On representations of linear dynamic networks

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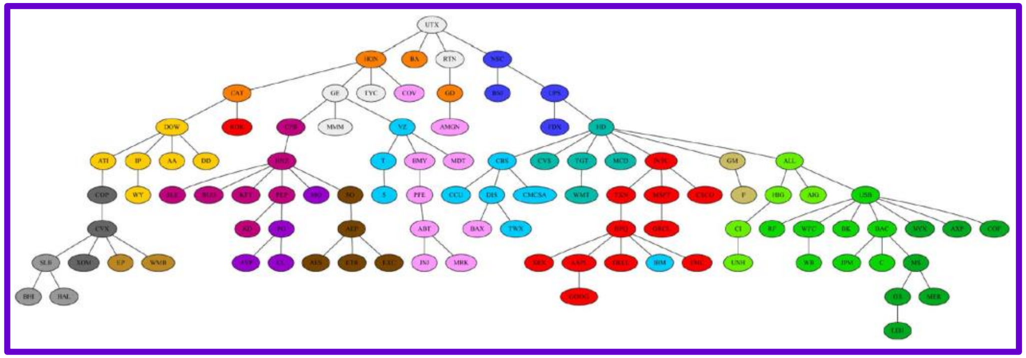
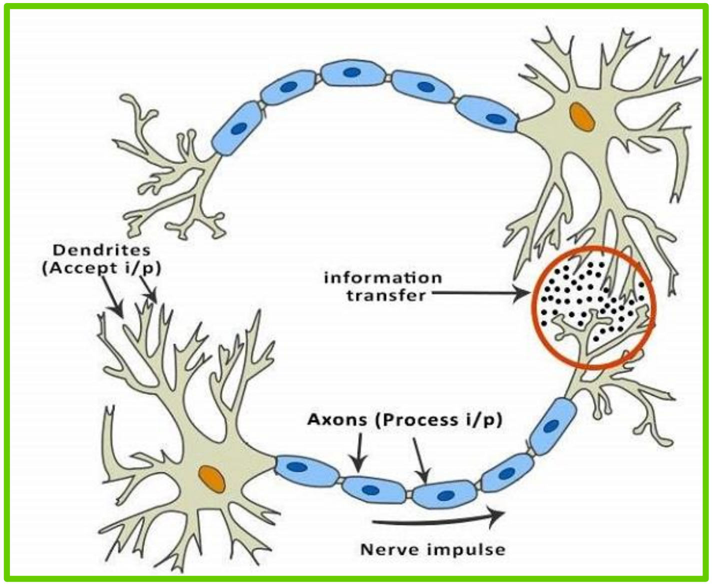
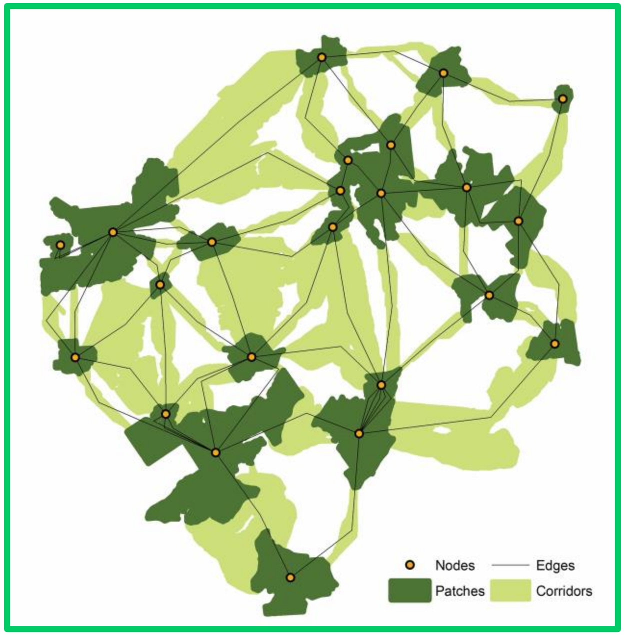
Where innovation starts

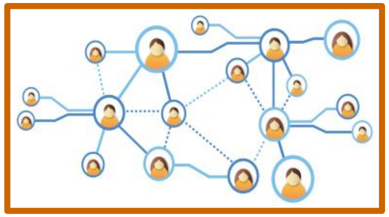
Introduction

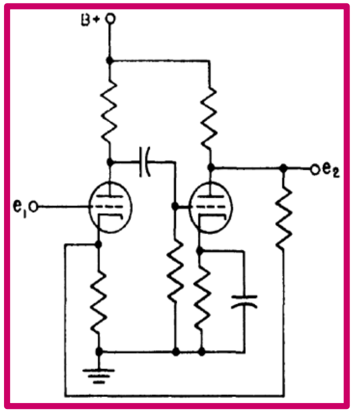
Linear dynamic networks











■ D. Materassi and G. Innocenti (2010): Stock market. ■ TutorialsPoint.com: Artificial neural network.
■ D.A. Rudnick & et al (2012): Ecology, habitat patches. ■ StockPhotoSecrets.com: Social network grid.
■ J. Farrell (2011): Democratised electricity system. ■ S.J. Mason (1956): Electric circuit.

