Challenges in System Identification

From closed-loop to dynamic network identification

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Where innovation starts

Introduction

Model-based design and optimization
is de dominant paradigm in the
design – operation – maintenance
of complex engineering systems

- **Interacting robotics**
- **Smart (energy) grids**
- **Intelligent transportation systems**
- **Nano-positoning systems**
- **…….**

Introduction

Future Requirements:

Handling of highly complex
interacting
distributed **systems that**

operate autonomously
in a changing environment
with changing objectives
in a ``learning'' mode
adapting to changing circumstances

and maintain a verifiable high performance

Introduction

Some required capabilities of models

1. **Accuracy assessment**
   on-line assessment of model validity
2. **Adaptability**
   flexible on-line updating of models (dynamics and interconnection structure)
3. **Active data-driven learning**
   demands on accuracy, autonomy, robustness
   → active probing for information

all relating to phenomena of data-driven modeling

Data-driven modeling becomes an integral part in virtually all complex engineering systems

Introduction

Example of current limitations:

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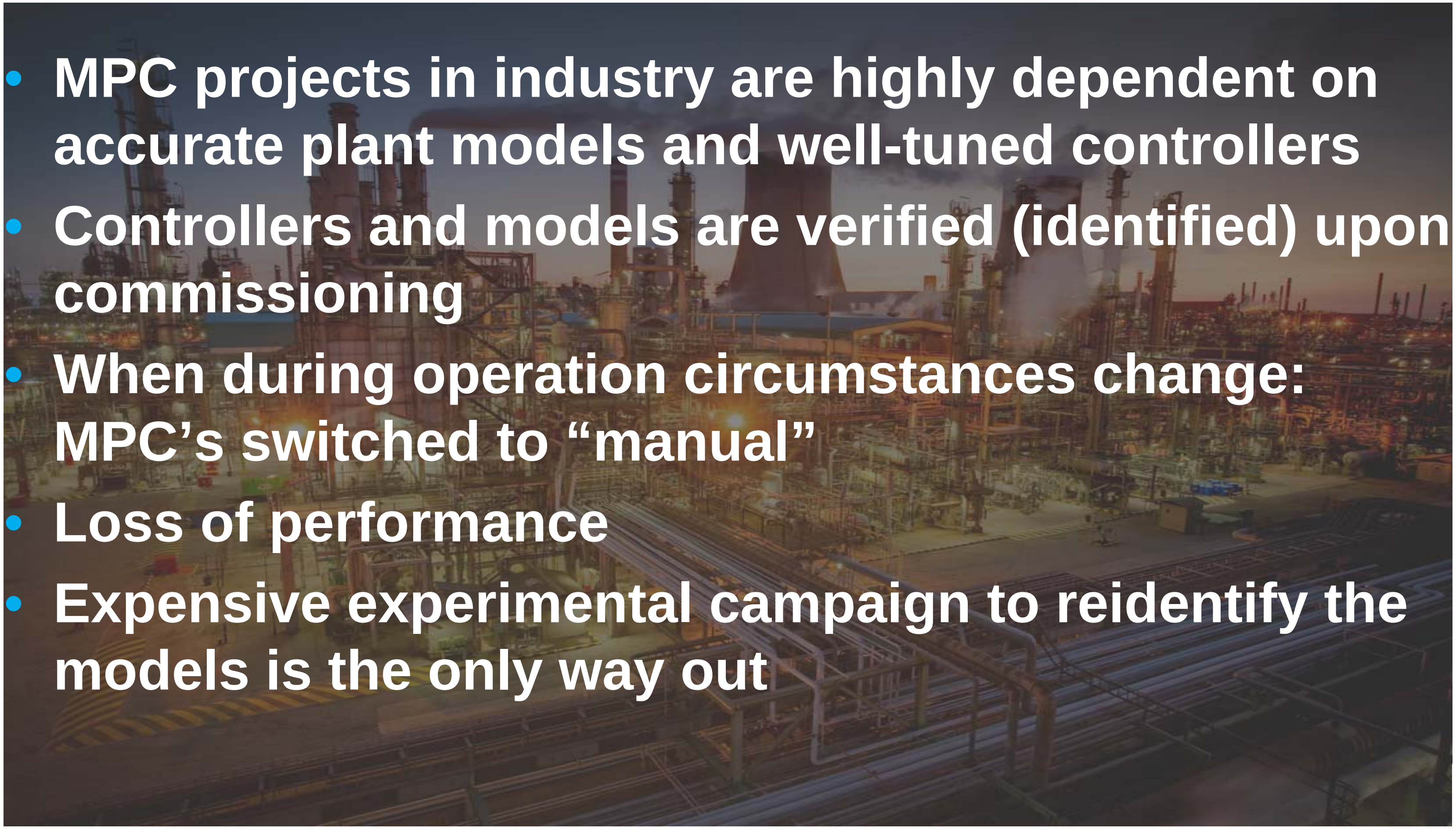
Introduction

Example of current limitations:



- MPC projects in industry are highly dependent on accurate plant models and well-tuned controllers
- Controllers and models are verified (identified) upon commissioning
- When during operation circumstances change: MPC’s switched to “manual”
- Loss of performance
- Expensive experimental campaign to reidentify the models is the only way out

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Introduction

Next step in the development:

