3rd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production
Esbjerg, Denmark, May 30 , 2018

Scenario-based robust optimization of water flooding in oil reservoirs enjoys probabilistic guarantees

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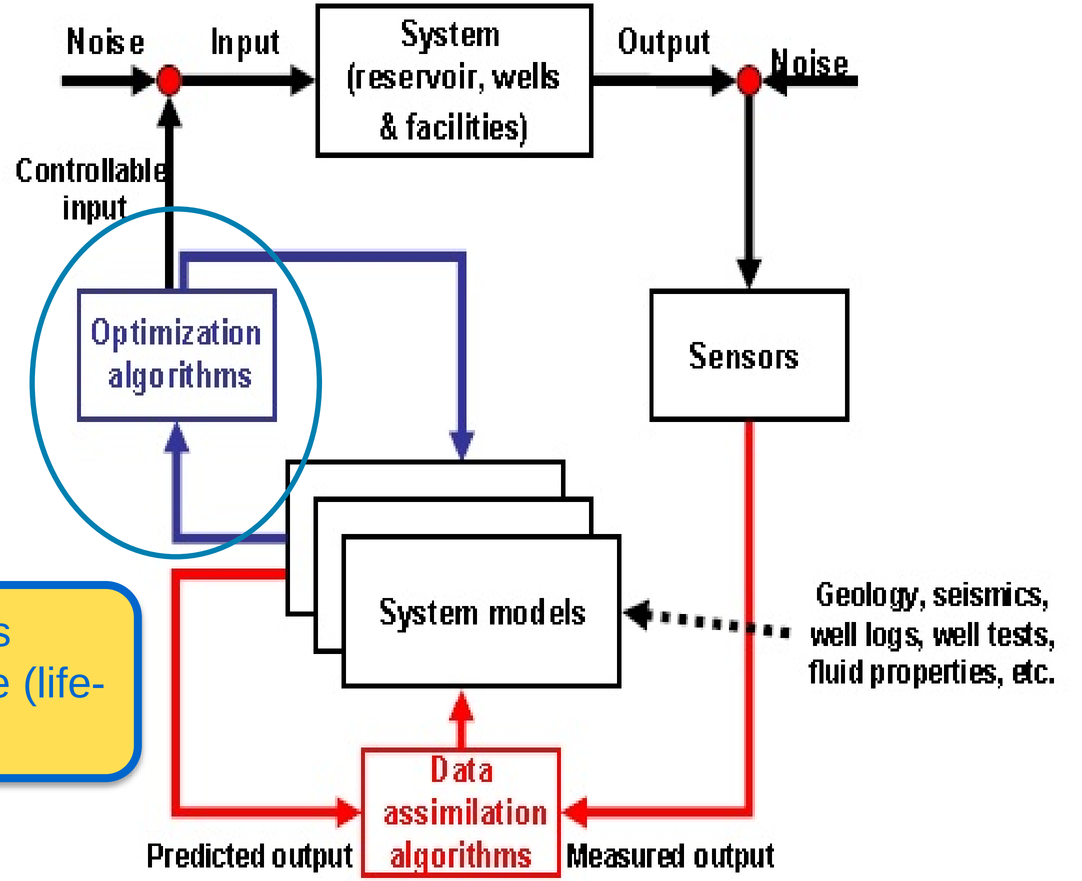
Where innovation starts

Closed-Loop Reservoir Management (CLRM)

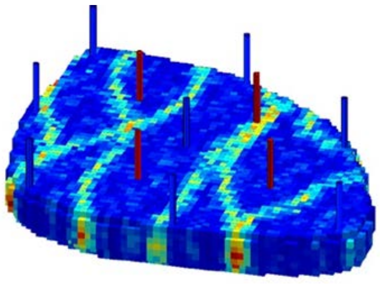


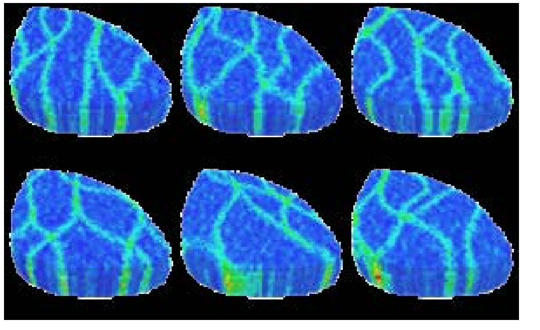
Strategy that optimizes economic performance (life-cycle)

