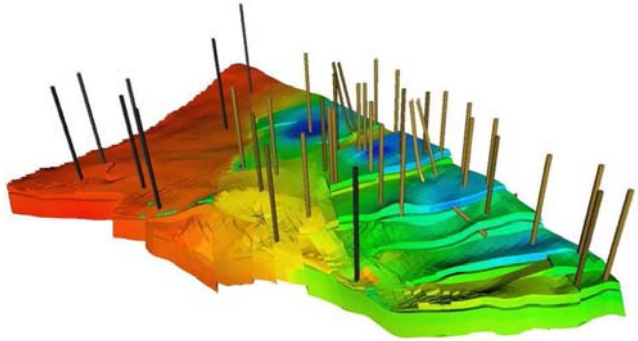

Model-Based Control and Optimization of Large Scale Physical Systems

Challenges in reservoir engineering

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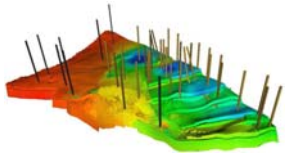
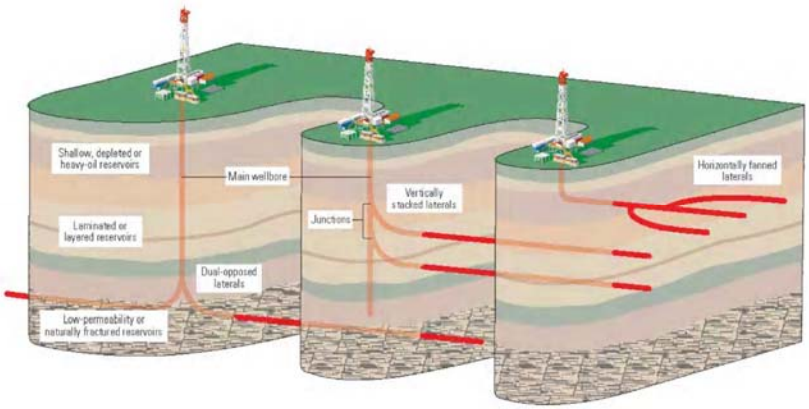
- Introduction – problems in reservoir engineering
- The models
- Model-based optimization – OL & CL
- Parameter estimation, identifiability, model structure approximation
- 2D Example
- Prospects and conclusions

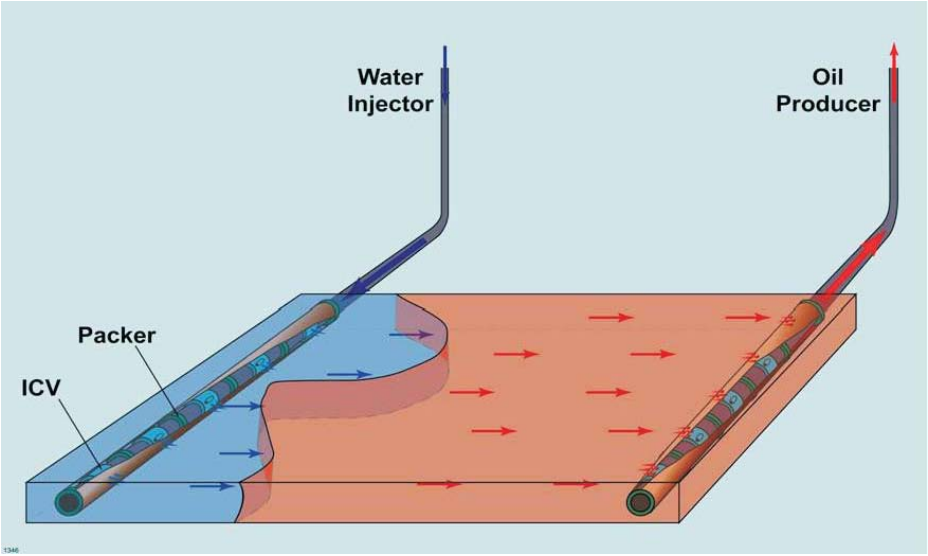
Introduction: reservoir engineering



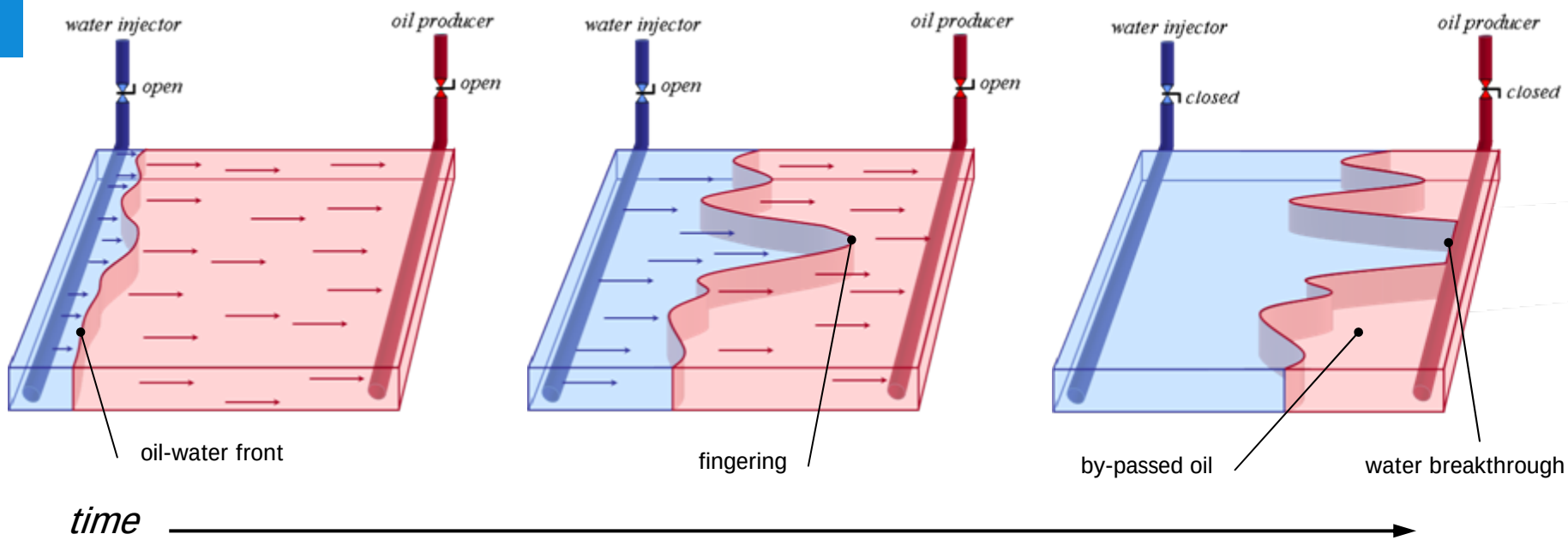
Water flooding

- Involves the injection of water through the use of injection wells
- Goal is to displace oil by water
- Production is terminated when (too much) water is being produced