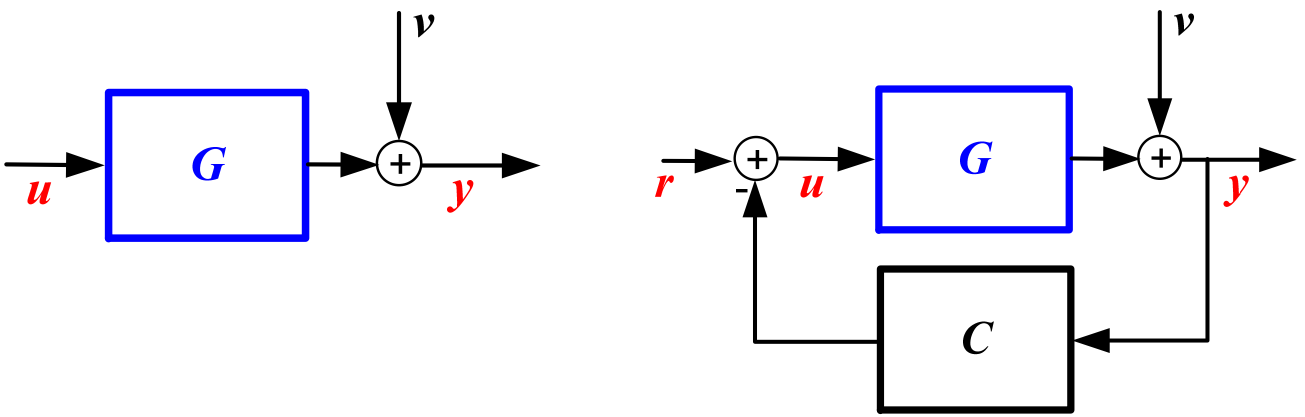
- **Bring plant operation / automation to higher level of autonomy**
- **Monitor plant performance and detect changes on-line**
- **Generate probing signals when necessary and based on economic considerations (least costly experiments)**
- **Reidentify models and retune controllers on-line**
- **Keep high performance control**
- **Use economic performance criteria**

Autoprofit

Autonomous economic model-based operation of industrial process systems

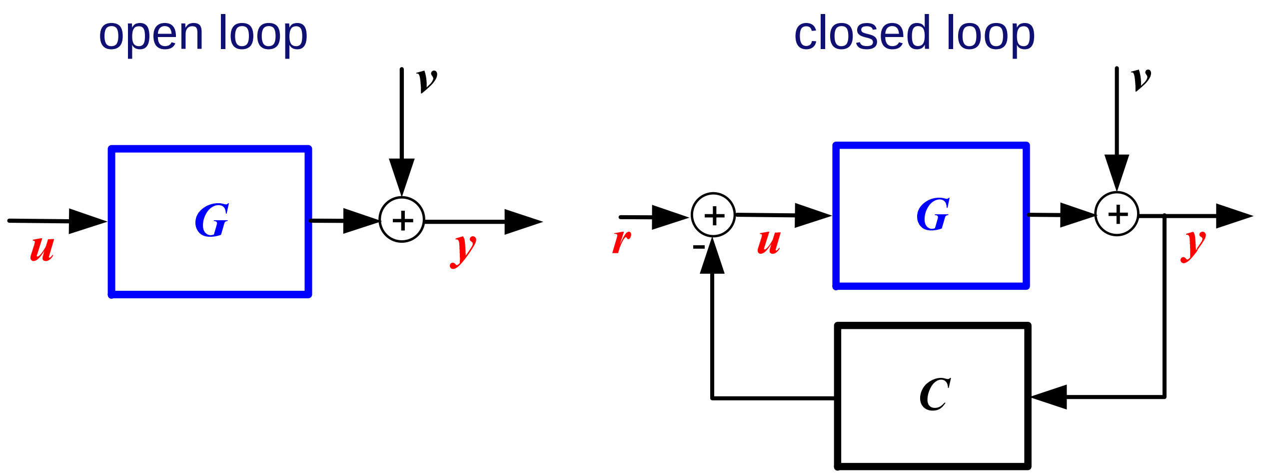
Introduction

Back to the core of the problem of data-driven modelling / identification of LTI models



Introduction

The classical identification problems:



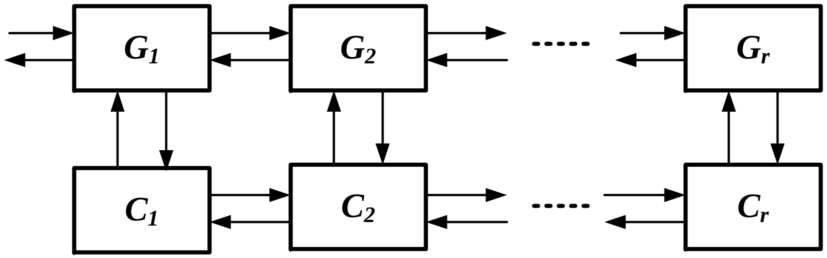
Identify a plant model $\hat{G}$ on the basis of measured signals *u, y* (and possibly *r*)

- **Several classical methods available (PE, subspace, nonparam,..)**
- **Well known results for identification *in known structure* (open loop, closed-loop, possibly known controller)**

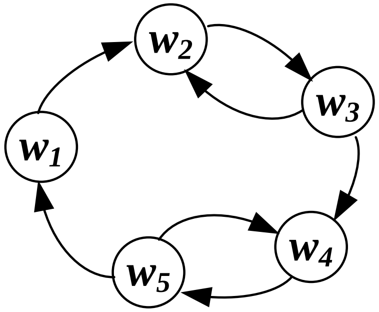
Introduction

Dynamical systems in emerging fields have a more complex structure:

distributed control system



dynamic network



(distributed systems, multi-agent systems, biological networks, smart grids,…..)

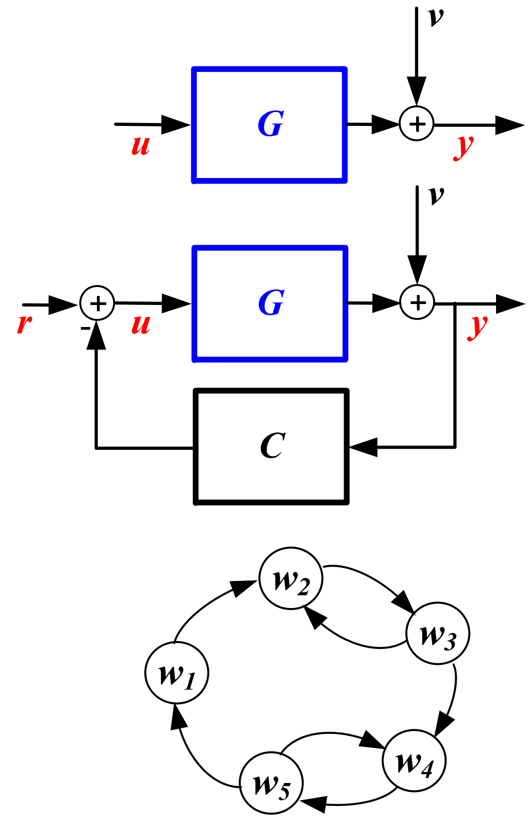
Questions to be adressed:

- **How to identify ``single'' transfers in a known (complex) structure?**
- **Can currently available tools from (closed-loop) identification be used for this purpose?**
- **How to identify the structure?** **(not focussed on here)**

Contents

From open-loop and closed-loop identification to dynamic network identification

- Methods for (classical) closed-loop ID
- Dynamic network setup
- Network identification – Direct method
- Network identification – Two-stage method
- Illustrative example
- Two-stage method: generalizations
- Discussion



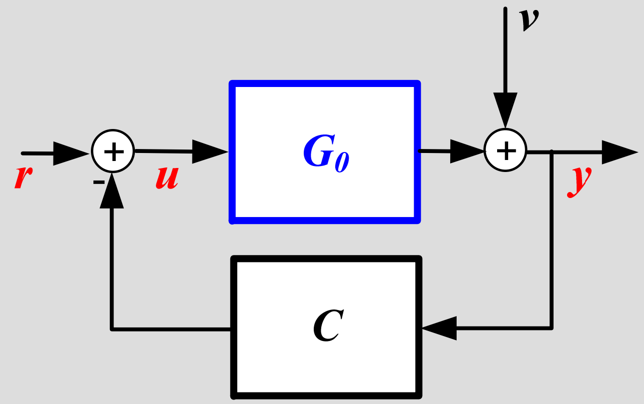
Closed-loop identification

Methods for closed-loop identification:

- **Relying on full-order noise modelling (direct method)**
  $(\mathcal{S} \in \mathcal{M})$

or:

- **Relying on external excitation (indirect, two-stage, projection, IV)**
  $(G_0 \in \mathcal{G})$



Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

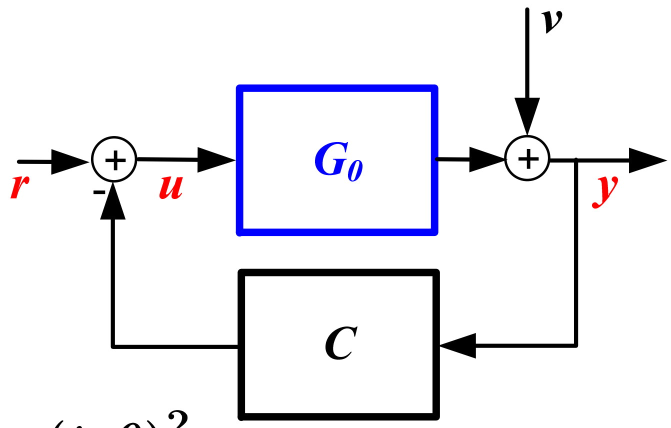
e white noise

r and v uncorrelated

Closed-loop identification – Direct method

Model parametrization:

plant model $G(\theta)$, noise model $H(\theta)$



Prediction error:

$$\varepsilon(t,\theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)]$$

Parameter estimate: $\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t,\theta)^2$

Under weak regularity conditions: $\hat{\theta}_N \to \theta^*$ for $N \to \infty$ w.p. 1

The plant model is consistently estimated ($G(\theta^*) = G_0$) if:

- **Plant and noise model are correctly parametrized** $(\mathcal{S} \in \mathcal{M})$
- **The feedback loop has at least one delay (no algebraic loop)**
- **For $z := vec(y,u)$ the power spectral density $\Phi_z(\omega) > 0 \quad \forall\omega$**

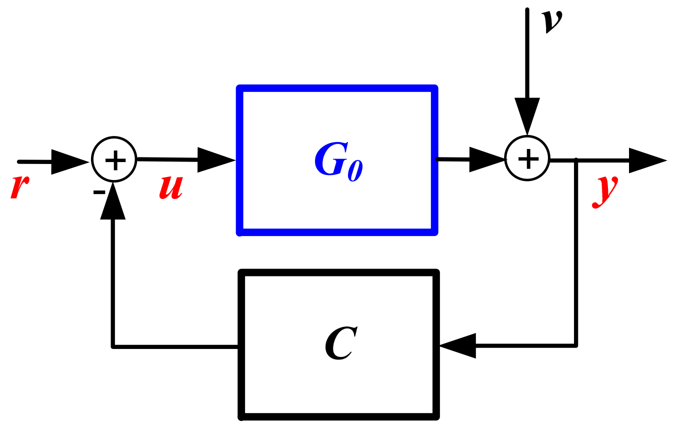
[Ljung, 1987]

Closed-loop identification – Two-stage method

Decompose the input signal:

$$u = u^r + u^v$$

such that u^r and v are uncorrelated



Prediction error:

$$\varepsilon(t,\theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

A similar ``standard'' (LS) identification approach then leads to:

The plant model is consistently estimated ($G(\theta^*) = G_0$) if:

- **Plant model is correctly parametrized** $(G_0 \in \mathcal{G})$
- **Plant and noise model are independently parametrized**
- u^r **is persistently excitation of sufficient order, e.g.**

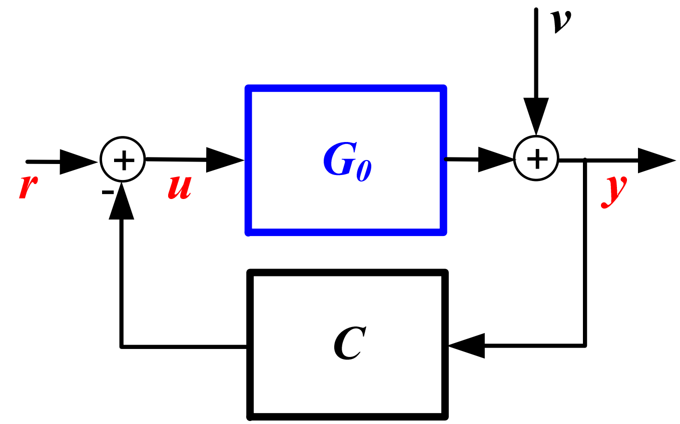
$$\Phi_{u^r}(\omega) > 0 \quad \forall\omega$$

Closed-loop identification – Two-stage method

Decompose the input signal:

$$u = u^r + u^v$$

such that u^r and v are uncorrelated



How to do this decomposition?

