

Scenario-based robust optimization of water flooding in oil reservoirs enjoys probabilistic guarantees

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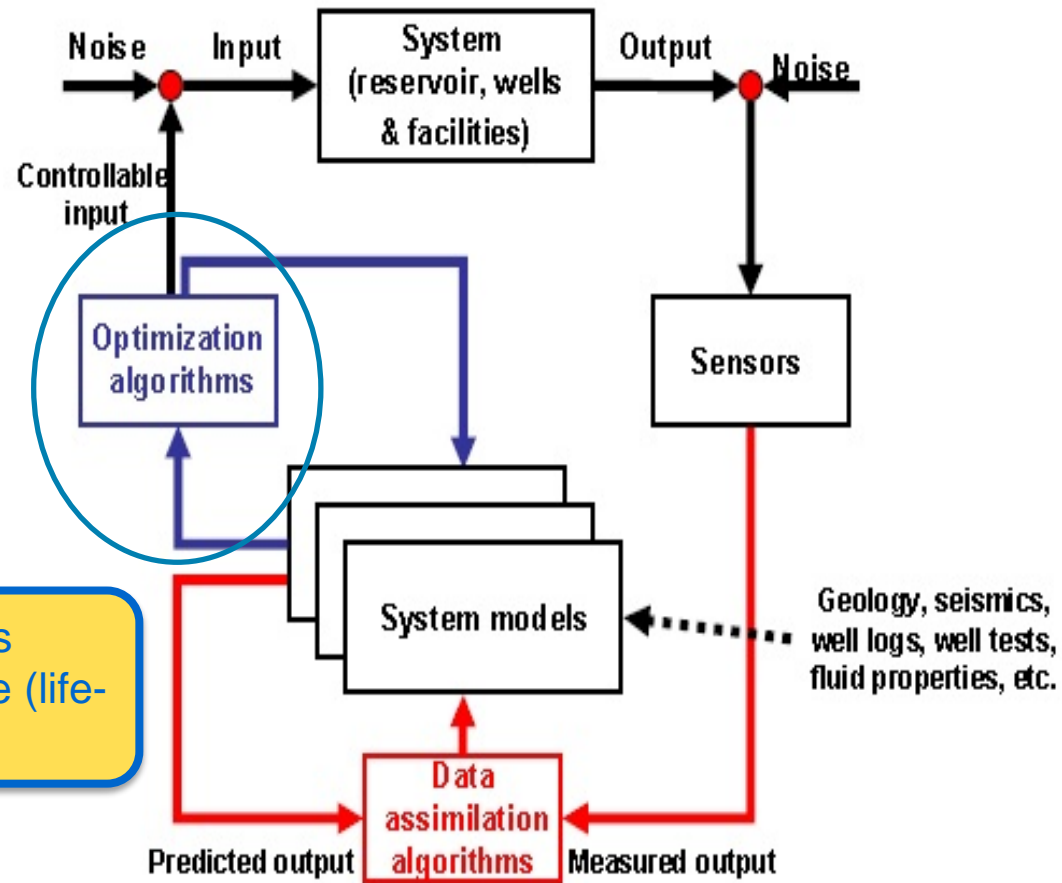


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Where innovation starts

Closed-Loop Reservoir Management (CLRM)

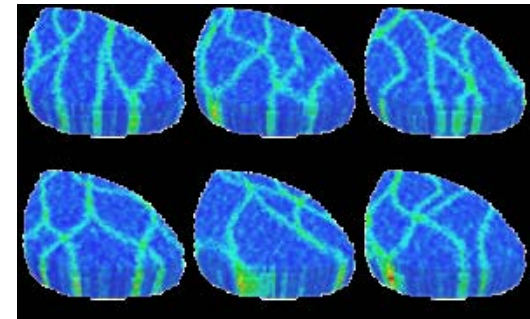
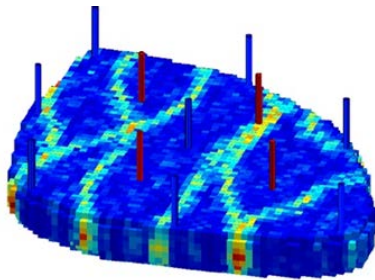