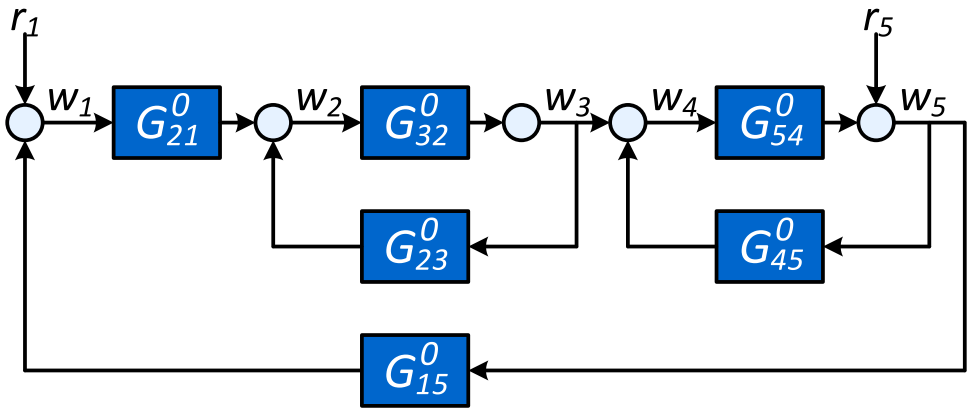
Representations

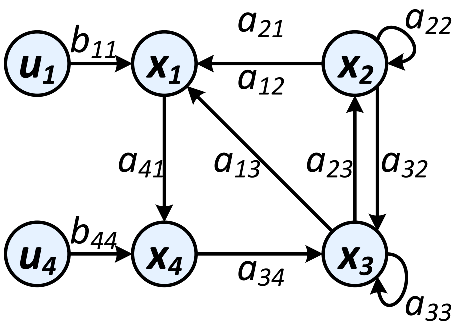
Module representation:

$$w_j(t) = \sum_{i \neq j} G^0_{ji} w_i(t) + r_j(t)$$

State-space form:

$$x(t+1) = Ax(t) + Bu(t)$$





Research question & approach

Are the module representation and the state-space form equivalent?

Can they be transformed into one another without losing any information?

- Module dynamic network
- Abstraction
- Realisation

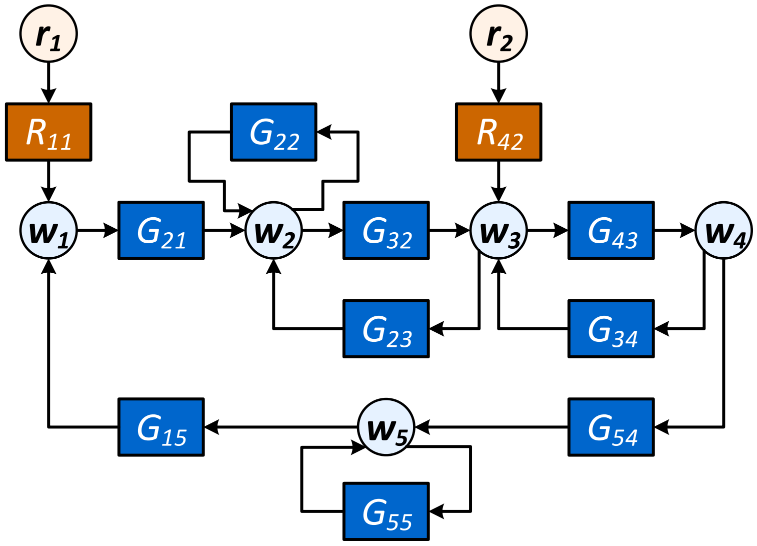
Module dynamic network

Module dynamic network

$$w_j(t) = \sum_i G_{ji}(q) w_i(t) + \sum_k R_{jk}(q) r_k(t)$$

$$\begin{bmatrix} w_1(t) \\ \vdots \\ w_L(t) \end{bmatrix} = \begin{bmatrix} G_{11} & \cdots & G_{1L} \\ \vdots & & \vdots \\ G_{L1} & \cdots & G_{LL} \end{bmatrix} \begin{bmatrix} w_1(t) \\ \vdots \\ w_L(t) \end{bmatrix} + \begin{bmatrix} R_{11} & \cdots & R_{1K} \\ \vdots & & \vdots \\ R_{L1} & \cdots & R_{LK} \end{bmatrix} \begin{bmatrix} r_1(t) \\ \vdots \\ r_K(t) \end{bmatrix}$$

$$w(t) = G\, w(t) + R\, r(t)$$

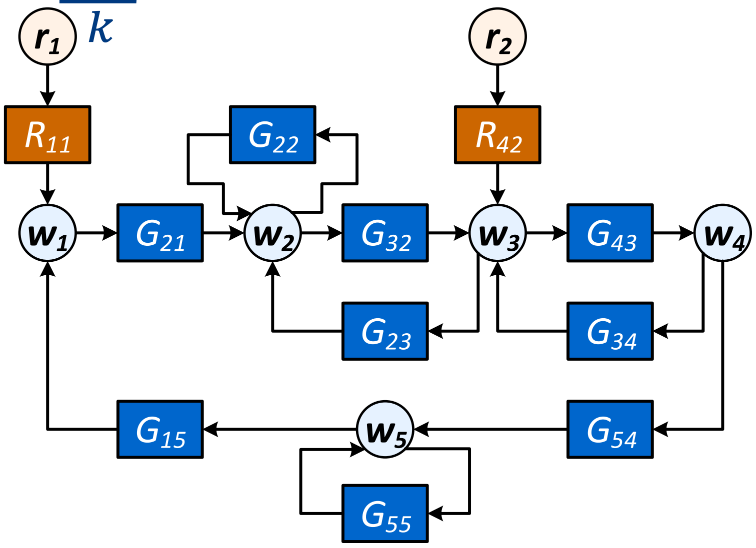


Module dynamic network

$$w(t) = G\, w(t) + R\, r(t)$$
$$x(t+1) = A\, x(t) + B\, r(t)$$
$$w_j(t) = \sum_i G_{ji}(q) w_i(t) + \sum_k R_{jk}(q) r_k(t)$$
$$x_j(t) = \sum_i q^{-1} A(j,i) x_i(t) + \sum_k q^{-1} B(j,k) r_k(t)$$

