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Low-dimensional tensor representations for the estimation of petrophysical reservoir parameters

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Introduction

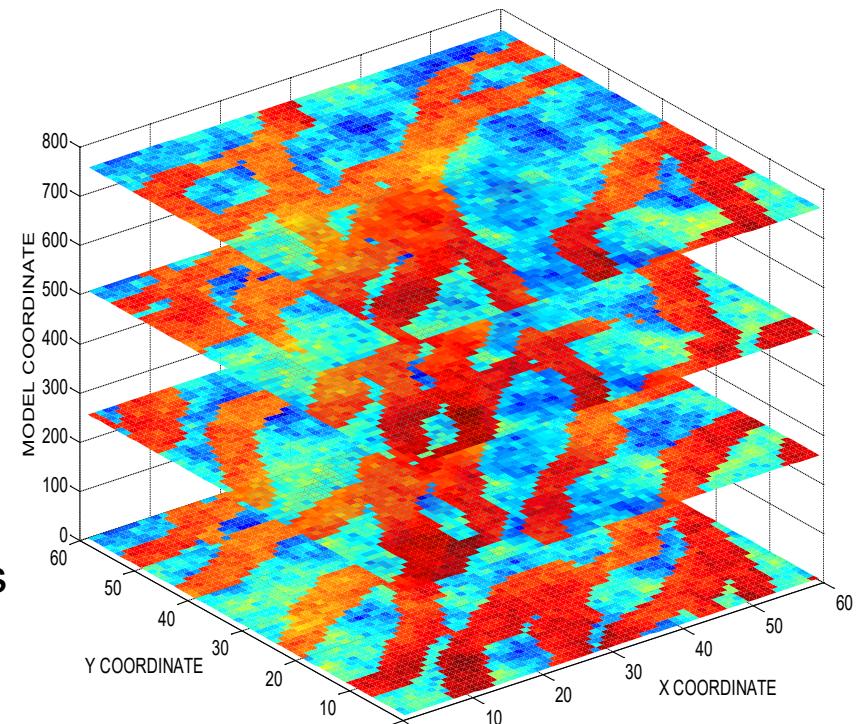
- Setting: Computer-assisted history matching (CAHM)
- Topic: Reparameterization of petrophysical gridblock properties (permeabilities, porosities)
- Motivation:
 - CAHM of grid block parameters is an ill-posed problem; therefore we need regularisation (usually in Bayesian framework with a prior ensemble)
 - Reparameterization to reduce the number of parameters to be matched is an alternative to regularization
 - Reparameterization in terms of the dominant spatial patterns of petrophysical parameters helps to build better priors

Earlier Reparameterization Studies (Incomplete List)

- Zonation (Jacquard & Jain 1965; Jahns 1966)
- Pilot points (de Marsily et al. 1984; Bissell et al. 1997)
- Principal Component Analysis (PCA) (Gavalas et al. 1976, Oliver 1996; Reynolds et al. 1996; Tavakoli et al. 2010)
- Kernel PCA (Sarma et al. 2008; Ma & Zabaras, 2011)
- Optimization-based PCA (Vo & Durlofsky, 2014, 2015)
- Level sets (Dorn & Villegas, 2008)
- Wavelets (Lu & Horne, 2000; Sahni & Horne, 2005; Awotunde & Horne, 2013)
- Discrete Cosine Transform (DCT) (Jafarpour et al. 2009, 2010; Zhao et al. 2016)
- Tensor decompositions (Afra & Gildin, 2013, 2014, 2016; Afra et al. 2014)

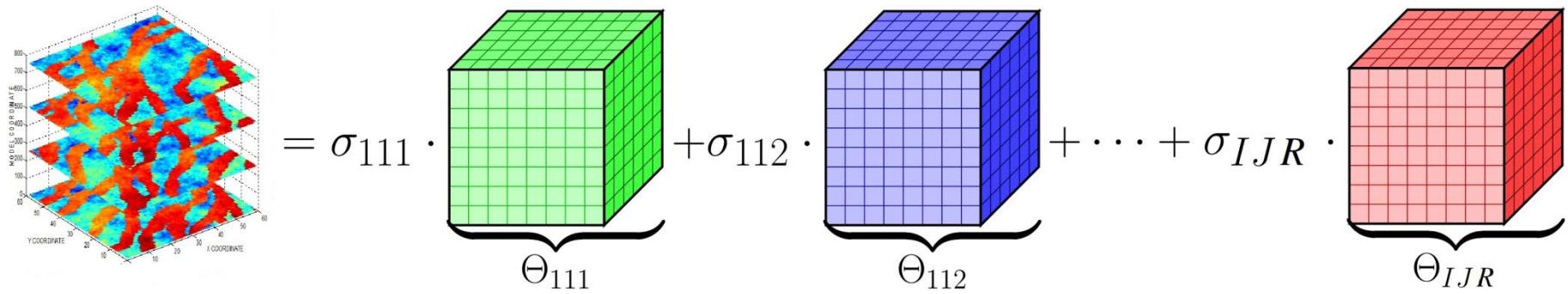
Tensor Representation of Gridblock Properties

