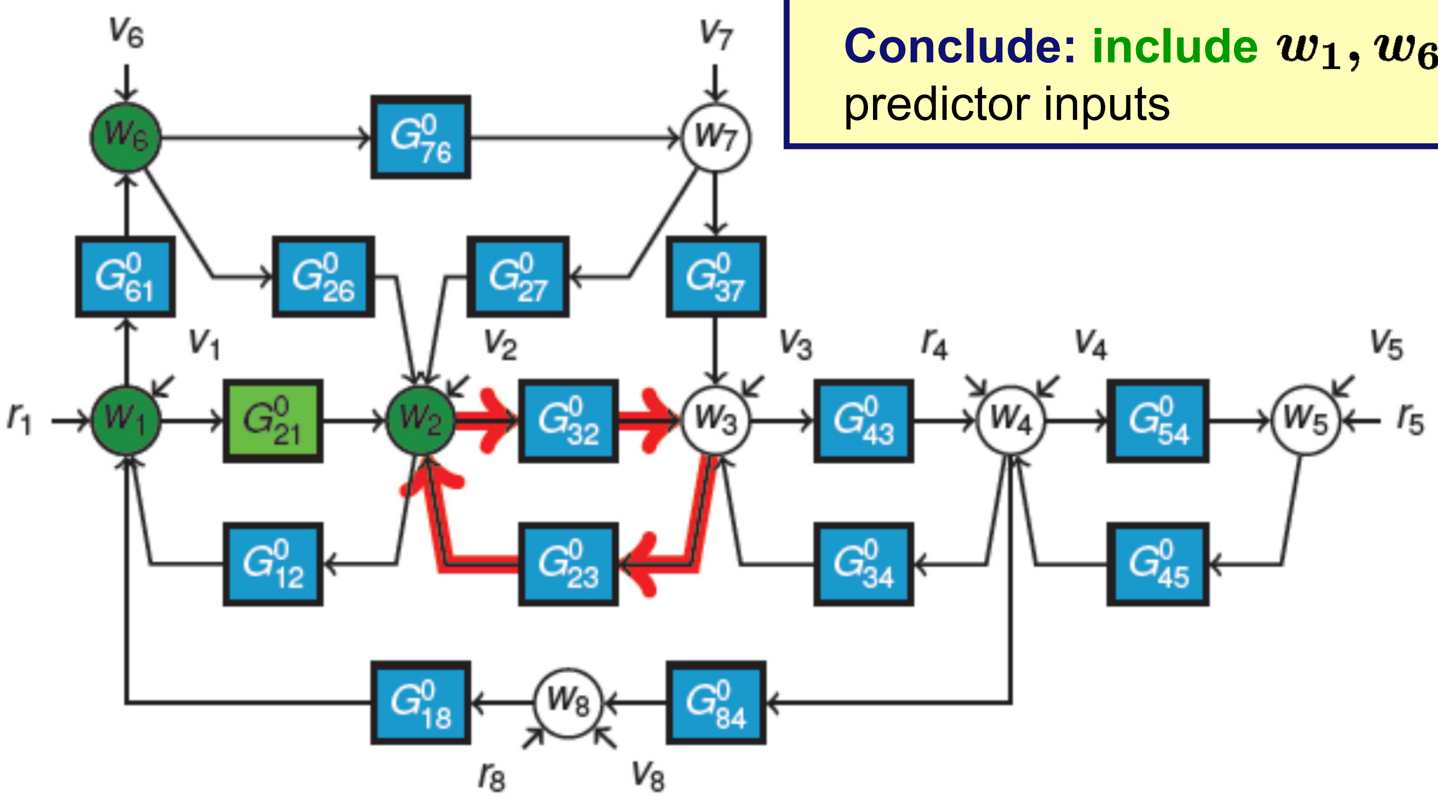
- ○ $w_1 \rightarrow w_2$ ➡ **Include w_6 in predictor**
- ○ $w_2 \rightarrow w_2$

Conclude: include w_1, w_6 and **…** as predictor inputs



Example with predictor input conditions

Objective: Estimate G_{21}^0.



