Local identification in dynamic networks using a multi-step least squares method

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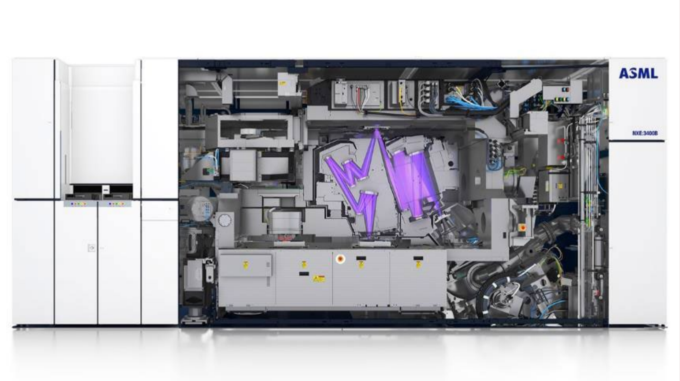
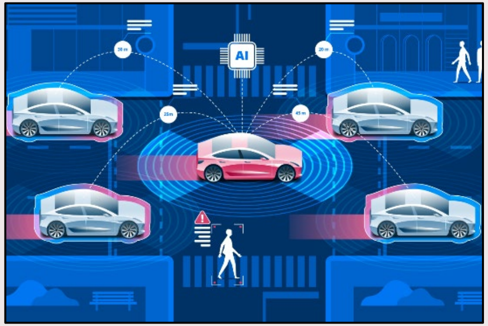
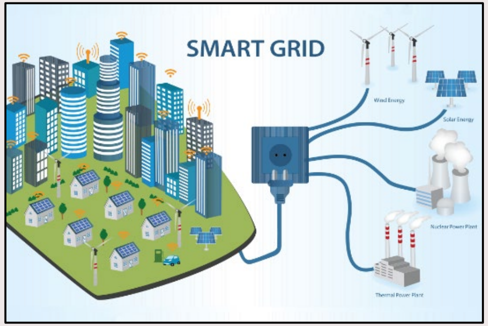
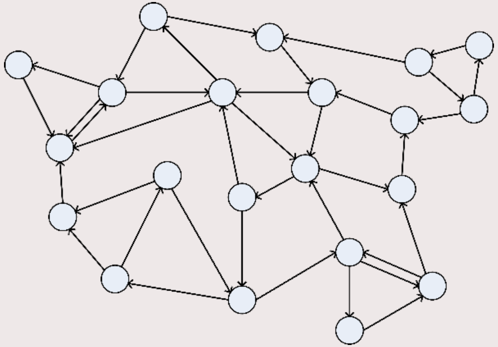
Motivation – Modern Engineering







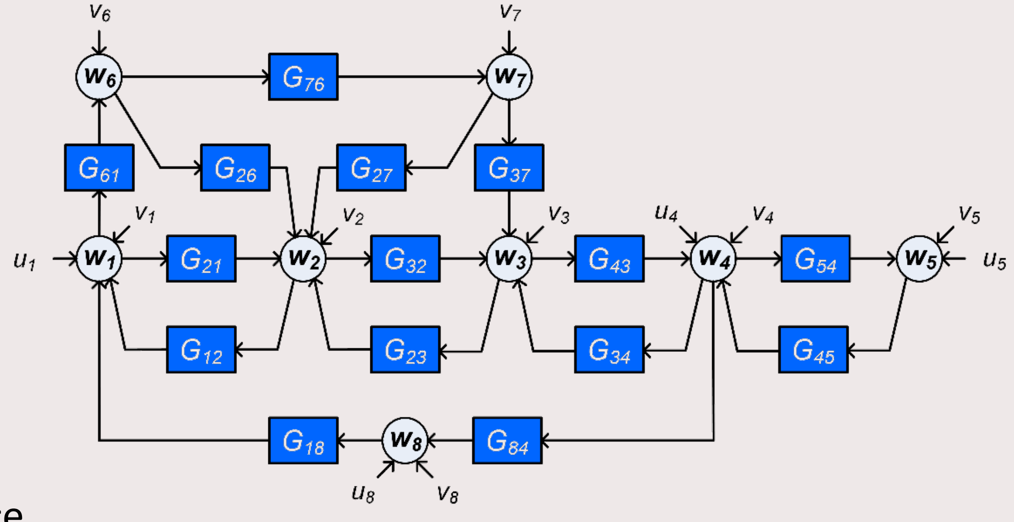


asml.com, smartcitiesworld.net, innovationatwork.ieee.org

Dynamic networks

Dynamic network framework [1]