- **Write:** $u = F_{ur}r + F_{uv}v$
- **Identify a model $\hat{F}_{ur}$ on the basis of measured signals r and u (this is an open-loop problem)**
- **Construct $\hat{u}^r = \hat{F}_{ur}r$**
- **Use $\hat{u}^r$ as an estimate of u^r**

u^r is the projection of u onto the space of causally time-shifted versions of r

[Van den Hof & Schrama, 1993]

Closed-loop identification

Direct method

- Plant and noise model need to be identified simultaneously
- No algebraic loops

Two-stage/projection

- Plant model can be consistently identified without noise model
- External excitation is necessary

- Controller information is not required / utilized

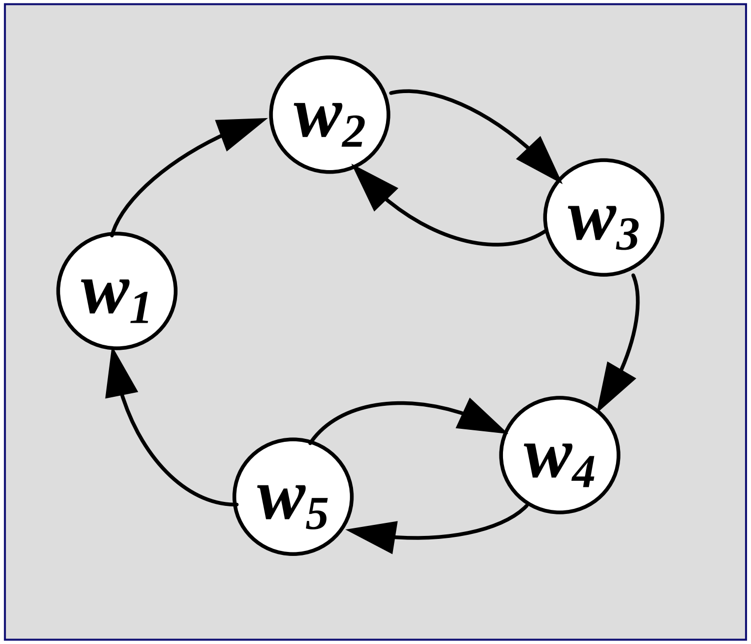
Many more variants of closed-loop PE identification methods:

- **Indirect, Joint IO** [Ljung, 1999]
- **IV methods** [Gilson&Van den Hof, Automatica, 2005]
- **Dual-Youla** [Van den Hof, Annual Rev. Control,1998]
- **Virtual Closed-Loop** [Aguero, Goodwin & Van den Hof, Automatica 2011]

see also [Forssel & Ljung, Automatica, 1999]

Question

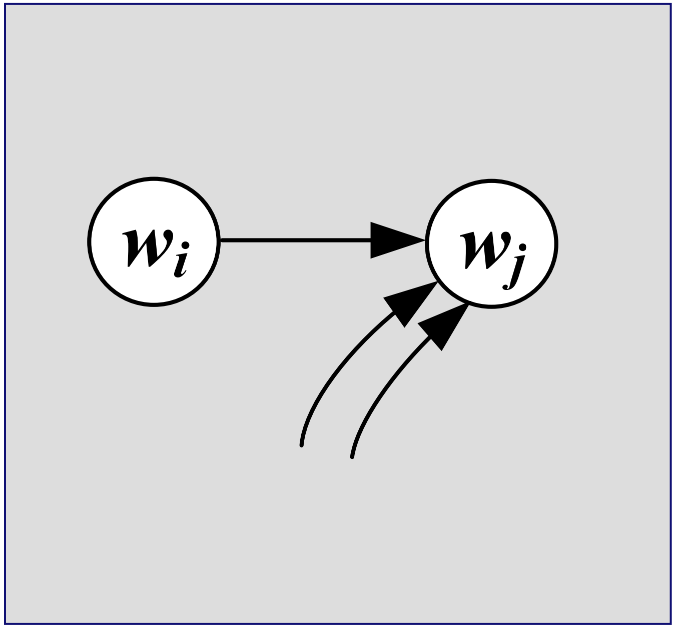
- **Can we utilize these tools for identification of transfer functions in a (complex) dynamic network ?**