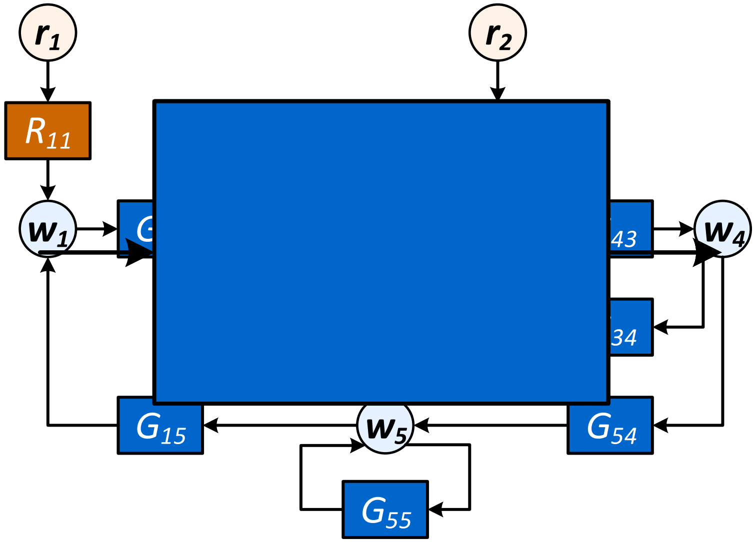
State-space dynamic network

- all $x_j(t)$ are $w_j(t)$
- $G_{ji}(q) = q^{-1} A(j,i)$
- $R_{jk}(q) = q^{-1} B(j,k)$



Module dynamic network

- Node = measurable state
- Module = transfer
- Hidden state
  - Unmeasurable state
  - Hidden in a module
- Shared hidden state[1]
  - Single physical state
  - Hidden in multiple modules

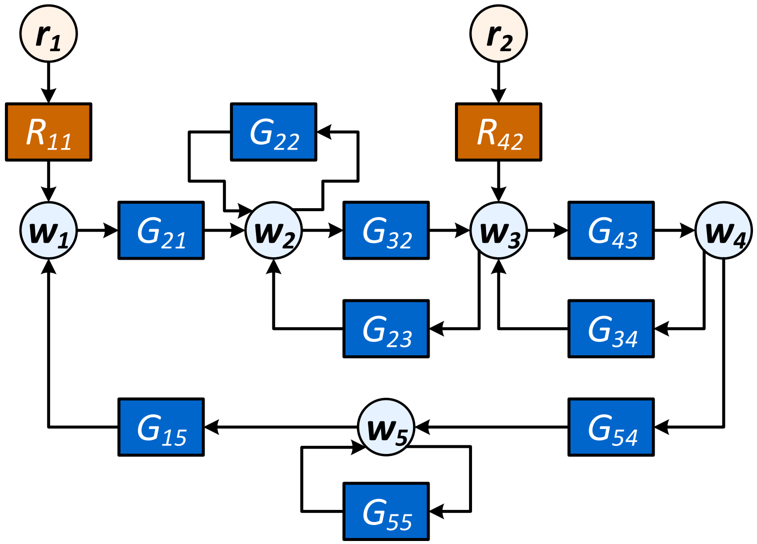


[1] S. Warnick. (2015). Shared hidden state and network representations of interconnected dynamical systems. In 2015 53rd Allerton Conference on Communication, Control, and Computing, 25-32.

Module dynamic network

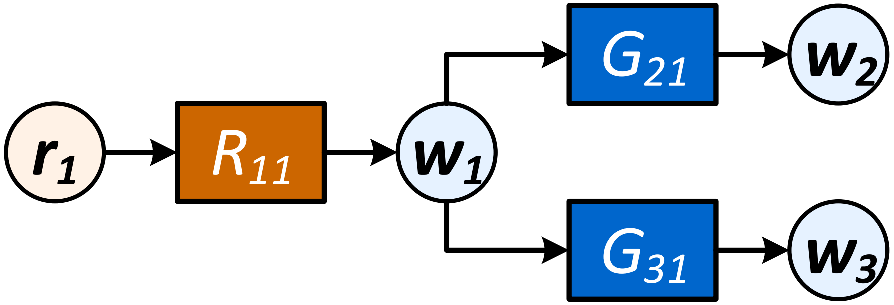
$$w(t) = G\, w(t) + R\, r(t)$$
$$w(t) = (I - G)^{-1} R\, r(t)$$

- Equal: $(G_1, R_1) = (G_2, R_2)$
- Equivalent: $(I - G_1)^{-1} R_1 = (1 - G_2)^{-1} R_2$



Abstraction

Abstraction[1]



$$w_1(t) = R_{11} r_1(t)$$ Immersion

$$w_2(t) = G_{21} w_1(t)$$

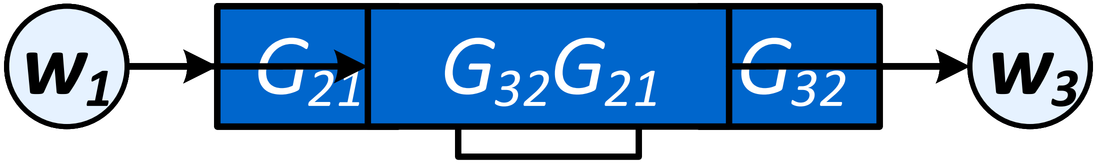
$$w_3(t) = G_{31} w_1(t)$$

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ G_{21} & 0 & 0 \\ G_{31} & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} + \begin{bmatrix} R_{11} \\ 0 \\ 0 \end{bmatrix} r_1(t)$$

[1] N. Woodbury, A. Dankers and S. Warnick. (2017). On the well-posedness of lti networks. In 2017 56th IEEE Conference on Decision and Control (CDC), 4813-4818.