- $w(t)$: Nodes
- G_{ji}^{0} : Modules
- External signals
  - $u(t)$: Excitation
  - $v(t) = H^0(q)e(t)$: Disturbance

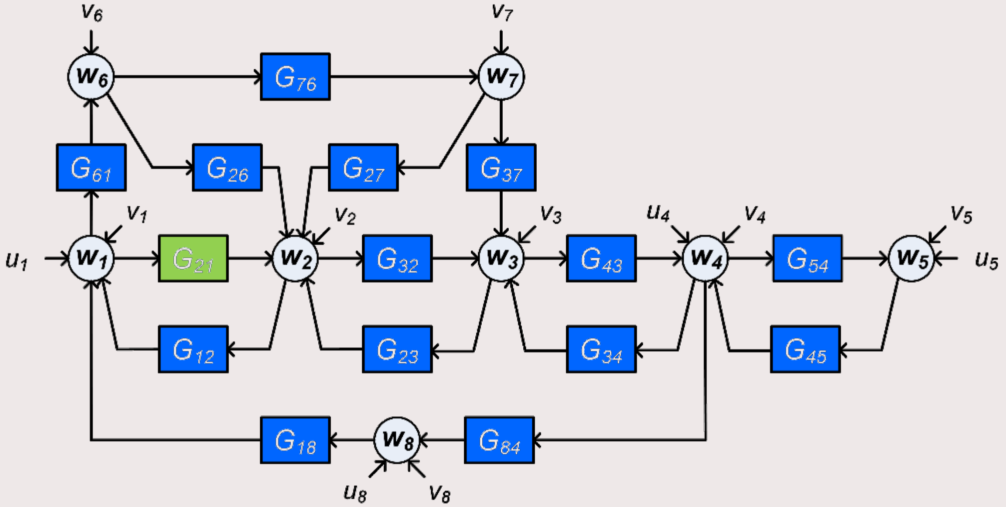


$$w = G^0 w + u + H^0 e$$

[1] Van den Hof, et al., Automatica, 2013.

Challenges in Dynamic networks

- Topology [1]
- Identifiability [2]
- Identification [3]
  - Full network
  - **Local identification**



[1] Materassi, et al., TAC 2010, TAC 2012. Chiuso et al., Automatica 2012. Shi et al., ECC, 2019
[2] Goncalves, Warnick, TAC, 2008. Weerts et al., IFAC, 2015. Bazanella et al., CDC 2017. Weerts et al., Automatica 2018. Hendrickx et al., TAC 2019, Gevers et al., TAC, 2019. Shi et al., Automatic 2022, TAC 2023.
[3] Van den Hof et al., Automatica, 2013. Materassi et al., CDC, 2015. Dankers et al., Automatica, 2016. Galrinho et al., IFAC, 2018. Weerts et al., IFAC 2018, Automatica 2018. Ramaswamy et al., TAC, 2021.

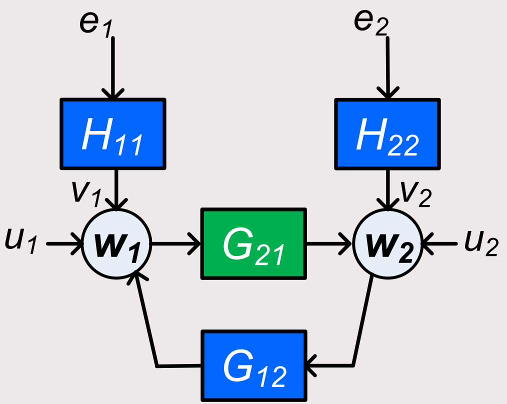
Content

Local identification

Single module identifiability [1]