Closed-loop reservoir management Jansen et al. (2005)

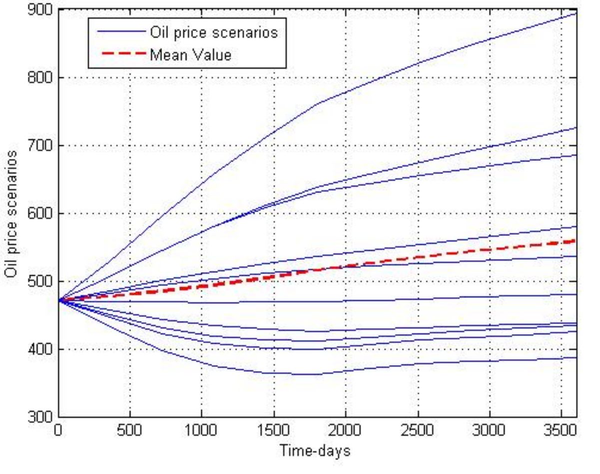
Challenges

- Large-scale, non-convex and non-linear optimization
- Uncertainty
  - Parametric uncertainty





  - Economic uncertainty
    - Varying oil prices



Decision making (model-based economic optimization) under geological parametric and economic uncertainty

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- Introduction
- Decision making under uncertainty (robust optimization)
- Worst-case optimization (WCO)
- Scenario-based worst-case optimization
  - Performance robustness probability
  - Support constraint
- Scenario-based WCO under Economic uncertainty
- Scenario-based WCO under Geological uncertainty

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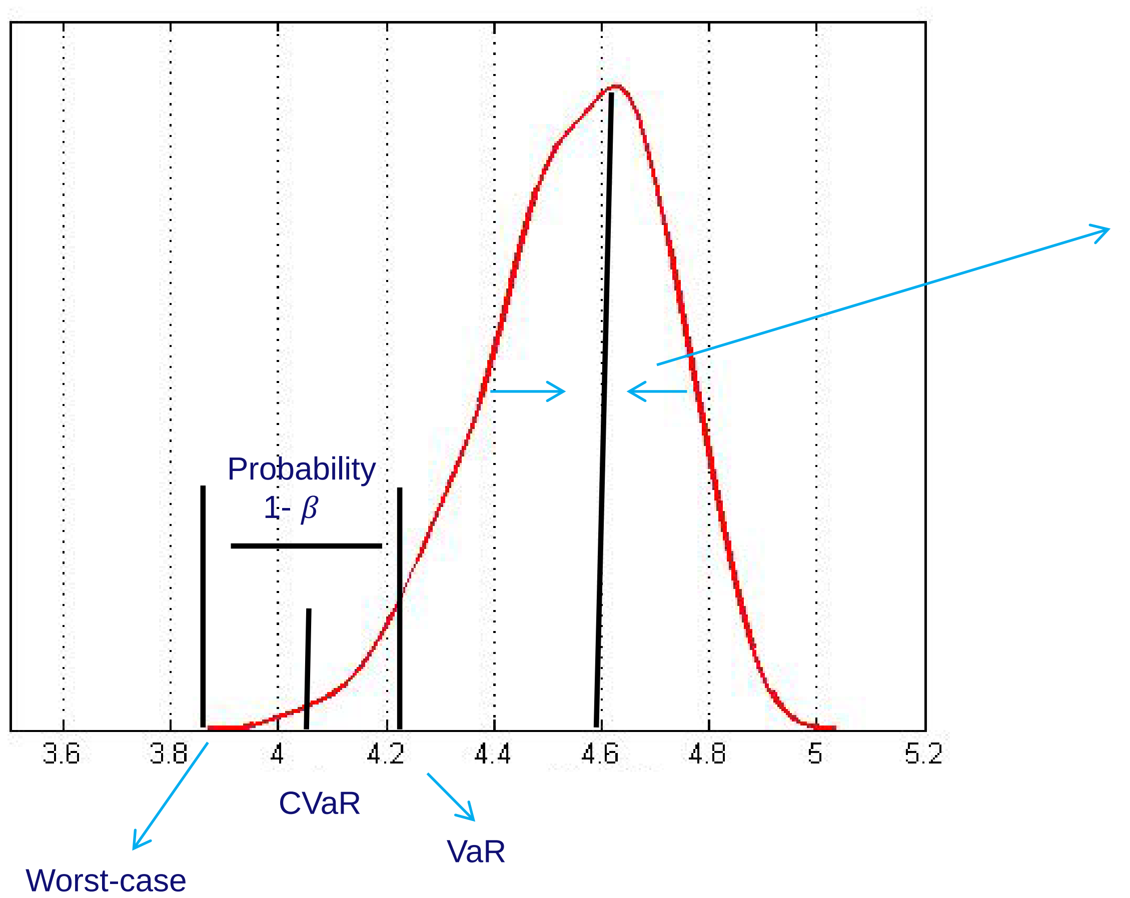
Decision making under uncertainty
robust optimization