- **Classical representation:**
 - Gridblock parameters vectorized
 - Vectors combined in a matrix
 - Spatial structure is destroyed
- **Tensor representation:**
 - Reservoir realizations are stacked in a separate dimension, without vectorizing the gridblock parameters
 - For Cartesian grids



Tensor Decompositions

Linear combination of rank-1 tensors. Multilinear generalization of the SVD



Tucker decomposition:

$$\mathcal{S} = \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R \sigma_{ijr} \varphi_i \otimes \Psi_j \otimes \vartheta_r = \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R \sigma_{ijr} \Theta_{ijr}$$

Tensor Decompositions – Matrix Example

Example: Tucker decomposition of a 3×2 matrix via SVD:

$$\begin{aligned}
 \mathbf{Z} &= \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ z_{31} & z_{32} \end{bmatrix} = \Phi \Sigma \Psi^T = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \\ \varphi_{31} & \varphi_{32} \end{bmatrix} \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix} \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^T \\
 &= [\Phi_1 \quad \Phi_2] \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} [\Psi_1 \quad \Psi_2]^T = [\Phi_1 \quad \Phi_2] \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix} \begin{bmatrix} \Psi_1^T \\ \Psi_2^T \end{bmatrix} \\
 &= \sigma_{11} \underbrace{\Phi_1 \Psi_1^T}_{3 \times 2} + \sigma_{22} \underbrace{\Phi_2 \Psi_2^T}_{3 \times 2} = \sigma_{11} \underbrace{\Phi_1 \otimes \Psi_1}_{3 \times 2} + \sigma_{22} \underbrace{\Phi_2 \otimes \Psi_2}_{3 \times 2}
 \end{aligned}$$

Multilinear mapping $Z_{ij} : \square^3 \times \square^2 \rightarrow \square$ (Here $\sigma_{12} = \sigma_{21} = 0$; not always so)

Tensor Decompositions – Low Rank Approximations

- Low-rank approximation of a matrix using a truncated SVD:

$$\mathbf{Z} = \Phi \Sigma \Psi^T = \sum_{i=1}^I \sigma_i \phi_i \otimes \psi_i^T$$

$$\hat{\mathbf{Z}} = \hat{\Phi} \hat{\Sigma} \hat{\Psi}^T = \sum_{i=1}^{\hat{I}} \sigma_i \phi_i \otimes \psi_i^T = \arg \min_{\Phi_i, \Psi_i} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F$$

where $\|\mathbf{Z}\|_F = \sqrt{\sum_{i=1}^I \sum_{j=1}^J z_{ij}^2}$ is the Frobenius norm of matrix \mathbf{Z}

Tensor Decompositions – Low Rank Approximations

- Many “optimal” tensor decompositions and low-rank approximations.
- We use the High-Order SVD (HOSVD) decomposition
(De Lathauwer et al. 2000):

$$S = \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R \sigma_{ijr} \Phi_i \otimes \Psi_j \otimes \Theta_r$$

$$\hat{S} = \sum_{i=1}^{\hat{I}} \sum_{j=1}^{\hat{J}} \sum_{r=1}^{\hat{R}} \sigma_{ijr} \Phi_i \otimes \Psi_j \otimes \Theta_r = \arg \min_{\Phi_i, \Psi_j, \Theta_r} \|S - \hat{S}\|_F$$

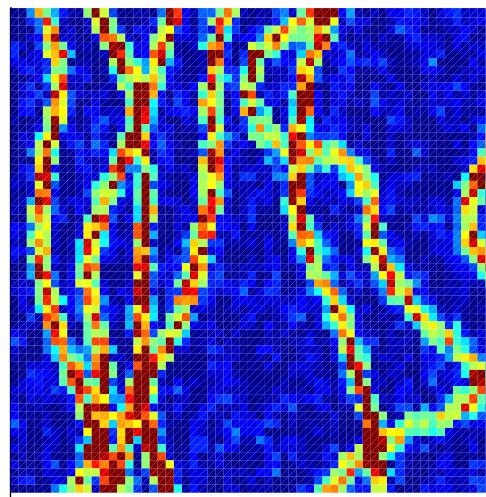
subject to the constraint that $\Phi_i, 1 \leq i \leq \hat{I}$, $\Psi_j, 1 \leq j \leq \hat{J}$, $\Theta_r, 1 \leq r \leq \hat{R}$,
are orthonormal

- Note: Results in a full core tensor; not diagonal

Low-Rank Approximation: SVD (Rank 30) and Tensor Decomposition

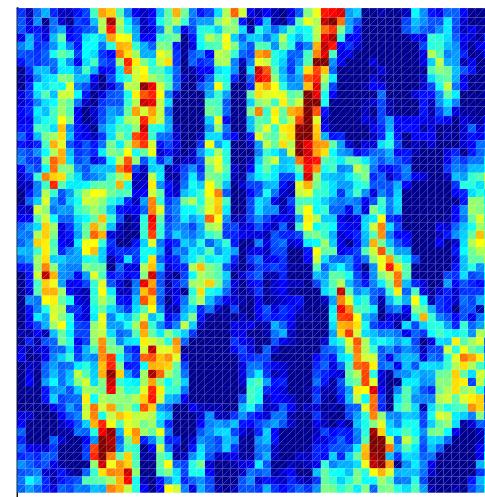
Example: $60 \times 60 \times 7 = 25200$ gridblocks; 100 realizations of permeability