Strategy that optimizes economic performance (life-cycle)

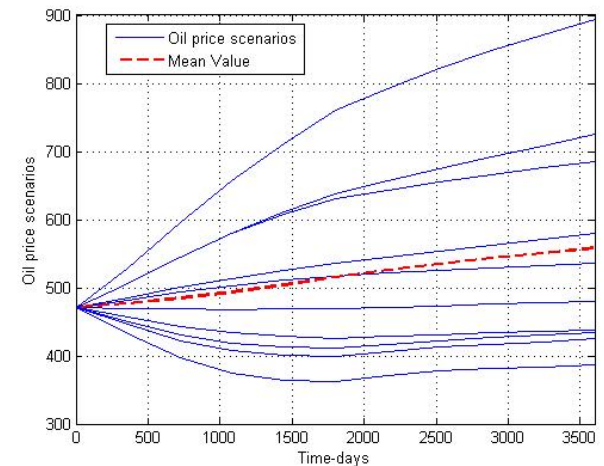
Closed-loop reservoir management Jansen et al. (2005)

Challenges

- Large-scale, non-convex and non-linear optimization
- Uncertainty
 - Parametric uncertainty



- Economic uncertainty
 - Varying oil prices



Decision making (model-based economic optimization) under geological parametric and economic uncertainty