Conditions: Include variable on every path

- $w_1 \rightarrow w_2$ ➡ **Include w_6 in predictor**
- $w_2 \rightarrow w_2$

Conclude: include w_1, w_6 and **…** as predictor inputs

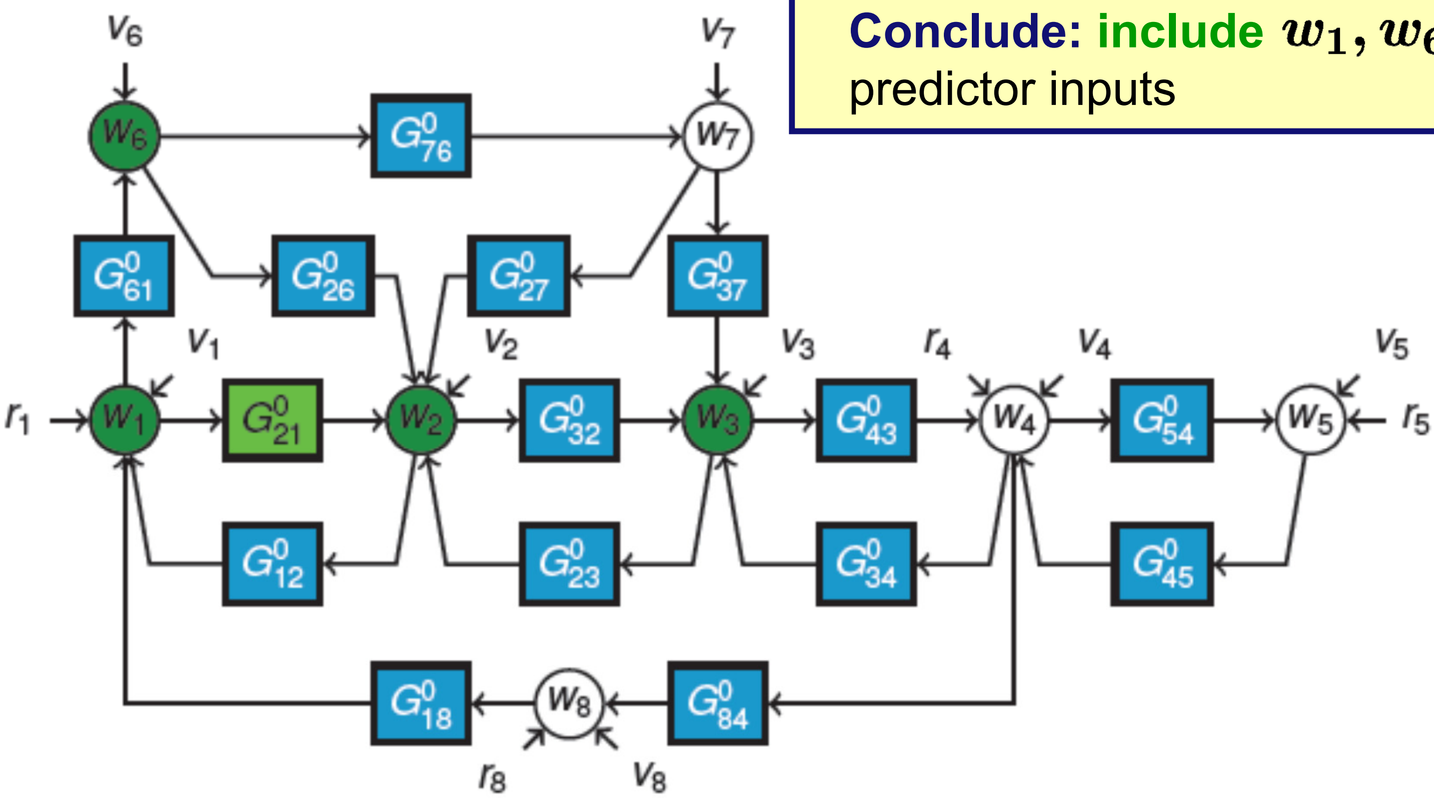
Example with predictor input conditions

Objective: Estimate G_{21}^0.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2$ ➡ **Include w_6 in predictor**
- $w_2 \rightarrow w_2$ ➡ **Include w_3 in predictor**

Conclude: **include w_1, w_6** and w_3 as predictor inputs



Predictor input selection

Result:

The consistency results of both direct and 2s/projection method remain principally valid when the predictor inputs satisfy the formulated conditions on *parallel paths* and *loops* around w_j

In the "full" MISO case: consistent estimates of all $G_{jk}^0, \;\; k \in \mathcal{U}_j$

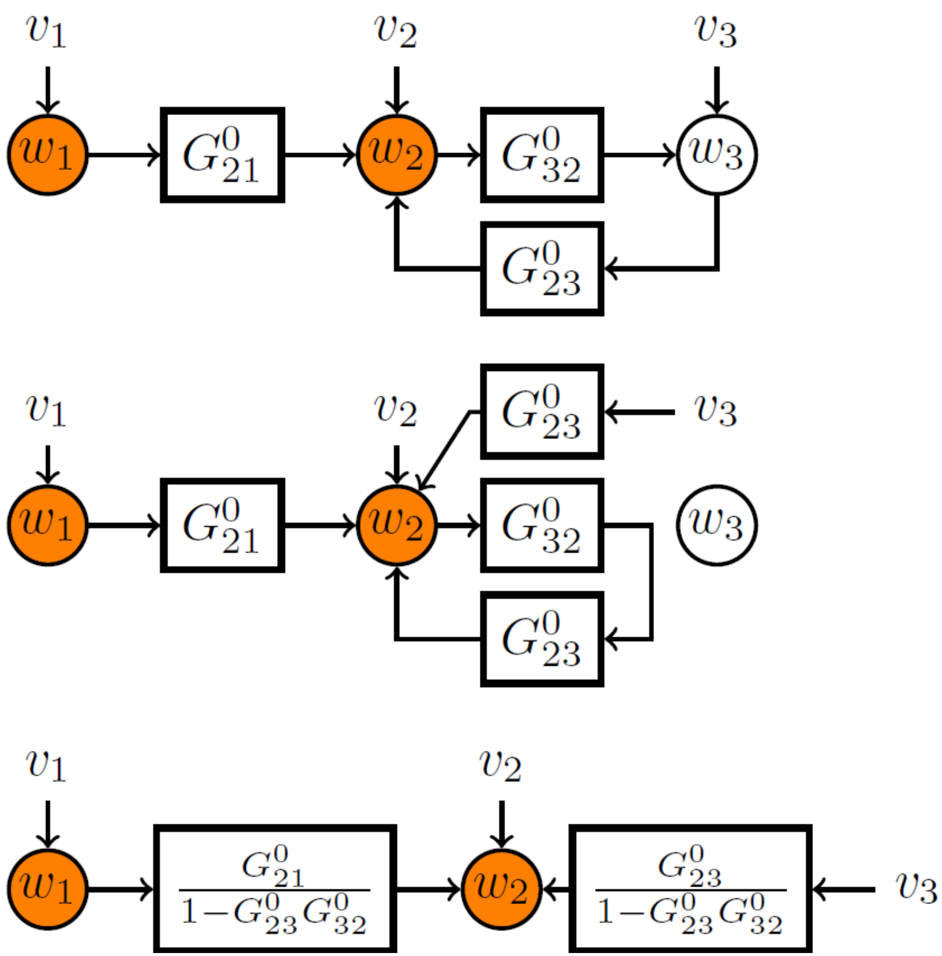
In the "selected" predictor input case: consistent estimates of G_{ji}^0