- Method independent
- Identifiability: no u-signal required

Method	Transfer		# u-signals needed
Local Direct [2]	$w \to w$	Consistency & ML	0
Indirect[3]	$u \to w$	Consistency	1



[1] Shi, et al. 2021, 2022, 2023
[2] Ramaswamy, et al., TAC, 2021.
[3] Gevers, et al., IFAC, 2018. Hendrickx, et al., TAC, 2019. Bazanella, et al., CDC, 2019.

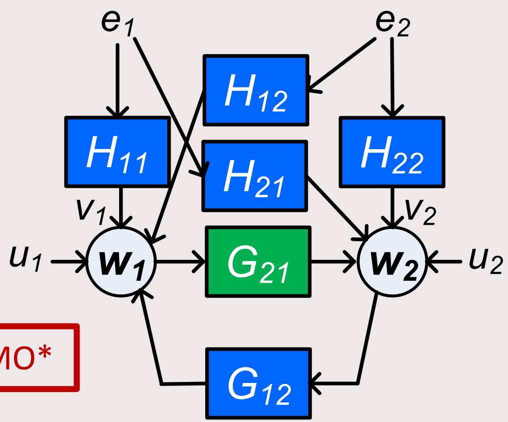
Local identification – confounding variables

Single module identifiability

- Method independent
- Identifiability: one u-signal required

Method	Transfer		# u-signals needed
Local Direct [1]	$w \to w$	Consistency & ML	2 Conservative
Indirect	$u \to w$	Consistency	1

MIMO*



*MIMO: Multivariate noise model to model confounding variables as correlated noise

[1] Ramaswamy, et al., TAC, 2021. Van den Hof, et al., 2017,2020,2023.

Research question

How can we obtain relaxed conservatism compared to current direct methods in local identification?

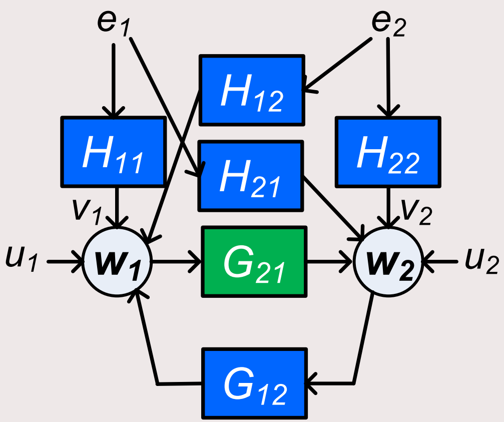
- Requires fewer u-signals
- Keep advantages of the current direct methods

Local identification – confounding variables

Single module identifiability

- Method independent
- Identifiability: one u-signal

Method	Transfer		# u-signals needed
Local Direct	$w \rightarrow w$	Consistency & ML	2
Indirect	$u \rightarrow w$	Consistency	1
Multi-step Least squares [1]			1



Full network identification

[1] Fonken, et al., Automatica, 2022.

Multi-step method [1,2]

Full network identification $w = Gw + He + u$

1. High order ARX model to reconstruct the innovation

Estimate $\underline{u \rightarrow w}$ with high-order ARX and reconstruct $\hat{e}$ ← Indirect method

$$w = Gw + (H - I)e + Ie + u$$

$$w = Gw + (H - I)\hat{e} + Ie + u$$

→ No confounding variables

2. Parametric target module estimate

Use $\hat{e}$ as additional measured input

$$w_j = \sum_k \underline{G_{jk} w_k} + \sum_\ell \underline{(H_{j\ell} - I_{jj})\hat{e}_\ell} + e_j + u_j$$

← Direct method

Estimate MISO models for each row $j \in \mathcal{L}$ using Weighted Null Space Fitting[3] → convex

[1] Fonken, et al., Automatica, 2022
[2] Dankers, et al., technical note, 2019
[3] Galrinho, et al., TAC, 2019.