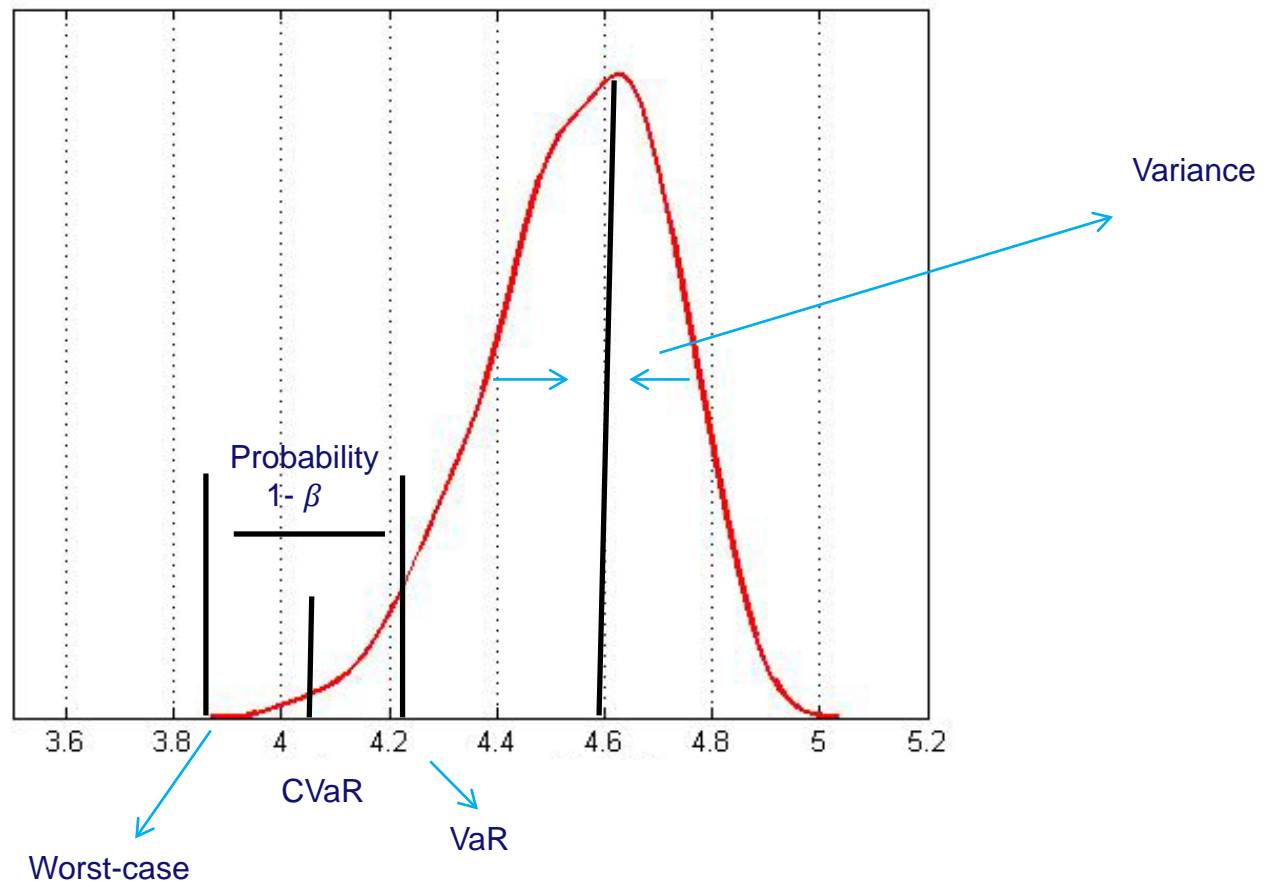
Contents

- Introduction
- Decision making under uncertainty (robust optimization)
- Worst-case optimization (WCO)
- Scenario-based worst-case optimization
 - Performance robustness probability
 - Support constraint
- Scenario-based WCO under Economic uncertainty
- Scenario-based WCO under Geological uncertainty

Decision making under uncertainty

robust optimization

$$\max (\text{Mean} - \gamma \text{CVaR})$$



Worst-case optimization (WCO)

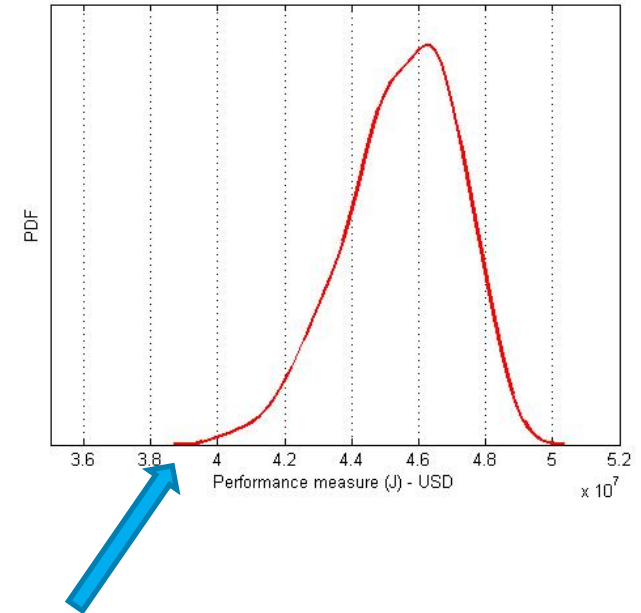
Worst-case (robust) optimization:

$$\max_{\mathbf{u} \in \mathcal{U}} \min_{\theta \in \Theta} J(\mathbf{u}, \theta)$$

Reformulation

$$\text{WCO : } \begin{cases} \max_{\mathbf{u} \in \mathcal{U}, z \in \mathbb{R}} & z \\ \text{s.t.} & z \leq \min_{\theta \in \Theta} J(\mathbf{u}, \theta) \end{cases}$$

$\theta \in \Theta$ is a representation of uncertainty space



Scenario-based worst-case optimization

Introduction

Scenario-based worst-case optimization

- sample the uncertainty space
- a finite number of realizations of the uncertain parameters

$$\{\theta_1, \theta_2, \dots, \theta_N\}$$

$$\max_{\mathbf{u} \in \mathcal{U}} \min_{\theta_i \in \Theta} J(\mathbf{u}, \theta_i) \quad i = 1, 2, \dots, N$$

$$\text{WCO}_N : \begin{cases} \max_{\mathbf{u} \in \mathcal{U}, z \in \mathbb{R}} & z \\ \text{s.t.} & z \leq \min_{i \in \{1, \dots, N\}} J(\mathbf{u}, \theta_i) \end{cases}$$

Scenario-based worst-case optimization

Introduction

What can we claim about the achieved objective function value, when u_N^* is applied in the presence of the unseen scenarios for θ ?