Realization 3



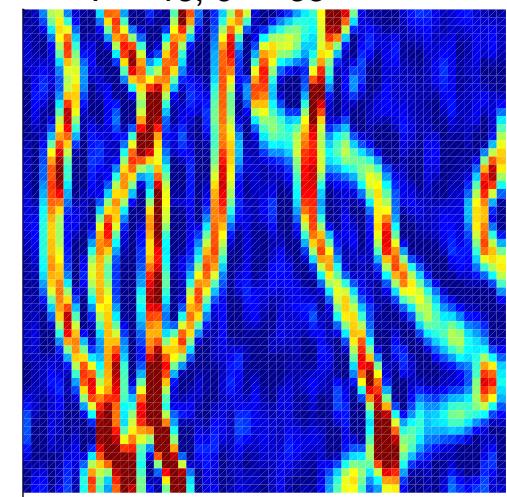
20.16 MB

SVD approximation r=30



12.17 MB (60.3%)

Tensor approximation
 $\hat{I} = 15, \hat{J} = 35$

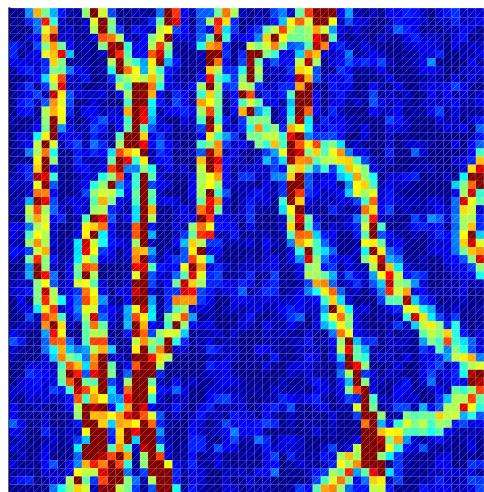


0.53 MB (2.6%)

Low-Rank Approximation: SVD (Rank 60) and Tensor Decomposition

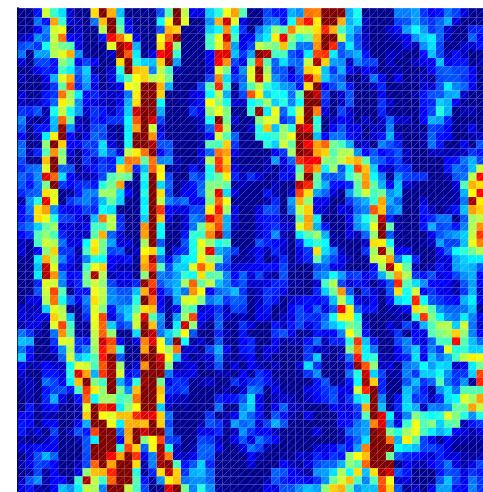
Example: $60 \times 60 \times 7 = 25200$ gridblocks; 100 realizations of permeability