Immersion[1]

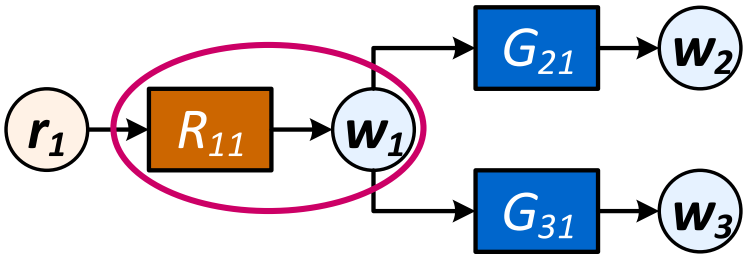
- Lift path
- Delete node
- Merge modules

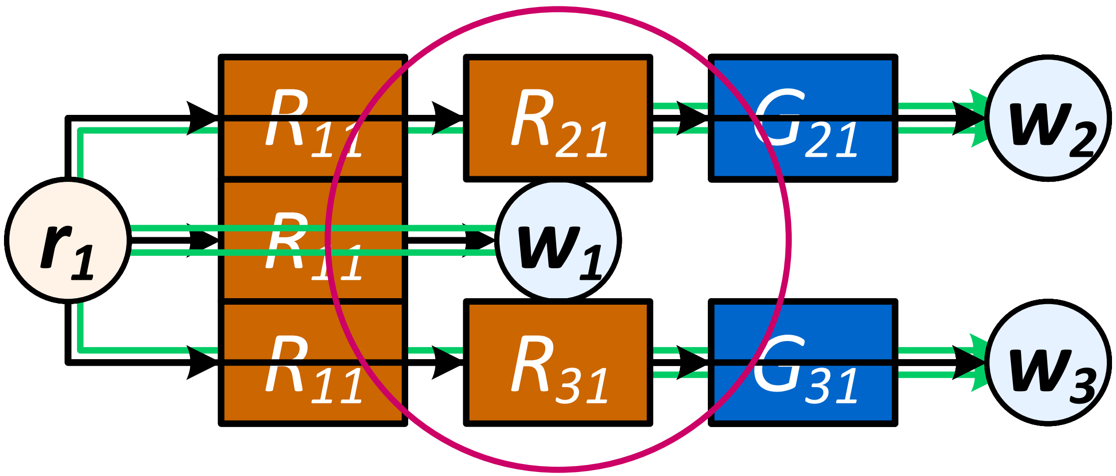


[1] A. Dankers, P.M.J. van den Hof, X. Bombois and P.S.C. Heuberger (2016). Identification of dynamic models in complex networks with prediction error methods: Predictor input selection. IEEE Transactions on Automatic Control, 61(4), 937-952.

Single-path immersion

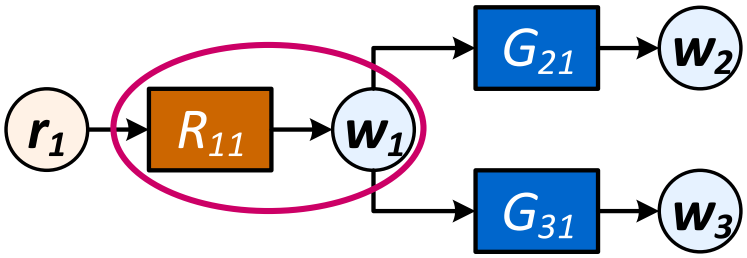
- New module per path
- SISO structure preserved
- Shared hidden states
- Network structure changed

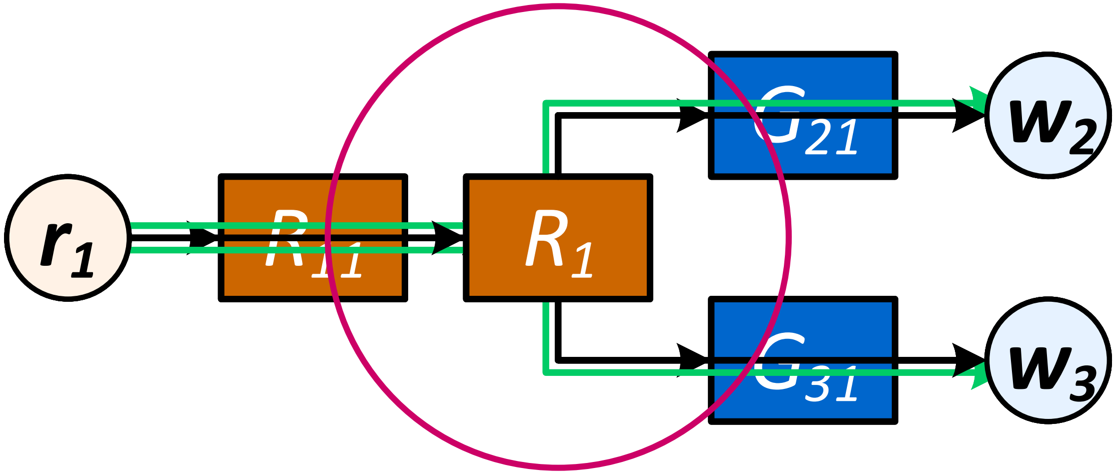




Multi-path immersion

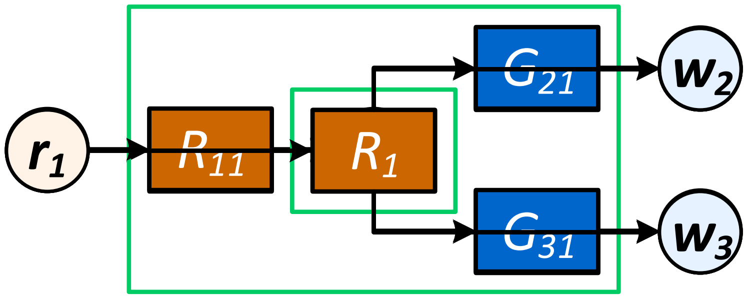
- Single new module
- MIMO structure
- No shared hidden states
- Network structure preserved

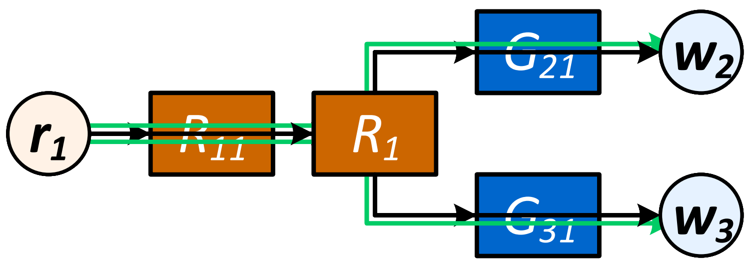




Multi-path immersion

- Single new module
- MIMO structure
- No shared hidden states
- Network structure preserved
- Constructing sub-networks