Given a number of samples N , can we quantify, e.g., in probabilistic terms, the robustness of the performance, i.e., the measure of the subset of Θ such that $J(u_N^*) \geq \max_{\theta \in \Theta} J(u_N^*, \theta)$?

Scenario-based worst-case optimization

Performance robustness probability

Performance robustness probability

Definition: Let u_N^* be an optimal solution to WCO_N . The performance robustness probability of u_N^* in terms of NPV is defined as:

$$V(\mathbf{u}_N^*) := \mathbb{P}_{\Theta}[J(\mathbf{u}_N^*) \geq J(\mathbf{u}_N^*, \theta \in \Theta)].$$

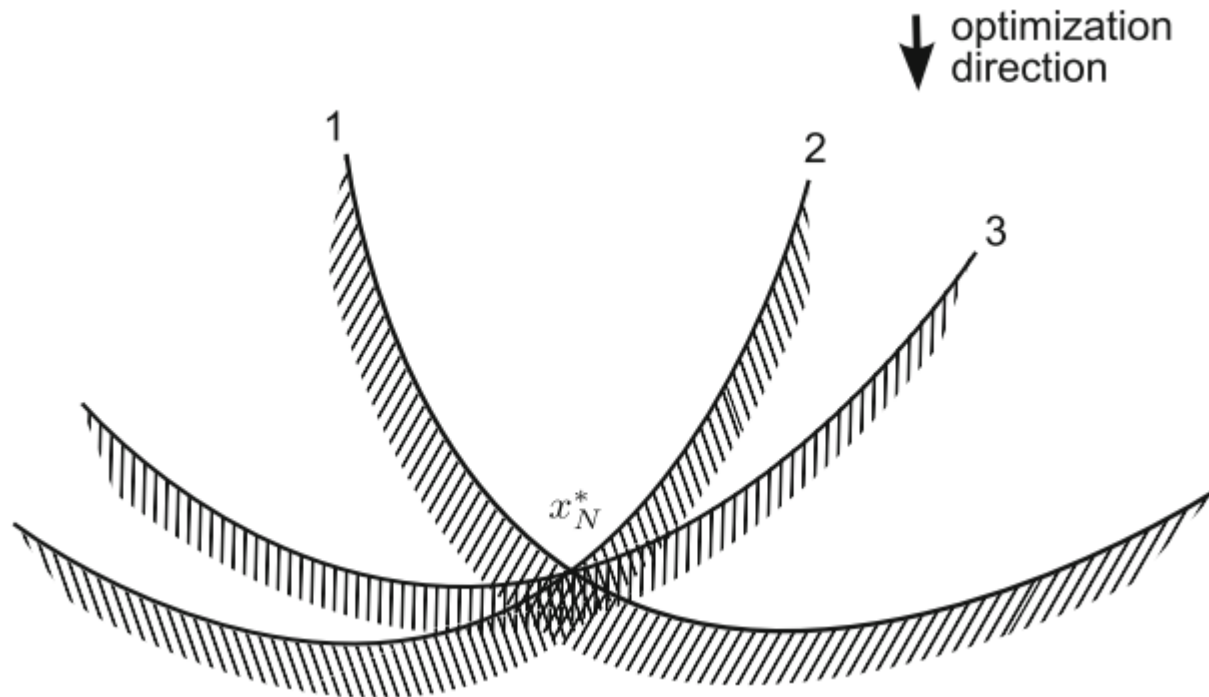
ϵ – level robust solution: if $\epsilon \in (0,1)$, $V(\mathbf{u}_N^*) \leq \epsilon$

- A-priori guarantees (convex optimization)
- A-posteriori guarantees (wait-and-judge for non-convex optimization)