TU/e

Local Identification using a multi-step method

1. Selection of nodes
2. High order ARX model to reconstruct the innovation signal
3. Parametric target module estimate

Data Informativity:

sufficient number of u-signals?

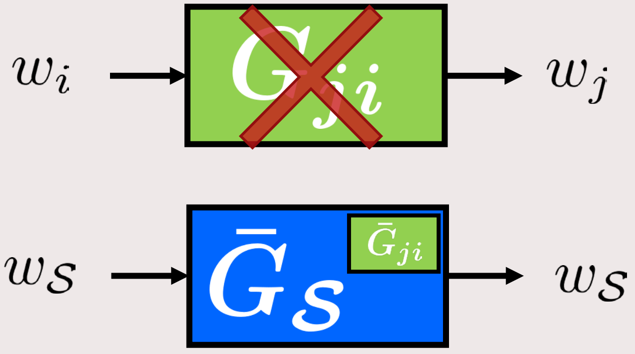
Local Identification – 1. Selection of nodes

Select nodes

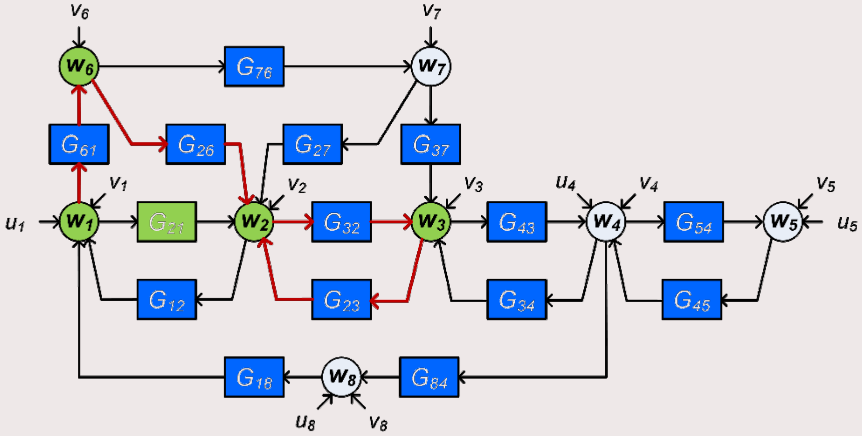
Immerse [1]

w_k Block parallel paths and loops around the output [1]

w_k



$$\bar{G}_{ji} = G_{ji}$$ Module invariance



[1] Dankers, et al., Automatica, 2016

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Local identification [1,2] – 1. Selection of nodes

Decompose $w_{\mathcal{S}}$ in[1]

$j \in \mathcal{Y}$

$$\left[\begin{array}{c} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{array}\right] = \begin{bmatrix} \bar{G} \\ \bar{G}_{\mathcal{U}} \end{bmatrix} \left[\begin{array}{c} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{array}\right] + \left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}} \end{array}\right] \left[\begin{array}{c} \xi_{\mathcal{Y}} \\ \hline \xi_{\mathcal{U}} \end{array}\right] + \left[\begin{array}{c} u_{\mathcal{Y}} \\ \hline u_{\mathcal{U}} \end{array}\right]$$ [2]

→ No confounding variables $w_{\mathcal{U}} \to w_{\mathcal{Y}}$

- Confounding variable → Correlated noise

$$w_{\mathcal{Y}} = \bar{G} \left[\begin{array}{c} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{array}\right] + \bar{H}\xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$