Introduction: reservoir engineering

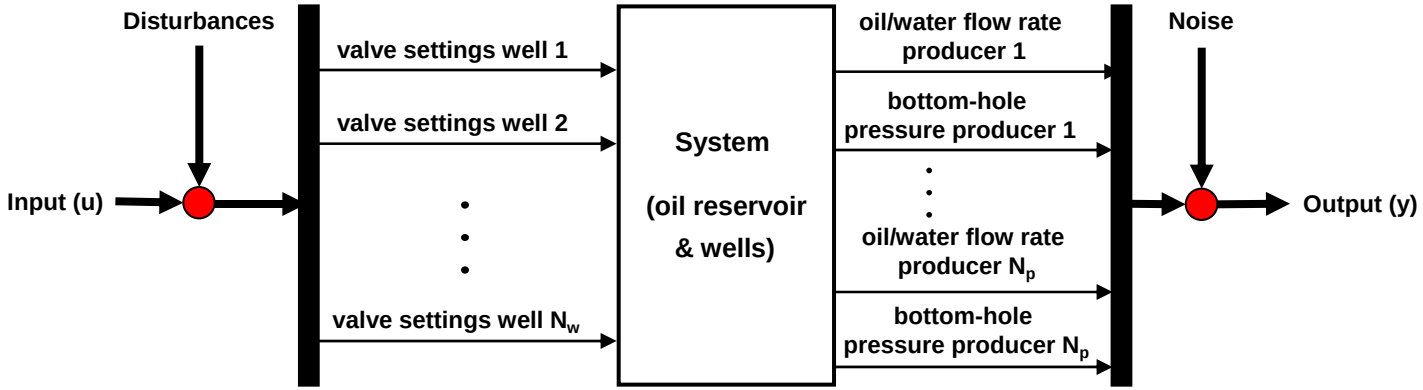


Oil production from a reservoir can last for periods of 15-25 years.

Introduction: reservoir engineering

System involves the reservoir, wells and sometimes surface facilities

- Inputs: control valve settings of the wells (injectors and producers)
  - Smart wells: multiple (subsurface) valves
- Outputs: (fractional) flow rates and/or bottomhole pressures
  - Smart wells: multiple (subsurface) measurement devices



Introduction: reservoir engineering

Objective

Optimize Net Present Value (NPV):

- Optimize economic revenues related to oil recovery, as a function of dynamic valve settings

$$J = \sum_{k=1}^{N} \frac{\Delta(t_k)[r_o q_{o,k} - r_w q_{w,k} - r_i q_{i,k}]}{(1+b)^{\frac{t_k}{\tau}}}$$

Under constraints: $c(x_k, u_k) \leq 0$

typically limits on water injection capacity, and max/min pressures in injection/production wells

- Rationalise the reservoir operational decisions

The Model

isothermal two-phase (oil-water) flow

Mass balance:

$$\nabla(\rho_i u_i) + \frac{\partial}{\partial t}(\phi \rho_i S_i) = 0 \qquad i = \{o, w\}$$

Momentum (Darcy's law):

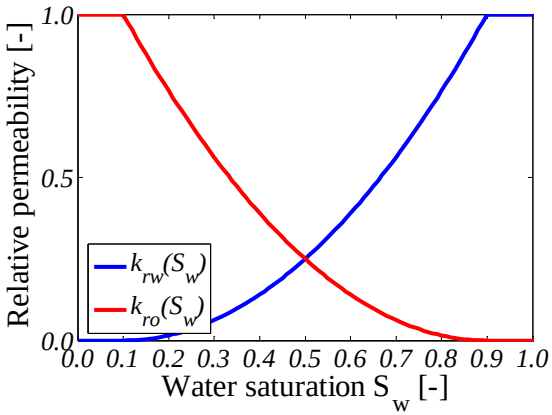
$$u_i = -k \frac{k_{ri}}{\mu_i} \nabla p_i \qquad i = \{o, w\}$$

Variables: p_o, p_w, S_o, S_w

Saturations satisfy: $S_o + S_w = 1$

Simplifying assumptions, a.o.: $p_o = p_w$

nonlinearity



Discretization in space and time

State space model:

$$\begin{aligned} V(x_t)\dot{x}_t &= T(x_t)x_t + q_t; \qquad x_0 \\ y_t &= h(x_t) \end{aligned}$$

$$\begin{aligned} y^T &= [p_{well}^T \; q_{well,o}^T \; q_{well,w}^T] \\ x^T &= [p_o^T \; S_w^T] \end{aligned}$$

After discretization in space (and time):

$$g(x_{k+1}, x_k, u_k, \theta) = 0 \qquad dim(x) \approx 10^4 - 10^6$$

$$y_k = h(x_k)$$

and θ typically the permeabilities in each grid block

Characteristic of process

- Nonlinear batch process (one-go), of which the dynamics is essentially dependent on the location of the –moving- oil/water front

Model-based Optimization

Optimization problem:

$$\max_q J(q) = \max_q \sum_{k=1}^{N} L(x_k, q_k)$$

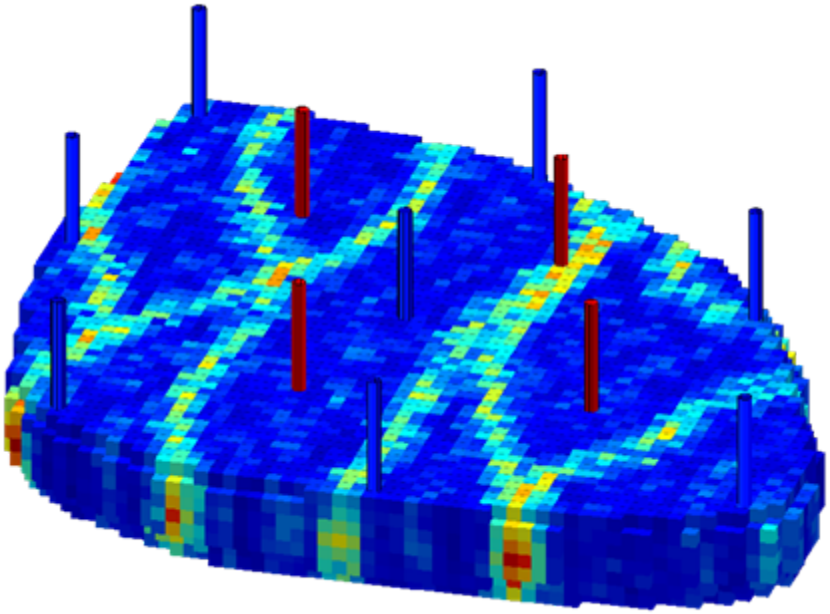
such that: $g(x_{k+1}, x_k, q_{i,k}) = 0, \quad x_0 = x(0)$