Scenario-based worst-case optimization

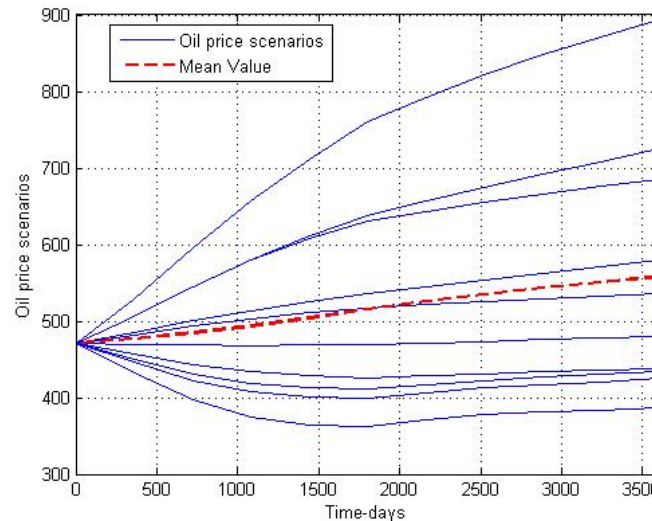
Support constraint

A sampled constraint is a support constraint for WCO_N if its removal would alter the optimal solution of the problem.



Scenario-based WCO under economic uncertainty

Oil price variation r_k^{oil} as source of uncertainty



$$J(\mathbf{u}, \theta) = \sum_{k=1}^K \frac{\Delta t_k}{(1+b)^{t_k/\tau}} \left(r_k^{oil} q_k^{oil} - r_k^{water} q_k^{water} - r_k^{inj} q_k^{inj} \right)$$

r_k^{oil} appears linearly in $J(u, \theta)$

Scenario-based WCO under economic uncertainty

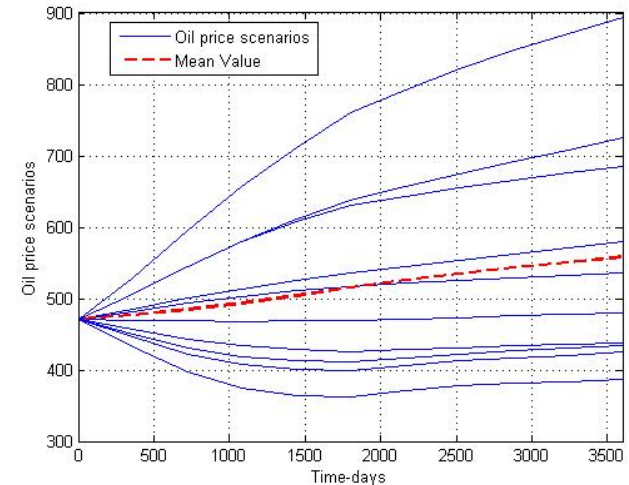
Main result: Suppose that, for all $\theta = r_k^{oil} \in \Theta$, the function $u \rightarrow J(u, \theta)$ is concave. Let u_N^* be the optimal solution to WCO_N in,

$$N \geq \frac{2}{\epsilon} \left(K + \ln \left(\frac{1}{\beta} \right) \right).$$

Then, it holds

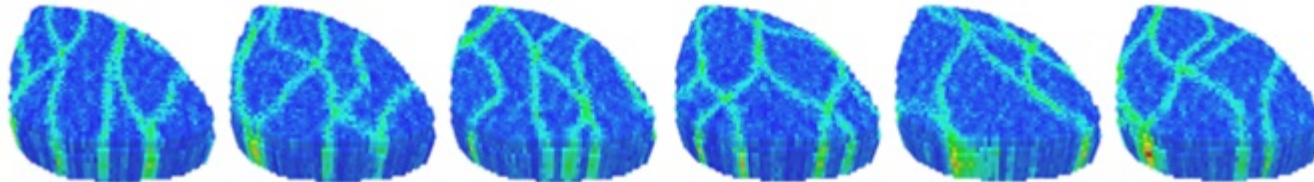
$$\mathbb{P}_{\Theta}^N [V(u_N^*) > \epsilon] \leq \beta$$