[1] Shi, et al., IFAC, 2023
[2] Ramaswamy, et al., TAC, 2021

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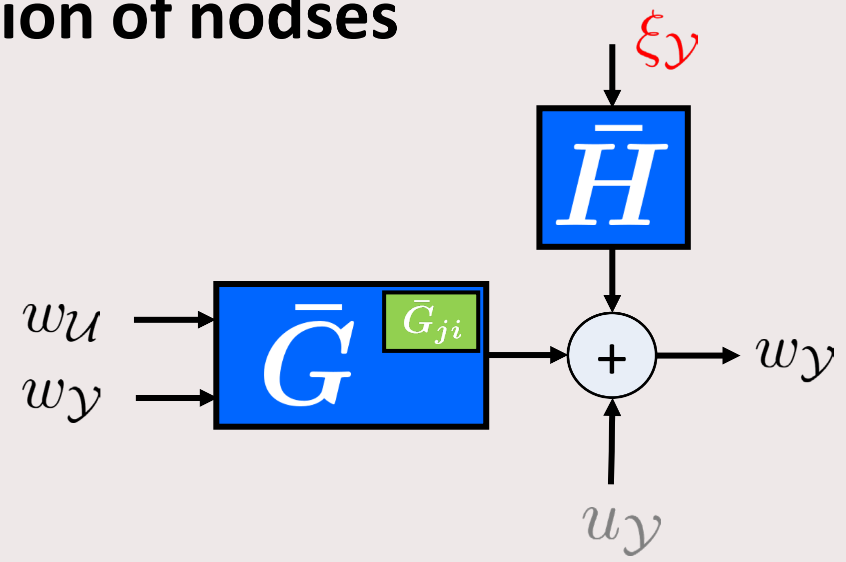
Local identification – 1. Selection of nodses

$$w_{\mathcal{Y}} = \bar{G}\begin{bmatrix} w_{\mathcal{Y}} \\ w_{\mathcal{U}} \end{bmatrix} + \bar{H}\xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$



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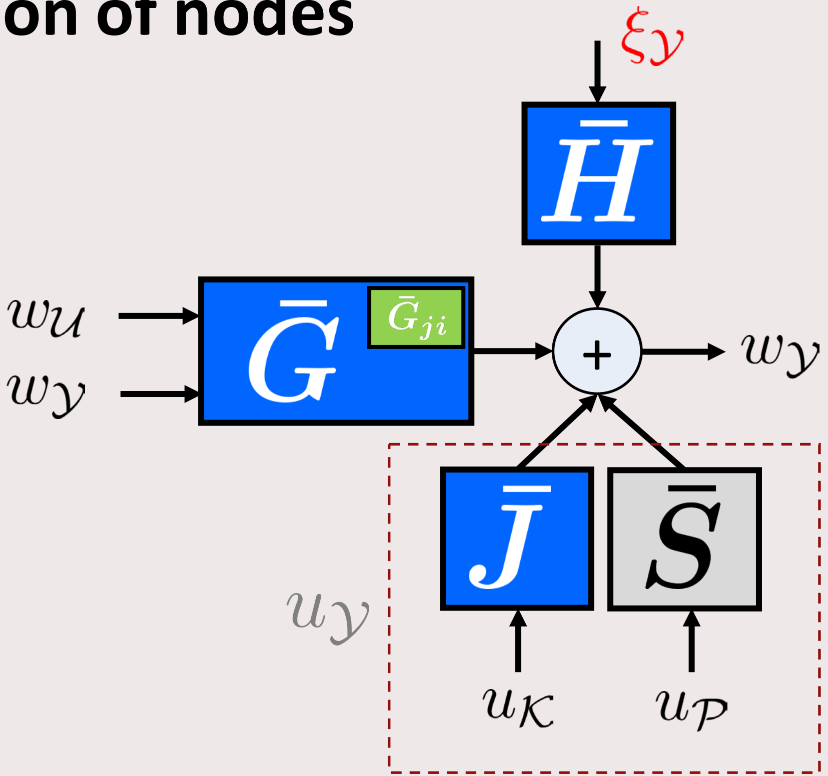
Local identification – 1. Selection of nodes

Immersed network: $u_{\mathcal{Y}}$ [1]

$$\underbrace{u_{\mathcal{Y}} = \bar{J}(q)u_{\mathcal{K}}}_{\text{Dynamic}} + \underbrace{\bar{S}u_{\mathcal{P}}}_{\text{Known}}$$