$$q_{min} \leq q_k \leq q_{max}$$

$$q_{o,k} + q_{w,k} = q_{i,k}$$

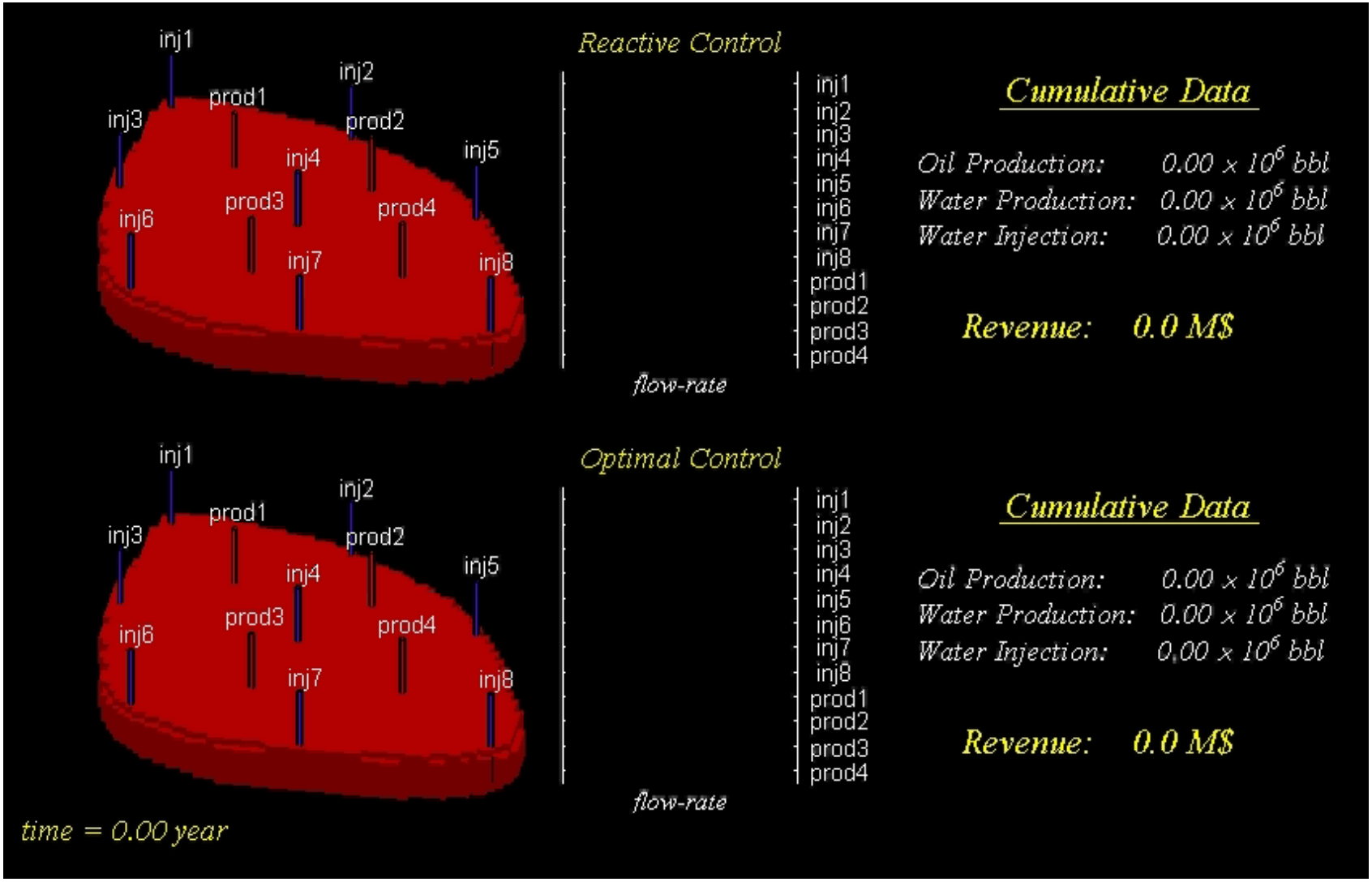
Non-convex optimization, solved by gradient-based method:
Adjoint-variables calculation through backward integration of the related (Hamiltonian based adjoint) equation.
(feasible for systems of this size)

12-well example



(Gijs van Essen et al., CAA 2006)

- 3D reservoir
- 8 injection / 4 production wells
- Period of 10 years
- High-permeability channels
- 18.553 grid blocks
- Minimum rate of 0.1 *stb/d*
- Maximum rate of 400 *stb/d*
- No discount factor
- r_o = 20 *$/stb*, r_w = 3 *$/stb* and r_i = 1 *$/stb*
- Optimization of economic benefit


Reactive Control
inj1
inj2
inj3
inj4
inj5
inj6
inj7
inj8
prod1
prod2
prod3
prod4
flow-rate
Cumulative Data
Oil Production: 0.00 x 10^6 bbl
Water Production: 0.00 x 10^6 bbl
Water Injection: 0.00 x 10^6 bbl
Revenue: 0.0 M$
Optimal Control
inj1
inj2
inj3
inj4
inj5
inj6
inj7
inj8
prod1
prod2
prod3
prod4
flow-rate
Cumulative Data
Oil Production: 0.00 x 10^6 bbl
Water Production: 0.00 x 10^6 bbl
Water Injection: 0.00 x 10^6 bbl
Revenue: 0.0 M$
time = 0.00 year

Why this wouldn't work

- Model has some simplifying assumptions
- Optimization over life-time reservoir (changing economic circumstances)
- .....

- Open-loop strategy
- We do not know the reservoir model!

Closed-loop Reservoir Management

- Moving from (batch-wise) open-loop optimization to on-line closed-loop control
- However we need a model as a basis for e.g. a receding/shrinking horizon strategy

Obtaining a model

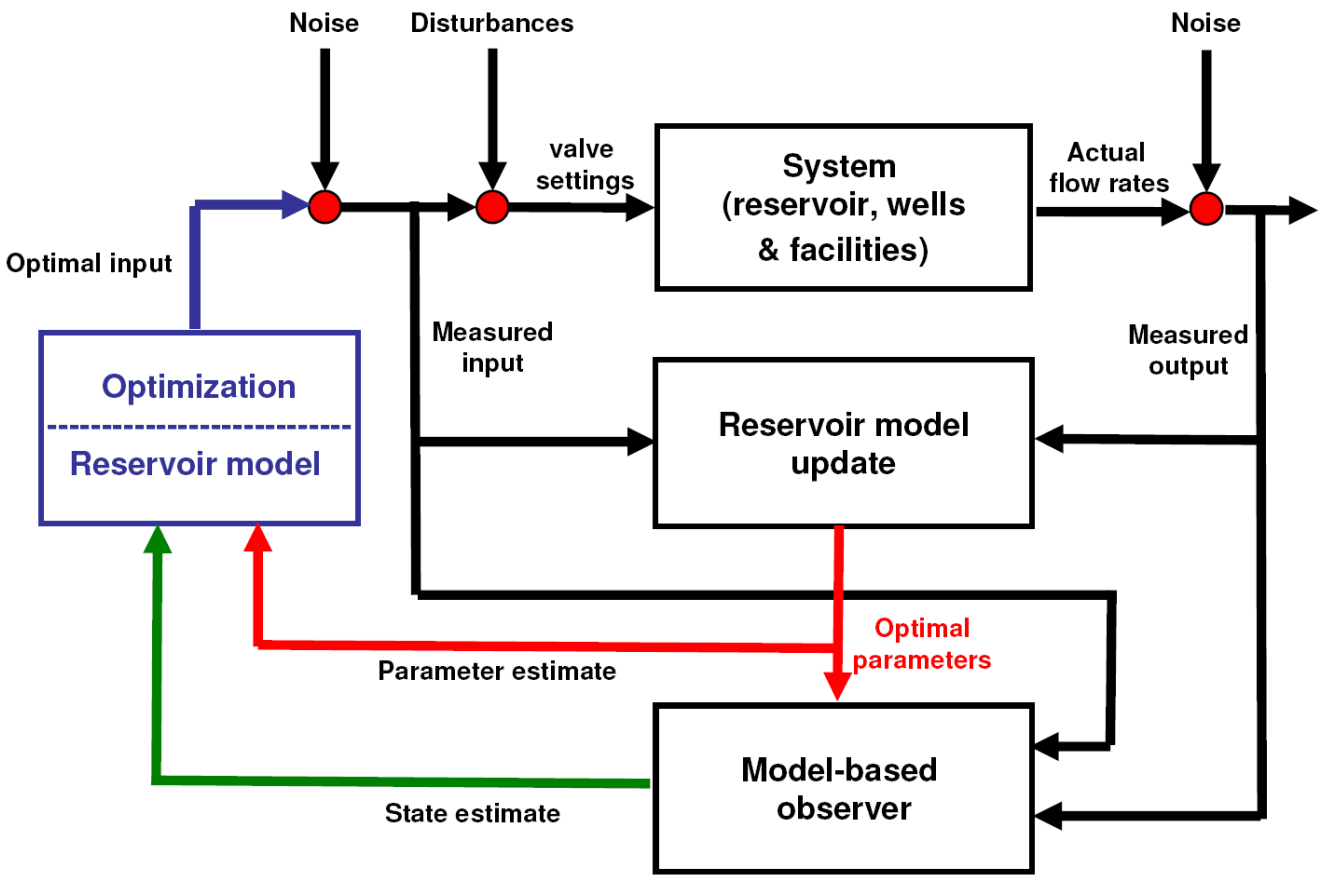
- First-principle models (geology) are very much uncertain
- Opportunities for identification are limited (nonlinear behaviour dependent on front-location, single batch process, experimental limitations)
- Option: estimate physical parameters (permeabilities) in first principles model; starting with initial guess