Background immersed network

- The two conditions (parallel paths and loops on output) result from an analysis of the so-called **immersed network**
- The **immersed network** is constructed on the basis of a reduced number of node variables only, and leaves present node signals invariant
- In the **immersed network** the module dynamics can change
- Whether dynamics in the **immersed network** is invariant can be verified with the graph theory/tools of separating sets.

[A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois. Identification of dynamic models in complex networks with predictior error methods - predictor input selection. IEEE Trans. Automatic Control, april 2016.]

Simple Example – Loops On Output



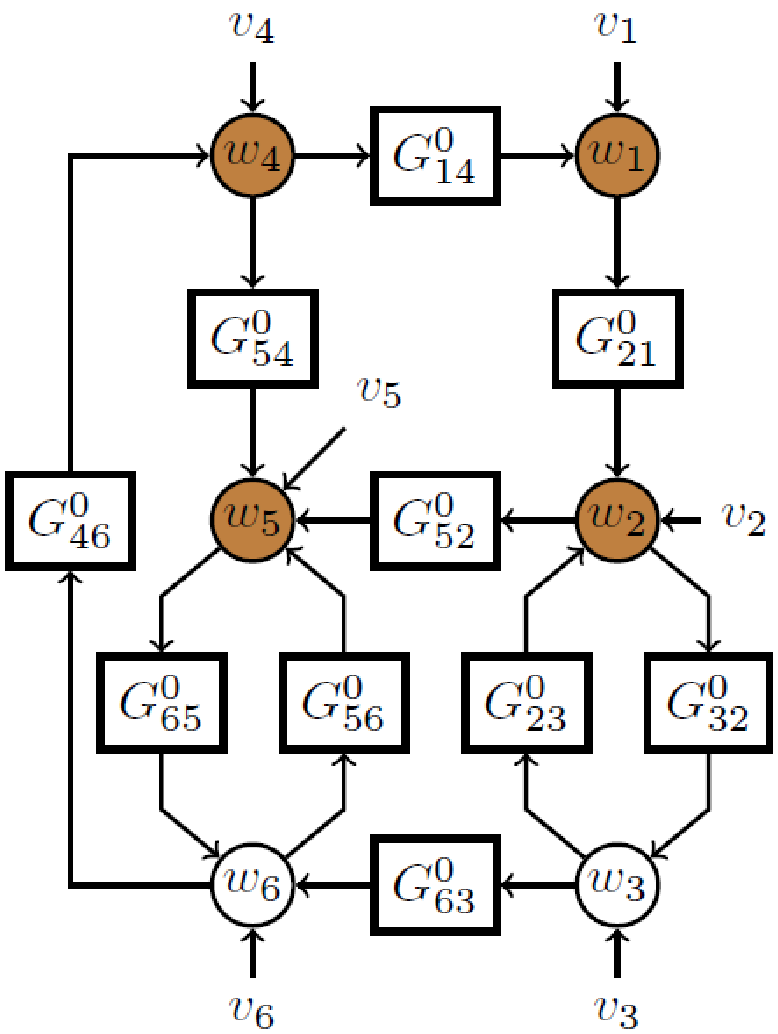
Removing path through w_3 called ***lifting a path***.

