EINDHOVEN UNIVERSITY OF TECHNOLOGY

Data-driven modeling in linear dynamic networks

Paul M.J. Van den Hof

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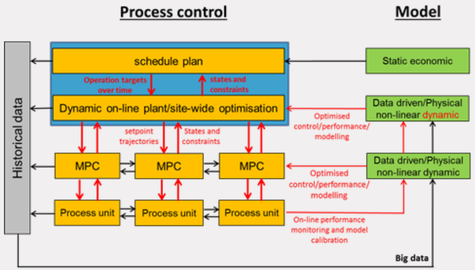
Control and Dynamical Systems, Caltech

www.sysdynet.eu
www.pvandenhof.nl
p.m.j.vandenhof@tue.nl

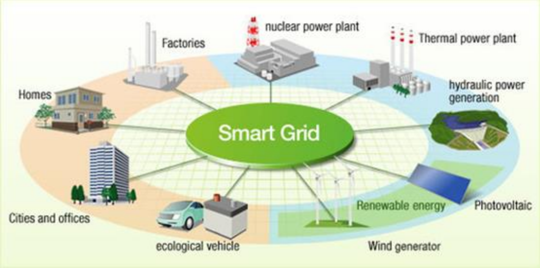
European Research Council

Introduction – dynamic networks

Decentralized process control



Smart power grid

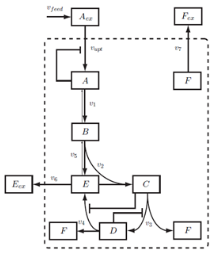


Pierre et al. (2012)

Autonomous driving

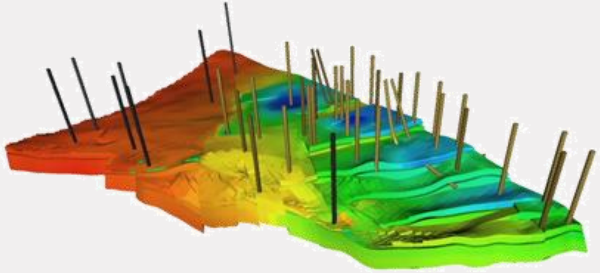
www.envidia.com

Metabolic network



Hillen (2012)

Hydrocarbon reservoirs



Mansoori (2014)

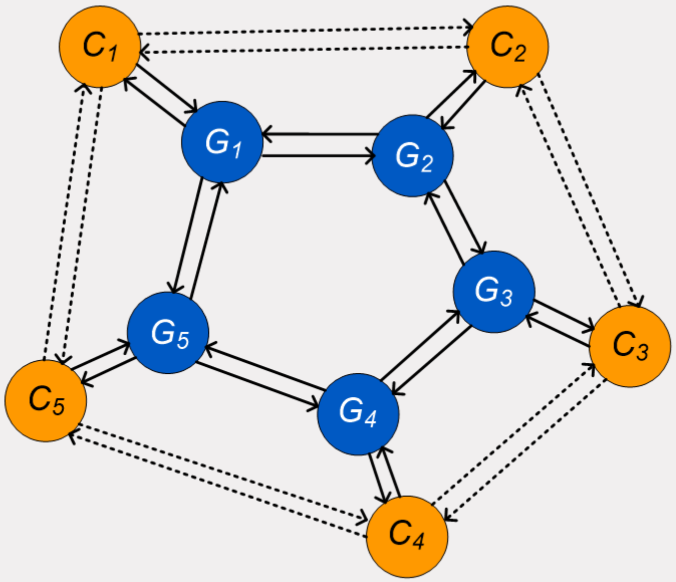
Introduction

Overall trend:

- (Large-scale) interconnected systems
- With hybrid dynamics (continuous / switching)
- Distributed / multi-agent type monitoring, control and optimization problems
- Data is “everywhere”, big data era
- Modelling problems will need to consider

Introduction

Distributed / multi-agent control:



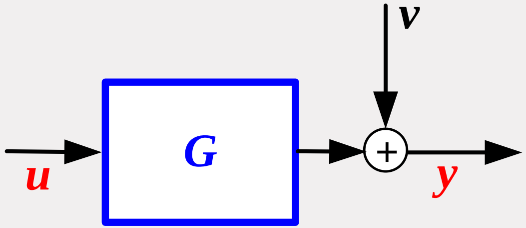
With both physical and communication links between systems G_i and controllers C_i

How to address data-driven modelling problems in such a setting?

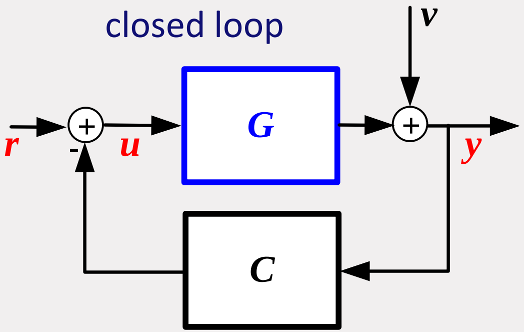
Introduction

The classical (multivariable) identification problems[1] :

open loop



closed loop



Identify a plant model $\hat{G}$ on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

We have to move from a fixed and known configuration to deal with ***structure*** in the problem.

[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

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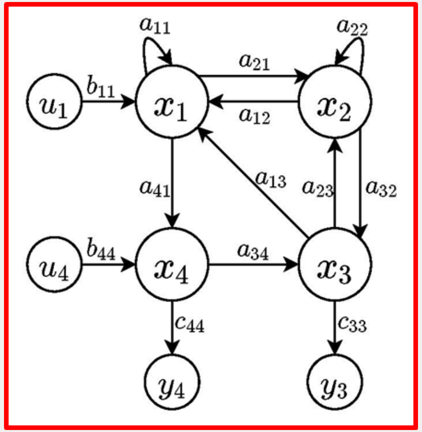
Contents

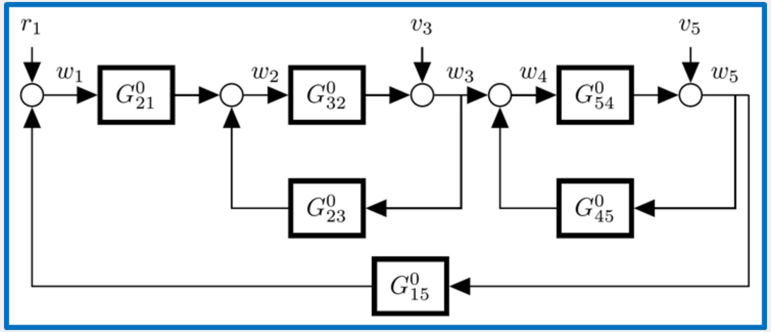
- Introduction and motivation
- How to model a dynamic network?
- Single module identification – known topology
- Network identifiability
- Extensions - Discussion

Dynamic networks for data-driven modeling

Dynamic networks

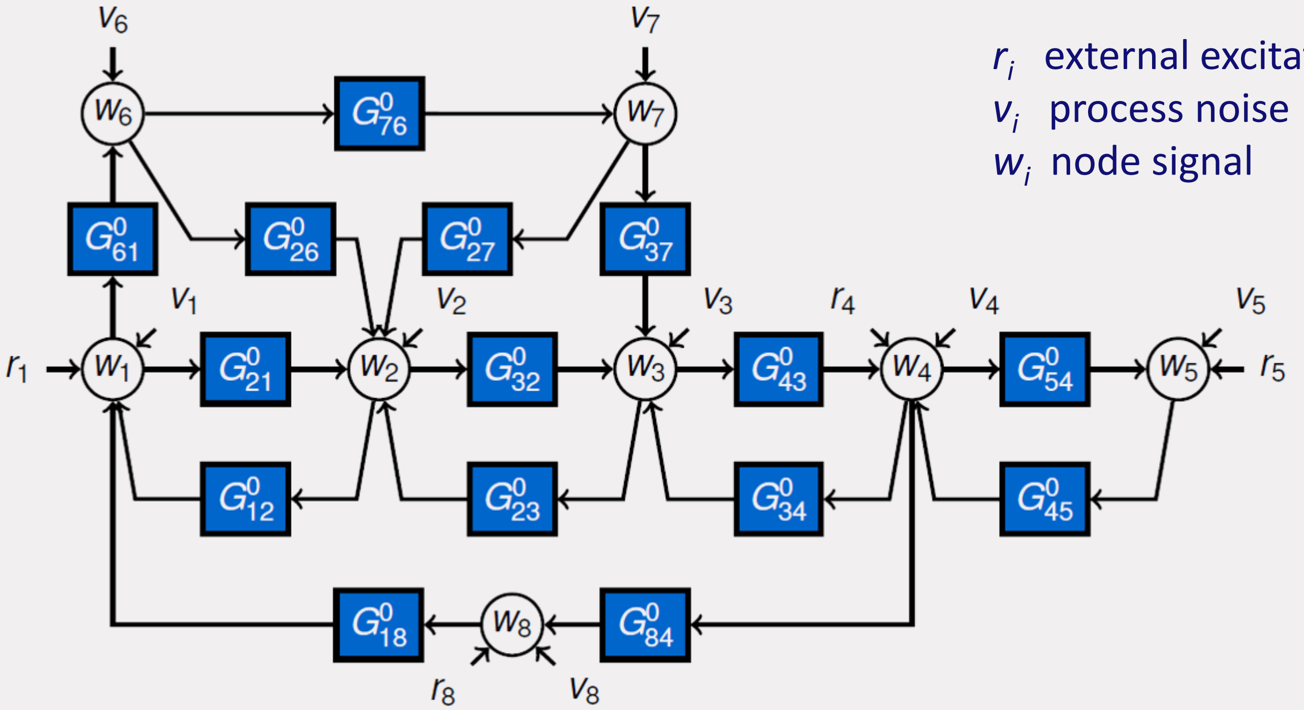


State space representations
(Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…)



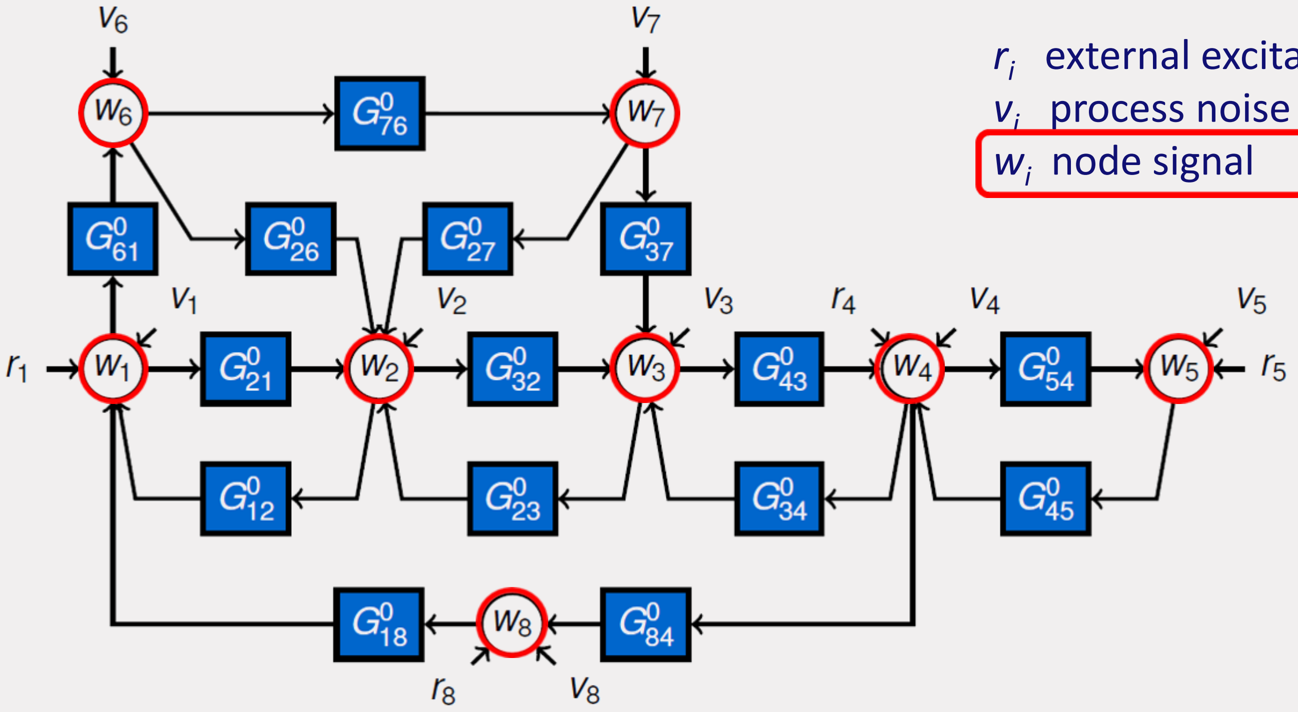
Module representation
(VdH, Dankers, Gevers, Bazanella,…)