[1] Ramaswamy, et al., Automatica, 2021

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Local identification

$$w_{\mathcal{Y}} = \bar{G}(q)\begin{bmatrix} w_{\mathcal{Y}} \\ w_{\mathcal{U}} \end{bmatrix} + \bar{H}(q)\xi_{\mathcal{Y}} + \bar{J}(q)u_{\mathcal{K}} + \bar{S}u_{\mathcal{P}}$$

2. High order ARX model to reconstruct the innovation

Estimate $u_{\mathcal{K}\cup\mathcal{P}} \rightarrow w$ with high-order ARX and reconstruct $\hat{\xi}_{\mathcal{Y}}$

3. Parametric target module estimate

Use $\hat{\xi}_{\mathcal{Y}}$ as additional measured input

$$w_j = \sum_{k\in\mathcal{N}_j^-} \bar{G}_{jk}w_k + \sum_{\ell\in\mathcal{Y}}(\bar{H}_{j\ell} - I_{jj})\hat{\xi}_\ell + \xi_j + \sum_{m\in\mathcal{K}_j} \bar{J}_{jm}u_m + u_{\mathcal{P}_j}$$

Estimate the j-th MISO model $j \in \mathcal{Y}$

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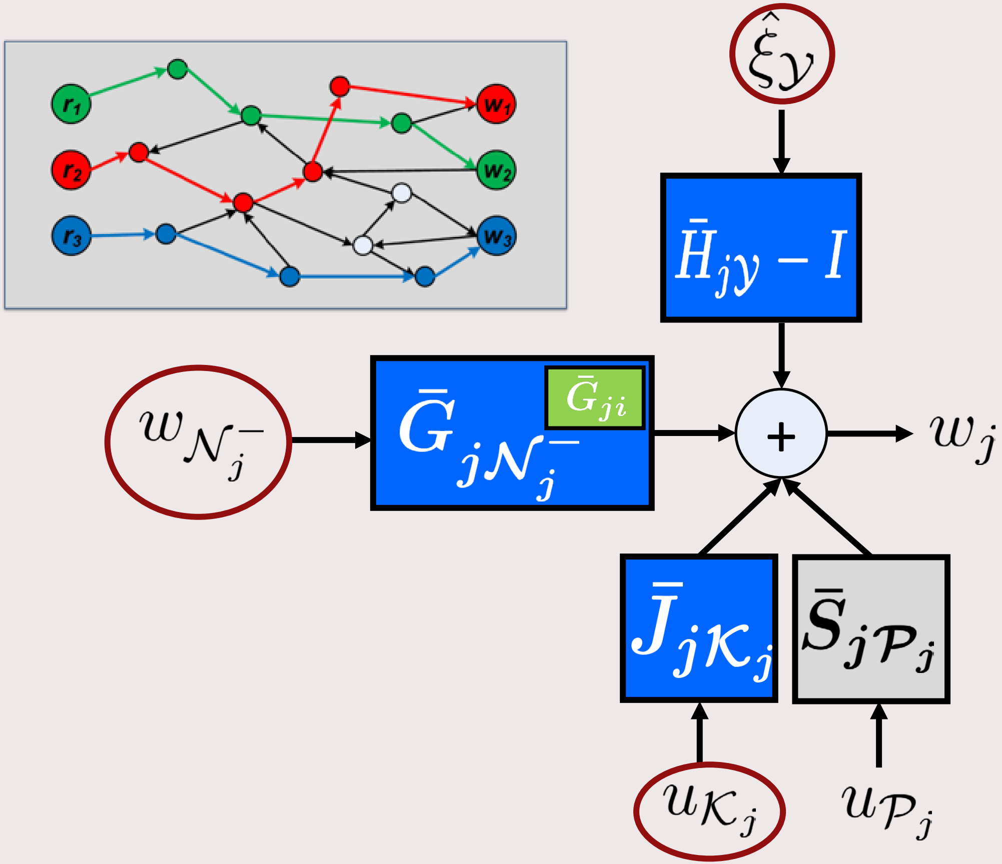
Local identification

Data informativity [1]

$$\Phi_\kappa(\omega) \succ 0 \quad \kappa = \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}_j} \end{bmatrix}$$