An a-priori probabilistic bound



Scenario-based WCO under geological uncertainty

- Non-linear relation between NPV and uncertainty (permeability field)
- Use wait-and-judge theory to derive a-posterior bound
- Simulation example:
 - An ensemble of standard egg-model realizations (N = 100 members)

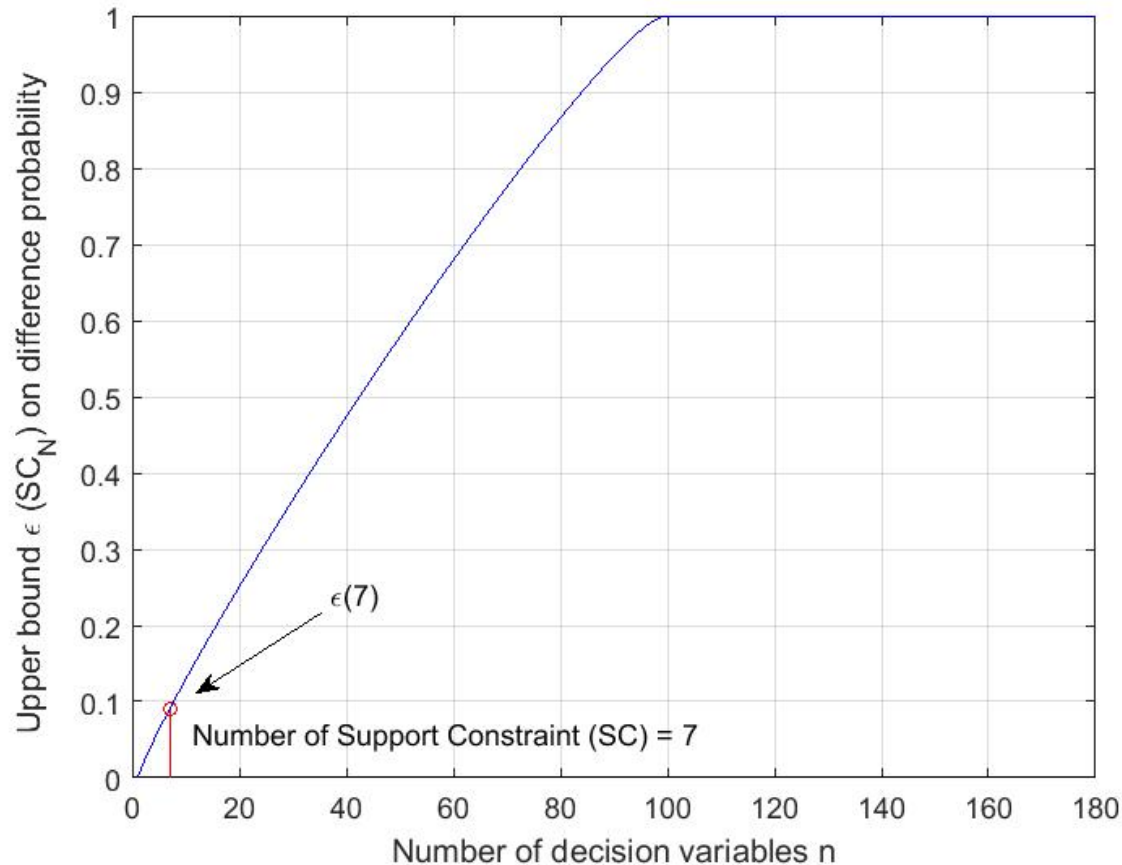


- Economic parameters fixed
 - Optimize injection rates
 - Worst-case robust optimization
- Wait (find the solution) and judge (observe the number of support constraints)

An a-posterior probabilistic bound

Scenario-based WCO under geological uncertainty

upper-bound ϵ as a function of SC and the confidence level $(1 - \beta) = 0.99\%$



Take-home office messages

- Scenario-based optimal control solutions do enjoy probabilistic guarantees.
- In case of economic uncertainty, probabilistic statements hold depending on the number of scenarios used (a-priori bounds).
- In case of geological uncertainty, probabilistic statements are derived based on the computed optimal solution (a-posterior bounds).
- Extendable to other constrained problems in the water-flooding optimization.