Dynamic network setup



r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup



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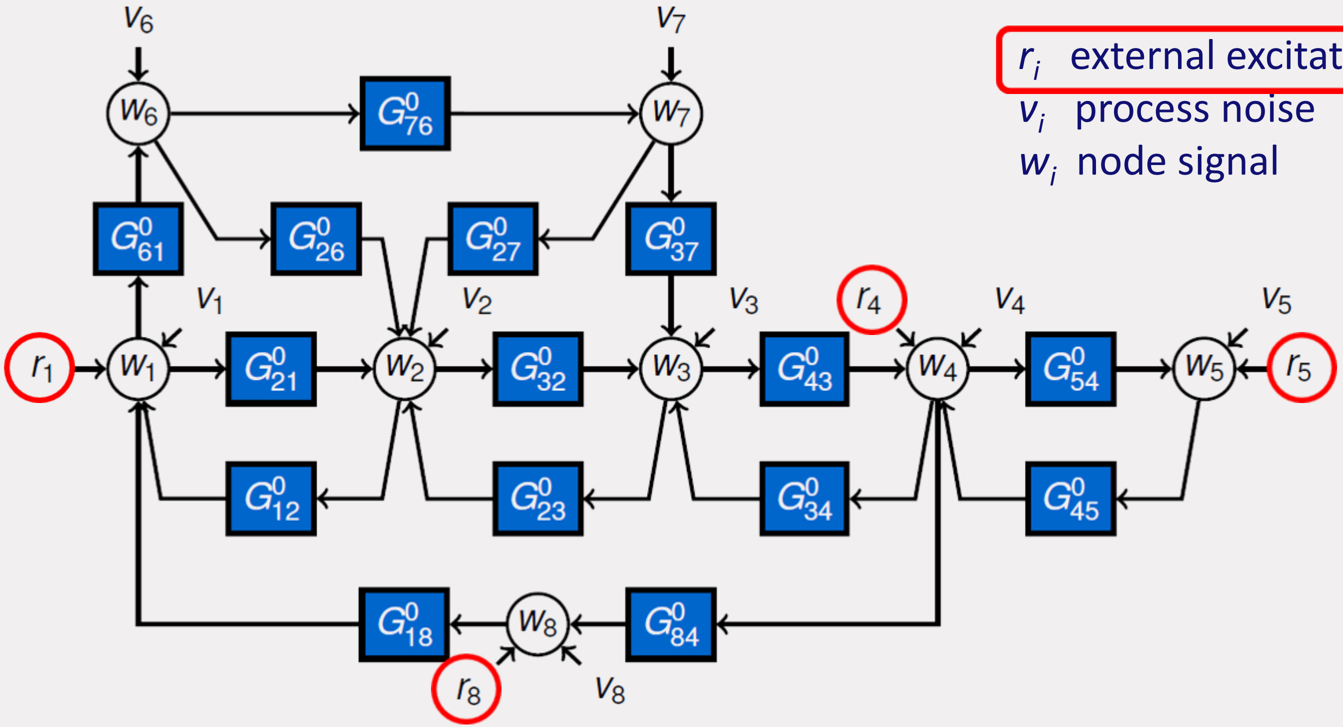

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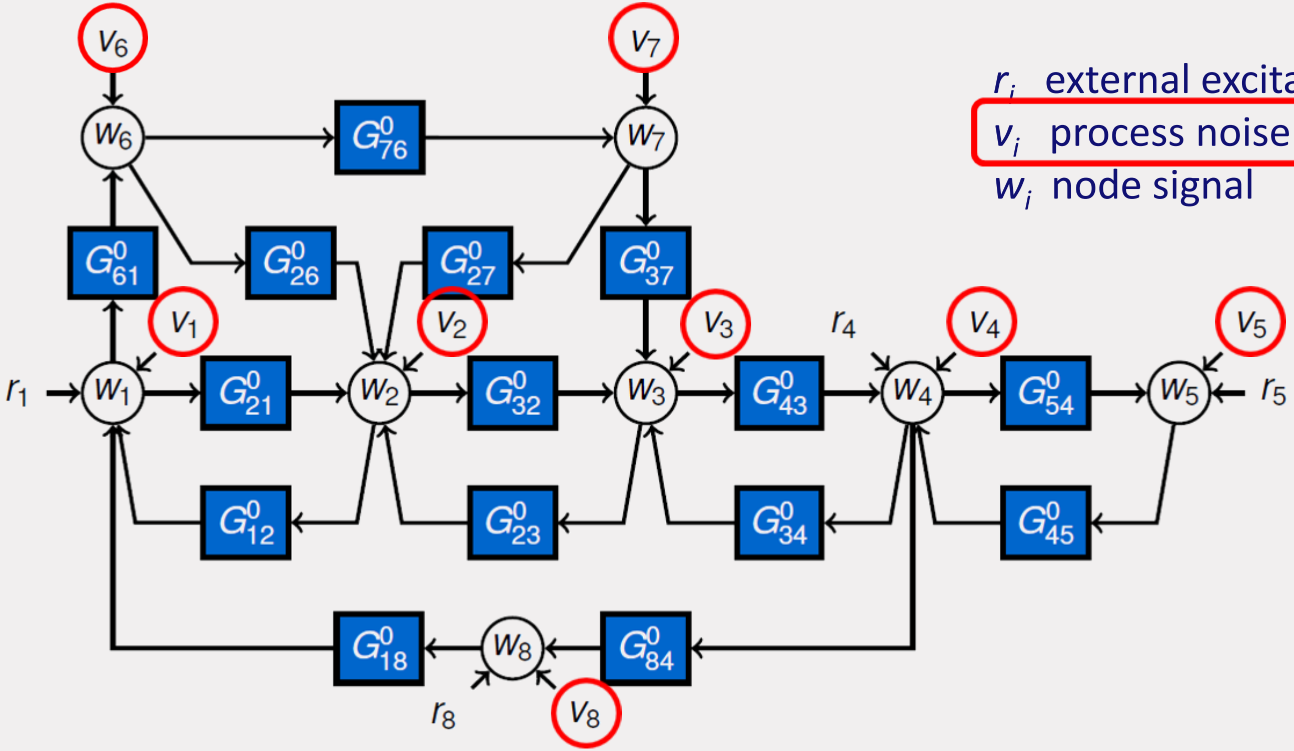

r_i external excitation

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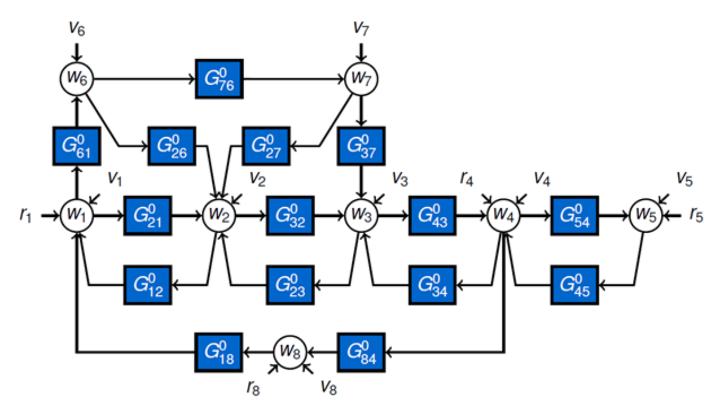
Dynamic network setup



Assumptions:

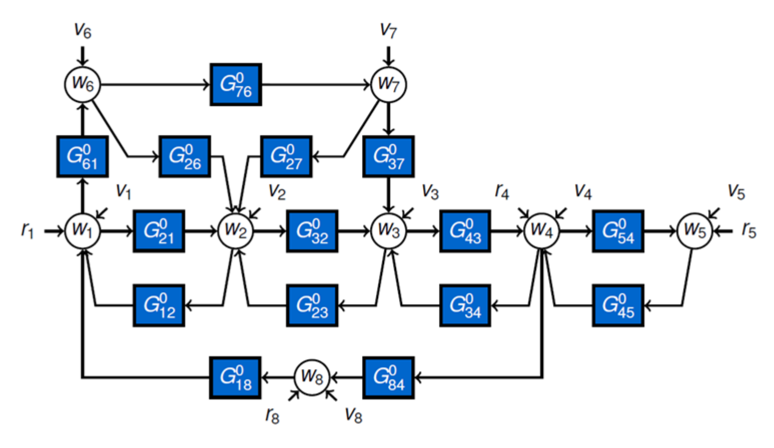
- Total of L nodes
- Network is well-posed and stable
- Modules are dynamic, may be unstable
- Disturbances are stationary stochastic and can be correlated

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t)$$

P.M.J. Van den Hof, A.G. Dankers, P.S.C. Heuberger and X. Bombois. Automatica, 2013.

Dynamic network setup



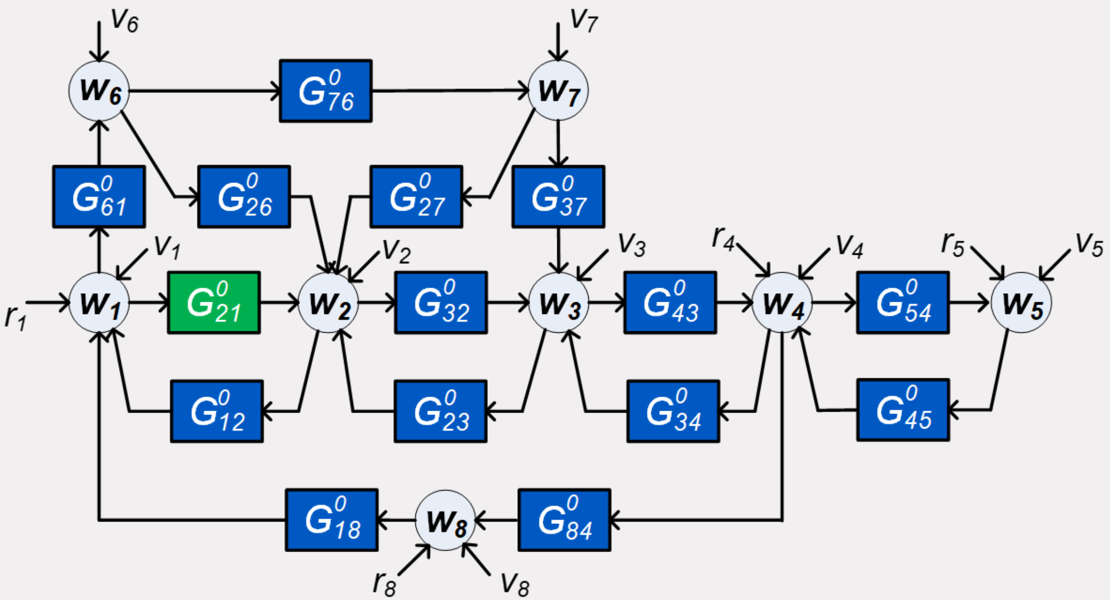
Here: focus on **prediction error methods**

Many new identification questions can be formulated:

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Sensor and excitation selection
- Fault detection
- Experiment design
- User prior knowledge of modules
- Scalable algorithms