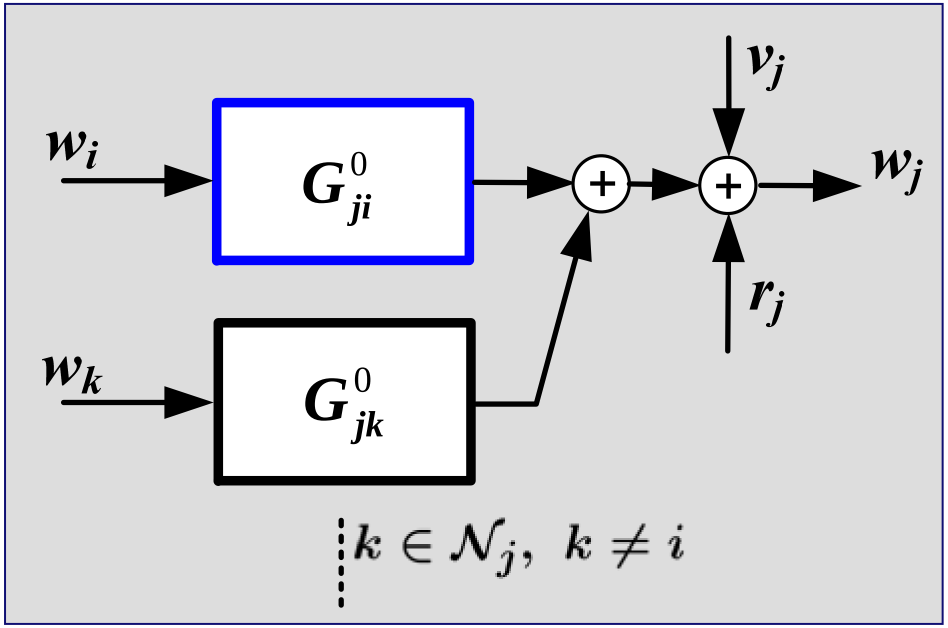


Node en link structure:
- Nodes are signals
- Directed links (edges) are causal transfers

Network Setup

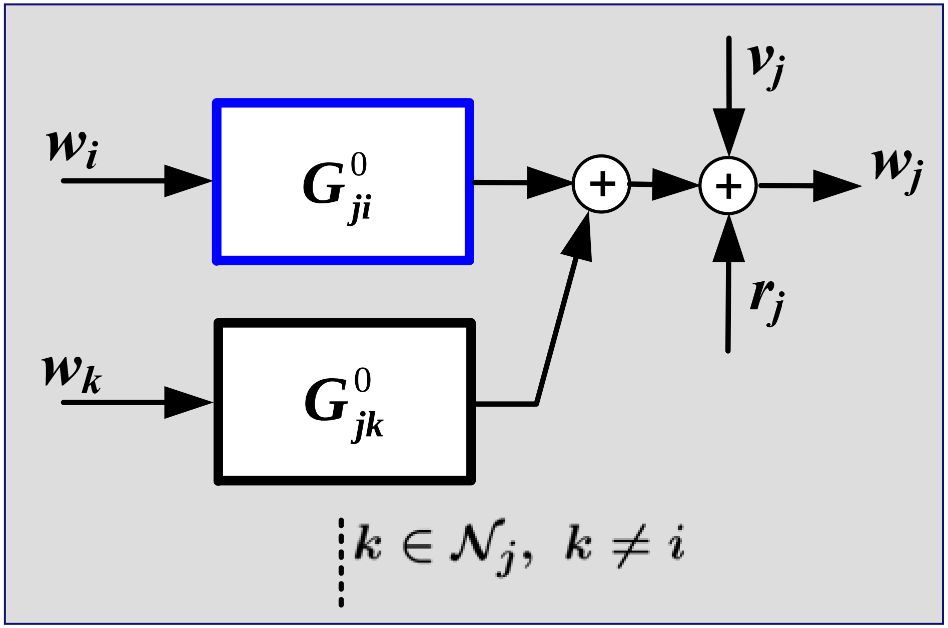
Formalizing one link (transfer between w_i and w_j)





- On each node a disturbance v_j and a reference r_j might be present
- Reference signals are uncorrelated to noise signals
- $\mathcal{N}_j$: set of nodes that has a direct causal link with node j

Network Setup

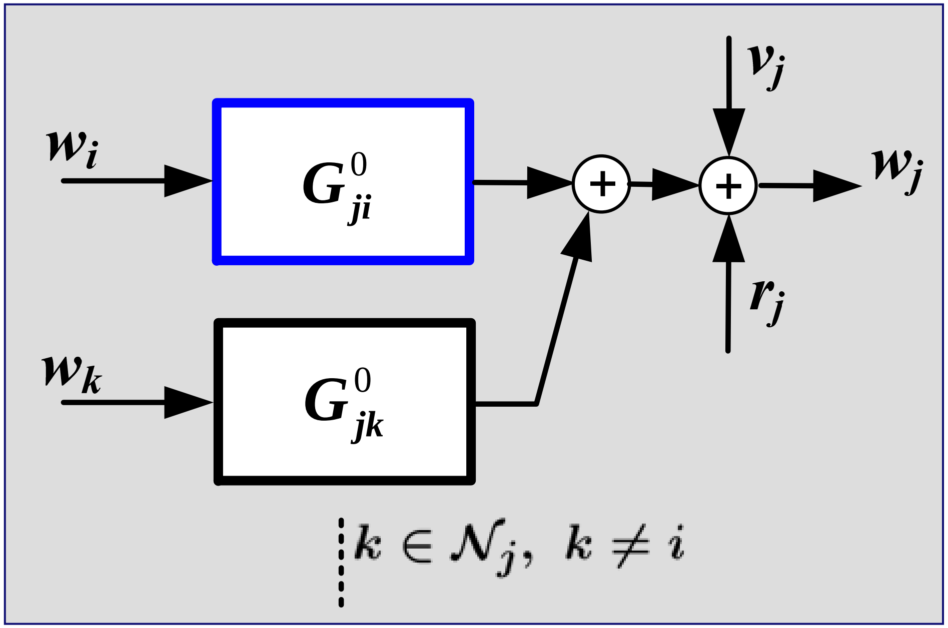


Assumptions:

- Total of L nodes
- Network is well-posed
  $I - G^0$ invertible
- Stable (all signals bounded)
- All $w_m, m = 1, \cdots L,$ measured, as well as all present r_m

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

Network Setup



Identification setting

- We focus on identifying a single transfer G^0_{ji}

- The set of nodes $\mathcal{N}_j$ is separated into
  - *i*: our plant input
  - $\mathcal{K}_j$: reflecting nodes *k* with known transfers G^0_{jk}
  - $\mathcal{U}_j$: reflecting nodes *k* with unknown transfers G^0_{jk}
  - $\mathcal{U}^i_j$: reflecting nodes *k≠i* with unknown transfers G^0_{jk}