Data informativity holds generically if there are $dim(\kappa)$ vertex disjoint paths from

$$\begin{bmatrix} \xi_{\mathcal{S}} \\ u_{\mathcal{L}} \end{bmatrix} \rightarrow \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}_j} \end{bmatrix}$$



[1] Van den Hof et al. (2020, 2023), Hendrickx, et al. (2019)

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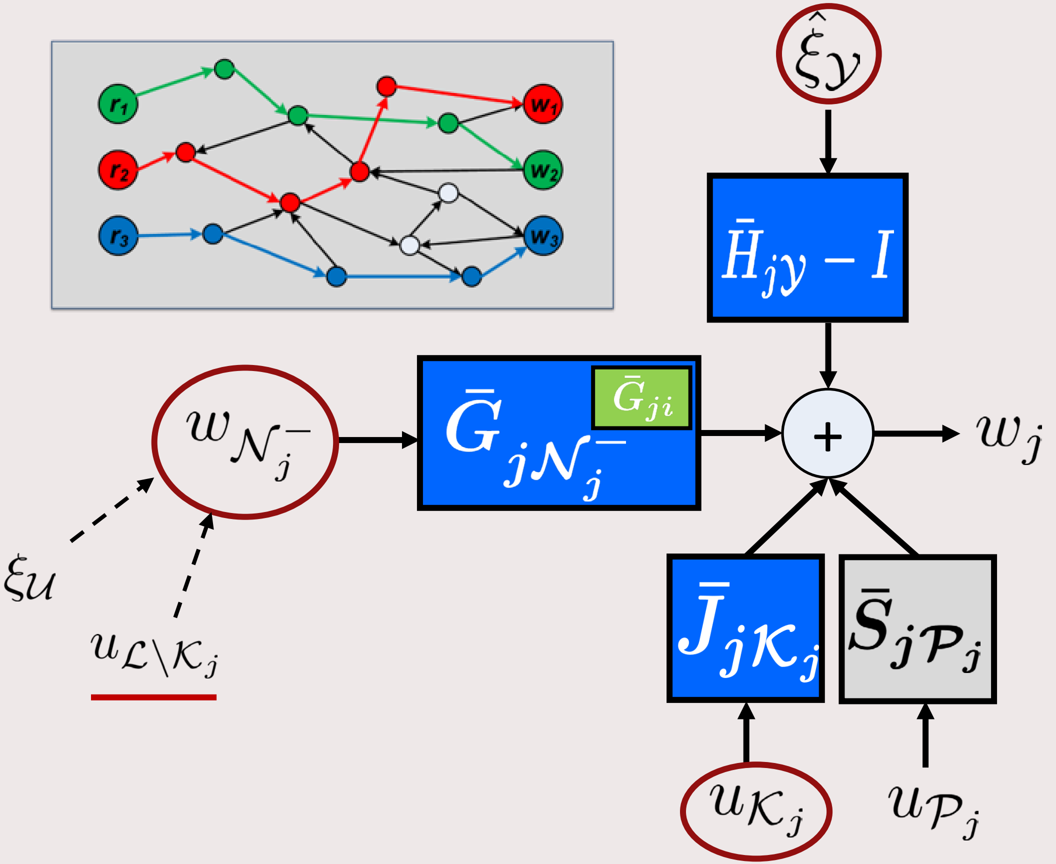
Local identification

Data informativity [1]

$$\Phi_{\kappa}(\omega) \succ 0 \quad \kappa = \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}_j} \end{bmatrix}$$

Data informativity holds generically if there are $dim(w_{\mathcal{N}_j^-})$ vertex disjoint paths from

$$\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\backslash\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-}$$



[1] Van den Hof et al. (2020, 2023), Hendrickx, et al. (2019)