Closed-loop Reservoir Management

Receding/shrinking horizon control strategy:

- Use a state-estimator to reconstruct the current state
- Run the optimization algorithm to evaluate future scenario's
- Implement the optimized valve settings until the next state update
- This is actually a NMPC in a shrinking horizon implementation
- However no trajectory following but trajectory finding, i.e. real-time dynamic optimization (RTO)

Closed-loop Reservoir Management



Closed-loop Reservoir Management

Several options for nonlinear state and parameter estimation:

Available from oceanographic domain:

Ensemble Kalman filter (EnKF) (Evensen, 2006)

- Kalman type estimator, with analytical error propagation replaced by Monte Carlo approach
  (error cov. matrix determined by processing ensemble of model realizations)
- Ability to handle model uncertainty (in some sense)
- In reservoir engineering used for estimation of states **and parameters** (history matching)

Ensemble Kalman Filter

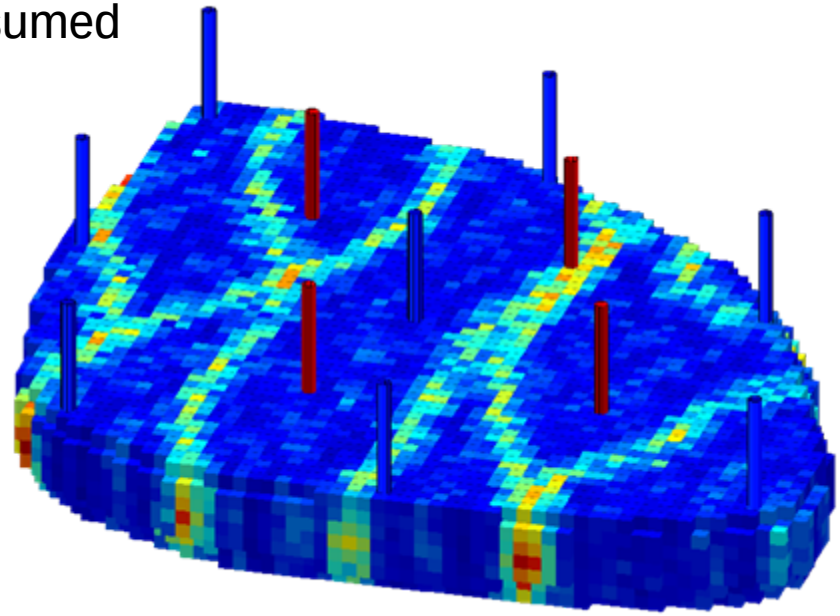
- As prior information an ensemble of initial states $\{\hat{x}_{k|k}\}$ is generated from a given distribution
- By simulating every ensemble member, corresponding ensembles $\{\hat{x}_{k+1|k}\}$ and $\{\hat{y}_{k+1|k}\}$ are generated, and stored as columns of matrices $\hat{X}$ and $\hat{Y}$ respectively
- The measurement update of a EKF is applied to every element of the ensemble, where the covariance matrices are replaced by sampled estimates on the basis of $\hat{X}$ and $\hat{Y}$.
- The update becomes: $\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}[y_{k+1} - \hat{y}_{k+1|k}]$, where K_{k+1} is given by:

$$K_{k+1} = \hat{X}\hat{Y}^T \cdot [\hat{Y}\hat{Y}^T + R]^{-1} \quad \text{(BLUE)}$$

- The result is a new ensemble $\{\hat{x}_{k+1|k+1}\}$

Closed-loop simulation example

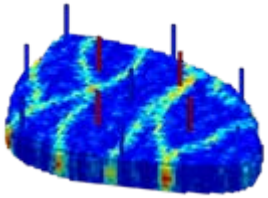
- Model with high-perm channels assumed to be 'reality'
- Permeabilities are unknown in closed-loop control
- Period of 8 years
- Objective function: NPV
  - r_o = 10 *$/stb*, r_w = 1 *$/stb* and r_i = 0 *$/stb*
  - Annual discount factor: 15%
- Measurements
  - Fractional flow rates (oil/water)
  - Bottom-hole pressures
- Yearly updates of parameters and control strategy