Optimization solver KNITRO

KNITRO:

A commercial solver for large-scale non-linear constraint optimization

Both interior-point (barrier) and active-set methods;

Programmatic interfaces: C/C++, Fortran, Java, Python;

Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;

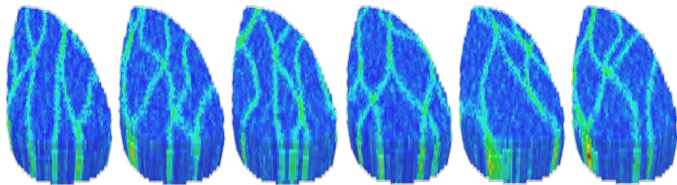
The screenshot displays the KNITRO website with the following content:

- Header:** ziena® OPTIMIZATION LLC Optimization Software, Modeling, and Consulting. EXPERTS IN NONLINEAR OPTIMIZATION.
- Navigation Menu:** PRODUCTS, SUPPORT, SERVICES, DISTRIBUTORS, COMPANY, SITE MAP.
- Left Sidebar:**
 - Home
 - PRODUCTS:
 - KNITRO® info
 - interfaces download documentation distributors presentations & case studies release history
 - AMPL® info
 - KNITRO-MATLAB® info
- Main Content:**
 - KNITRO Documentation**
 - USER MANUALS**
User Manuals are available for immediate download:
 - KNITRO 9.1 (PDF, HTML)
 - Ziena License Manager (PDF)
 - Older releases:
 - KNITRO 9.0 (PDF)
 - KNITRO 8.1 (PDF)
 - KNITRO 8.0 (PDF)
 - KNITRO 7.0 (PDF)
 - Discussion Forum** is at a Google group called "KNITRO". Anyone can view the discussions. To post a message you need to sign into Google, but Google accounts are free.
 - TECHNICAL REFERENCES**
 - The primary technical reference is:**
R. H. Byrd, J. Nocedal, and R. A. Waltz, "KNITRO: An Integrated Package for Nonlinear Optimization" in *Large-Scale Nonlinear Optimization*, G. di Pillo and M. Roma, eds, pp. 35-59 (2006), Springer-Verlag. The *KNITRO User Manual* also contains a comprehensive list of technical references.
 - The Interior Point algorithms are described in:**
R. Byrd, M. E. Hillstam, and J. Nocedal, "An Interior Point Method for Large Scale Nonlinear Programming" *SIAM J. Optimization*, 9,4, pp. 877-900 (1999).
 - Supporting theory is given in:**
R. Byrd, J.C. Gilbert, and J. Nocedal, "A Trust Region Method Based on Interior Point Techniques for Nonlinear Programming", *Mathematical Programming A*, 89: 149-185 (2000).
 - R. Byrd, Guanghui Liu, and J. Nocedal, "On the Local Behavior of an Interior Point Method for Nonlinear Programming" in *Numerical Analysis*, D. F. Griffiths and D. J. Higham, eds, pp. 37-56 (1997), Addison Wesley Longman.
- Right Sidebar:**
 - KNITRO User Manual**
Complete details for installing, using, and understanding Knitro.
 - Ziena License Manager User's Manual**
The manual for configuring and using Ziena licenses with KNITRO and AMPL.