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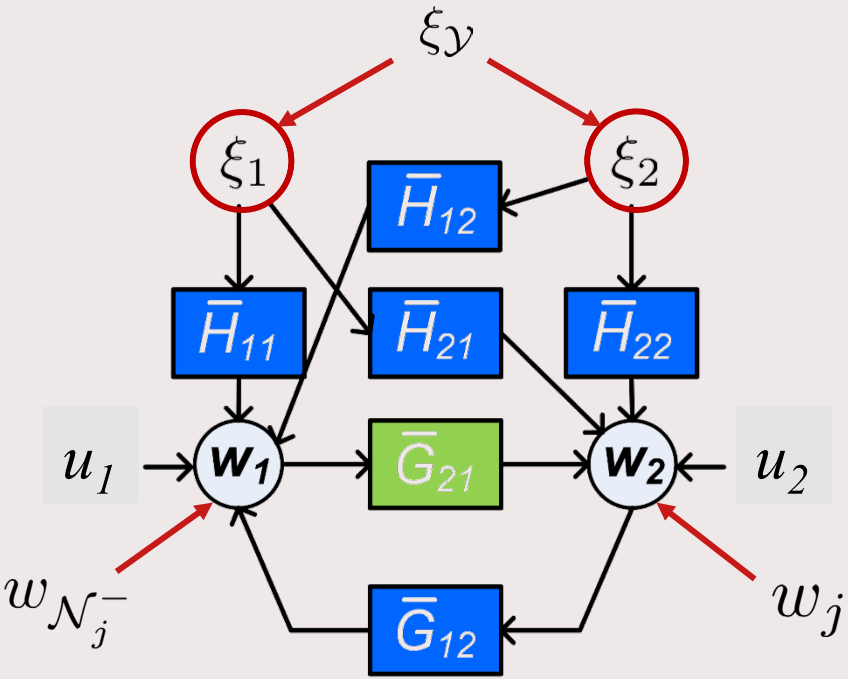
Local identification



$$\mathcal{Y} = \{1,2\}, \mathcal{U} = \emptyset$$

$$u_{\mathcal{L}\setminus\mathcal{K}_j} = u_1, u_2 \quad \notin \mathcal{K}_j = \emptyset$$

$$\Phi_\kappa \succ 0$$

$$\left.\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\setminus\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-} \right\} \quad \begin{bmatrix} \emptyset \\ u_1, u_2 \end{bmatrix} \rightarrow w_1$$

w_1 is persistently exciting holds generically if there is 1 vertex disjoint paths from u_1 or $u_2 \rightarrow w_1$

Local identification

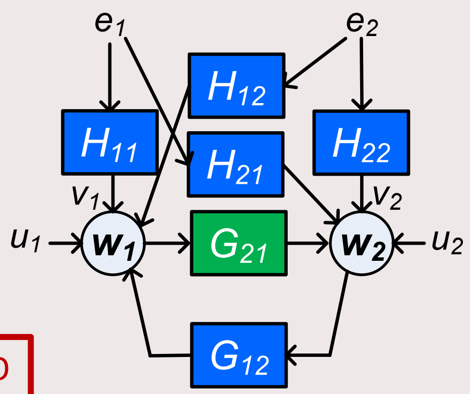
Single module identifiability

- Method independent
- Identifiability: 1 u-signal

Method	Transfer		# u-signals needed
Local Direct	$w \rightarrow w$	Consistency & ML	2
Indirect	$u \rightarrow w$	Consistency	1
Multi-step Least squares	1. Indirect 2. Direct	Consistency & ML?	1

MIMO

MISO



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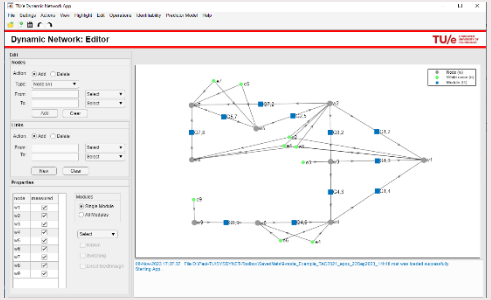
Conclusion

Combining indirect and direct methods:

- Requires fewer u-signals than current direct method
- Keeps advantages of the current direct methods
- Parametric estimation with Weighted Null Space Fitting → Convex

Available in Toolbox www.*sysdynet.net*



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