Network without w_3 is called ***immersed network***

Choosing w_1 as the predictor input results in an estimate of

$$\frac{G_{21}^0}{1-G_{23}^0 G_{32}^0}$$

Example – Immersed Network

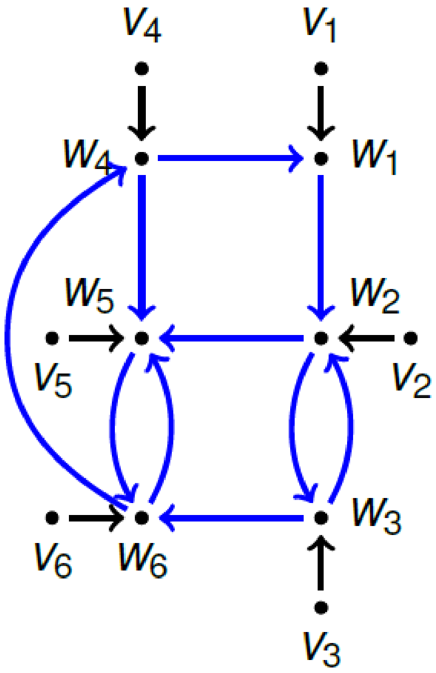


Given measurements of

w_1, w_2, w_4, and w_5

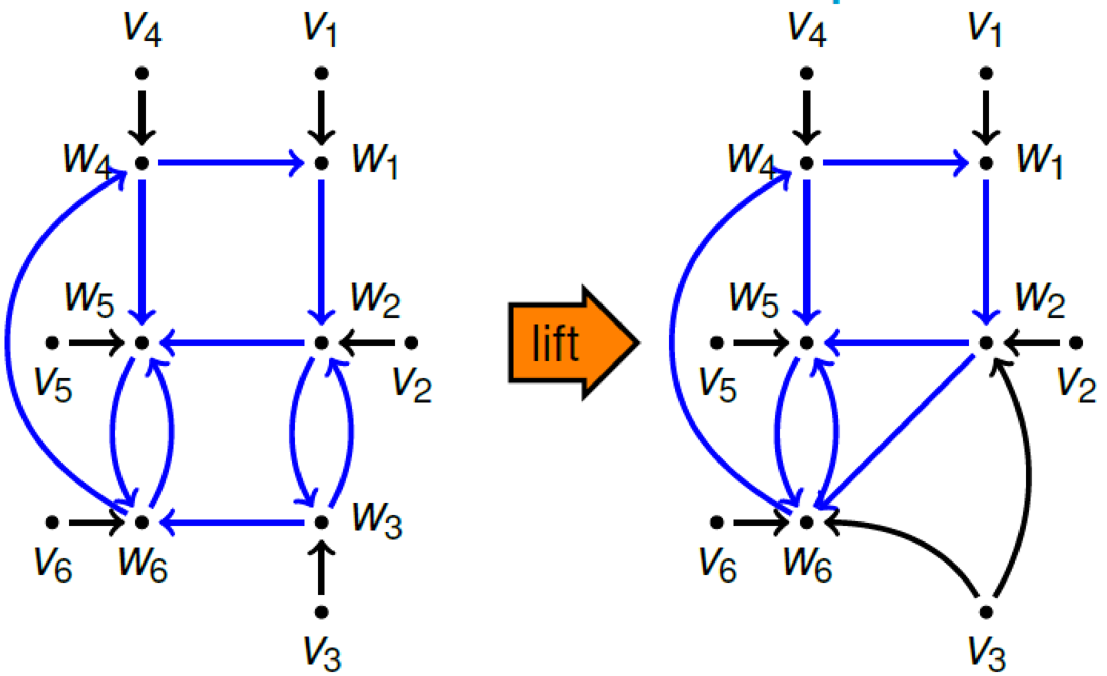
Immerse this network to contain these nodes only.

Example – Immersed Network



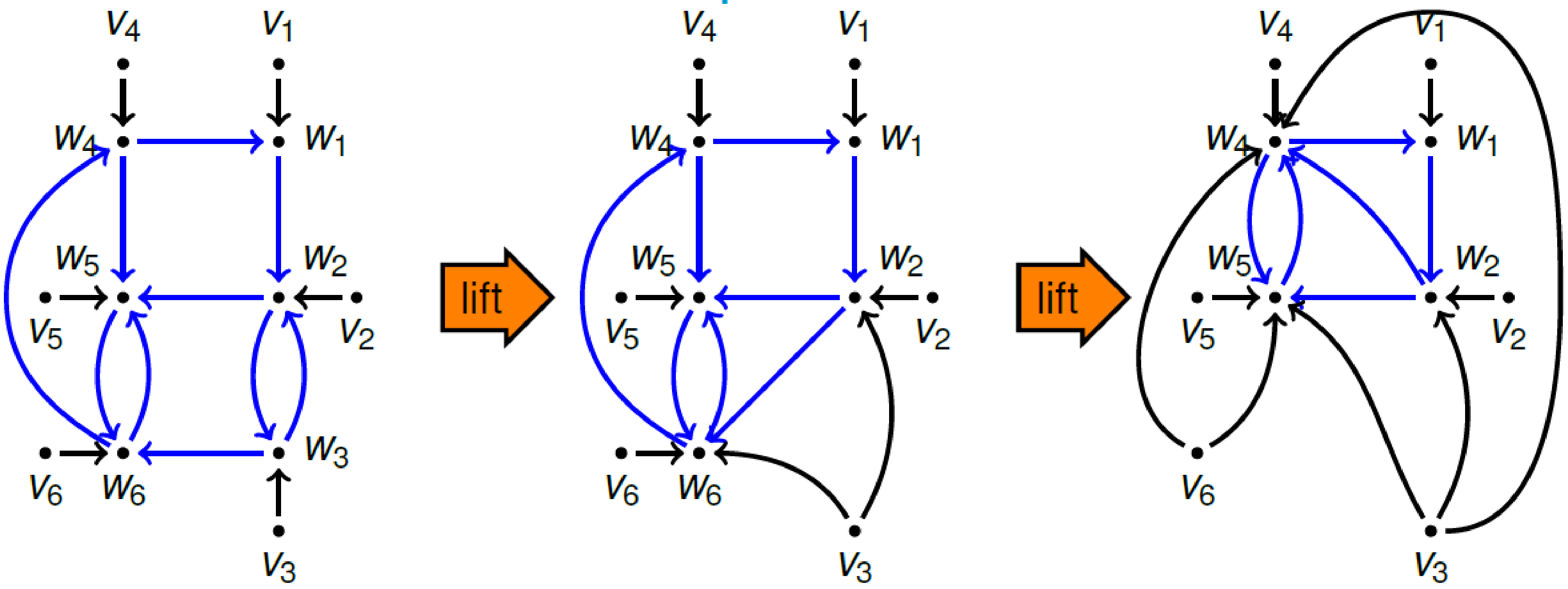
$$\begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & G^0_{14} & 0 & 0 \\ G^0_{21} & 0 & G^0_{23} & 0 & 0 & 0 \\ 0 & G^0_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G^0_{46} \\ 0 & G^0_{52} & 0 & G^0_{54} & 0 & G^0_{56} \\ 0 & 0 & G^0_{63} & 0 & G^0_{65} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