Single module identification - known topology

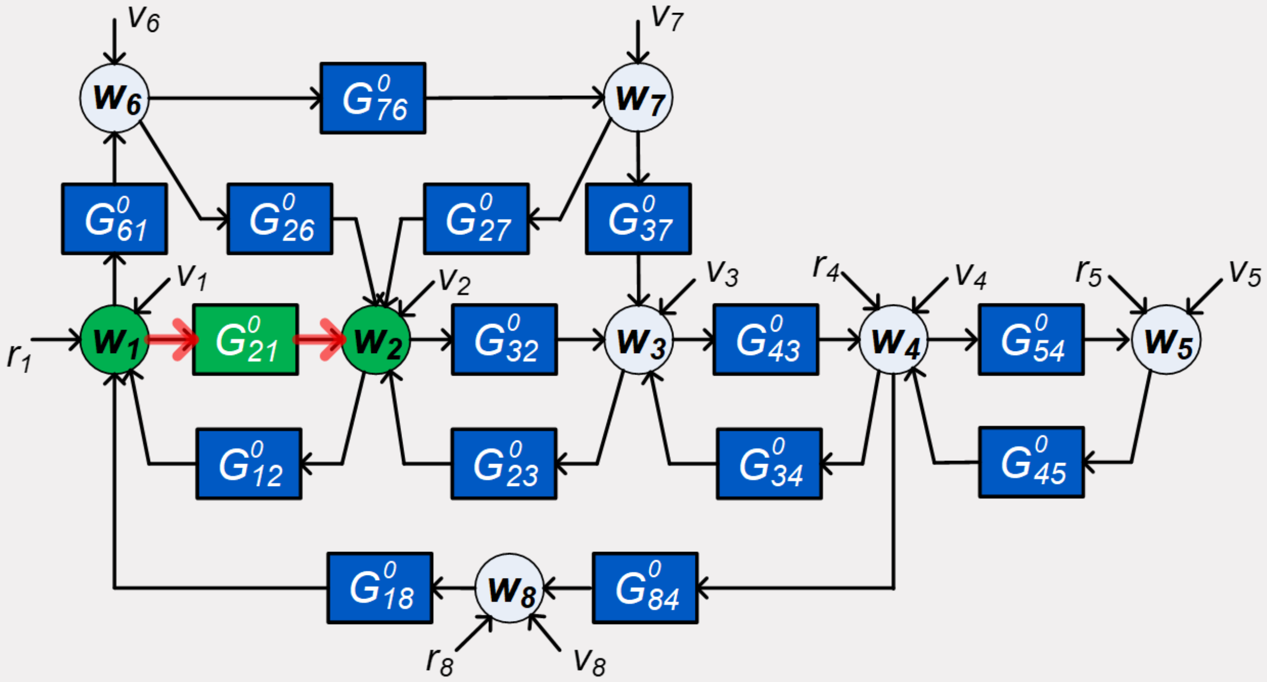
Single module identification



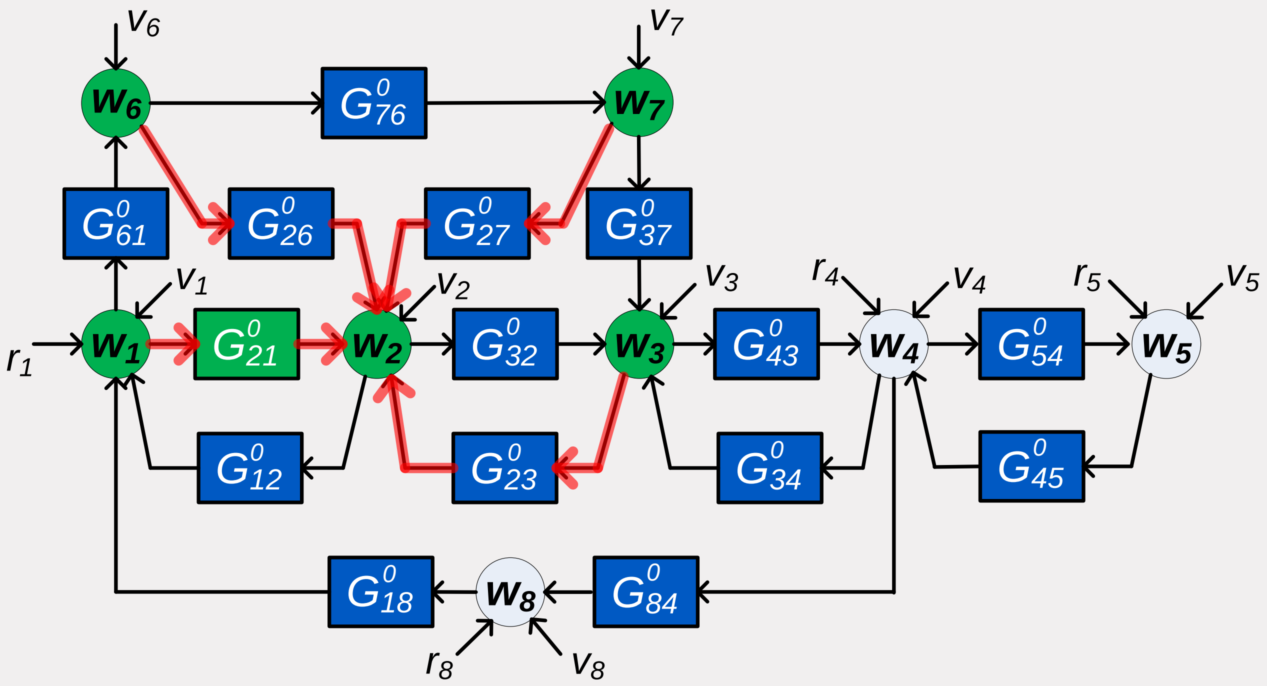
For a network with known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure? Preference for local measurements

Single module identification

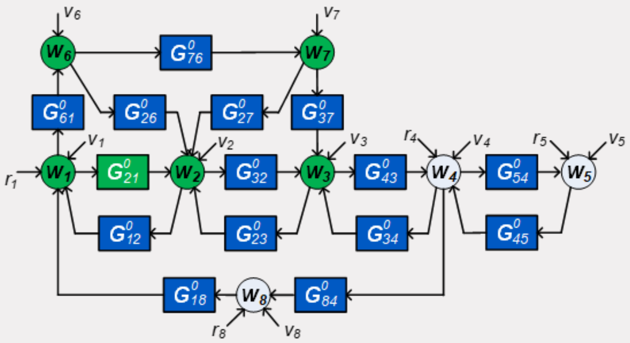


Single module identification



Identifiying G_{21}^{0} is part of a 4-input, 1-output problem

Identification methods



4-input 1-output problem

to be addressed by a closed-loop identification method

- **Direct PE method**

$$\varepsilon(t,\theta) = H(q,\theta)^{-1}[w_2(t) - \sum_{k\in\mathcal{D}_2} G_{2k}(q,\theta)w_k(t)]$$

ML properties
Disturbances v_i uncorrelated over channels
Excitation provided through r and v signals

- **2-stage/projection/IV (indirect) method**

$$\varepsilon(t,\theta) = H(q,\theta)^{-1}[w_2(t) - \sum_{k\in\mathcal{D}_2} G_{2k}(q,\theta)w_k^{\mathcal{R}}(t)]$$

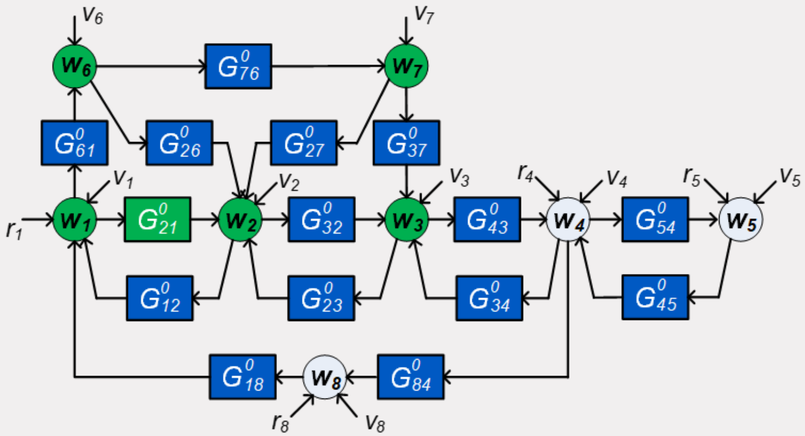
Consistency; no need for noise models; **no ML**
Excitation provided through r signals only

P.M.J. Van den Hof, A.G. Dankers, P.S.H. Heuberger and X. Bombois. Automatica, 49, 2994-3006, 2013.

Single module identification

4 input nodes to be measured:

Can we do with less?

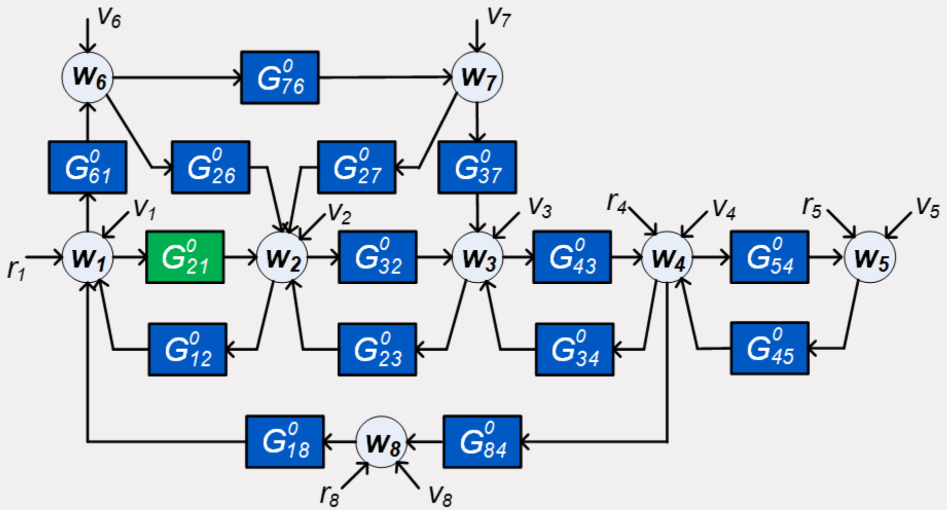


Network immersion [1]

- An **immersed network** is constructed by removing node signals, but leaving the remaining node signals invariant
- Modules and disturbance signals are adapted
- Abstraction through variable elimination (Kron reduction[2] in network theory).

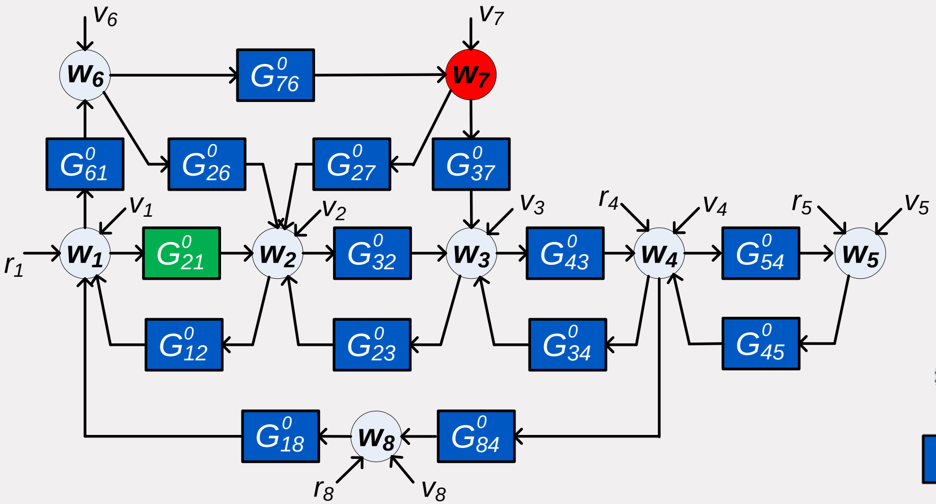
[1] A. Dankers. PhD Thesis, 2014.
[2] F. Dörfler and F. Bullo, IEEE Trans. Circuits and Systems I (2013)

Immersion

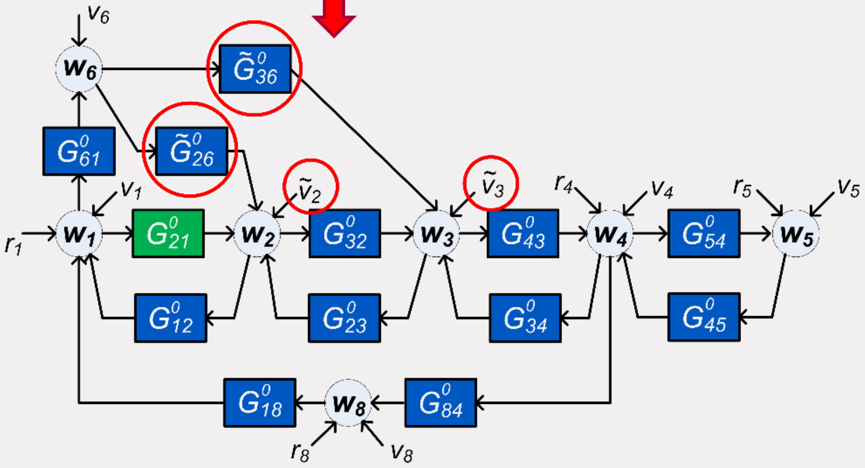


v_6
v_7
w_6
G^0_{76}
w_7
G^0_{61}
G^0_{26}
G^0_{27}
G^0_{37}
v_1
v_2
v_3
r_4
v_4
r_5
v_5
r_1
w_1
G^0_{21}
w_2
G^0_{32}
w_3
G^0_{43}
w_4
G^0_{54}
w_5
G^0_{12}
G^0_{23}
G^0_{34}
G^0_{45}
G^0_{18}
w_8
G^0_{84}
r_8
v_8

Immersion



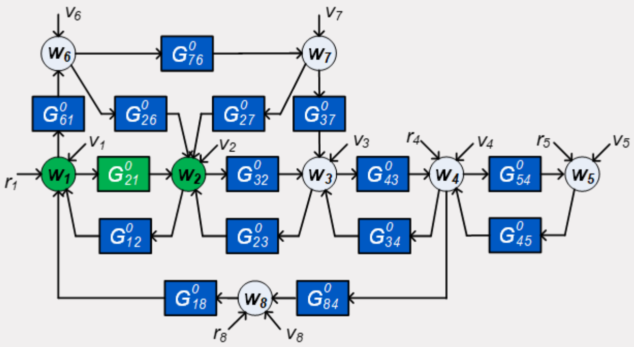
Immersing w_7



When does immersion leave G^0_{21} invariant?

Immersion

When does immersion leave G^0_{21} invariant?



Proposition

Consider an immersed network where w_1 and w_2 are retained.