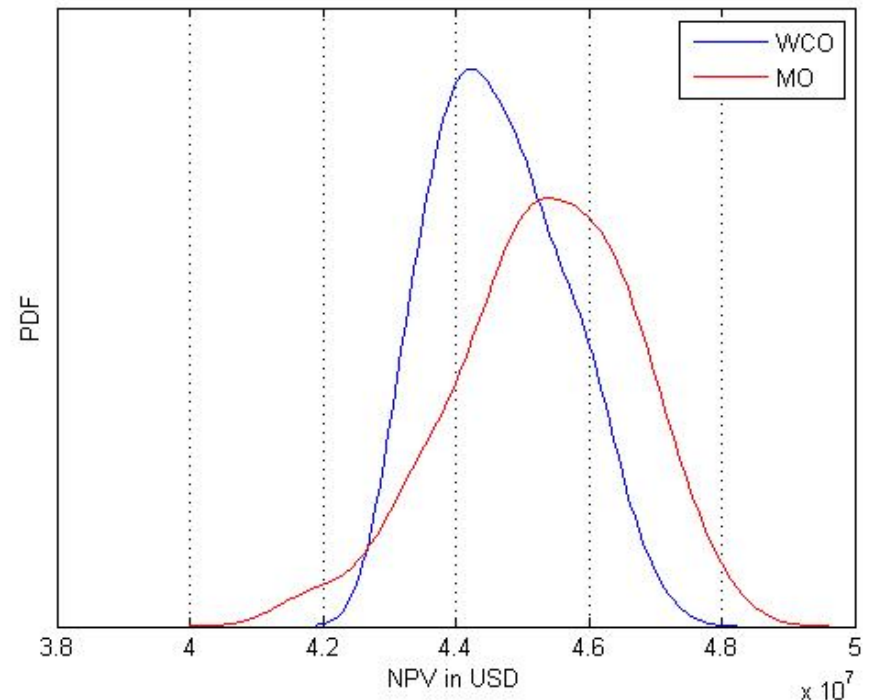
Worst-case robust optimization

Handling geological uncertainty

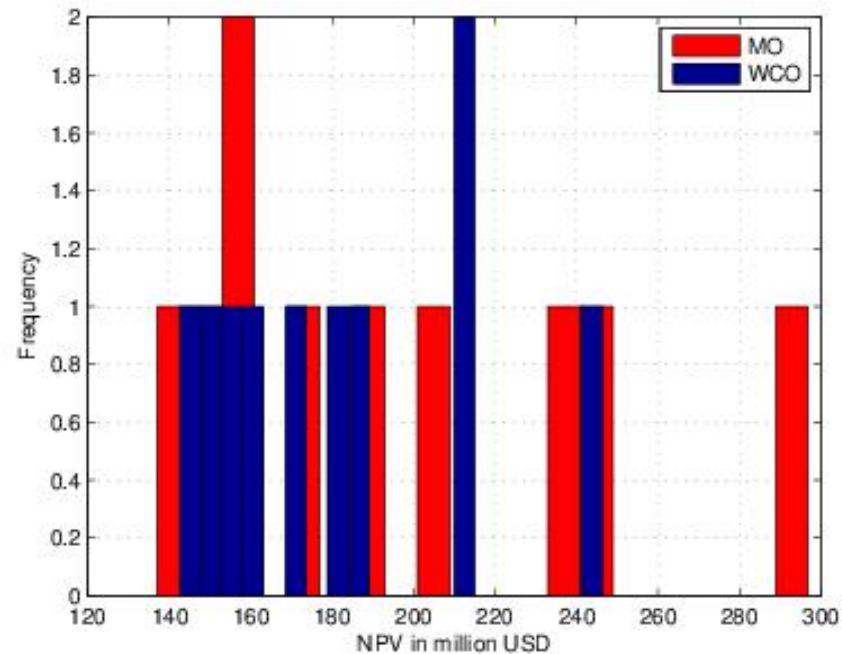
$$\begin{aligned} & \max_u z \\ \text{s.t.} \quad & z \leq J(u, \theta_i) \quad \forall i \end{aligned}$$



Worst-case increase: 3.60%
Average decrease: 1.54%



Worst-case robust optimization



Worst-case increase: 4.41%

Average decrease: 6.18%

Decision making under uncertainty

- A broad concept with ideas originating from various fields