Example – Immersed Network



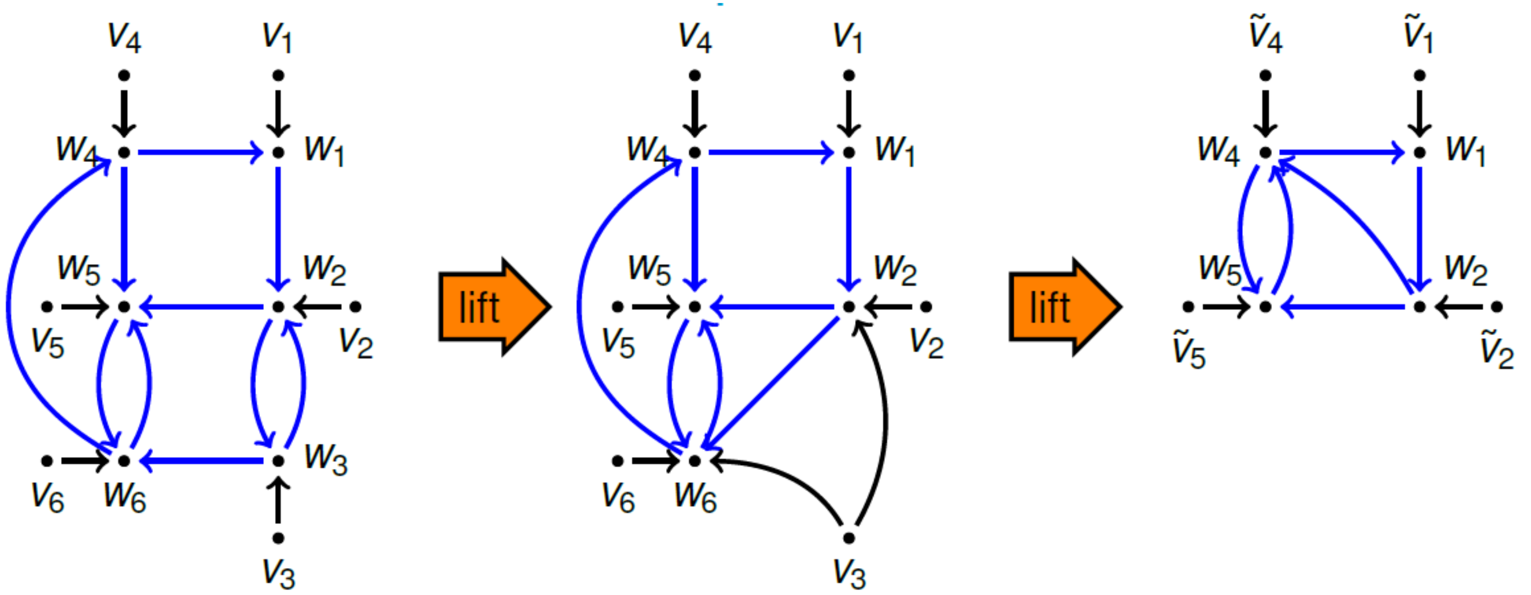
$$
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & G^0_{14} & 0 & 0 \\
\frac{G^0_{21}}{1-G^0_{23}G^0_{32}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & G^0_{46} \\
0 & G^0_{52} & G^0_{54} & 0 & G^0_{56} \\
0 & G^0_{63}G^0_{32} & 0 & G^0_{65} & 0
\end{bmatrix}
\begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}
+
\begin{bmatrix}
v_1 \\
\frac{1}{1-G^0_{23}G^0_{32}} v_2 + \frac{G^0_{23}}{1-G^0_{23}G^0_{32}} v_3 \\
v_4 \\
v_5 \\
v_6 + G^0_{63} v_3
\end{bmatrix}
$$