Then $\breve{G}^0_{21} = G^0_{21}$ if

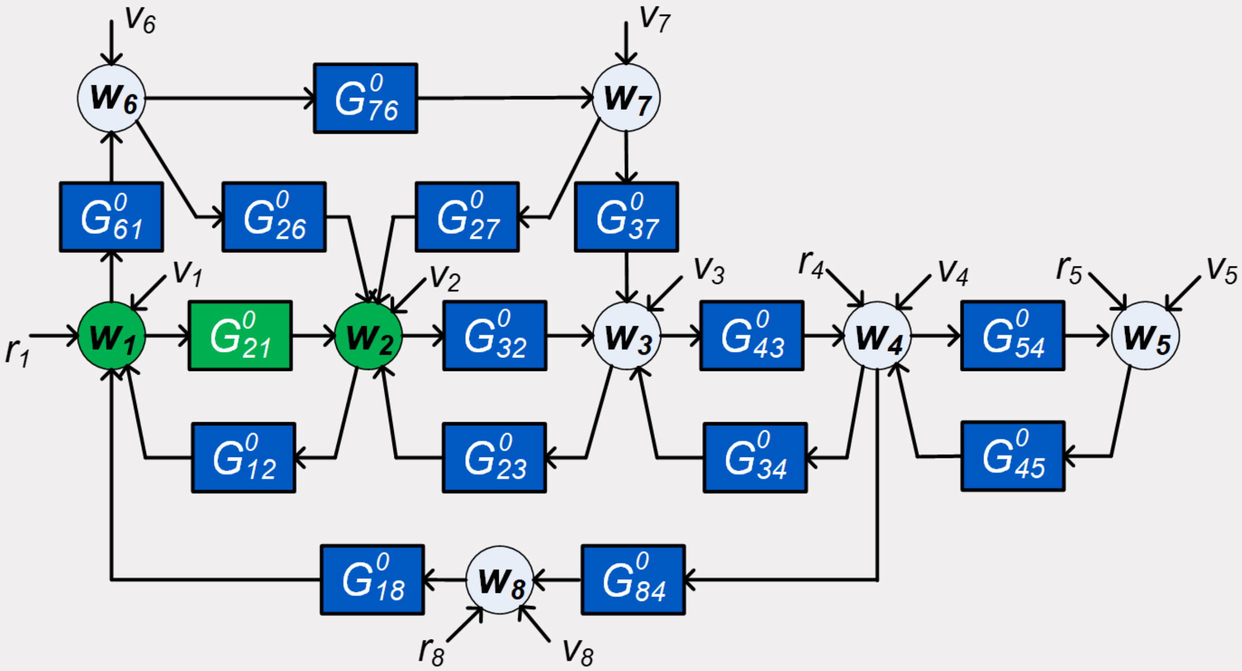
a) Every path $w_1 \rightarrow w_2$ other than the one through G^0_{21} goes through a node that is retained. (parallel paths)

b) Every path $w_2 \rightarrow w_2$ goes through a node that is retained. (loops around the output)

A.G. Dankers et al., *IEEE Trans. Automatic Control, 61*, 937-952, 2016.

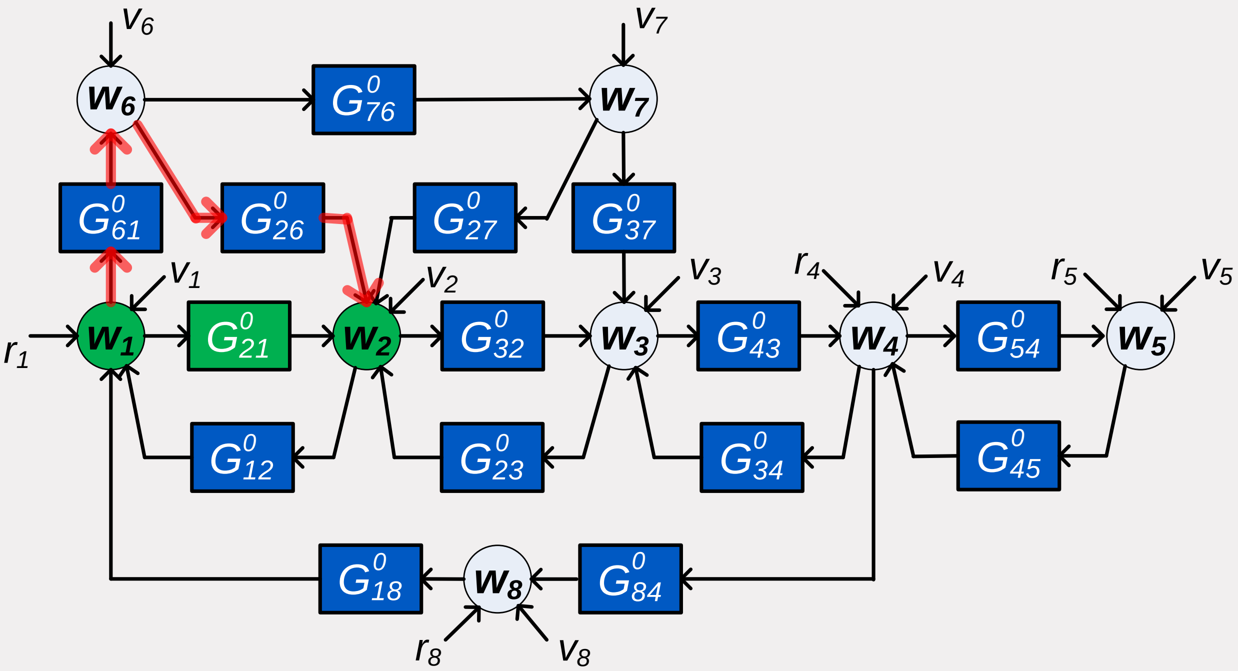
Single module identification

parallel paths, and loops around the output



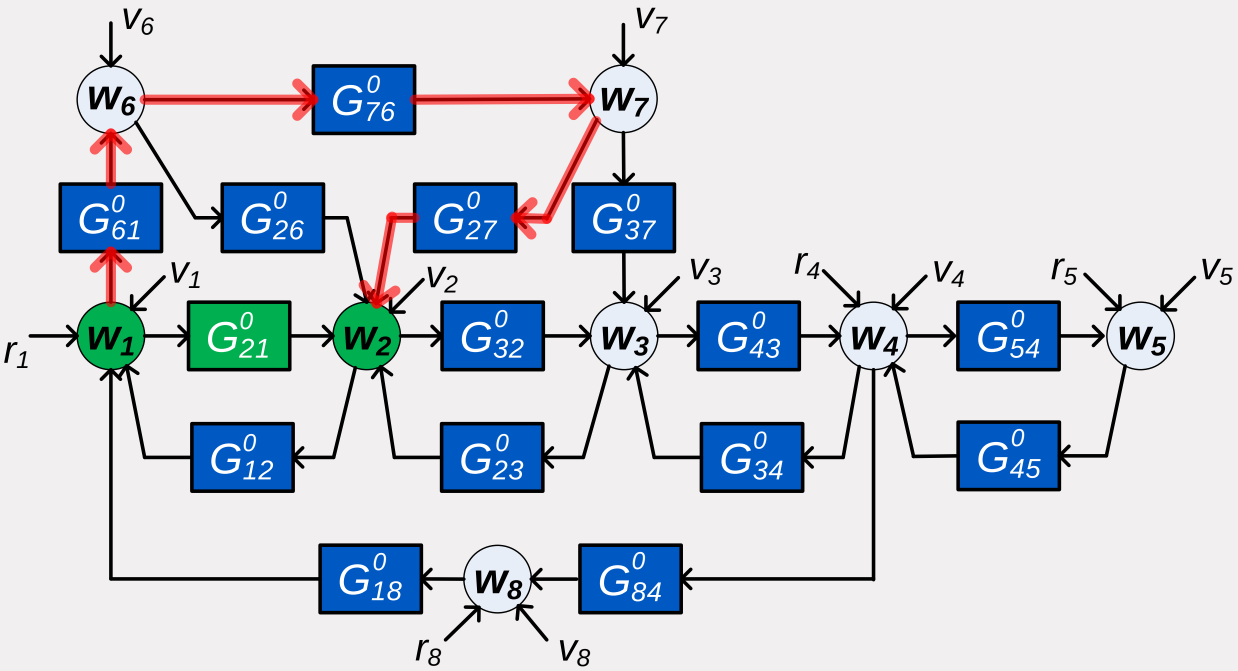
Single module identification

parallel paths, and loops around the output



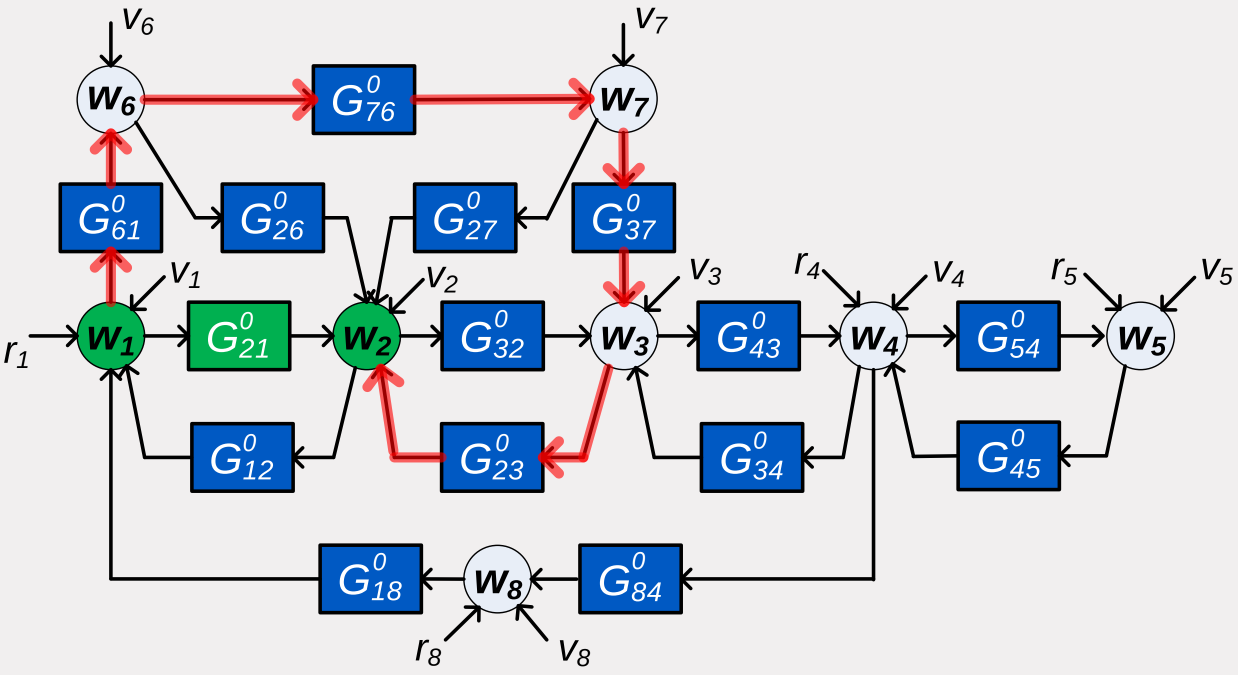
Single module identification

parallel paths, and loops around the output



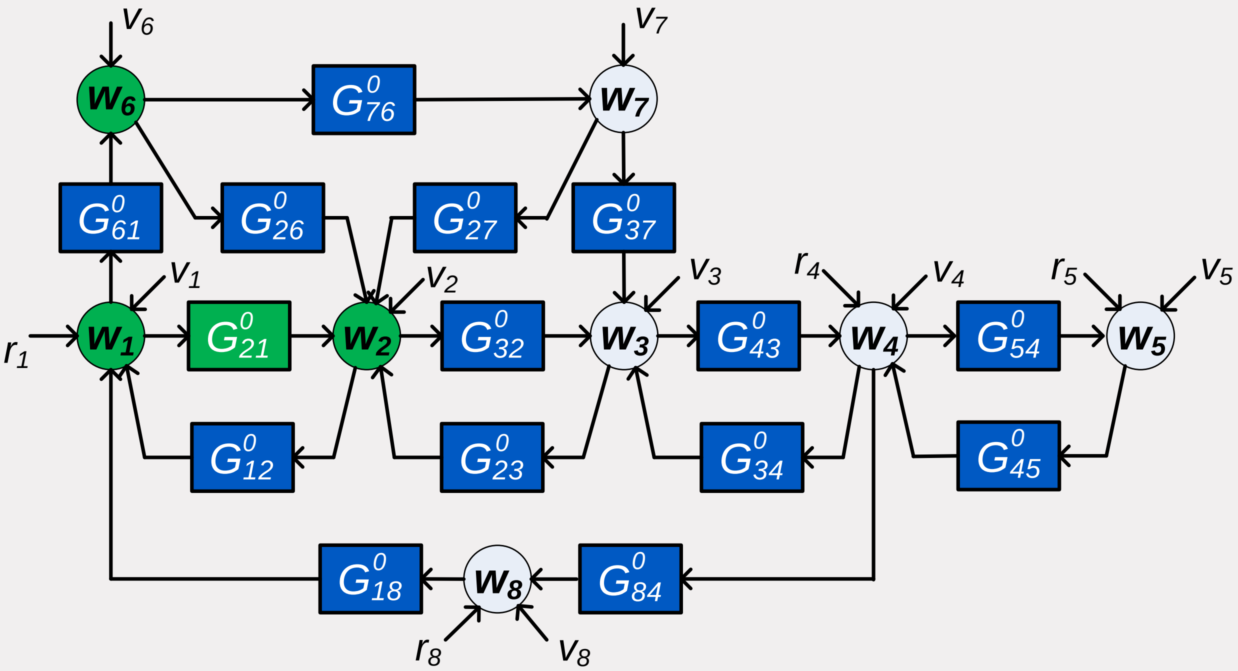
Single module identification

parallel paths, and loops around the output



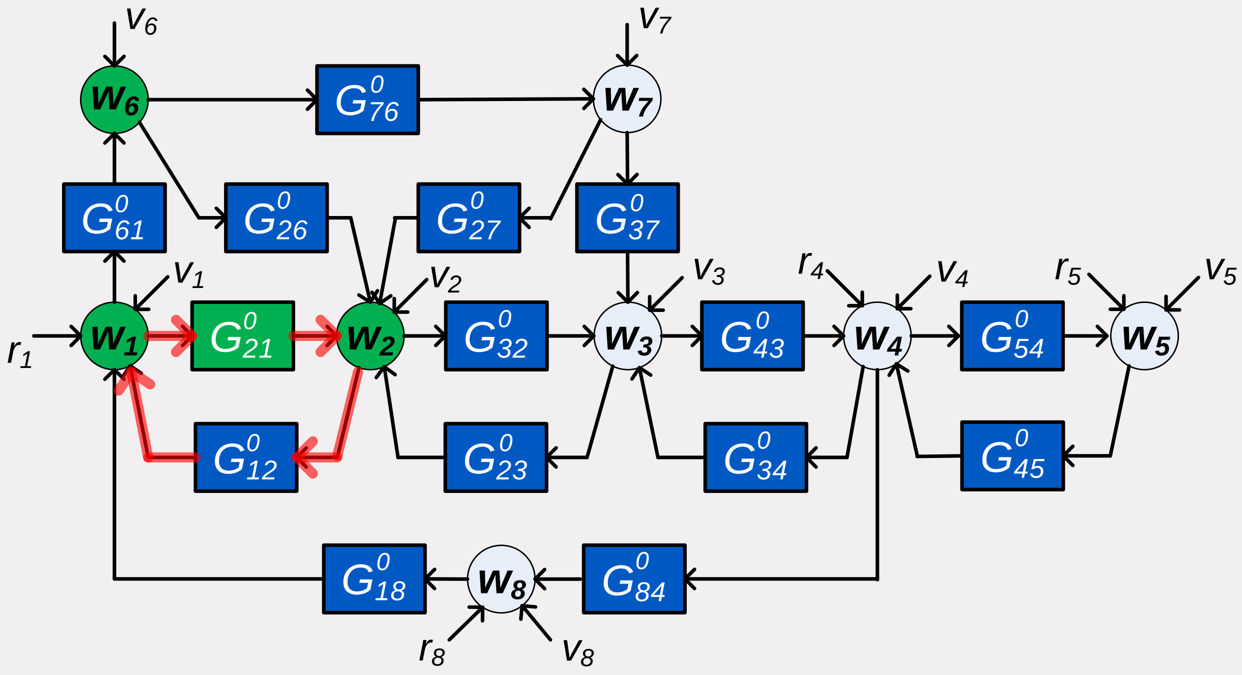
Single module identification

Choose w_6 as an additional input (to be retained)