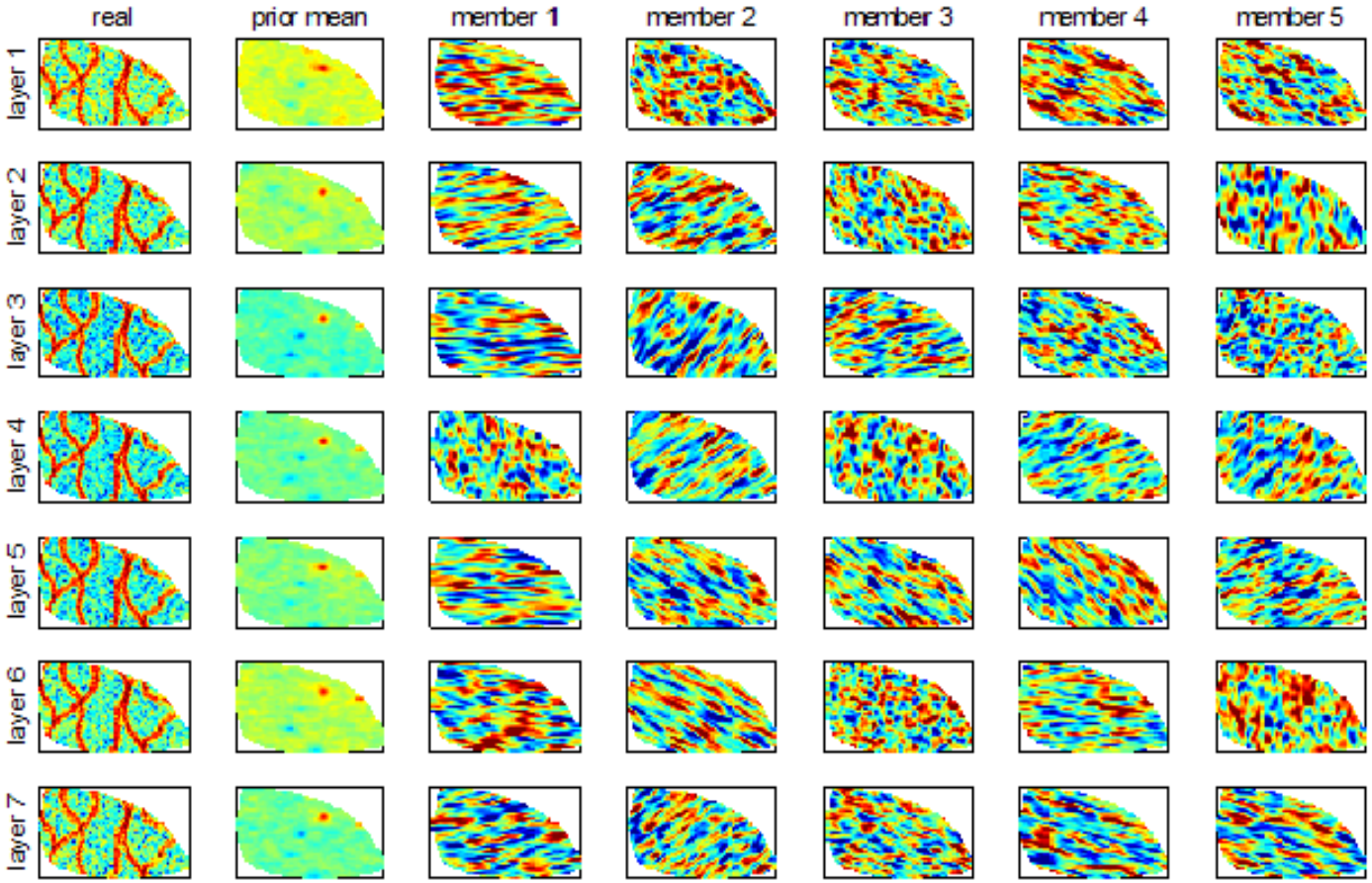


Gijs van Essen, 2006

Closed-loop simulation example

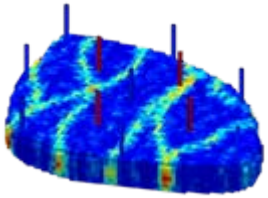
Initial ensemble

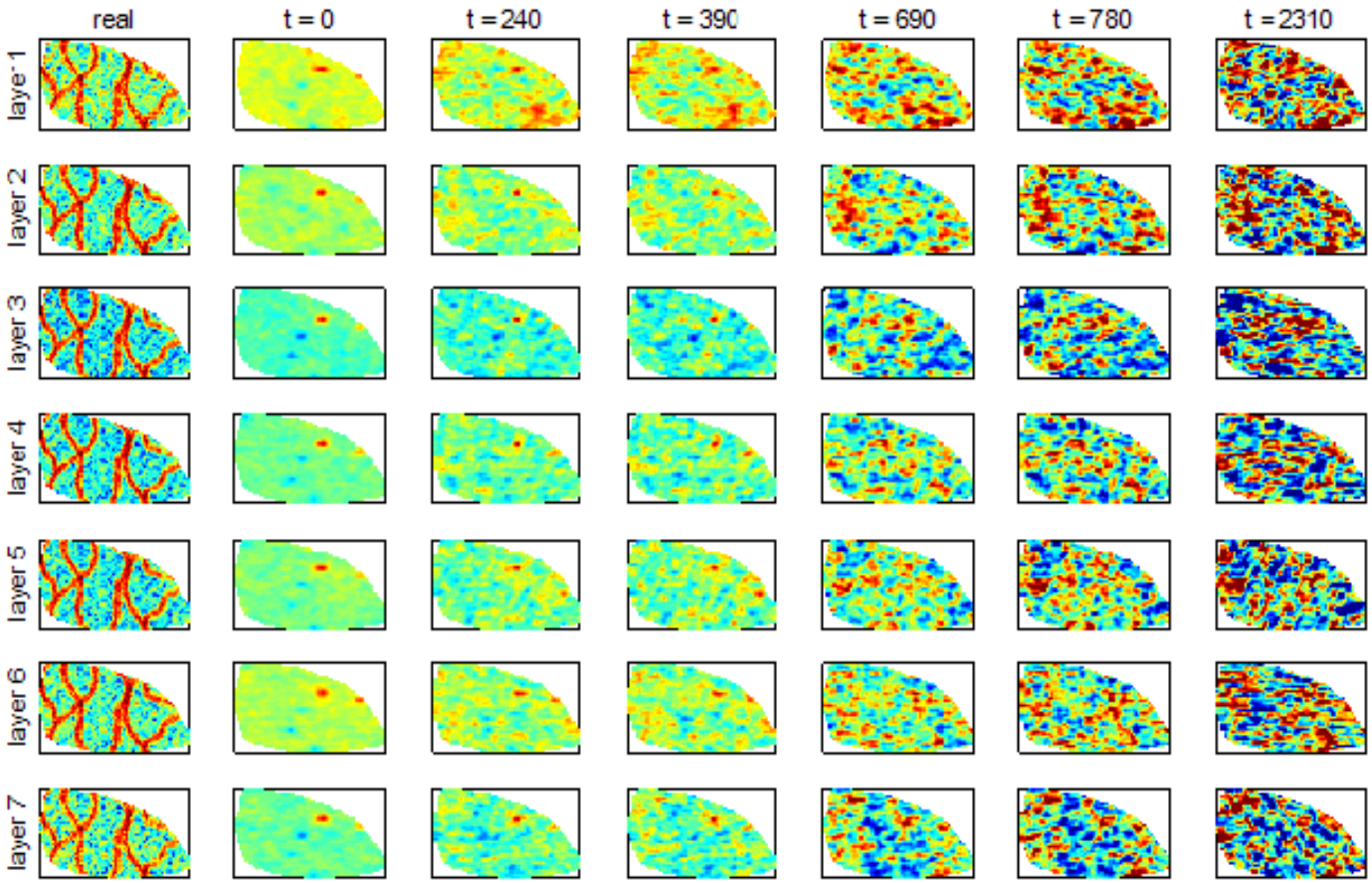




Closed-loop simulation example

Ensemble updates at different times

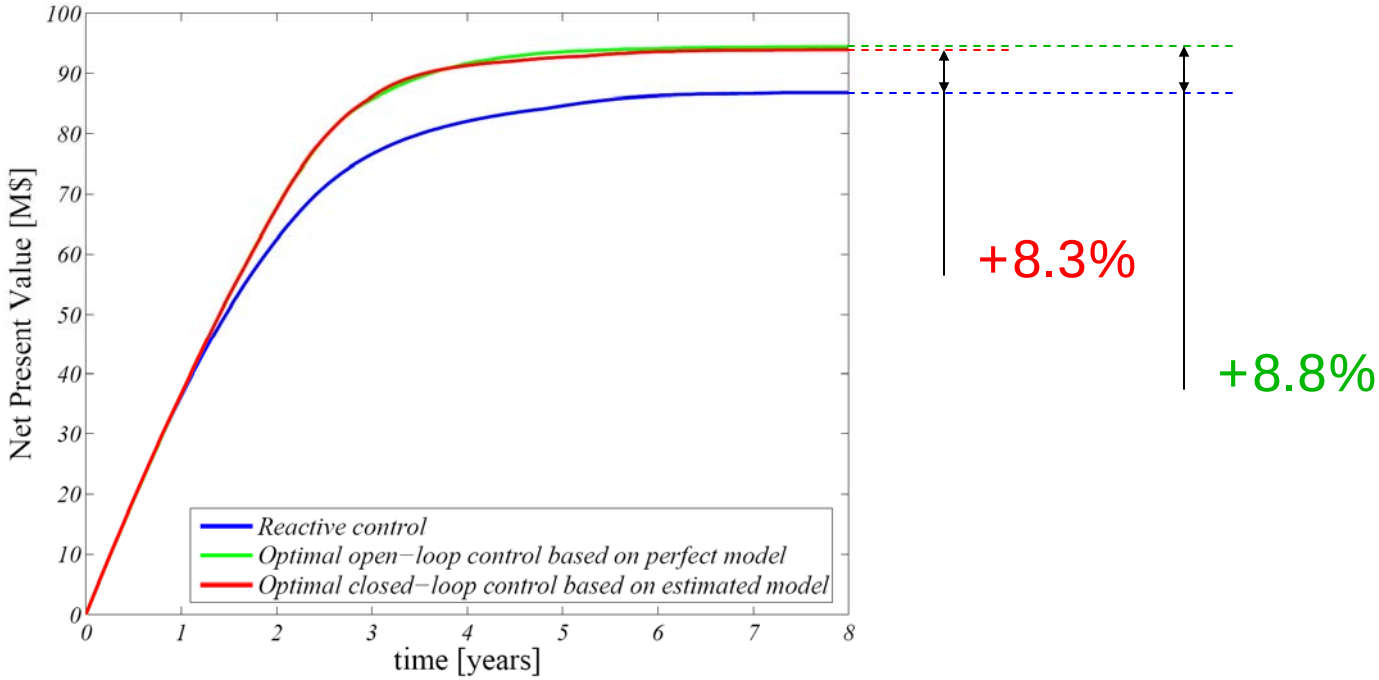




Closed-loop simulation example

Results

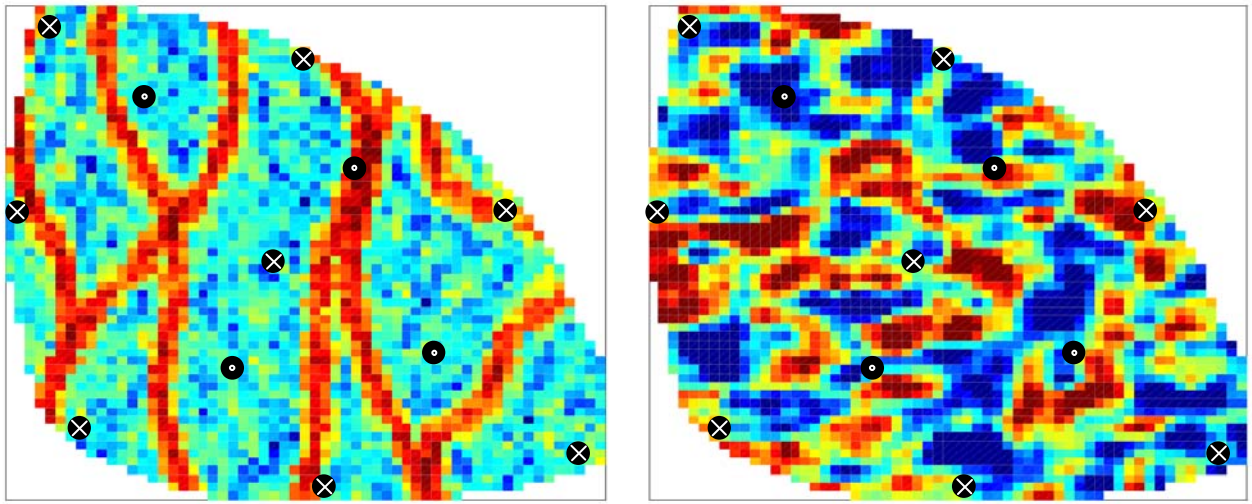
- 3 study cases: reactive control, optimal open-loop control based on perfect ('reality') model, optimal closed-loop control



Closed-loop reservoir management

Questions:

- Why are such poor models working so well?
- Does this mean that we don't need geology?



Reservoir dynamics live in low-order space

- Observation and control in the wells
  - Models will typically be poorly observable and/or poorly controllable
  - Real (local) input-output dynamics is of limited order

- Parameter estimation:
  - Physical parameters (permeabilities) determine predictive quality but one parameter per grid block leads to excessive over-parametrization (not to be validated)

- How to estimate the parameters in this model?
    - local dynamics versus global physics
    - non-linear batch process
    - extrapolate beyond what you have seen so far

Identifiability

- Consider nonlinear model structure $\hat{\mathbf{y}} = \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0)$ with $\hat{\mathbf{y}}$ being a prediction of $\mathbf{y} := [y_1^T \cdots y_N^T]^T$

- Locally identifiable in $\boldsymbol{\theta}_m$ for given $\mathbf{u}$ and $\mathbf{x}_0$ if in neighbourhood of $\boldsymbol{\theta}_m$:

$$\{\mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_1; \mathbf{x}_0) = \mathbf{h}(\mathbf{u}, \boldsymbol{\theta}_2; \mathbf{x}_0)\} \Rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$$

  [Grewal and Glover 1976]

- Global properties are generally hard to analyze

Identifiability

- Notion of *identifiability* is instrumental in analyzing model structure properties
- It determines whether it is feasible at all to relate unique values to the physical parameter variables, on the basis of measured data

Testing local identifiability in identification

- In Prediction Error framework, identification criterion