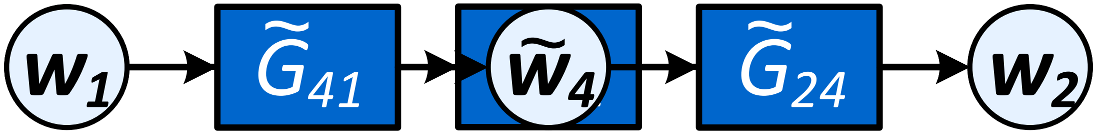




Realisation

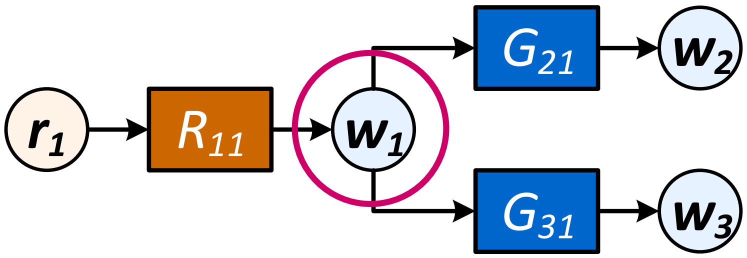
Realisation

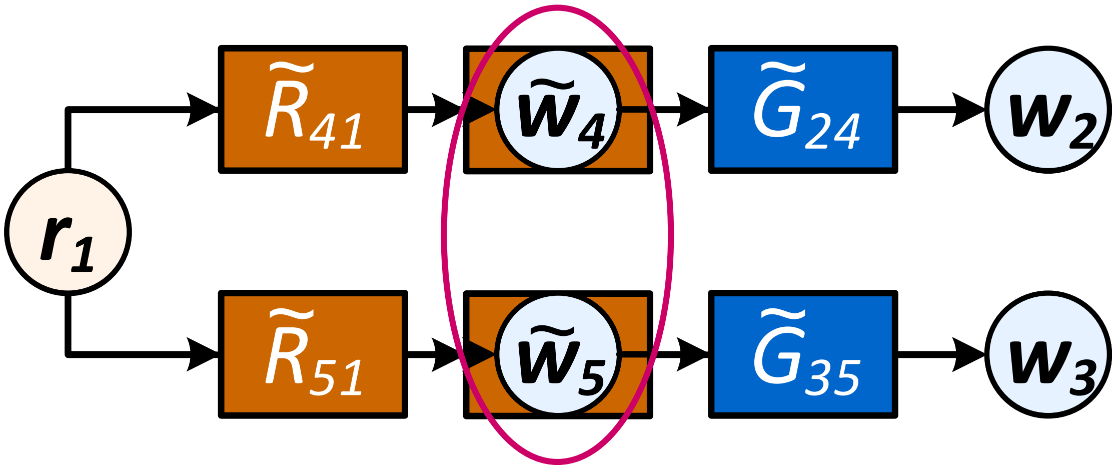
- State-space realisation
- Turn all states into nodes
- Find the new modules
- Non-unique