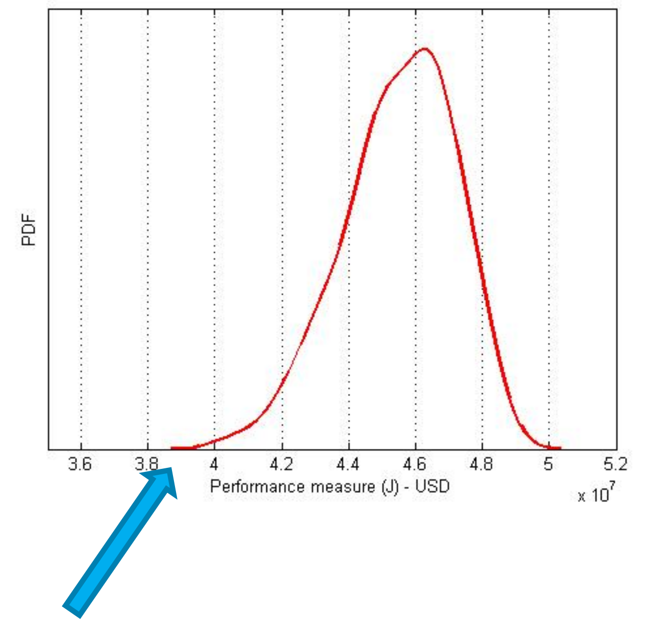
max (Mean - γ Variance)



Worst-case optimization (WCO)

Worst-case (robust) optimization:

$$\max_{\mathbf{u}\in\mathcal{U}} \min_{\theta\in\Theta} J(\mathbf{u},\theta)$$



Reformulation

$$\text{WCO}: \begin{cases} \max\limits_{\mathbf{u}\in\mathcal{U},z\in\mathbb{R}} & z \\ \text{s.t.} & z \leq \min\limits_{\theta\in\Theta} J(\mathbf{u},\theta) \end{cases}$$

$\theta \in \Theta$ is a representation of uncertainty space

Scenario-based worst-case optimization

- sample the uncertainty space
- a finite number of realizations of the uncertain parameters

$$\{\theta_1, \theta_2, \cdots, \theta_N\}$$

$$\max_{\mathbf{u}\in\mathcal{U}} \min_{\theta_i\in\Theta} J(\mathbf{u}, \theta_i) \quad i = 1, 2, \cdots, N$$

$$\mathrm{WCO}_N : \begin{cases} \displaystyle\max_{\mathbf{u}\in\mathcal{U}, z\in\mathbb{R}} & z \\ \text{s.t.} & z \leq \displaystyle\min_{i\in\{1,\ldots,N\}} J(\mathbf{u}, \theta_i) \end{cases}$$

Scenario-based worst-case optimization

Introduction

What can we claim about the achieved objective function value, when u_N^* is applied in the presence of the unseen scenarios for θ?

Given a number of samples N, can we quantify, e.g., in probabilistic terms, the robustness of the performance, i.e., the measure of the subset of Θ such that $\boldsymbol{J}\,(\boldsymbol{u}_N^*) \geq \max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \boldsymbol{J}(\boldsymbol{u}_N^*, \boldsymbol{\theta})$?