$$V(\boldsymbol{\theta}) := \frac{1}{2}\boldsymbol{\epsilon}(\boldsymbol{\theta})^T \mathbf{P}_v^{-1} \boldsymbol{\epsilon}(\boldsymbol{\theta}), \quad \boldsymbol{\epsilon}(\boldsymbol{\theta}) = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{h}(\boldsymbol{\theta}, \mathbf{u}; \mathbf{x}_0),$$

- Hessian given by

$$\frac{\partial^2 V(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} = \frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T + \mathbf{S}$$

- Local identifiability test in $\hat{\boldsymbol{\theta}} = \arg\min V(\boldsymbol{\theta})$: Hessian > 0

- With quadratic approximation of cost function around $\hat{\boldsymbol{\theta}}$:
  Hessian given by $\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left(\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \right)^T$

Testing local identifiability in identification

- Rank test on Hessian through SVD

$$\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}} \Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

- If $\boldsymbol{\Sigma}_2 = 0$ then lack of local identifiability

- SVD can be used to reparameterize the model structure through

$$\boldsymbol{\theta} = \mathbf{U}_1 \boldsymbol{\rho}, \quad \dim(\boldsymbol{\rho}) << \dim(\underline{\theta})$$

  in order to achieve local identifiability in $\boldsymbol{\rho}$

- Columns of $\mathbf{U}_1$ are basis functions of the identifiable parameter space

Testing local identifiability in identification

$$\frac{\partial \hat{\mathbf{y}}^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-\frac{1}{2}} \Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix}$$

- What if $\boldsymbol{\Sigma}_2 \neq 0$ but contains (many) small singular values ?

  No lack of identifiability, but possibly very poor variance properties

- Identifiability mostly considered in a yes/no setting: qualitative rather than quantitative [Bellman and Åström (1970), Grewal and Glover (1976)]

- Approach: *quantitative* analysis of appropriate parameter space, maintaining physical parameter interpretation

Model structure approximation

- How to reduce the model structure in terms of its ***parameter space?***
  (different from "classical" model reduction, in which the model dynamics of a single model is reduced)

- **Objective:** obtain a physical parametrization (model structure) in which the parameters can be reliably estimated from data.