Realisation

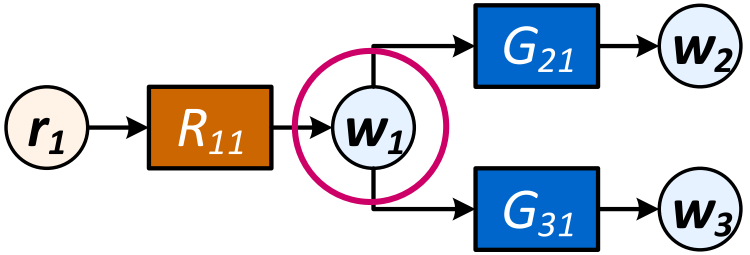
- More nodes

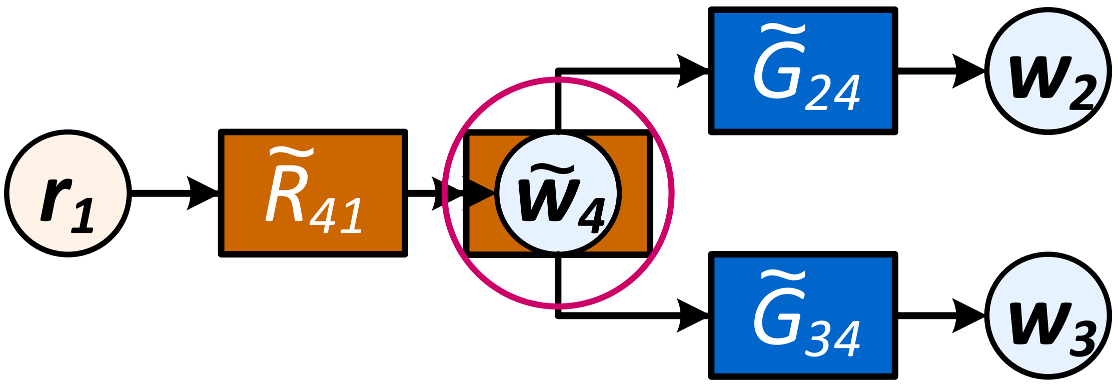




Realisation

- Same number of nodes





Generic McMillan degree

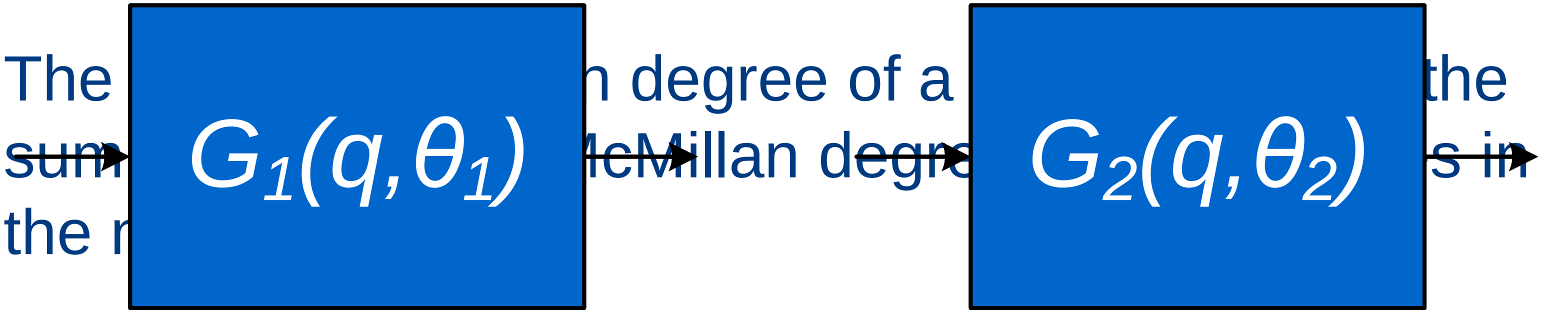
Generic McMillan degree

- McMillan degree for almost all choices of parameters
  (all choices except for a set of measure 0)
  - No special situations

$$G(q,\theta) = \frac{(q-\theta_1)}{(q-\theta_2)(q-\theta_3)}$$

- Independent modules
  - Shared hidden states not taken into account

- The … n degree of a … the
  sum … cMillan degr … s in
  the r …



Results

Results

Are the module representation and the state-space form equivalent?

Can they be transformed into one another without losing any information?

- Same framework
  - Module dynamic network

Results

Are the module representation and the state-space form equivalent?

Can they be transformed into one another without losing any information?

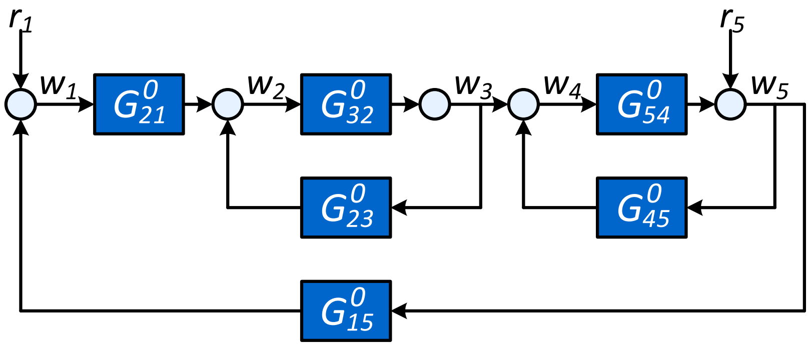
- State-space form → Equivalent module representation
  - Lose structural information between hidden states
  - Same generic McMillan degree if multi-path immersion

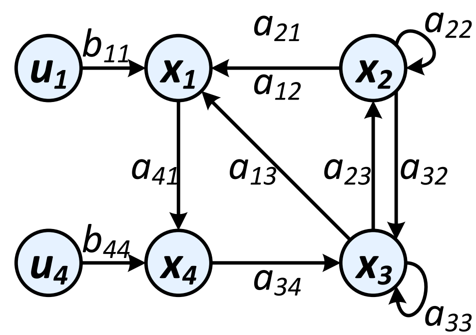
- Module representation → Equivalent state-space form
  - Non-unique
  - Same generic McMillan degree if minimal realisations

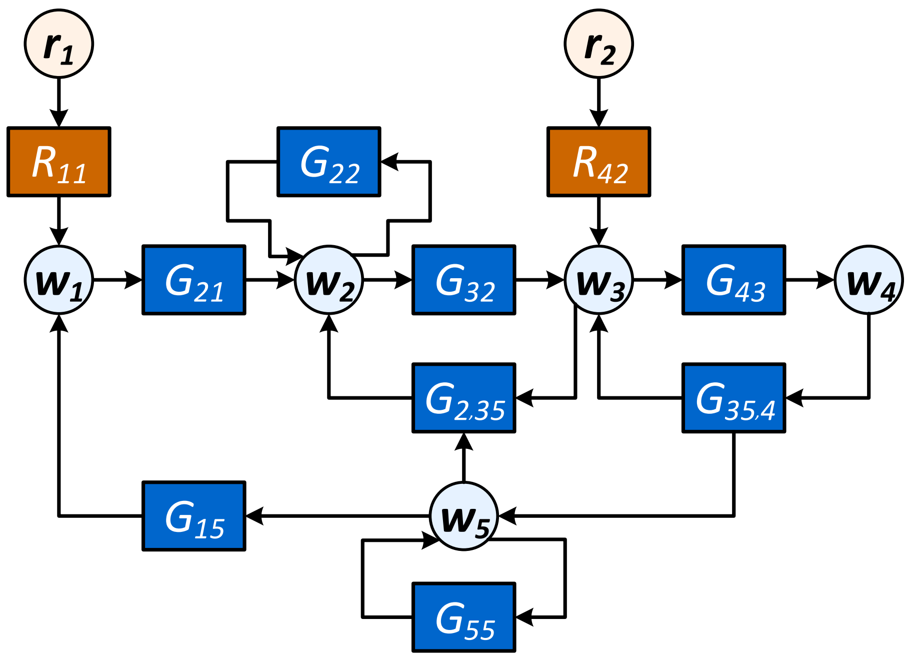
Identification

SISO modules	MIMO modules	
Hidden states		Topology not completely identifiable
Shared hidden states		Higher generic McMillan degree

Identification







Summary

Summary

- Module representation $\leftrightarrow$ state space form
- Module dynamic network
- Abstraction
  - State-space form $\rightarrow$ module representation
  - Multi-path immersion
- Realisation
  - Module representation $\rightarrow$ state-space form
  - Minimal realisations
- Result
  - Module dynamic network for system identification