Scenario-based worst-case optimization
Performance robustness probability

Performance robustness probability

Definition: Let u_N^* be an optimal solution to WCO_N. The performance robustness probability of u_N^* in terms of NPV is defined as:

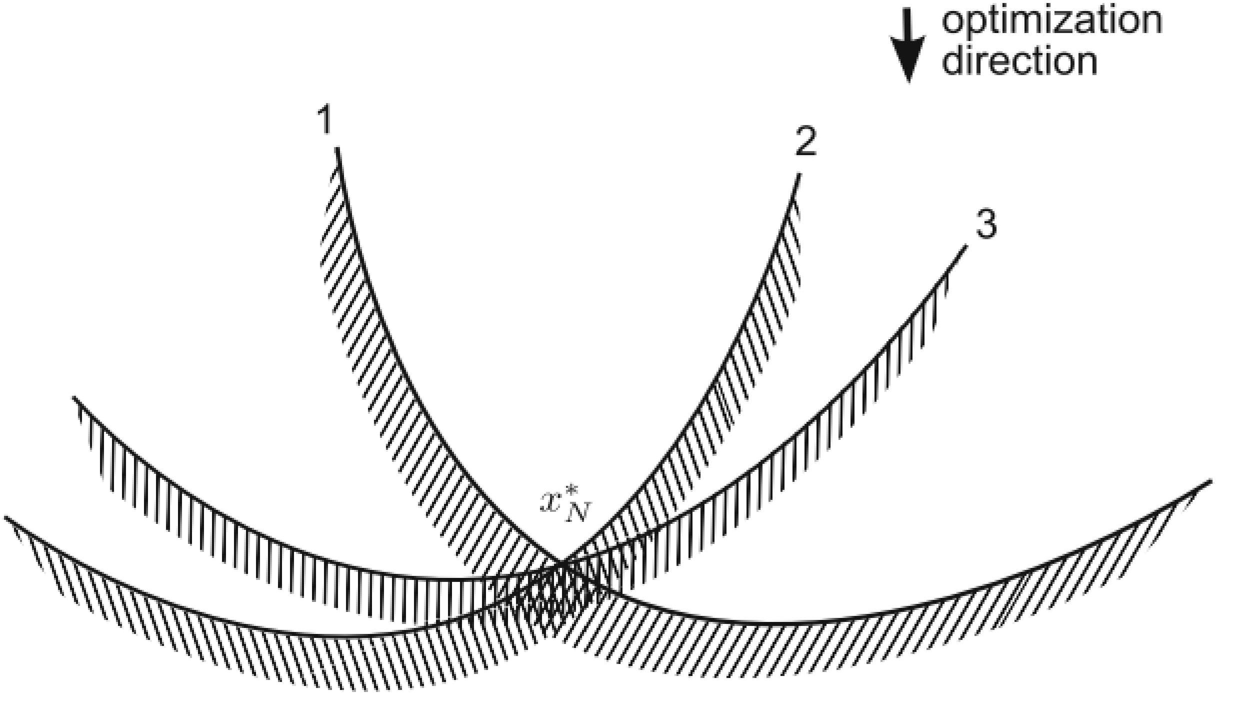
$$V(\mathbf{u}_N^*) := \mathbb{P}_\Theta[J(\mathbf{u}_N^*) \geq J(\mathbf{u}_N^*, \theta \in \Theta)].$$

ϵ – level robust solution: if $\epsilon \in (0,1)$, $V(\mathbf{u}_N^*) \leq \epsilon$

- A-priori guarantees (convex optimization)
- A-posterior guarantees (wait-and-judge for non-convex optimization)

Scenario-based worst-case optimization
Support constraint

A sampled constraint is a support constraint for WCO_N if its removal would alter the optimal solution of the problem.