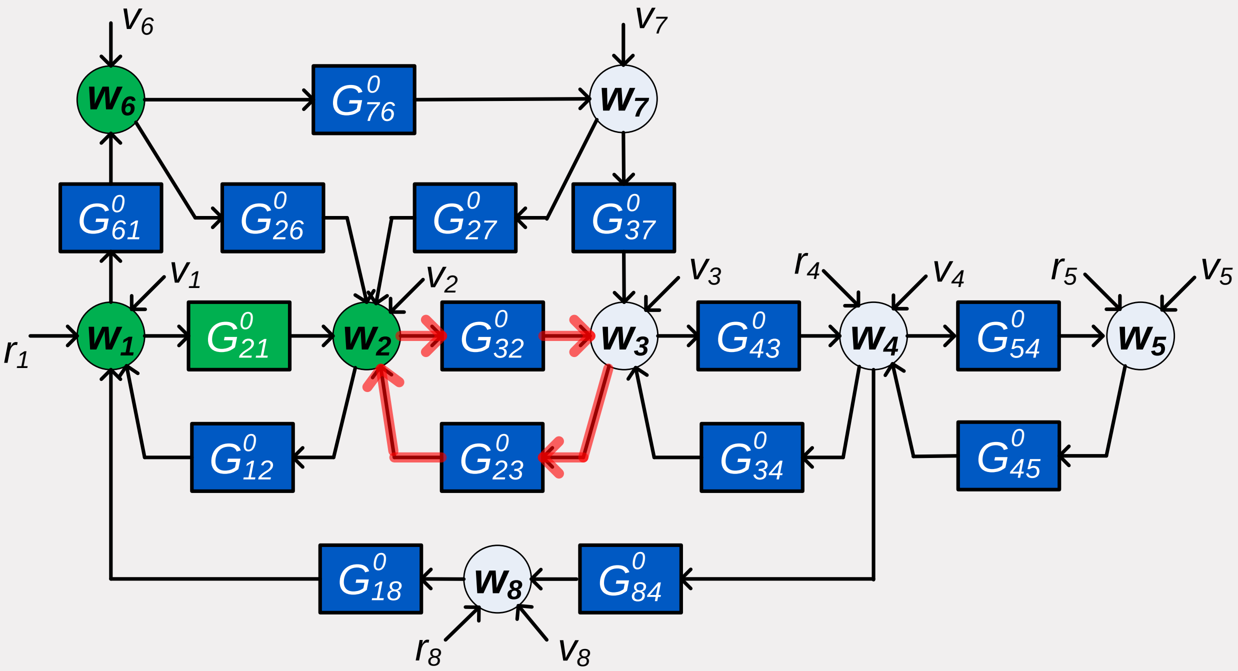
Single module identification

parallel paths, and **loops around the output**



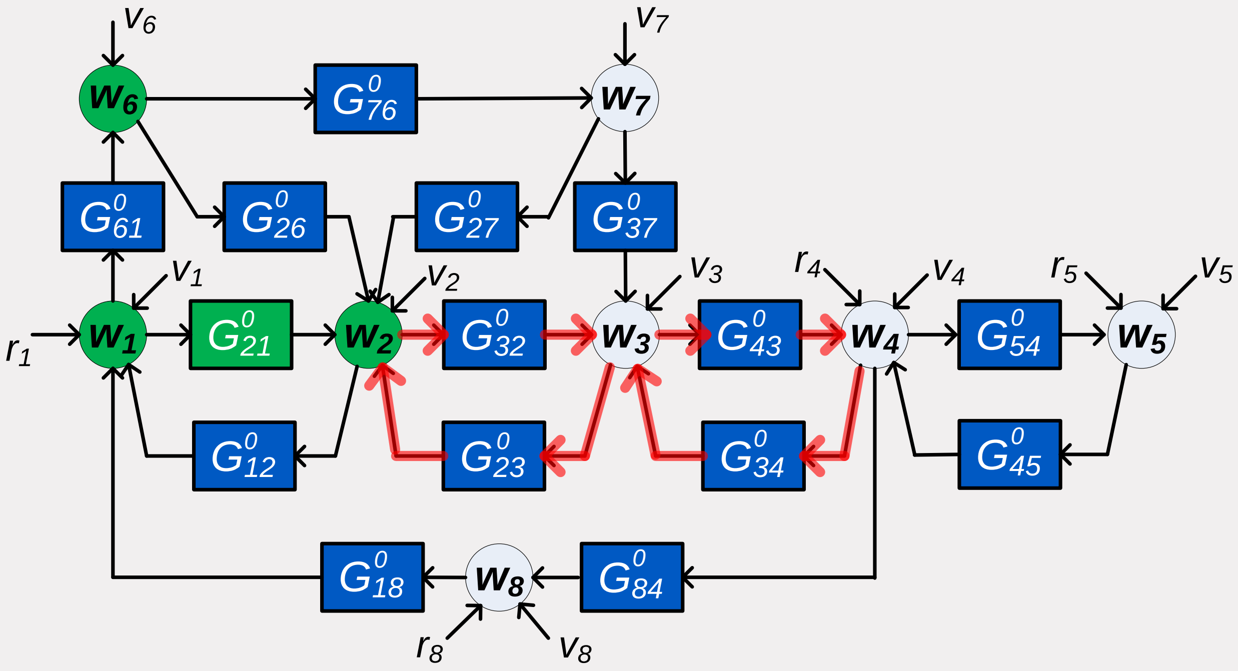
Single module identification

parallel paths, and **loops around the output**



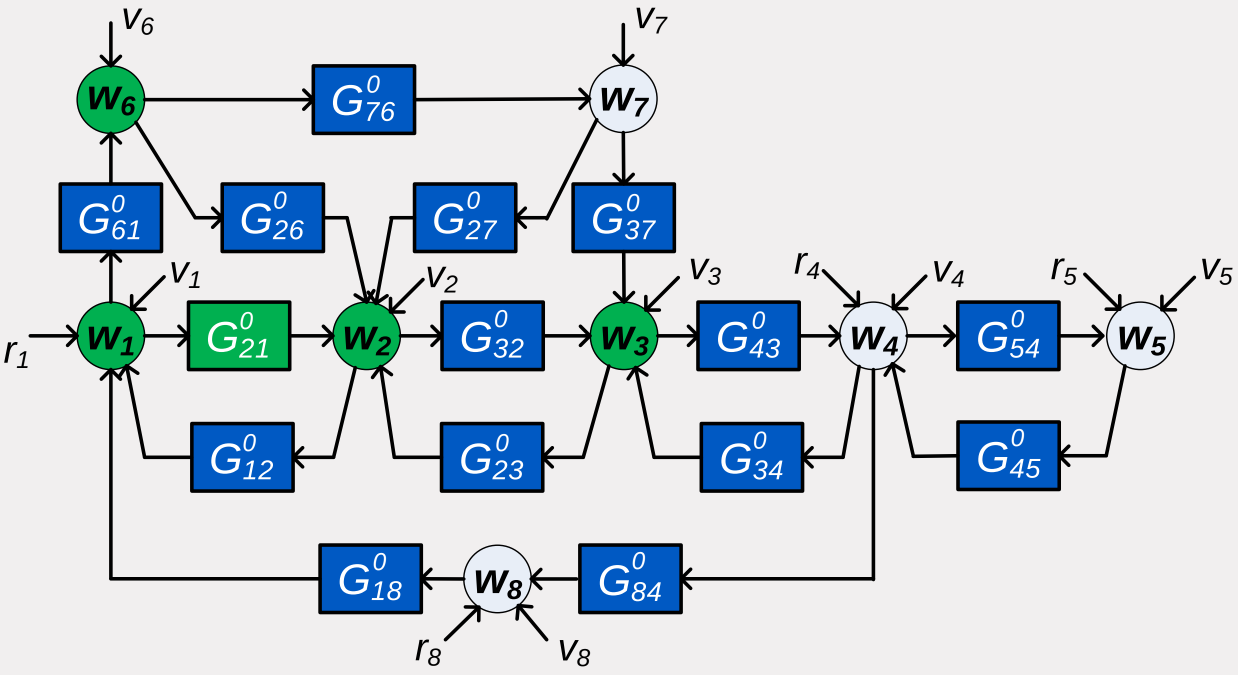
Single module identification

parallel paths, and **loops around the output**

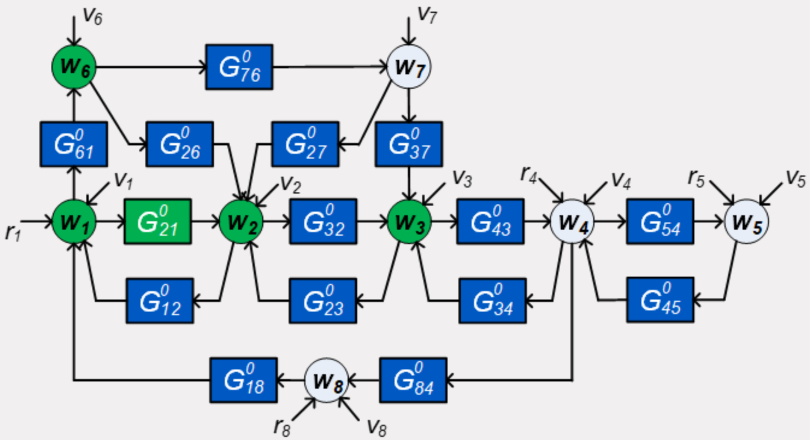


Single module identification

Choose w_3 as an additional input, to be retained



Single module identification



Conclusion:

With a 3-input, 1 output model we can consistently identify G^0_{21}

The immersion reasoning is *sufficient* but not *necessary* to arrive at a consistent estimate, see e.g. Linder and Enqvist [1] and Gevers et al. [2]

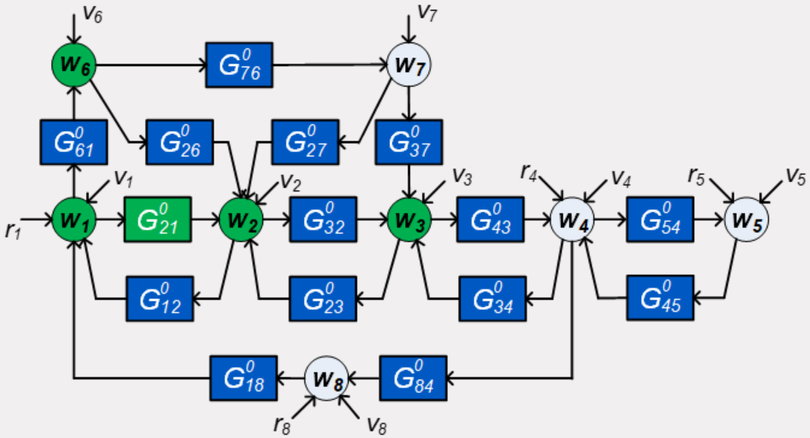
[1] J. Linder and M. Enqvist. *Int. J. Control*, 90(4), 729-745, 2017.
[2] A. Bazanella, M. Gevers et al., CDC 2017.

Single module identification

Conclusion:

With a 3-input, 1 output model we can consistently identify G^0_{21}

with an indirect method



For a consistent and minimum variance estimate (direct method) there is one additional condition:

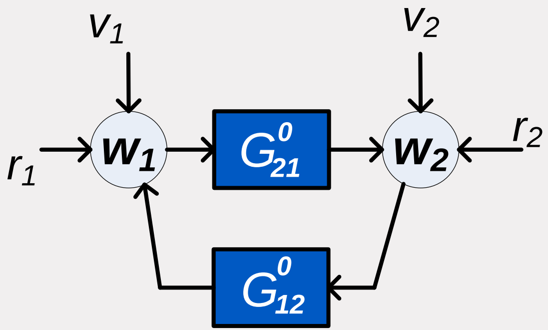
- absence of **confounding variables,** [1][2] i.e. correlated disturbances on inputs and outputs

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.