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Appendix

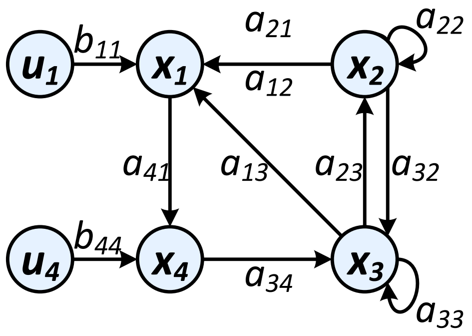
Continuous time

- Discrete time

$$x(t+1) = Ax(t) + Bu(t)$$

$$qx(t) = Ax(t) + Bu(t)$$

$$x(t) = q^{-1}Ax(t) + q^{-1}Bu(t)$$



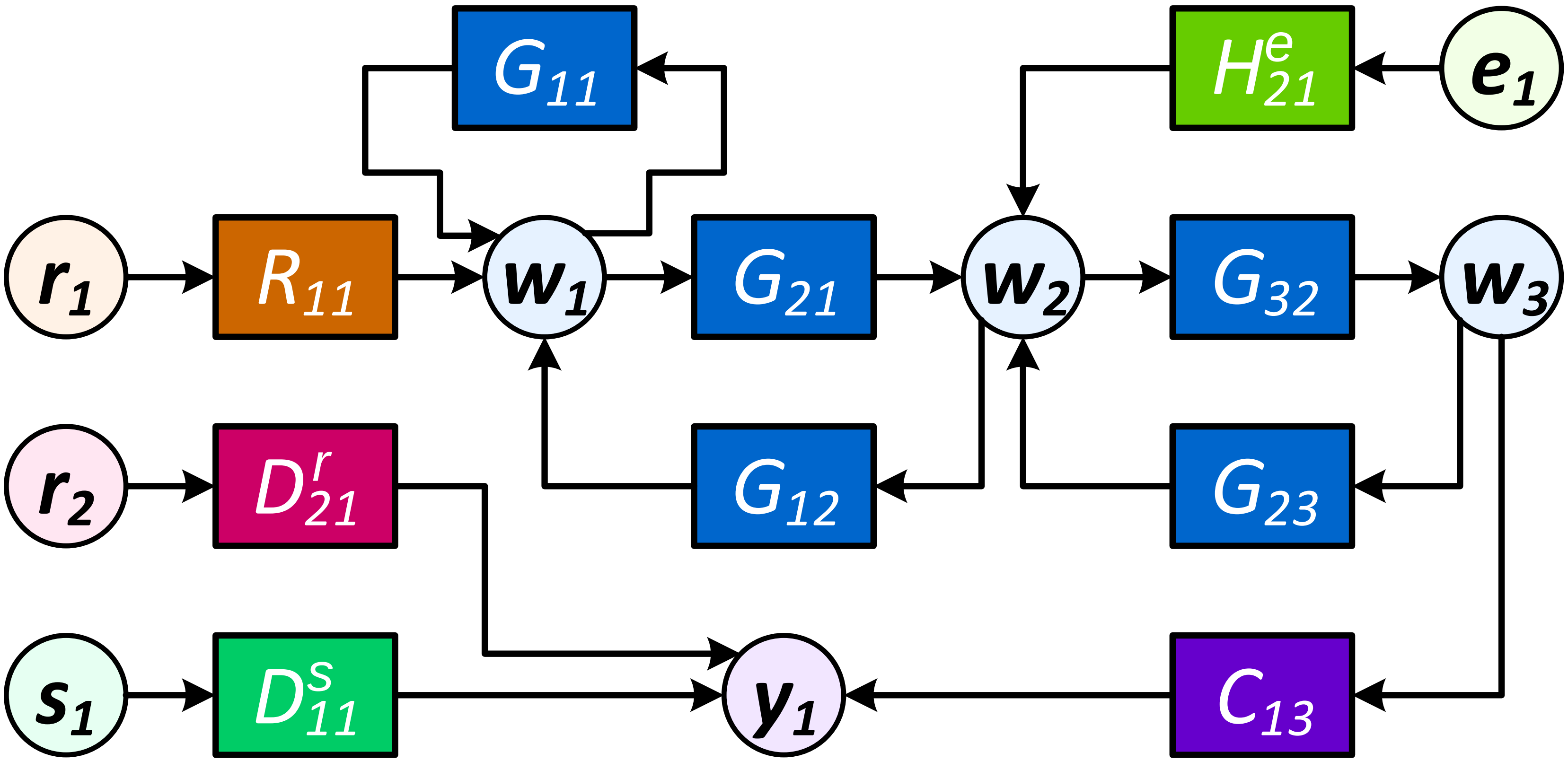
- Continuous time

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$sX(s) = AX(s) + BU(s)$$

$$X(s) = s^{-1}AX(s) + s^{-1}BU(s)$$

Disturbances & General measurements



Disturbances & General measurements

$$w(t) = G\, w(t) + R\, r(t) + H\, e(t)$$
$$y(t) = C\, w(t) + D^r\, r(t) + D^s\, s(t)$$
$$w_j(t) = \sum_{i=1}^{L} G_{ji}(q) w_i(t) + \sum_{k=1}^{K} R_{jk}(q) r_k(t) + \sum_{p=1}^{P} H_{jp}(q) e_p(t)$$
$$y_m(t) = \sum_{i=1}^{L} C_{mi}(q) w_i(t) + \sum_{k=1}^{K} D_{mk}^r(q) r_k(t) + \sum_{n=1}^{N} D_{mn}^s(q) s_n(t)$$

$$x(t+1) = A\, x(t) + B^r\, r(t) + B^e\, e(t)$$
$$y(t) = C\, x(t) + D^r\, r(t) + D^s\, s(t)$$
$$x_j(t) = \sum_{i=1}^{L} q^{-1} A_{ji}(q) x_i(t) + \sum_{k=1}^{K} q^{-1} B_{jk}^r(q) r_k(t) + \sum_{p=1}^{P} q^{-1} B_{jp}^e(q) e_p(t)$$
$$y_m(t) = \sum_{i=1}^{L} C_{mi}(q) x_i(t) + \sum_{k=1}^{K} D_{mk}^r(q) r_k(t) + \sum_{n=1}^{N} D_{mn}^s(q) s_n(t)$$