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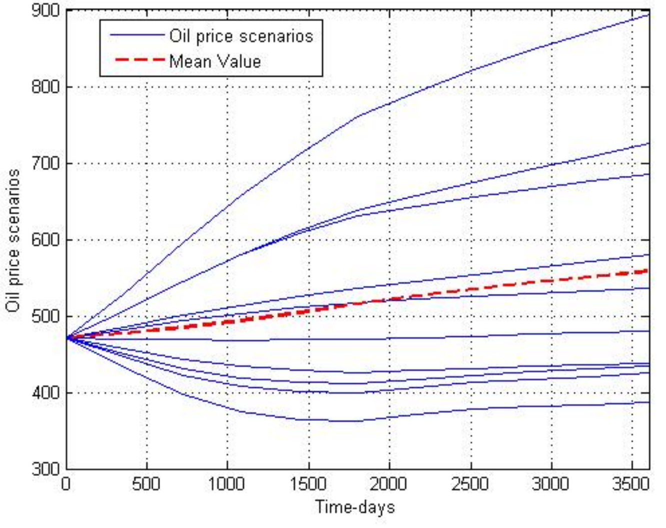
Scenario-based WCO under economic uncertainty

Oil price variation r_k^{oil} as source of uncertainty



$$J(\mathbf{u},\theta)=\sum_{k=1}^{K}\frac{\Delta t_k}{(1+b)^{t_k/\tau}}\left(r_k^{\text{oil}}q_k^{\text{oil}}-r_k^{\text{water}}q_k^{\text{water}}-r_k^{\text{inj}}q_k^{\text{inj}}\right)$$

r_k^{oil} appears linearly in $J(u,\theta)$

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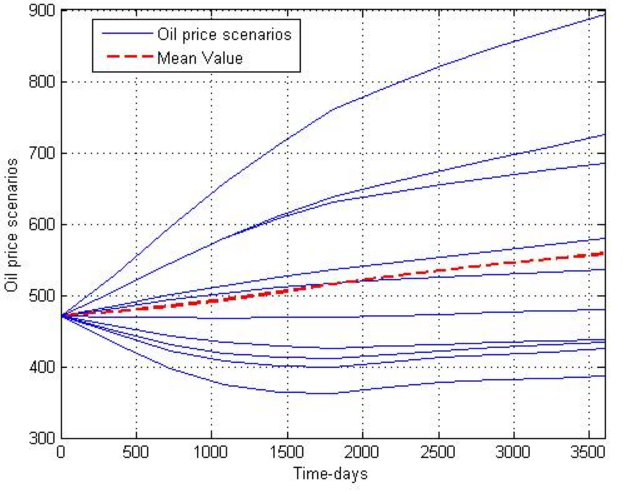
Scenario-based WCO under economic uncertainty

Main result: Suppose that, for all $\theta = r_k^{oil} \in \Theta$, the function $u \rightarrow J(u,\theta)$ is concave. Let u_N^* be the optimal solution to WCO_N in,

$$N \geq \frac{2}{\epsilon}\left(K + \ln\left(\frac{1}{\beta}\right)\right).$$

Then, it holds

$$\mathbb{P}_{\Theta}^{N}\left[V(u_N^*) > \epsilon\right] \leq \beta$$



An a-priori probabilistic bound

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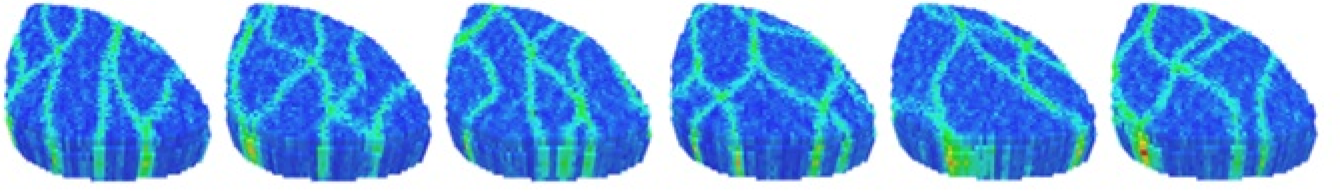
Scenario-based WCO under geological uncertainty

- Non-linear relation between NPV and uncertainty (permeability field)
- Use wait-and-judge theory to derive a-posterior bound

- Simulation example:
  - An ensemble of standard egg-model realizations (N = 100 members)