Confounding variables

Back to the (classical) closed-loop problem:

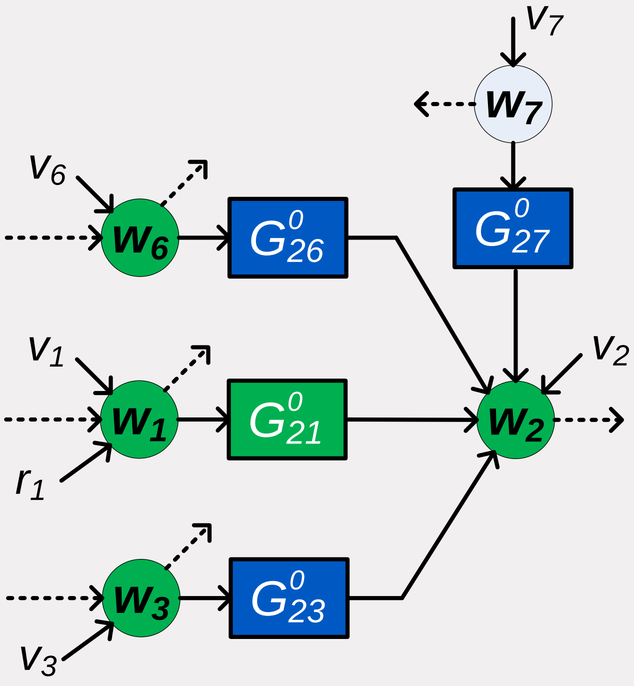
Direct identification of G_{21}^0 can be consistent provided that v_1 and v_2 are uncorrelated



In case of correlation between v_1 and v_2:

Special attention is required

Confounding variables in the MISO case

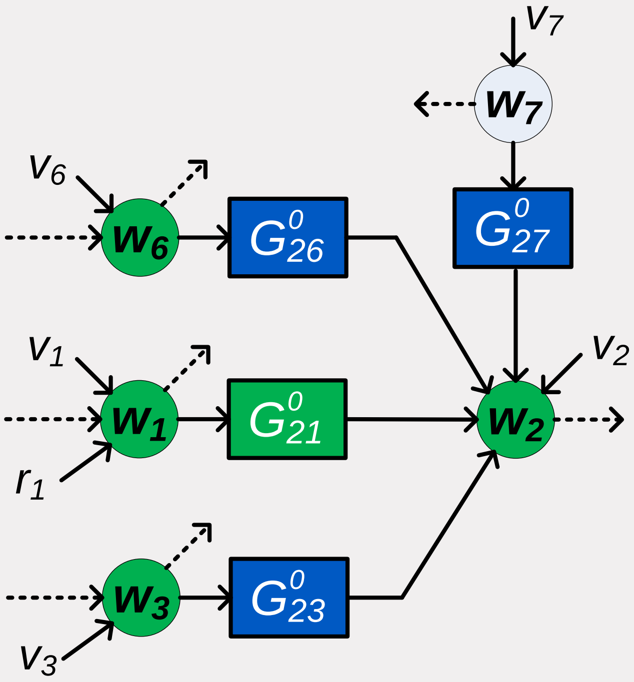


- w_7 (not measured) now acts as a disturbance
- For minimum variance: **MISO direct method** loses consistency if there are confounding variables
- This requires:

$$\begin{bmatrix} v_2 \\ v_7 \end{bmatrix} \text{ uncorrelated with } \begin{bmatrix} v_1 \\ v_3 \\ v_6 \end{bmatrix}$$

and no path from w_7 to an input

Confounding variables in the MISO case



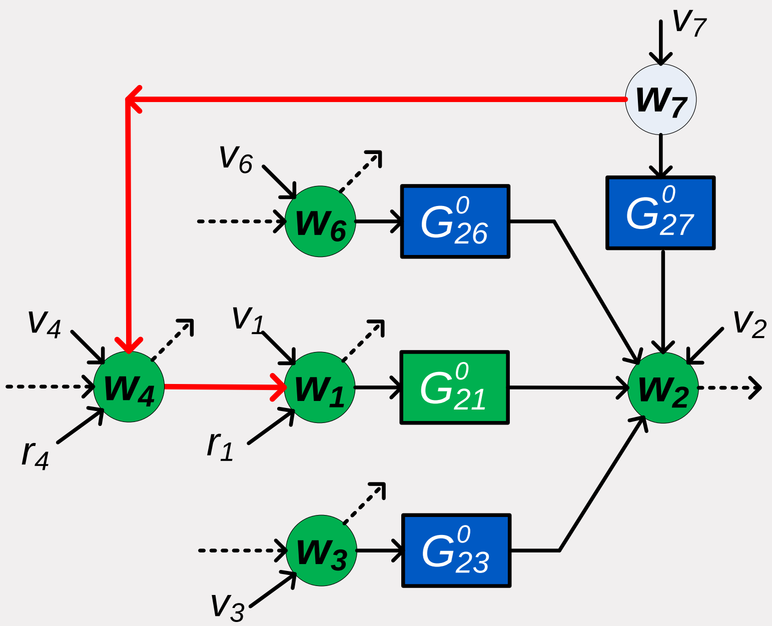
Solutions while restricting to MISO models:

(a) Including the node w_7 as additional input, or

(a) Block the paths from w_7 to inputs/outputs by measured nodes, to be used as additional inputs.

TU/e

Confounding variables in the MISO case



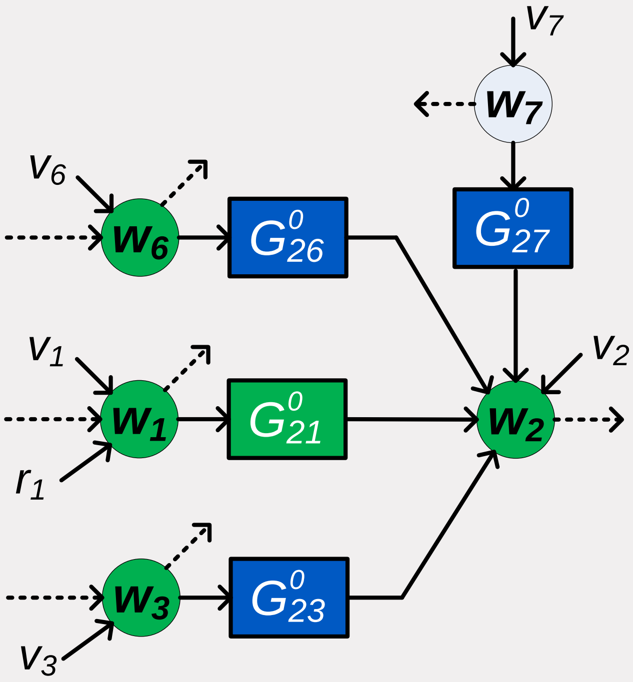
Solutions:

b) Block the paths from w_7 to input w_1 by measured node w_4 to be used as additional input.

Relation with d-separation in graphs (Materassi & Salapaka)

A.G. Dankers et al., *IFAC World Congress*, 2017.

Confounding variables in the MISO case



Can we always address confounding variables in this way?

No

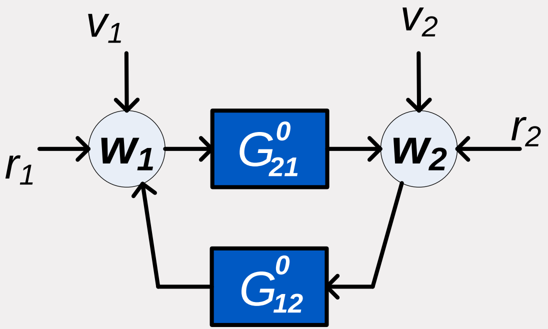
If v_2 and v_1 are correlated then:

A MIMO approach with predicted outputs w_2 and w_1 can solve the problem

P.M.J. Van den Hof et al., ACC 2019, submitted.

Confounding variables

Back to the (classical) closed-loop problem:



In case of correlation between v_1 and v_2: MIMO approach
joint prediction of w_1 and w_2 leads to ML results,

$$\begin{bmatrix}\varepsilon_1(t,\theta)\\ \varepsilon_2(t,\theta)\end{bmatrix} = H(q,\theta)^{-1}\begin{bmatrix}w_1(t) - G_{12}(q,\theta)w_2(t)\\ w_2(t) - G_{21}(q,\theta)w_1(t)\end{bmatrix}$$

Joint estimation of G^0_{21} and G^0_{12} : Joint–direct method [1,2] related to the classical joint-io method [3,4]

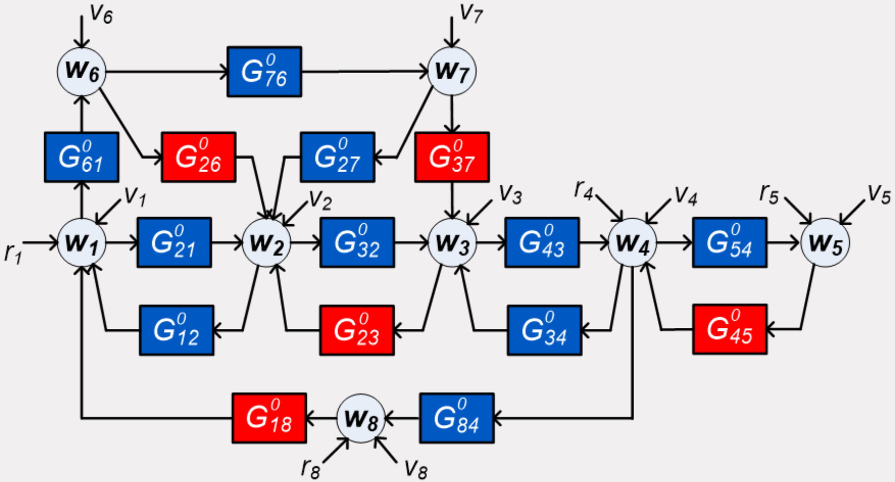
[1] P.M.J. Van den Hof et al. *Proc. 56th IEEE CDC*, 2017
[2] H.H.M. Weerts et al., *Automatica*, Dec. 2018.
[3] T.S. Ng, G.C. Goodwin, B.D.O. Anderson, Automatica, 1977
[4] B.D.O. Anderson and M. Gevers, Automatica 1982.

Summary single module identification

- Methods for **consistent** and **minimum variance** module estimation
- For direct method / ML results: treatment of confounding variables / correlated disturbances
- Degrees of freedom in selection of measured signals – sensor selection
- A priori known modules can be accounted for

Network Identifiability

Network identifiability



blue = unknown
red = known

Question: Can the dynamics/topology of a network be *uniquely determined* from measured signals w_i, r_i ?

Required: Can different dynamic networks be *distinguished* from each other from measured signals w_i, r_i ?

Starting assumption: all signals w_i, r_i that are present are measured.

Network identifiability

Network: $$w = G^0 w + R^0 r + H^0 e \qquad cov(e) = \Lambda^0,$$ rank p

$$w = (I - G^0)^{-1}[R^0 r + H^0 e]$$ dim(r) = K

The network is defined by: $(G^0, R^0, H^0, \Lambda^0)$

a network model is denoted by: $M = (G, R, H, \Lambda)$

and a **network model set** by:

$$\mathcal{M} = \{M(\theta) = (G(\theta), R(\theta), H(\theta), \Lambda(\theta)), \theta \in \Theta\}$$

represents **prior knowledge** on the network models:

- topology
- disturbance correlation
- known modules
- the signals used for identification

Network identifiability

$$w = (I - G^0)^{-1}[R^0 r + H^0 e]$$

Denote: $w = T^0_{wr} r + \bar{v}$

$$\bar{v} = T^0_{we} e$$

$$\Phi^0_{\bar{v}} = T^0_{we}(e^{i\omega})\Lambda^0 T^0_{we}(e^{i\omega})^*$$

Objects that are uniquely identified from data r, w: T^0_{wr}, $\Phi^0_{\bar{v}}$

How to define identifiability?