Example – Immersed Network



$$
\begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \end{bmatrix} =
\begin{bmatrix}
0 & 0 & G^0_{14} & 0 \\
\frac{G^0_{21}}{1-G^0_{23}G^0_{32}} & 0 & 0 & 0 \\
0 & G^0_{32}\,G^0_{46}\,G^0_{63} & 0 & G^0_{46}\,G^0_{65} \\
0 & \frac{G^0_{52}+G^0_{56}G^0_{63}G^0_{32}}{1-G^0_{56}G^0_{65}} & \frac{G^0_{54}}{1-G^0_{56}G^0_{65}} & 0
\end{bmatrix}
\begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \end{bmatrix} +
\begin{bmatrix}
v_1 \\
\frac{v_2+G^0_{23}v_3}{1-G^0_{23}G^0_{32}} \\
v_4 + G^0_{46}\,G^0_{63}\,v_3 + G^0_{46}\,v_6 \\
\frac{v_5+G^0_{56}G^0_{63}v_3+G^0_{56}v_6}{1-G^0_{56}G^0_{65}}
\end{bmatrix}
$$

Example – Immersed Network



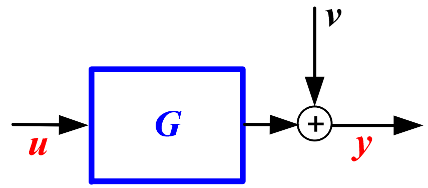
$$\begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & G_{14}^0 & 0 \\ \frac{G_{21}^0}{1-G_{23}^0 G_{32}^0} & 0 & 0 & 0 \\ 0 & G_{32}^0 G_{46}^0 G_{63}^0 & 0 & G_{46}^0 G_{65}^0 \\ 0 & \frac{G_{52}^0 + G_{56}^0 G_{63}^0 G_{32}^0}{1-G_{56}^0 G_{65}^0} & \frac{G_{54}^0}{1-G_{56}^0 G_{65}^0} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_4 \\ w_5 \end{bmatrix} + \begin{bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{v}_4 \\ \tilde{v}_5 \end{bmatrix}$$

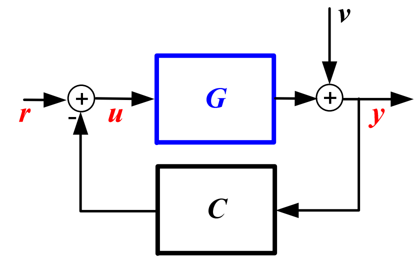
Conclude: only G_{14}^0 from the original network is identifiable given this data set

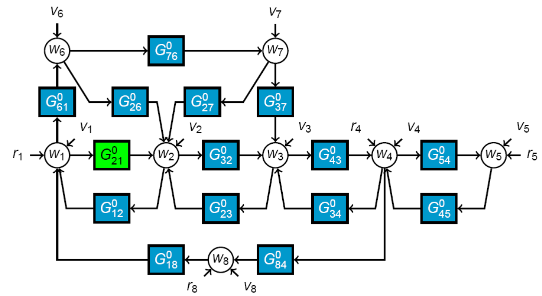
Contents

Towards dynamic network identification

- The basic (prediction error) tools: direct and 2s
- Dynamic network setup
- Single module identification - consistency
  - full MISO models
  - predictor input (sensor) selection
- Sensor noise – the errors-in-variables problem
- Discussion / Wrap-up