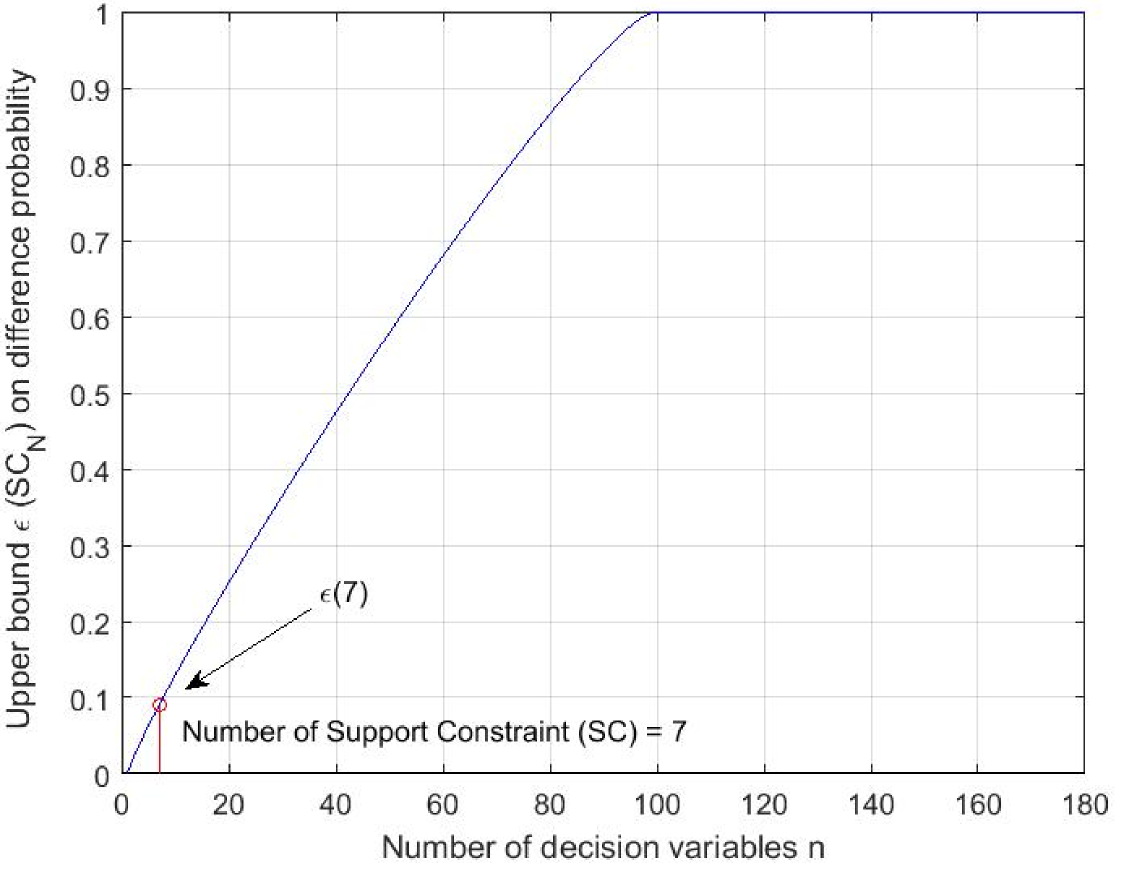
  - Economic parameters fixed
  - Optimize injection rates
  - Worst-case robust optimization

- Wait (find the solution) and judge (observe the number of support constraints)

An a-posterior probabilistic bound

Scenario-based WCO under geological uncertainty

upper-bound ϵ as a function of SC and the confidence level $(1 - \beta) = 0.99\%$



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Take-~~home~~ office messages

- Scenario-based optimal control solutions do enjoy probabilistic guarantees.
- In case of economic uncertainty, probabilistic statements hold depending on the number of scenarios used (a-priori bounds).
- In case of geological uncertainty, probabilistic statements are derived based on the computed optimal solution (a-posterior bounds).
- Extendable to other constrained problems in the water-flooding optimization.

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Optimization solver
KNITRO

KNITRO:

A commercial solver for large-scale non-linear constraint optimization

Both interior-point (barrier) and active-set methods;

Programmatic interfaces: C/C++, Fortran, Java, Python;

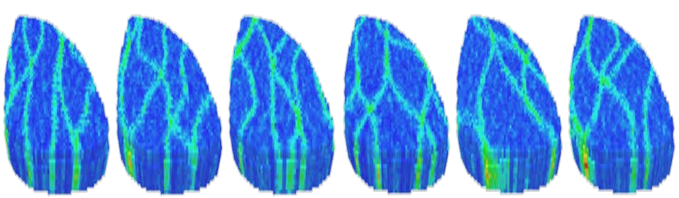
Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;