Clasically:

- Property of a model set
- Unique mapping between **parameters** and models

In the **network** situation:

- Property of a model set
- Unique mapping between **models** and identified objects

Network identifiability

Definition

A network model set $\mathcal{M}$ is network identifiable from (w, r) at $M_0 = M(\theta_0)$ if for all models $M(\theta_1) \in \mathcal{M}$:

$$\left.\begin{array}{c} T_{wr}(q,\theta_1) = T_{wr}(q,\theta_0) \\ \Phi_{\bar{v}}(\omega,\theta_1) = \Phi_{\bar{v}}(\omega,\theta_0) \end{array}\right\} \Longrightarrow M(\theta_1) = M(\theta_0)$$

$\mathcal{M}$ is network identifiable if this holds for all models $M_0 \in \mathcal{M}$

Weerts et al., SYSID2015; Weerts, Van den Hof and Dankers, Automatica, March 2018

Network identifiability

Theorem – identifiability for general model sets

If:

a) Each unknown entry in $M(\theta)$ covers the set of all proper rational transfer functions
b) All unknown entries in $M(\theta)$ are parametrized independently

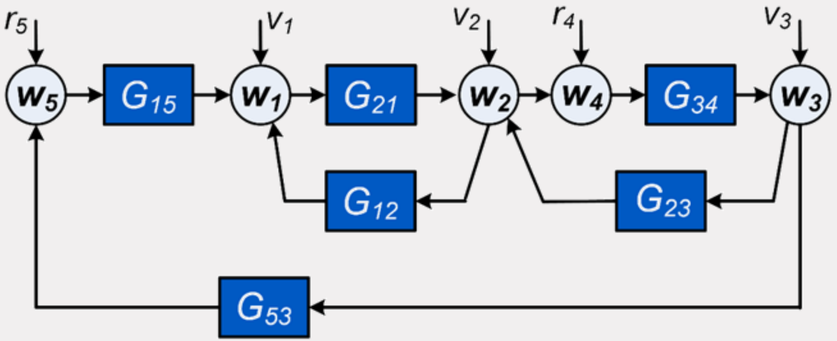
Then $\mathcal{M}$ is network identifiable from (w, r) at $M_0 = M(\theta_0)$ if and only if

- Each row of $[G(\theta)\ H(\theta)\ R(\theta)]$ has at most *K+p* parametrized entries
- For each row *i* the transfer matrix $\check{T}_i(q, \theta_0)$ has full row rank, with $\check{T}_i(q, \theta_0)$:

$$[G_{i*}(\theta)\ H_{i*}(\theta)\ R_{i*}(\theta)] = \begin{array}{ccccc|cccc|cc} & & & & & & & [v_3 & v_4 & r_1 & r_2] \\ [0 & * & 0 & * & * & * & * & 0 & 0 & 1 & 0] \\ & [w_2 & & w_4 & w_5] & & & & & & \end{array}$$

Weerts et al, SYSID2015; Weerts et al., Automatica, March 2018

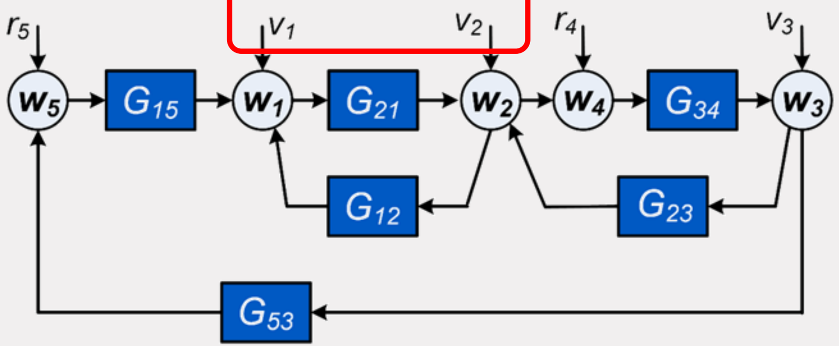
Example 5-node network



There are noise-free nodes, and v_1 and v_2 are expected to be correlated

$$\mathcal{M} \text{ with } H(\theta) = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 \\ 0 & 0 & H_{33}(\theta) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Example 5-node network



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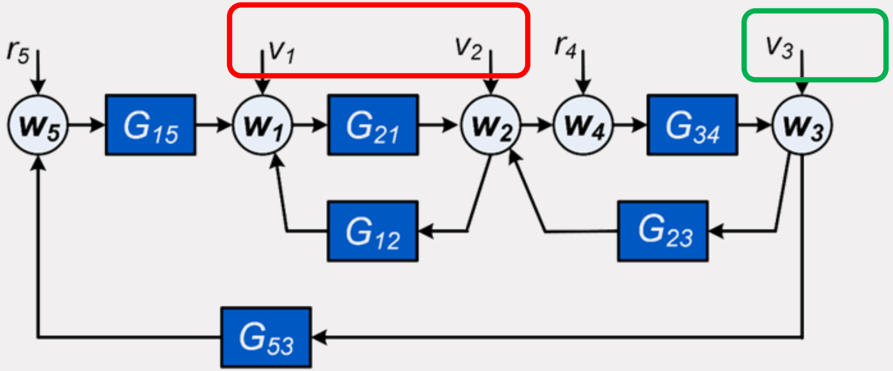

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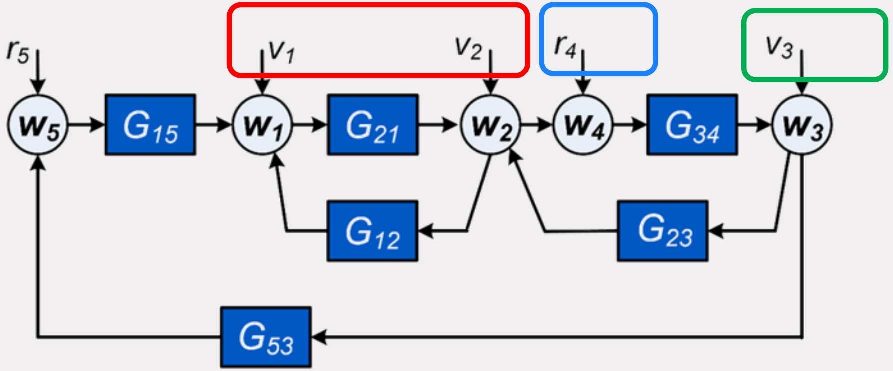

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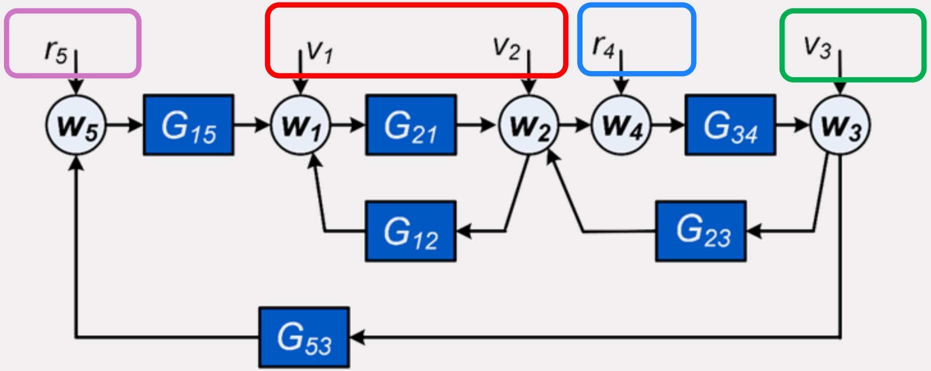
Example 5-node network



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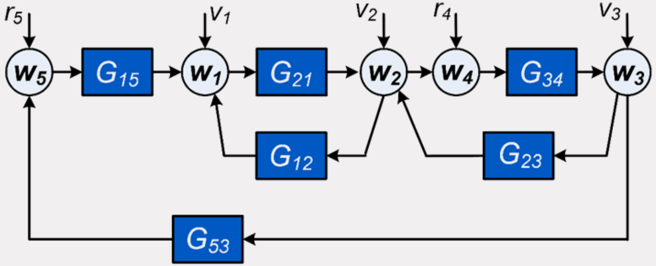
Example 5-node network

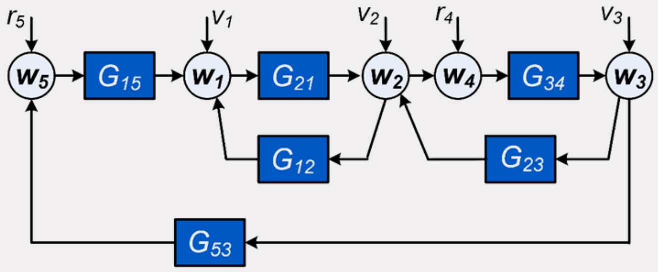


If we restrict the structure of $G(\theta)$:

$$G(\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta) \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H \;\; R] = \underbrace{\begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{K+p=5}$$

First condition:
Number of parametrized entries in each row < K+p = 5

Example 5-node network



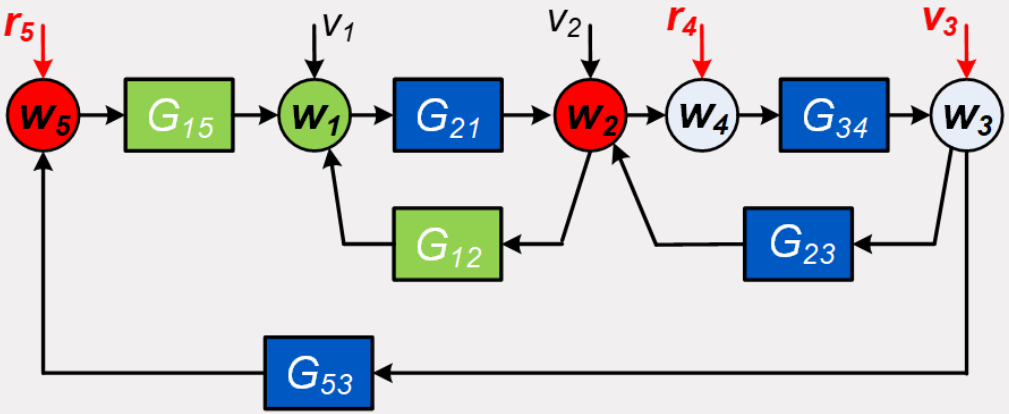
$$G(\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta) \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H\ \ R] = \underbrace{\begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{K+p=5}$$

Rank condition:

Row 1: Full row rank of transfer: $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

Example 5-node network

Verifying the rank condition for $\check{T}_1(q, \theta_0)$



$i = 1$: Evaluate the rank of the transfer matrix $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

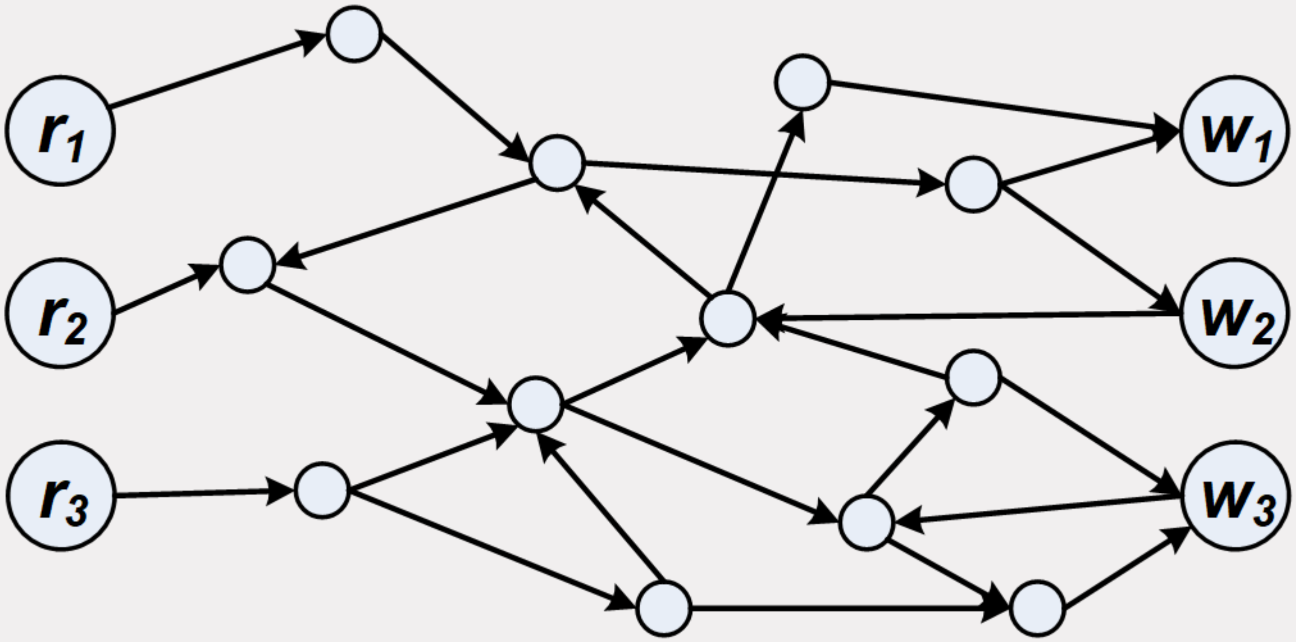
Vertex-disjoint paths

Theorem (Van der Woude, 1991; Hendrickx et al. 2017; Weerts et al., 2018)

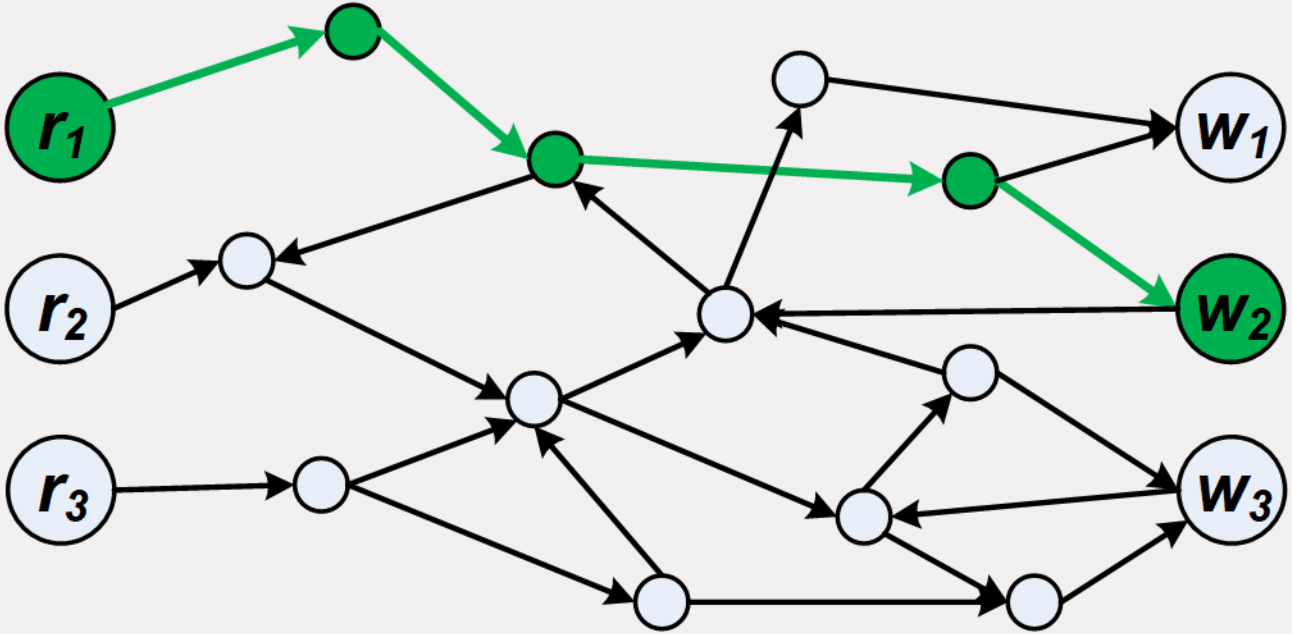
The **generic rank** of a transfer function matrix between
inputs r and nodes w
is equal to the maximum number of **vertex-disjoint paths** between the sets of inputs and outputs.