Approximating the identifiable parameter space

Asymptotic variance analysis: $\mathrm{cov}(\hat{\boldsymbol{\theta}}) = J^{-1} = \left(\mathbb{E}\left[\left.\frac{\partial^2 V(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2}\right|_{\hat{\boldsymbol{\theta}}}\right]\right)^{-1}$

with J = Fisher Information Matrix.

- Sample estimate of parameter variance, on the basis of $V(\boldsymbol{\theta})$:

$$cov(\hat{\boldsymbol{\theta}}) = \begin{cases} \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_1^{-2} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2^{-2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} & \text{for } \boldsymbol{\Sigma}_2 > 0 \\ \infty & \text{for } \boldsymbol{\Sigma}_2 = 0 \end{cases}$$

$$cov(\boldsymbol{U_1}\hat{\boldsymbol{\rho}}) = \mathbf{U_1}\boldsymbol{\Sigma}_1^{-2}\mathbf{U_1}^T$$

$$cov(\hat{\boldsymbol{\theta}}) > cov(\boldsymbol{U_1}\hat{\boldsymbol{\rho}}) \qquad \text{if } \boldsymbol{\Sigma}_2 > 0$$

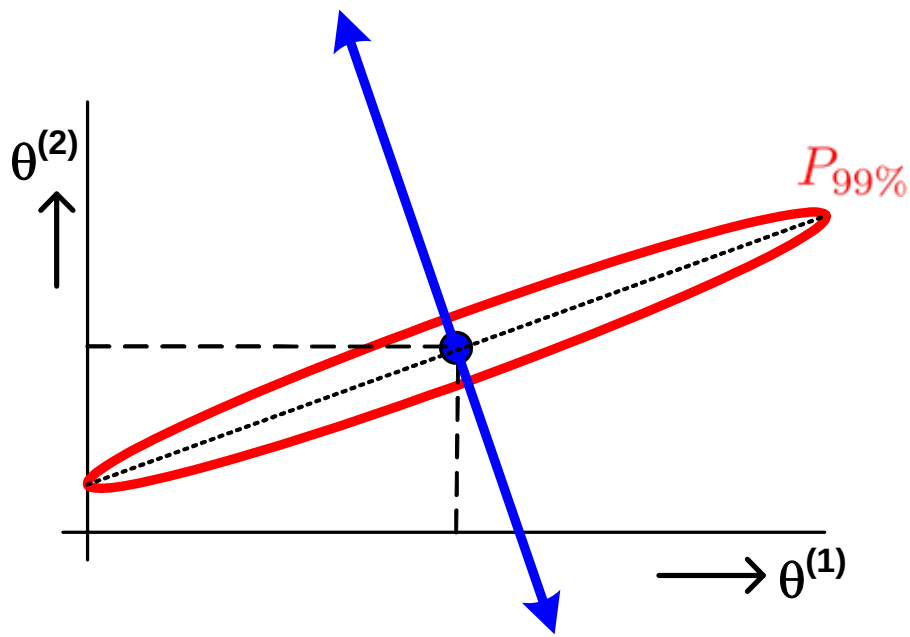
Approximating the identifiable parameter space

$$cov(\hat{\boldsymbol{\theta}}) > cov(\boldsymbol{U_1}\hat{\boldsymbol{\rho}}) \qquad \text{if } \boldsymbol{\Sigma}_2 > 0$$

- Discarding singular values that are small, reduces the variance of the resulting parameter estimate
- Particularly important in situations of (very) large numbers of small s.v.'s
- Model structure approximation (local)
- Quantified notion of identifiability – related to parameter variance

Approximating the identifiable parameter space

- Interpretation:
  Remove the parameter directions that are poorly identifiable (have large variance)



- This is different from removing the (separate) parameters for which the value 0 lies within the confidence bound [Hjalmarsson, 2005]

A Bayesian approach

- Often applied method for dealing with overdetermination in parameter space:
- Incorporate prior knowledge term (regularization) in cost function

$$V_p(\boldsymbol{\theta}) := V(\boldsymbol{\theta}) + \frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}_p)\mathbf{P}_{\boldsymbol{\theta}_p}^{-1}(\boldsymbol{\theta} - \boldsymbol{\theta}_p)$$

where $\boldsymbol{\theta}_p$ is the prior parameter vector (with covariance $\mathbf{P}_{\boldsymbol{\theta}_p}$).

- Model output approximated with first-order Taylor expansion. Hessian is

$$\frac{\partial^2 V_p(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} = \frac{\partial \mathbf{h}(\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}} \mathbf{P}_v^{-1} \left(\frac{\partial \mathbf{h}(\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}} \right)^T + \mathbf{P}_{\boldsymbol{\theta}_p}^{-1}$$

- “Always” identifiable, since $\mathbf{P}_{\boldsymbol{\theta}_p}$ full rank by construction!!

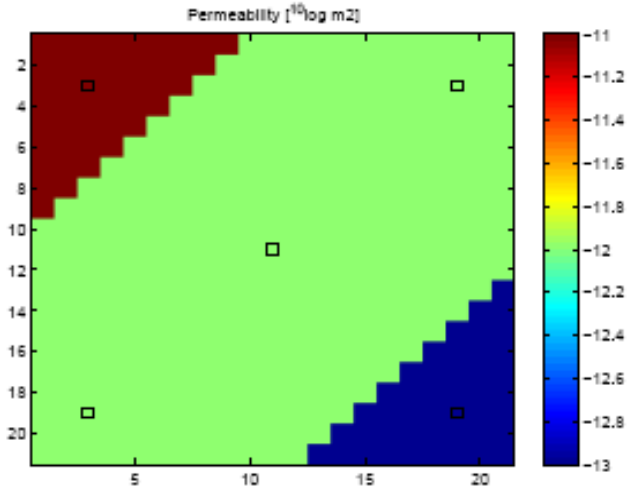
A Bayesian approach

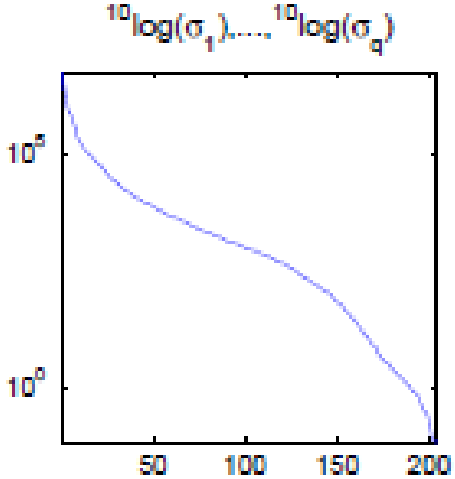
Implications

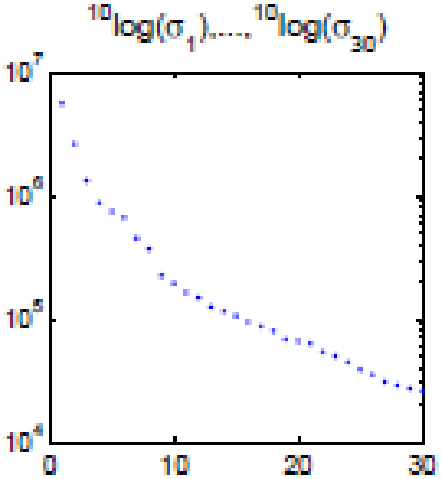
- Bayesian methods seem not to suffer from identifiability problems.....
- This includes all (extended) Kalman filter type algorithms. Where parameters are recursively estimated by augmenting the states
- Unique parameter estimates usually result, but
- In the parameter subspace that is poorly identifiable, estimated parameters will be heavily dominated by the prior information.
- Analysis of $V(\theta)$ can show identifiable directions (locally)