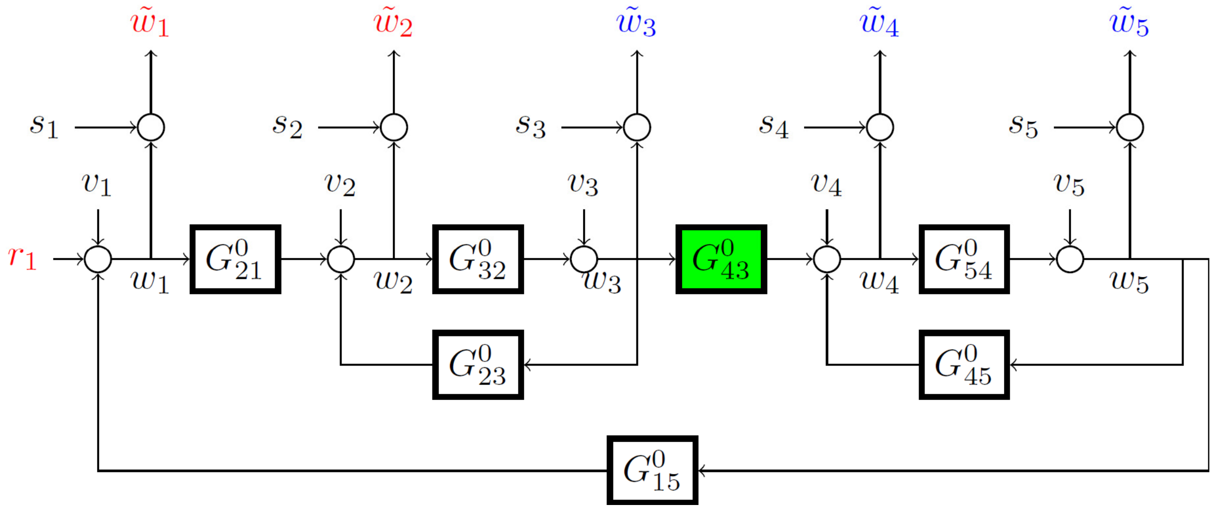






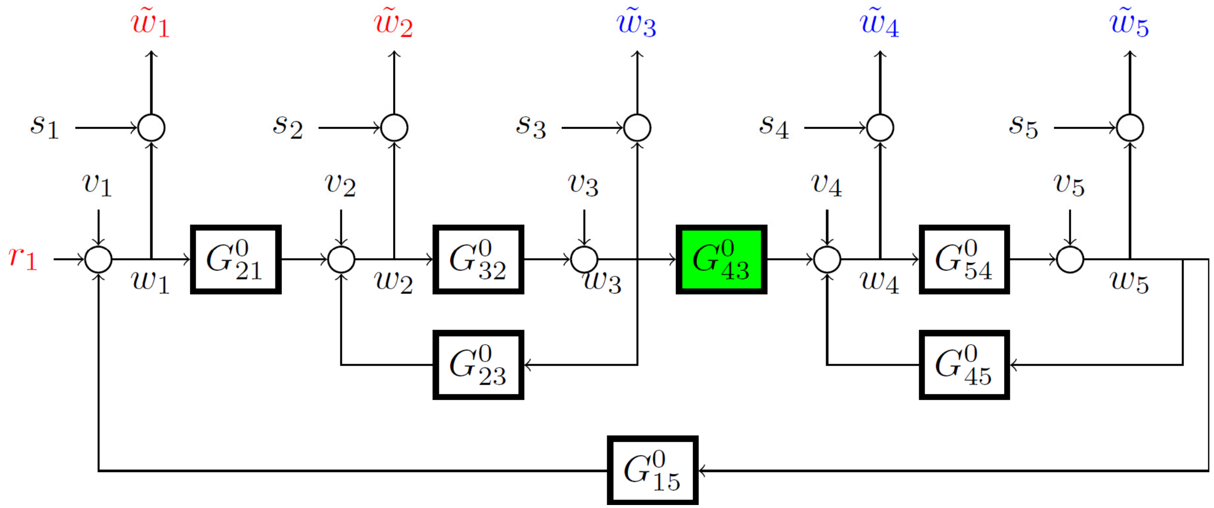
Sensor noise – the errors-in-variables problem

What if node variables are measured with (sensor) noise?



- Classical (tough) problem in open-loop identification
- In dynamic networks this may become *more simple* due to the presence of multiple (correlated) node signals

Sensor noise – the errors-in-variables problem

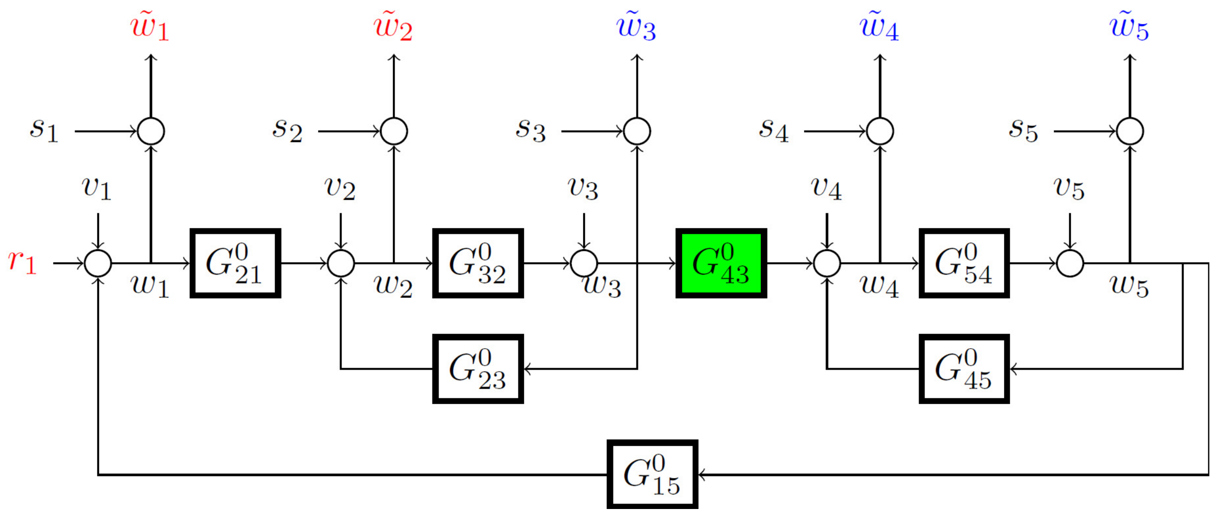


Two solution strategies:

1. Use *external signals* in combination with 2s/projection/IV method
2. Apply an *Instrumental Variable (IV)* method with generalized options for selecting IV signals

Sensor noise – the errors-in-variables problem

1. **Use *external signals* in combination with 2s/projection/IV method**



- If measured predictor input signals ($\tilde{\boldsymbol{w}}_3, \tilde{\boldsymbol{w}}_5$) are projected onto $\boldsymbol{r}_1$ and then applied in a 2s-PE criterion, the sensor noise on the inputs is effectively removed
- when assuming that r-signals and s-signals are uncorrelated.