A (path-based) check on the topology of the network can decide whether the conditions for identifiabiltiy are satisfied generically.

Vertex-disjoint paths



Vertex-disjoint paths



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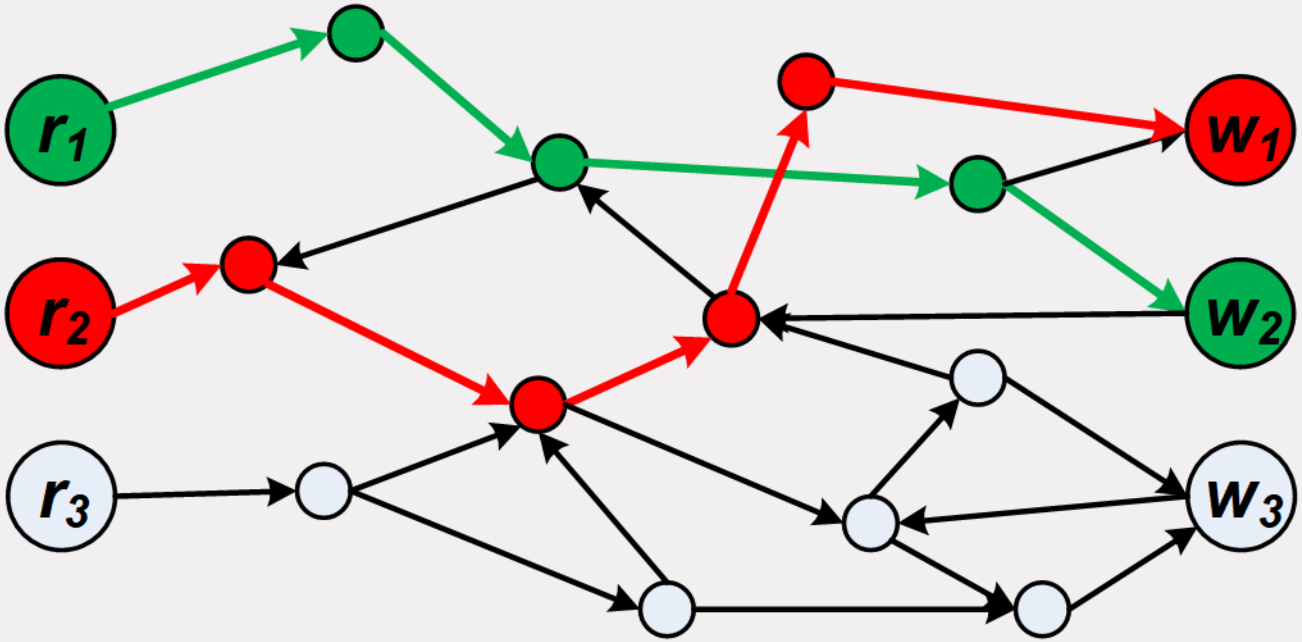
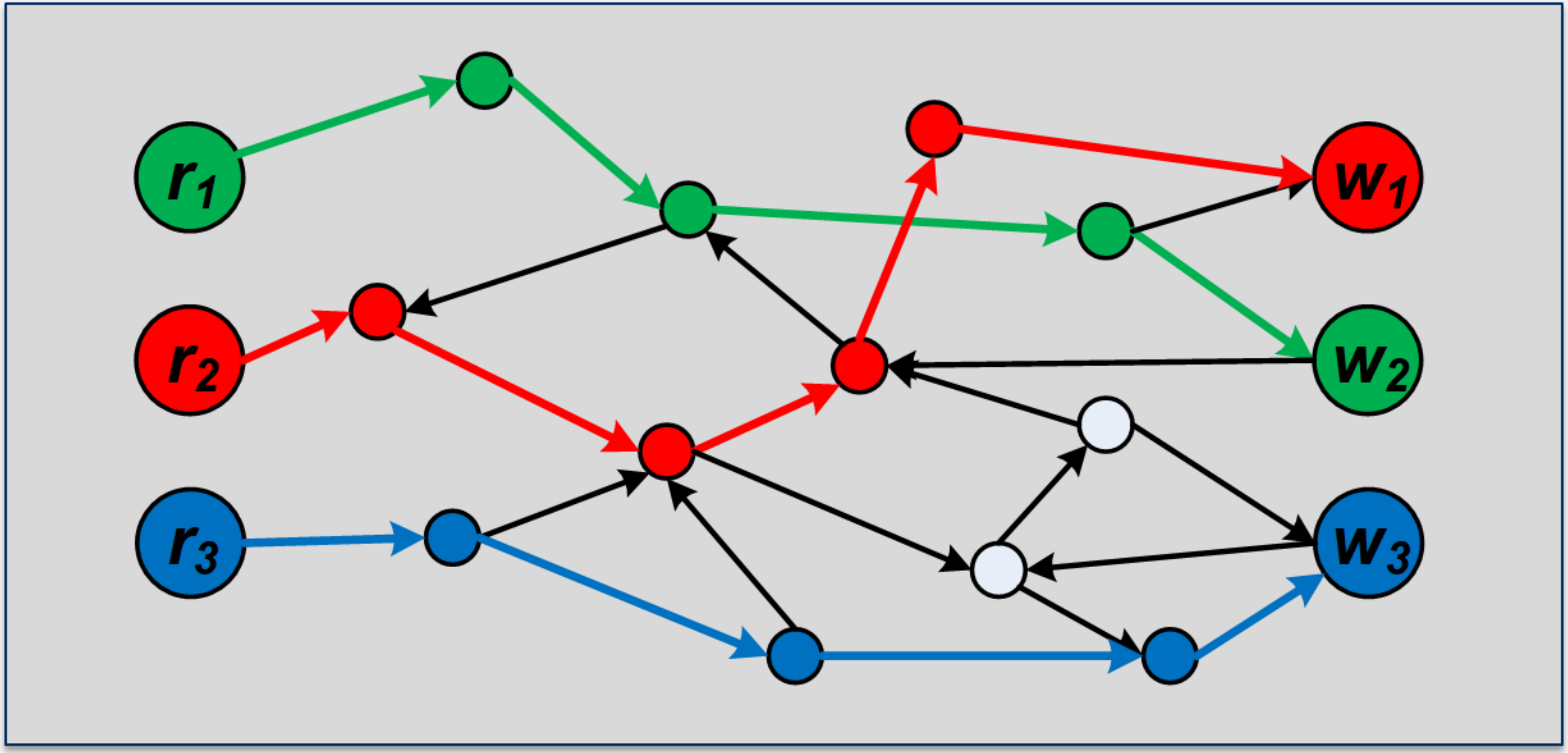
Vertex-disjoint paths



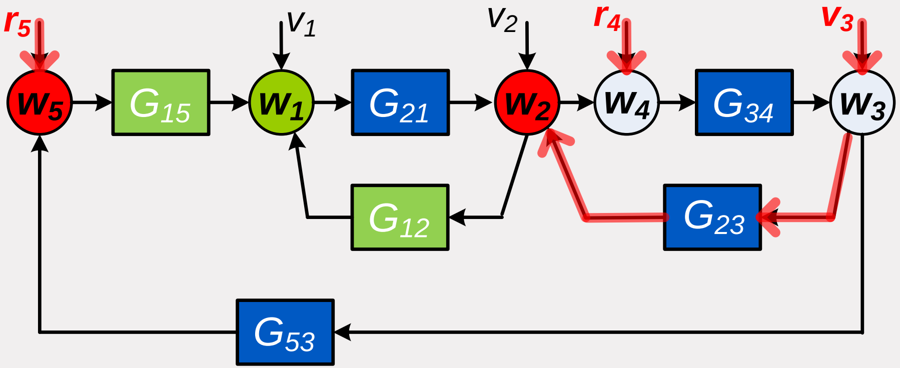
Vertex-disjoint paths



Generic rank = 3

Example 5-node network

Verifying the rank condition for $\check{T}_1(q, \theta_0)$

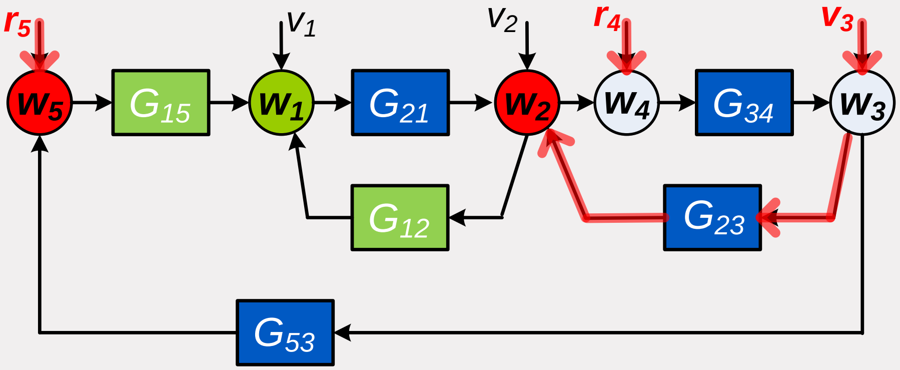


$i = 1$: Evaluate the rank of the transfer matrix $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix}$ to $\begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

2 vertex-disjoint paths → full row rank 2

Example 5-node network

Verifying the rank condition for $\check{T}_1(q,\theta_0)$



$i=1$: Evaluate the rank of the transfer matrix $\begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix}$ to $\begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$

For each row i : $\boxed{\text{\# unknown modules } G_{ik}(q,\theta) \leq \text{\# external signals uncorrelated with } v_i}$

Summary identifiability

Identifiability of network model sets is determined by

- Presence and location of external signals, and
- Correlation of disturbances
- Prior knowledge on modules

So far:

- All node signals assumed to be measured
- Fully applicable to the situation $p < L$ (i.e. reduced-rank noise)
- Identifiability of the full network model – conditions per row/output node
- Extensions towards identifiability of a single module [1],[2]

[1] Hendrickx, Gevers & Bazanella, CDC 2017, IEEE-TAC 2019
[2] Weerts et al., CDC 2018

Extensions - Discussion

Extensions - Discussion

- **Identification algorithms to deal with reduced rank noise** [1]
  - number of disturbance terms is larger than number of white sources
  - Optimal identification criterion becomes a constrained quadratic problem with ML properties for Gaussian noise
  - Reworked Cramer Rao lower bound
  - Some parameters can be estimated variance free
- **Including sensor noise** [2]
  - Errors-in-variabels problems can be more easily handled in a network setting

[1] Weerts et al., Automatica, December 2018.
[2] Dankers et al., Automatica, 2015.

Extensions - Discussion

- **Machine learning tools for estimating large scale models** [1,2]
  - Choosing correctly parametrized model sets for all modules is impractical
  - Use of Gaussian process priors for kernel-based estimation of models

- **From centralized to distributed estimation (MISO models)** [3]
  - Communication constraints between different agents
  - Recursive (distributed) estimator converges to global optimizer (more slowly)

[1] Everitt et al., Automatica, 2018.
[2] Ramaswamy et al., CDC 2018.
[3] Steentjes et al., IFAC-NECSYS, 2018.

Discussion

- **Dynamic network identification:**
  intriguing research topic with many open questions
- The (centralized) LTI framework is only just the beginning
- Further move towards data-aspects related to distributed control
- As well as to physical networks

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Co-authors, contributors and discussion partners:

Arne Dankers   Harm Weerts

Xavier Bombois
Peter Heuberger
Donatelllo Materassi
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Michel Gevers
Jonas Linder
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Hakan Hjalmarsson
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Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
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- P.M.J. Van den Hof, K.R. Ramaswamy, A.G. Dankers and G. Bottegal. Local module identification in dynamic networks with correlated noise: the full input case. Submitted to 2019 ACC. ArXiv 1809.07502.

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The end