Realization 3



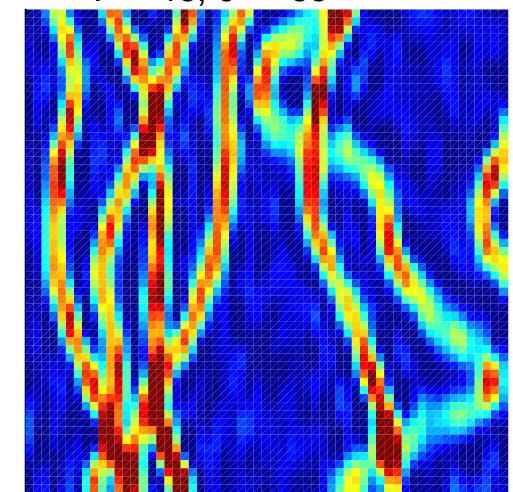
20.16 MB

SVD approximation r=60



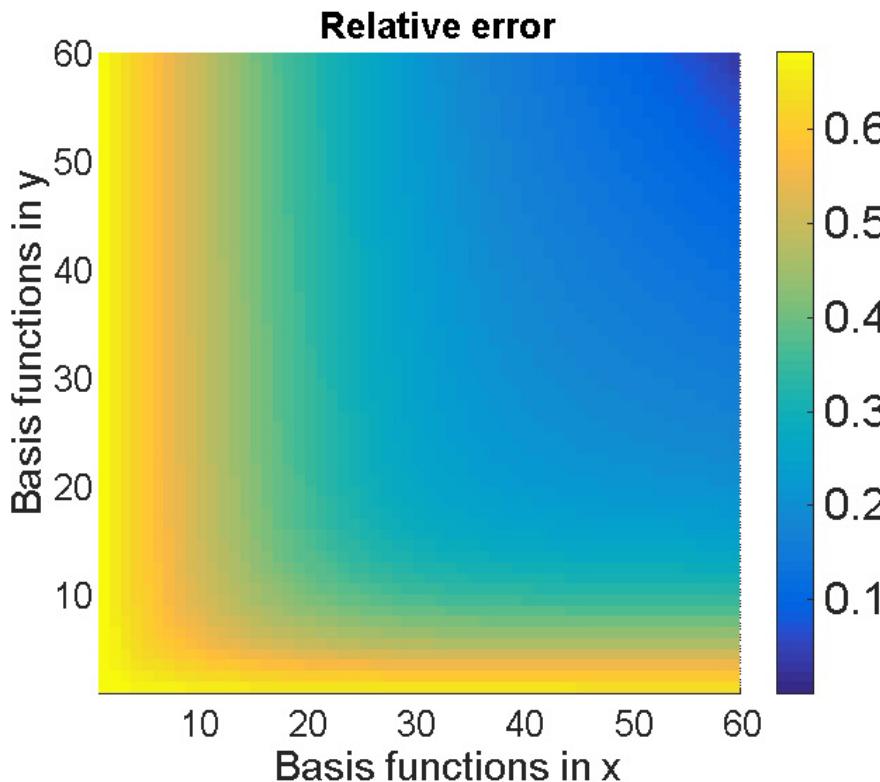
12.17 MB (60.3%)

Tensor approximation
 $\hat{I} = 15, \hat{J} = 35$



0.53 MB (2.6%)

Directional Approximation Errors



- Approximation of the y coordinate requires fewer basis functions than the one for the x coordinate to achieve the same error
- Related to channel orientation
- Gives opportunity to selectively truncate

Tensor Reparameterization of Grid Block Parameters

Low-rank tensor approximation of one realization:

$$\hat{\theta}_m = \hat{S}(\cdot, \cdot, \mathbf{e}_m) = \sum_{i=1}^{\hat{I}} \sum_{j=1}^{\hat{J}} \alpha_{ij}^m \varphi_i \otimes \Psi_j \quad \text{where} \quad \alpha_{ij}^m := \sum_{r=1}^{\hat{R}} \sigma_{ijr} \langle \vartheta_r, \mathbf{e}_m \rangle$$

All the realizations share the same basis functions!

Parameterization through letting the coefficients α_{ij} be free parameters.

General Formulation of EnKF With Tensor Parameterization

Reservoir model: $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \boldsymbol{\theta})$ and model output $\mathbf{d}_k^{sim} = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k)$,

\mathbf{x}_k : reservoir states (pressures, saturations)

\mathbf{u}_k : control variables (bottom hole pressures, well rates)

$\hat{\boldsymbol{\theta}} = \sum_{i=1}^I \sum_{j=1}^J \alpha_{ij} \varphi_i \otimes \psi_j$: petrophysical parameters (perms, porosities)

Augmented state vector

$$\mathbf{z}_{k+1} = \begin{bmatrix} \mathbf{x}_{k+1} \\ \mathbf{a} \\ \mathbf{d}_{k+1}^{sim} \end{bmatrix} = \bar{\mathbf{f}}(\mathbf{z}_k, \mathbf{u}_k) = \begin{bmatrix} \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \boldsymbol{\theta}) \\ \mathbf{a} \\ \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) \end{bmatrix}$$

where \mathbf{a} is the vector of parameters α_{ij}

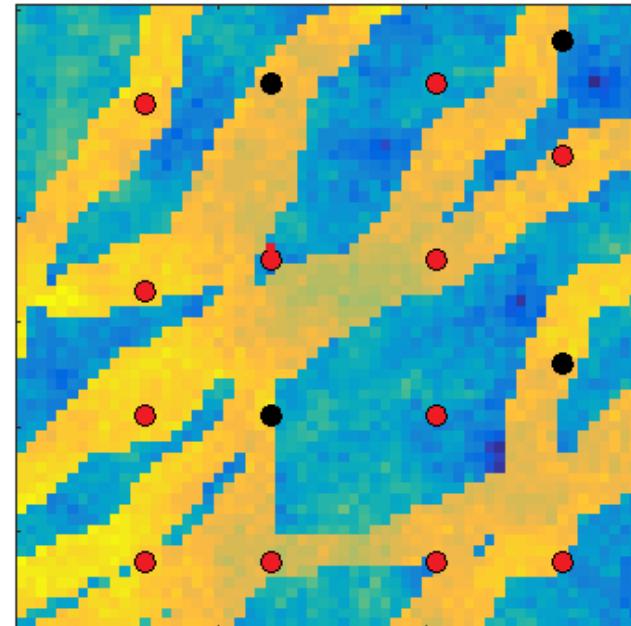
Example

Model:

- Stanford data set (3600 grid cells)
- Ensemble size: 100 realizations
- Waterflooding: 12 producers, 4 injectors

Computer Assisted History Matching:

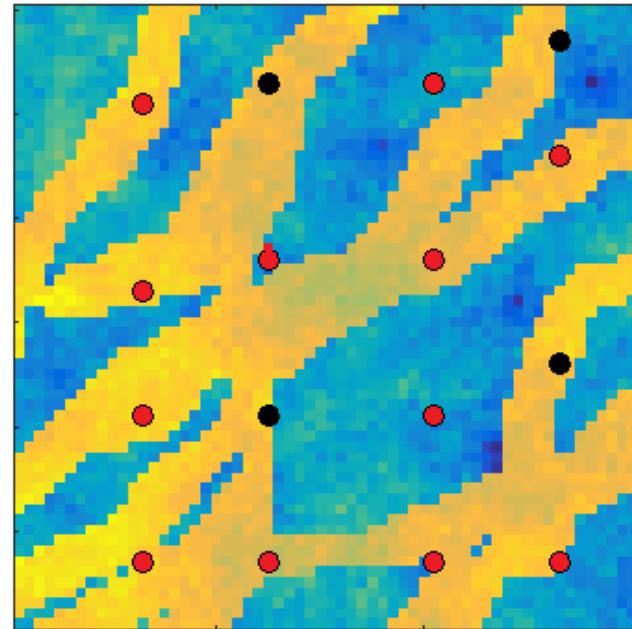
- Estimated parameter: Permeability
- Simulation time: 15 years
- Final update time: 6 year
- Update time interval: 2 year
- Measurement time interval: 3 month
(rates and pressures)



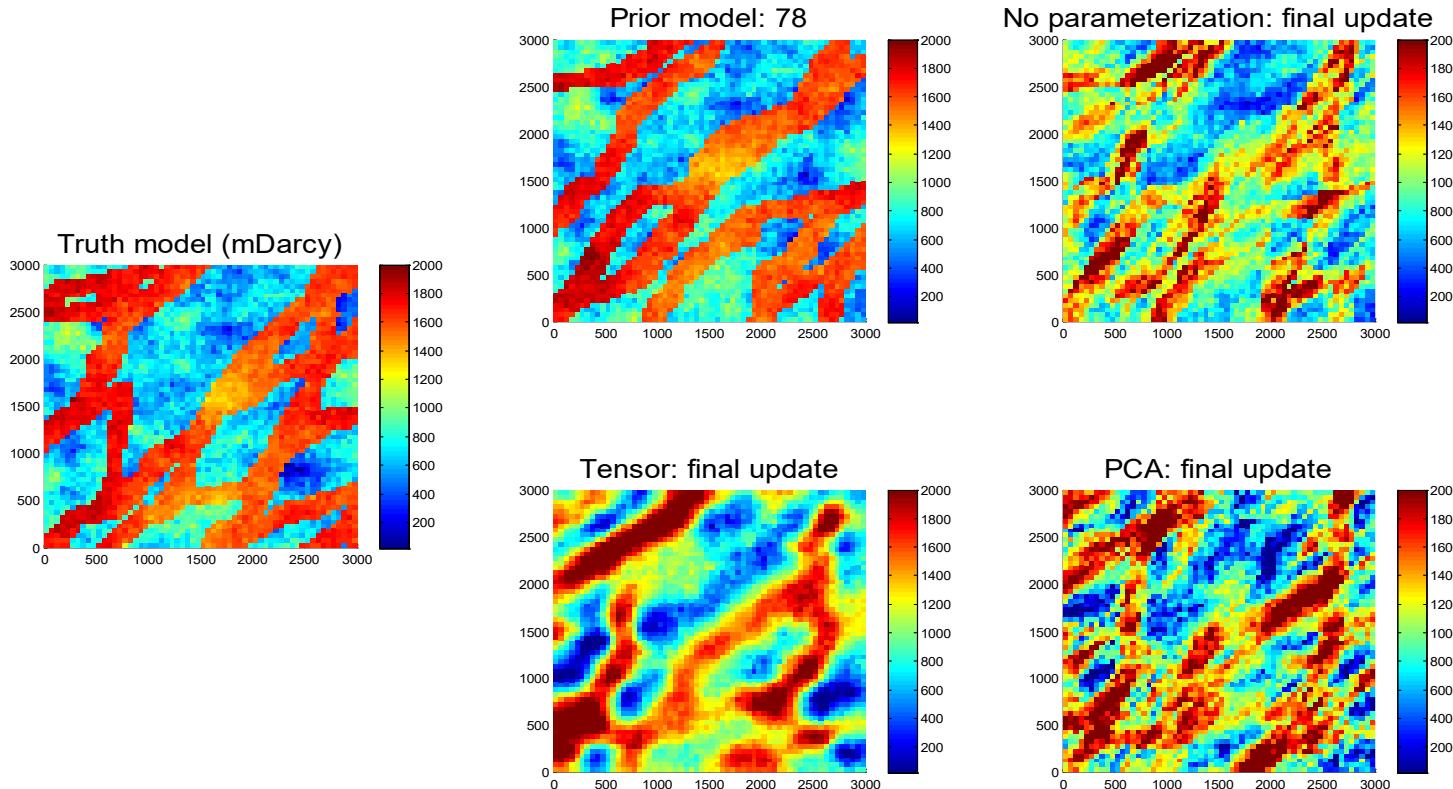
Example

Experiments:

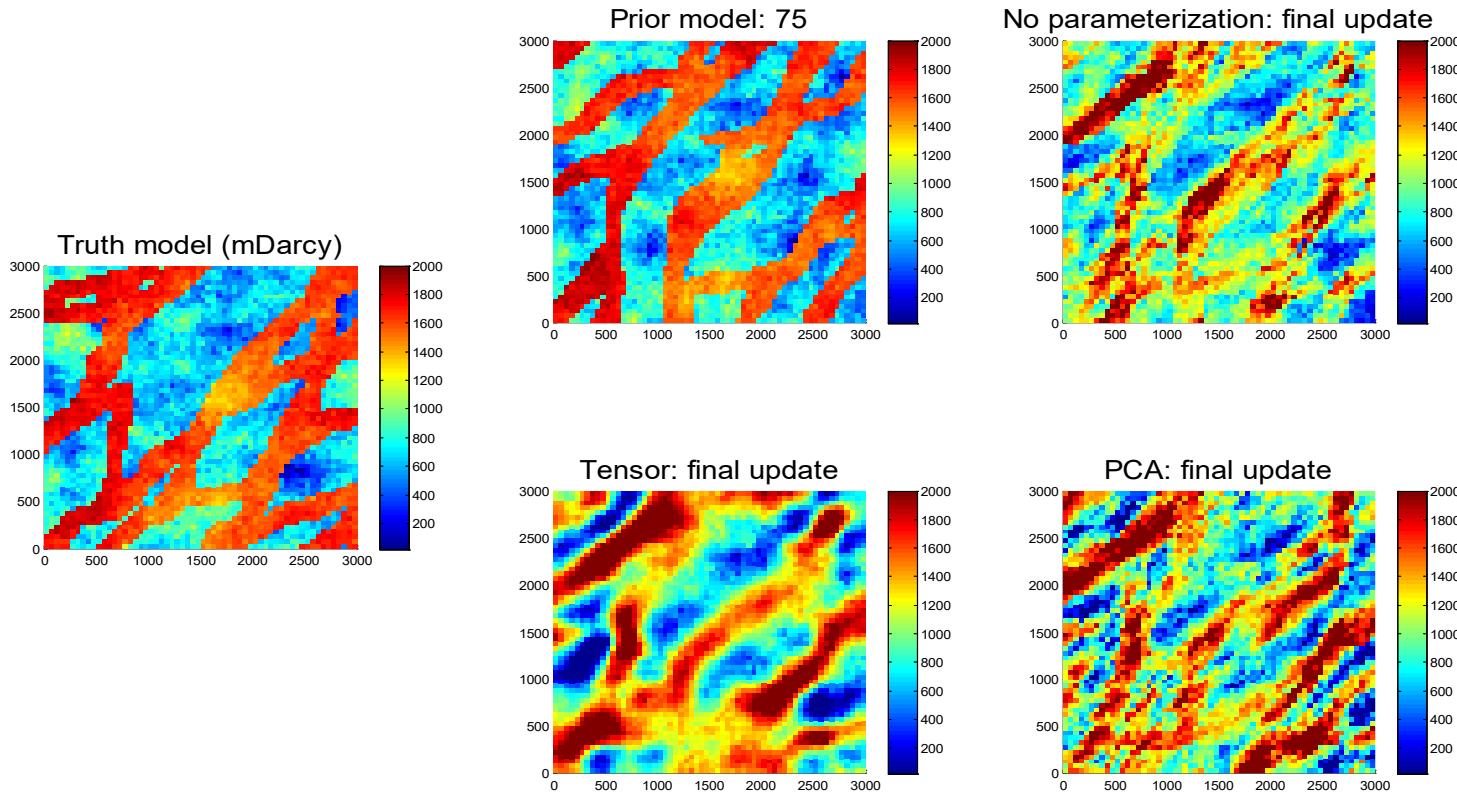
1. CAHM with grid-based permeability
(3600 parameters)
2. CAHM with PCA parameterization
(50 parameters)
3. CAHM with tensor parameterization
(50 parameters)