Sensor noise – the errors-in-variables problem

Result:

The consistency result of the 2s/projection method remains valid when sensor noise is present on measured variables, provided that

- Sufficient external excitation is present
- Sensor noise is uncorrelated to excitation signals

Extension of IV-approach to use node signals as IV signals, and including noise models, see:
[A. Dankers, P.M.J. Van den Hof, X. Bombois and P.S.C. Heuberger, Automatica, December 2015]

Discussion / Wrap-up

- So far: focus on (local) consistency results in networks with known structure

- Many additional questions/topics remain:

  Variance of estimates, influenced by

  - Additional (output) measurements
  - Excitation properties

  [See e.g. work of H. Hjalmarsson, B. Wahlberg, N. Everitt, B. Günes, M. Gevers, A. Bazanella]

Discussion / Wrap-up

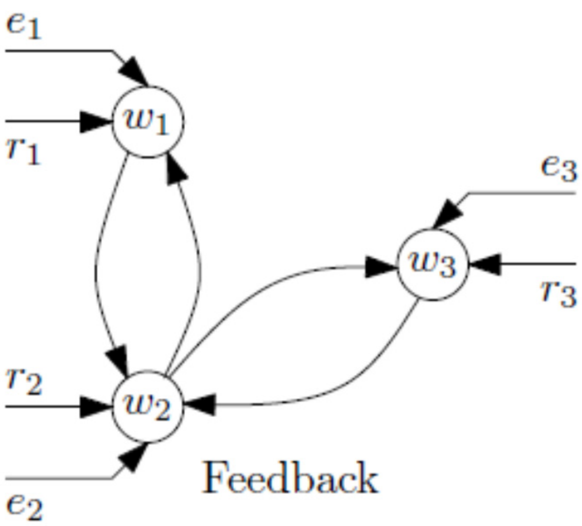
- Identification of the structure/topology addressed in the literature, in particular forms:
  - Tree-like structures (no loops)
  - Nonparametric methods (Wiener filter)
  - Mostly networks without external excitation and uncorrelated process noises on every node

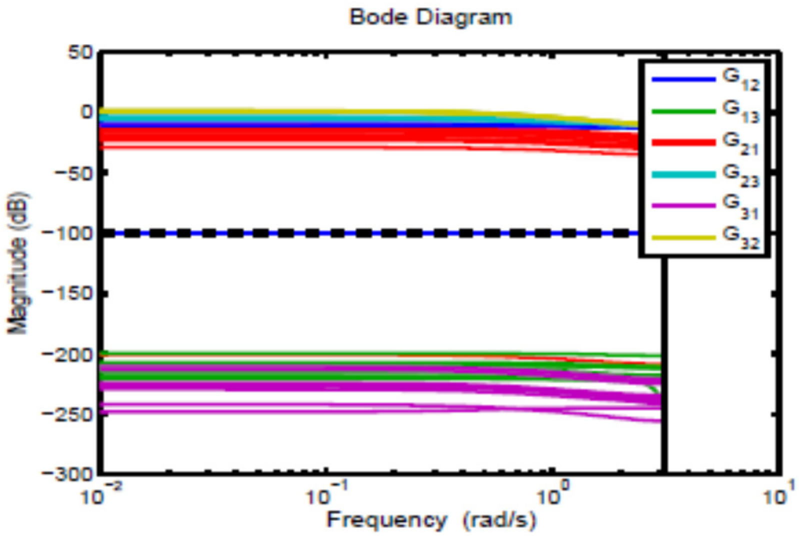
  see e.g. Materassi, Innocenti (TAC-2010), Chiuso and Pillonetto (Automatica, 2012)

- New identifiability concepts apply to the unique determination of a network topology
  see e.g. Goncalves & Warnick (TAC-2008), Weerts et al. (SYSID-2015).

- Sparse identification methods can be used in an PE identification setting to identify the topology (non-zero transfers)

Toplogy detection with sparse PE methods





$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t,\theta)$$
$$\text{subject to } \|\theta\|_1 \leq \lambda$$

- Detected: $\|G_{ij}\|_{\mathcal{H}_\infty} \geq 10^{-5}$
- 100 simulations

	Direct identification
G_{12}	100
G_{13}	0
G_{21}	99
G_{23}	100
G_{31}	0
G_{32}	100

[H. Weerts, 2014]

Network identifiability

Question:

When given measured node signals, can we consistently identify the network and its topology?

This will generally require conditions on
a) Informativity of the data (sufficient excitation), and
b) Ability to distinguish between different network models in the model set

Classical notion of identifiability is adressing a unique relationship between parameters θ and predictor filters that map measured signals to predicted values.

$$\left.\begin{array}{r} G(\theta_1) = G(\theta_2) \\ H(\theta_1) = H(\theta_2) \end{array}\right\} \Longrightarrow \theta_1 = \theta_2$$

Instead in dynamic networks we need to incorporate the structural issues in the representation of the network.

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Network identifiability

Discussion / Wrap-up

Many interesting –new- questions pop up!

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