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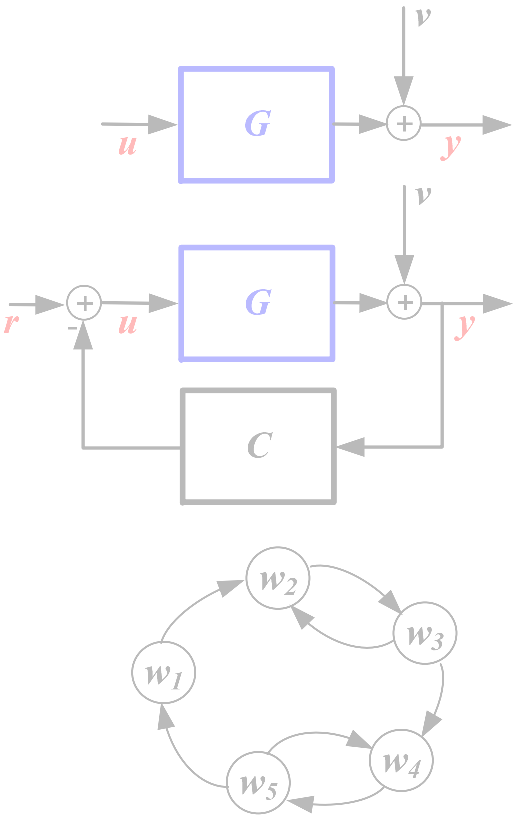

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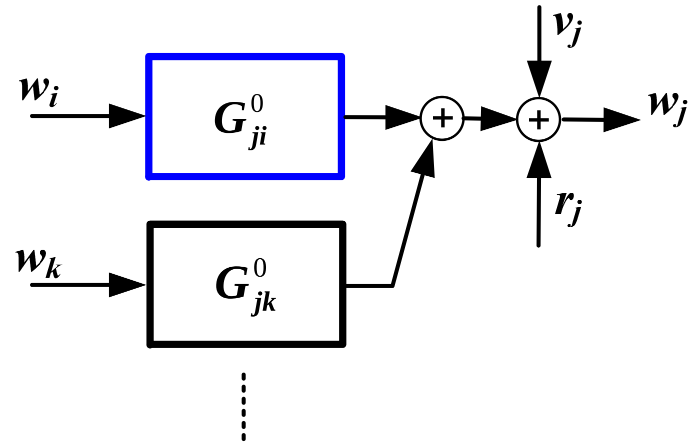
Network Identification – Direct method

Applying direct method to input w_i and output w_j will lead to biased results

- if the prediction error can not be whitened, or equivalently
- If there are nodes in $\mathcal{U}_j^i$ that affect w_j



A MISO approach:

$$\varepsilon(t,\theta) = H_j(\theta)^{-1}[\underbrace{w_j - r_j - \sum_{k\in\mathcal{K}_j} G_{jk}^0 w_k}_{\tilde{w}_j \text{ known}} - G_{ji}(\theta)w_i - \sum_{k\in\mathcal{U}_j^i} G_{jk}(\theta)w_k]$$

→ **Simultaneous identification of transfers $G_{jk}^0,\ k\in\mathcal{U}_j$ and a noise model for v_j**

Network Identification – Direct method

Result direct method

The plant model $G_{ji}(\theta)$ is consistently estimated ($G_{ji}(\theta^*) = G^0_{ji}$) if:

- **All parametrized plant and noise models are correctly parametrized, $G_{jk}(\theta),\ k \in \mathcal{U}_j;\ \ H_j(\theta)\ \ (\mathcal{S} \in \mathcal{M})$**
- **Every loop in the network that runs through node *j* has at least one delay (no algebraic loop)**
- **$\Phi_z(\omega) > 0 \ \ \forall\omega$, for $z := vec\{w_j, \{w_k\}_{k\in\mathcal{U}_j}\}$ (excitation condition)**
- **Noise source v_j is uncorrelated with all other noise terms in the network**

[Dankers et al., CDC2012 submitted]

Network Identification – Direct method

Observation:

- **Direct identification ``works''**
- **Requires full noise models (whitened residual)**
- **Require a MISO approach (more transfers to be simultaneously identified)**
- **and a "strict" excitation condition**

Next question:

- **Can we solve the problem without the full noise models and without the MISO approach?**