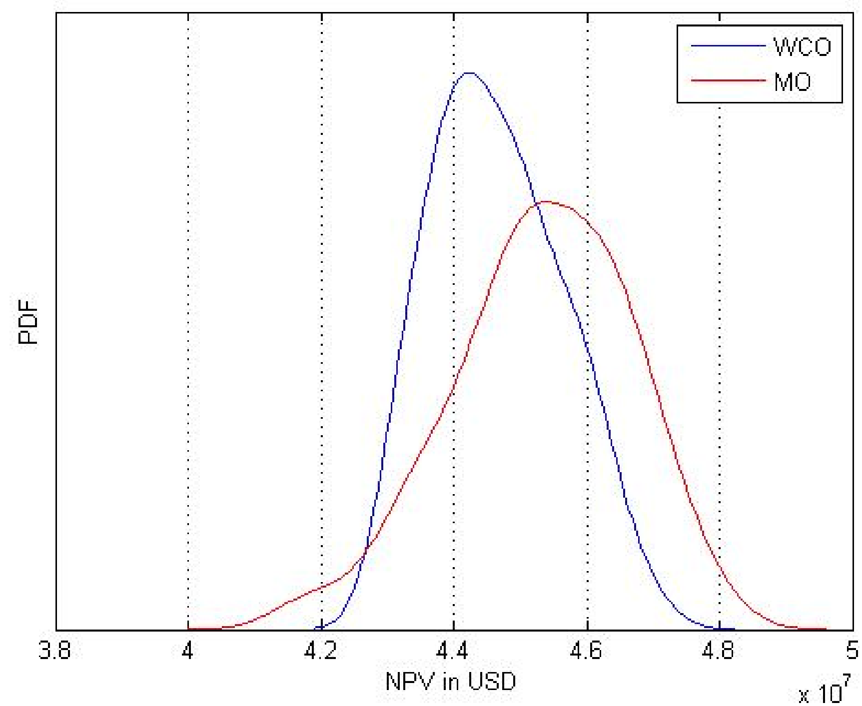
Worst-case robust optimization

Handling geological uncertainty

$$\max_{u} z$$

$$s.t \qquad z \leq J(u, \theta_i) \qquad \forall i$$

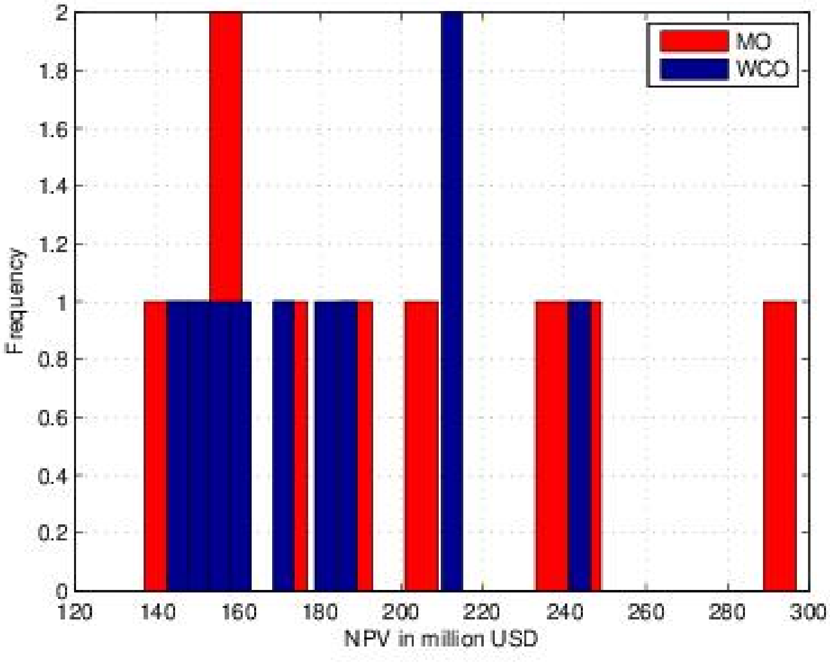




Worst-case increase: 3.60%

Average decrease: 1.54%

Worst-case robust optimization



Worst-case increase: 4.41%
Average decrease: 6.18%

Decision making under uncertainty

- A broad concept with ideas originating from various fields