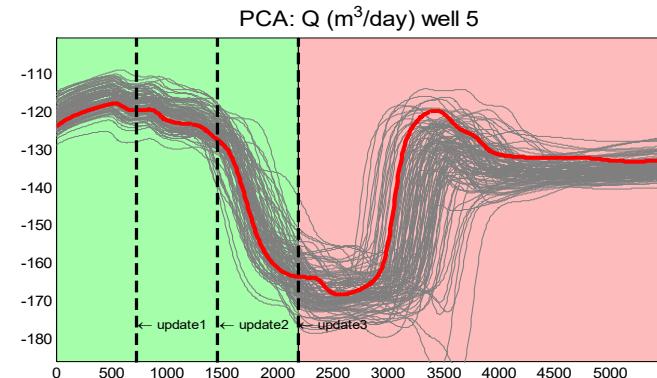
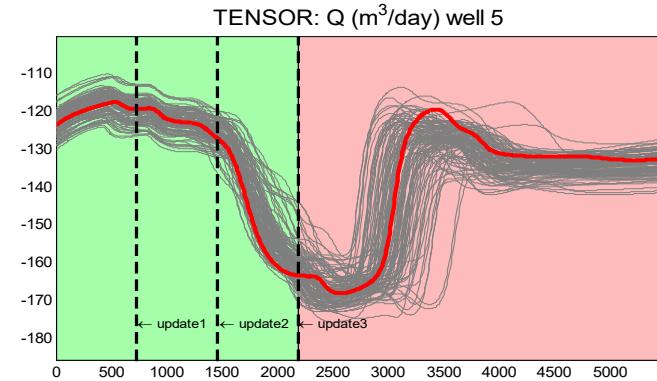
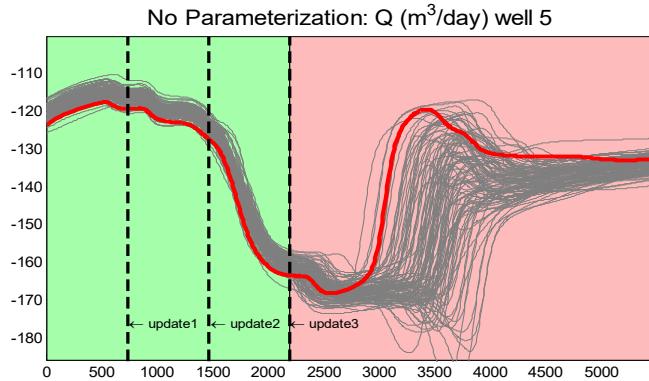
Model Updates



Model Updates

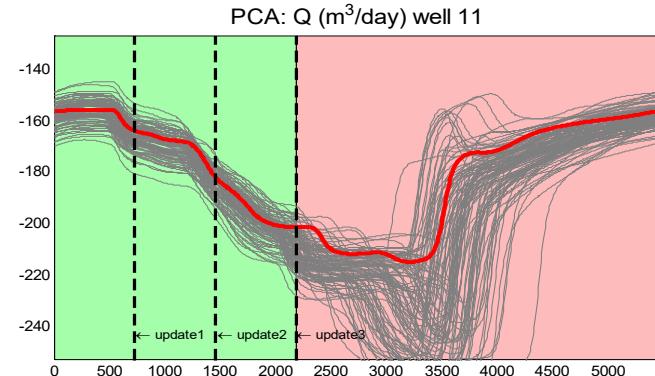
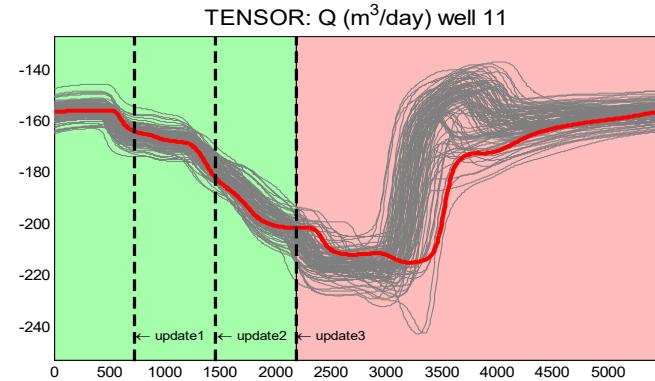
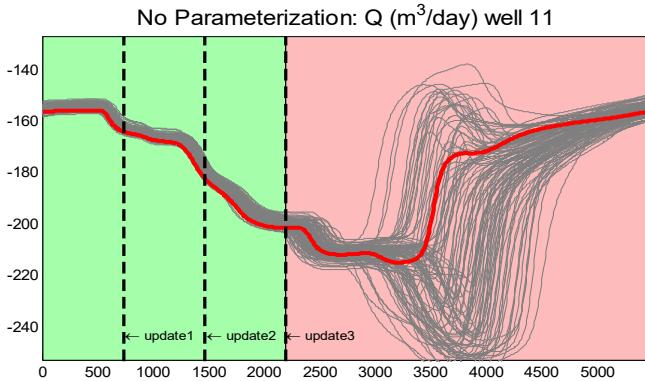