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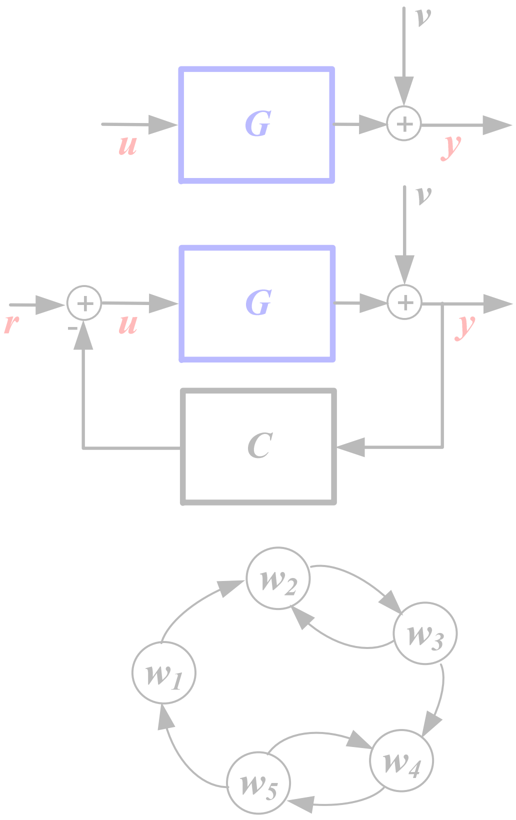
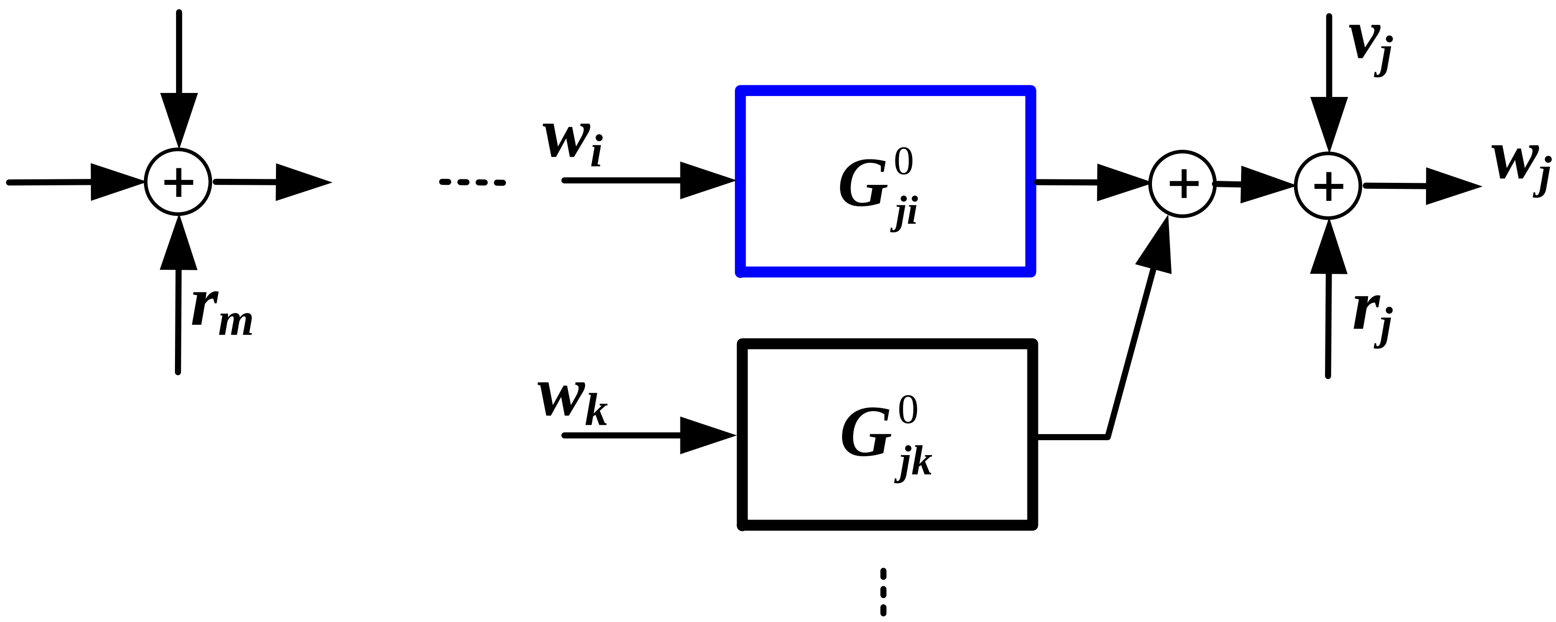


Network Identification – Two-stage method

Main approach:

- Look for an external reference signal that has a connection with w_i
- And that does not act as an unmodelled disturbance on w_j

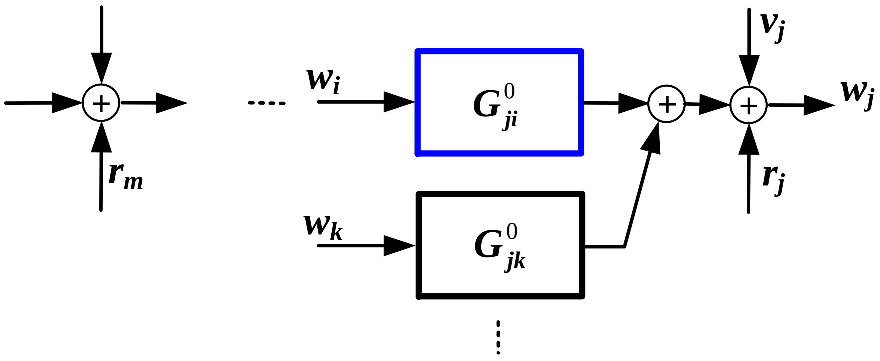


Network Identification – Two-stage method

Algorithm:

- **Determine whether there exists an r_m such that $w_i^{r_m}$ is sufficiently exciting**
- **Construct:**

$$\underbrace{\tilde{w}_j = w_j - r_j - \sum_{k \in \mathcal{K}_j} G^0_{jk} w_k}_{\text{known terms}}$$



- **Identify G^0_{ji} through PE identification with prediction error**

$$\varepsilon(t, \theta) = H_j(\rho)^{-1}[\tilde{w}_j - G_{ji}(\theta) w_i^{r_m}]$$

with the noise model fixed or parametrized independently from θ

- **The unmodelled terms $G^0_{jk} w_k, \; k \in \mathcal{U}^i_j$ appear as additional disturbance terms on the output w_j**

Network Identification – Two-stage method

Result two-stage method

The plant model $G_{ji}(\theta)$ is consistently estimated ($G_{ji}(\theta^*) = G^0_{ji}$) if:

- The plant model $G_{ji}(\theta)$ is correctly parametrized, $(G^0_{ji} \in \mathcal{G})$
- $w_i^{r_m}$ is sufficiently exciting for identification of G^0_{ji}, e.g. $\Phi_{w_i^{r_m}}(\omega) > 0 \quad \forall\omega$
- All $w_k, k \in \mathcal{U}^i_j$ are uncorrelated to r_m

[Van den Hof et al., CDC2012 submitted]