Disturbances & General measurements

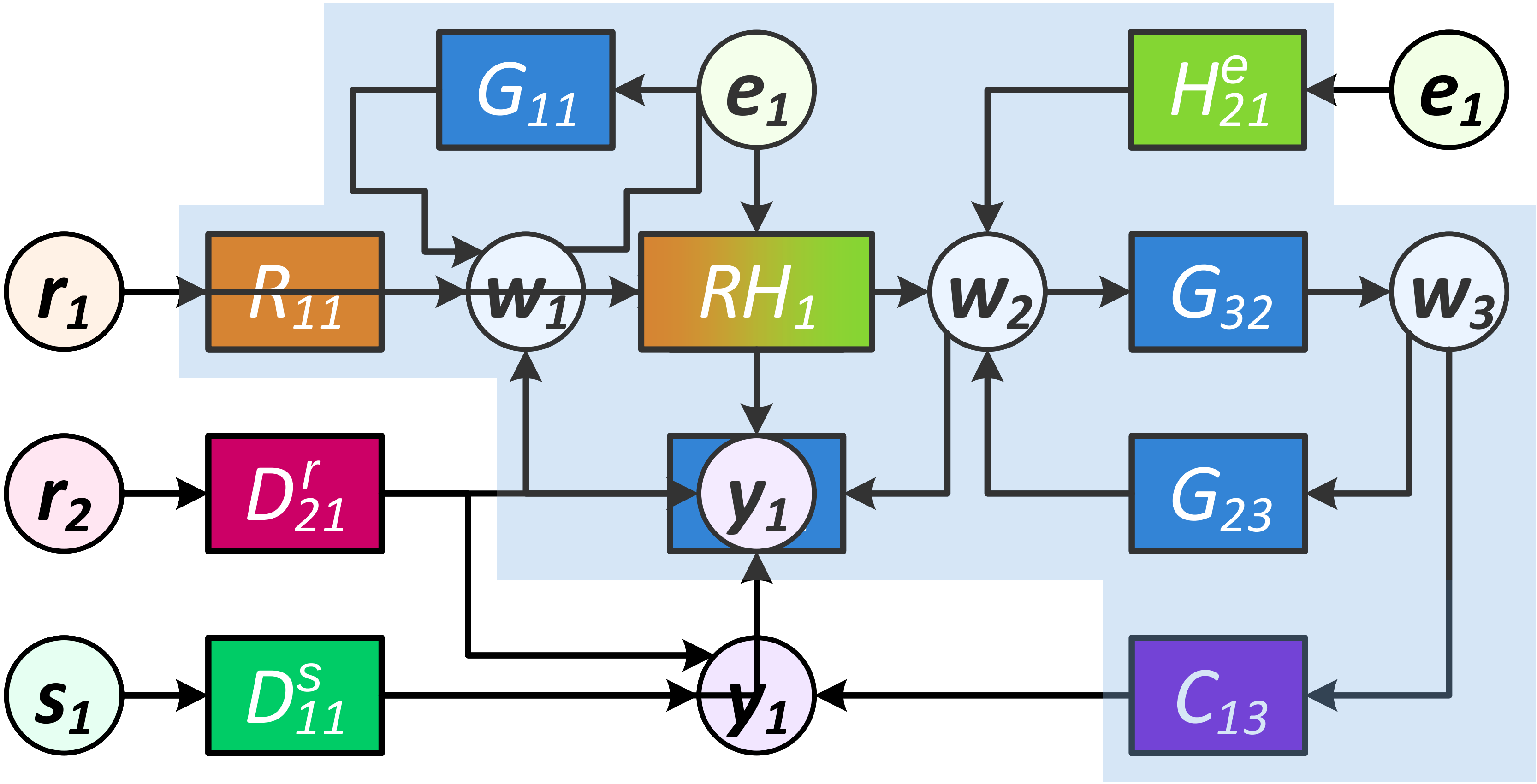
$$w_j(t) = \sum_{i=1}^{L} G_{ji}(q)w_i(t) + \sum_{k=1}^{K} R_{jk}(q)r_k(t) + \sum_{p=1}^{P} H_{jp}(q)e_p(t)$$
$$y_m(t) = \sum_{i=1}^{L} C_{mi}(q)w_i(t) + \sum_{k=1}^{K} D^r_{mk}(q)r_k(t) + \sum_{n=1}^{N} D^s_{mn}(q)s_n(t)$$

$$x_j(t) = \sum_{i=1}^{L} q^{-1}A_{ji}(q)x_i(t) + \sum_{k=1}^{K} q^{-1}B^r_{jk}(q)r_k(t) + \sum_{p=1}^{P} q^{-1}B^e_{jp}(q)e_p(t)$$
$$y_m(t) = \sum_{i=1}^{L} C_{mi}(q)x_i(t) + \sum_{k=1}^{K} D^r_{mk}(q)r_k(t) + \sum_{n=1}^{N} D^s_{mn}(q)s_n(t)$$

State-space dynamic network

- all $x_j(t)$ are $w_j(t)$ (*un*measurable nodes)
- $G_{ji}(q) = q^{-1}A(j,i)$
- $R_{jk}(q) = q^{-1}B^r(j,k)$
- $H_{jp}(q) = q^{-1}B^e(j,p)$
- all $y_j(t)$ are directly measurable nodes
- all $y_j(t)$ are static combinations of $x_j(t)$, $r_k(t)$, and $s_n(t)$

Disturbances & General measurements



Module dynamic network