Total rates: Well 5



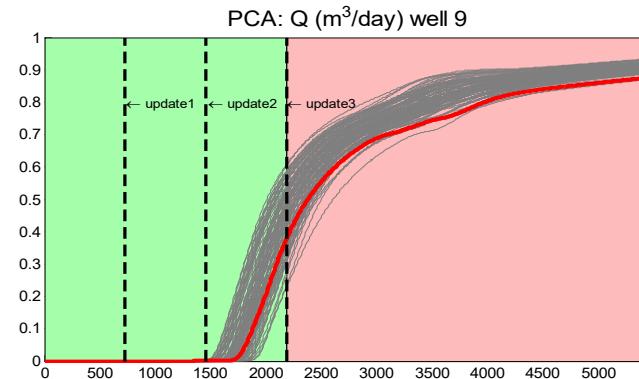
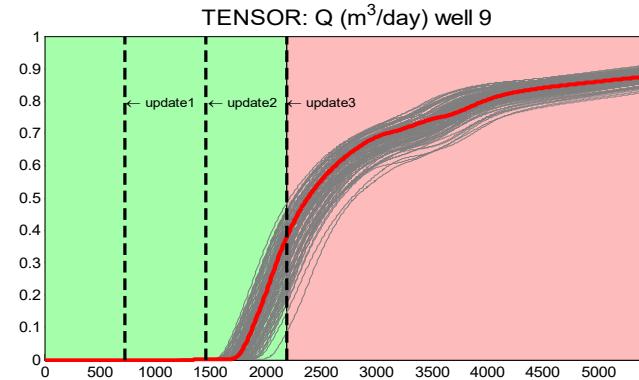
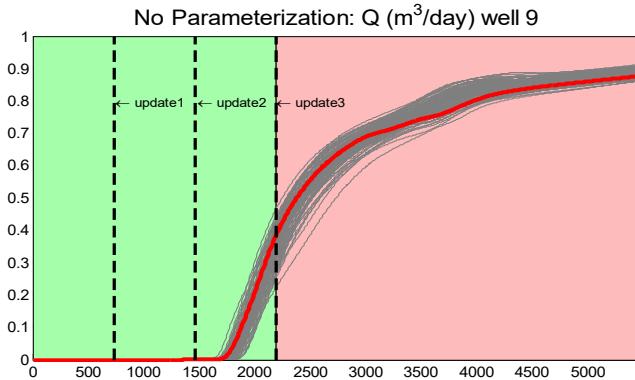
- Better predictions with parameterization
- No clear benefit of tensor over PCA

Total Rates: Well 11



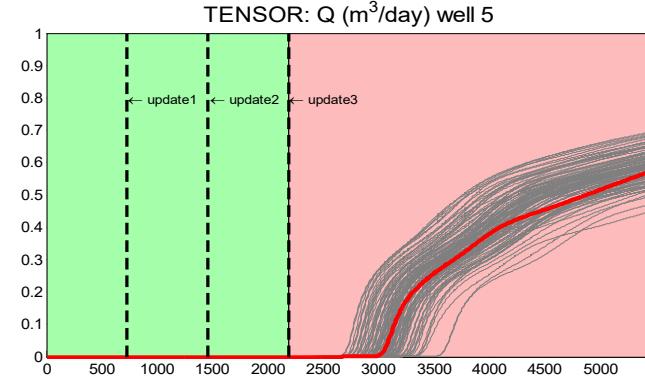
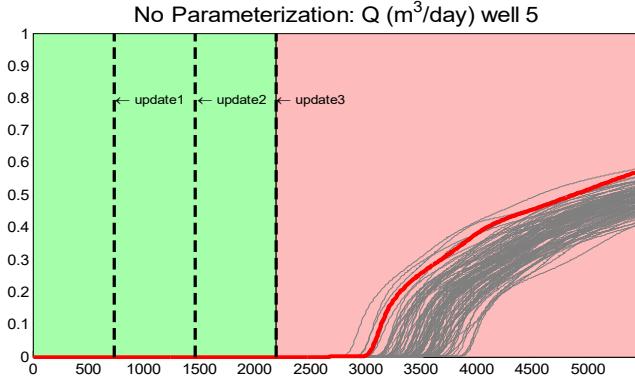
- Better predictions with parameterization
- No clear benefit of tensor over PCA

Water Cut: Well 9

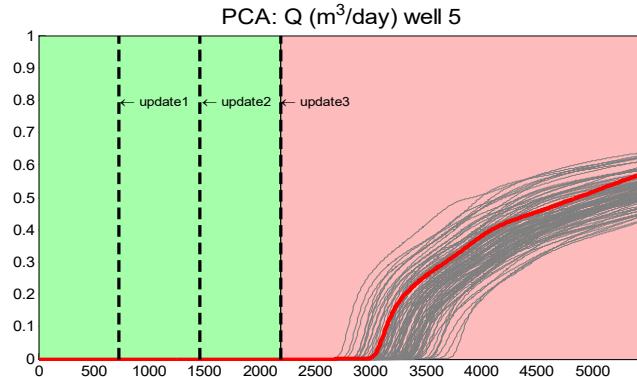


Water breakthrough information
in the measurements

Water cut: Well 5



No water breakthrough information
in the measurements



Conclusions

- Tensor representation of petrophysical grid block properties:
 - Very efficient in terms of storage
 - Maintains spatial properties much better than PCA
- EnKF history matching with tensor representation:
 - Channellized structure much better preserved than without parameterization or with PCA
 - Improved prediction capabilities of the updated models:
Some benefits compared to PCA, but not yet fully convincing
(Too optimistic conclusion in paper)
 - More experiments needed

SPE Reservoir Simulation Conference

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Acknowledgements / Thank You / Questions

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