Simple reservoir example

(top view)



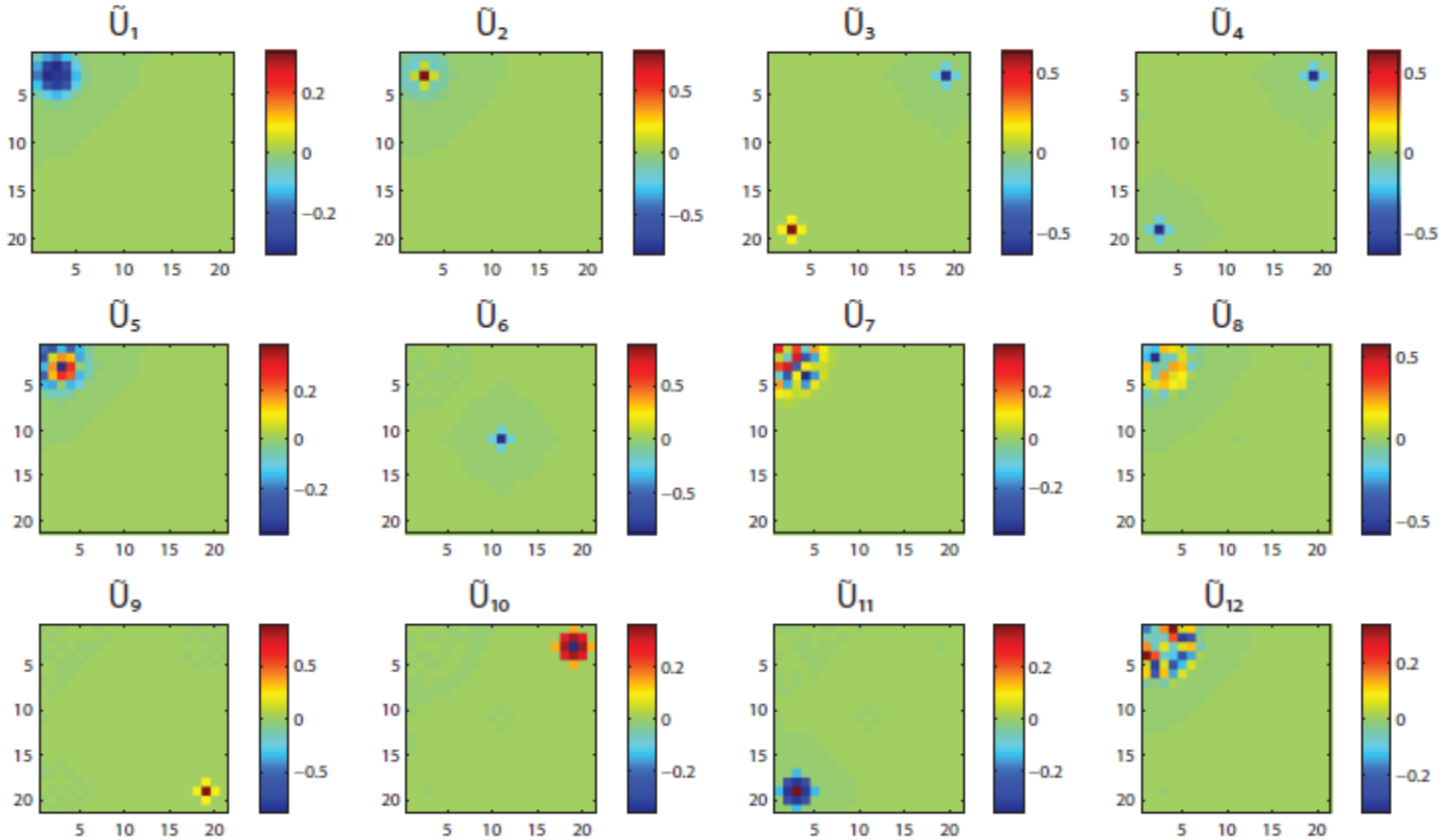




21 x 21 grid block permeabilities
1 injection well (center); 4 production wells (corners);
3 permeability strokes

Simple reservoir example

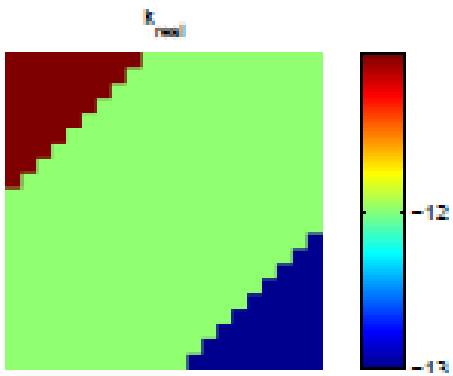
Singular vectors can be projected on the grid:



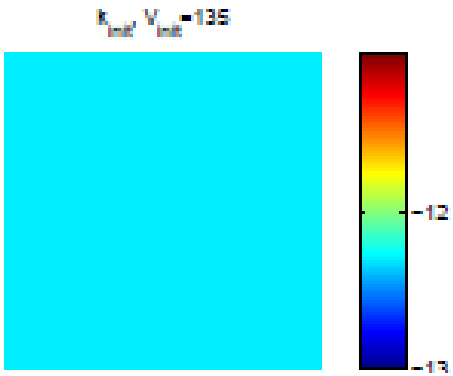
First 12 singular vectors; identifiability case

Simple reservoir example

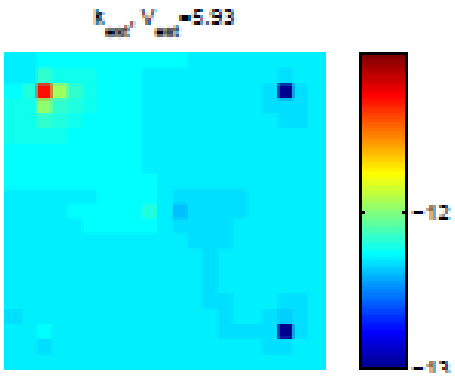
Using the reduced parameter space –iteratively-
in identification:



exact field



initial estimate
(local point)



final estimate

Observation:

Only grid block permeabilites around well are identifiable.

Simple reservoir example

- Grid block properties far away from wells are poorly identifiable
- There are indications that they might not be very important for the optimal control strategy.....

Discussion

- Estimating physical parameters in reservoirs is challenging and highly relevant
- Nonlinear (one-go) batch process complicates this
- Model structure approximation is required in order to guarantee identifiability
- Analysis can only be done locally linearized
- Identification of local linear models can serve a shorter time optimization (Van Essen, CDC 2010)
- Many model-based optimization challenges in this field

J.D. Jansen, O.H. Bosgra and P.M.J. Van den Hof, Model-based control of multiphase flow in subsurface oil reservoirs, *Journal of Process Control*, *18,* 846-855, 2008.

Cooperative project, together with:

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Jan Dirk Jansen
Dept. of Geotechnology TUD and Shell Intern. Exploration and Production

Sippe Douma, **Shell, and former PhD student**

Okko Bosgra, **DCSC, TUD**

Consortia: VALUE (Shell, TU Delft, MIT), ISAPP (Shell, TNO, TU Delft)

THE VALUE OF SMARTNESS