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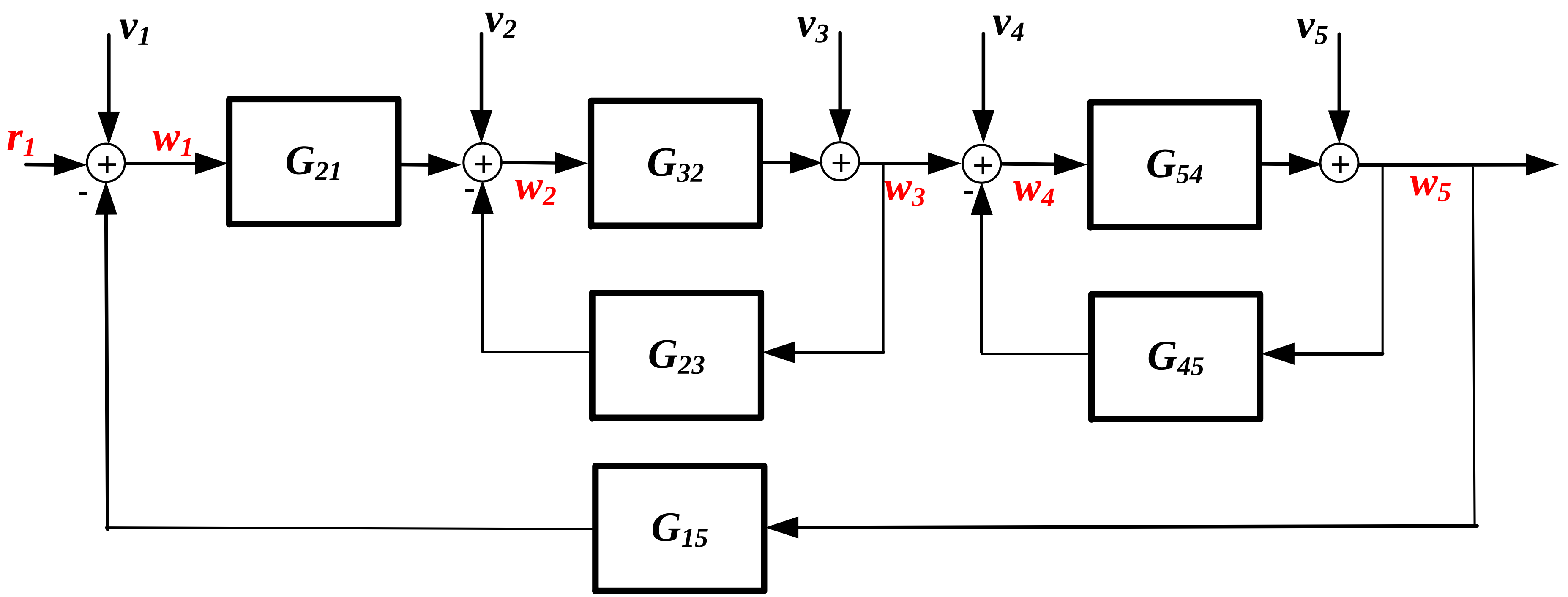
Network Identification – Two-stage method

Observation:

- **Consistent identification of single transfers is possible, dependent on network topology and reference excitation**
- **Full noise models are not necessary**
- **No conditions on uncorrelated noise sources, nor on absence of algebraic loops**
- **Excitation condition on single excitation signal**

Illustrative example

Network with 5 nodes – Configurations for direct identification



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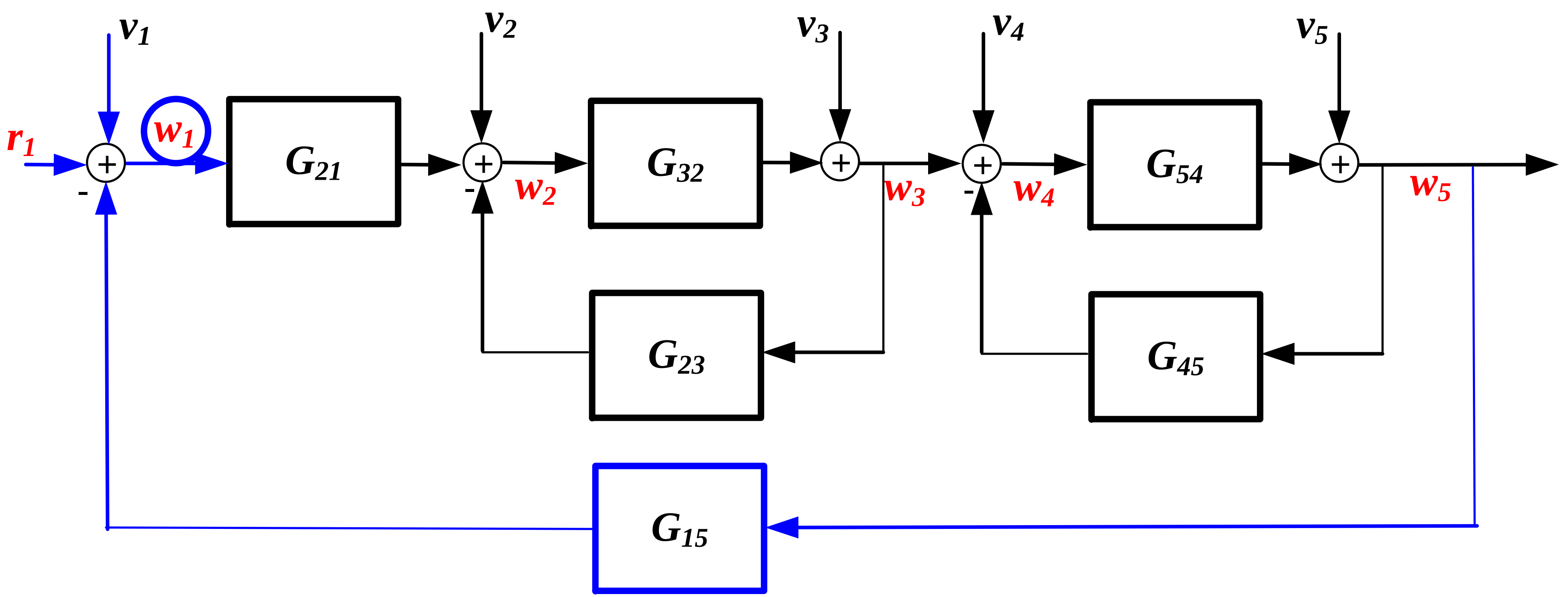

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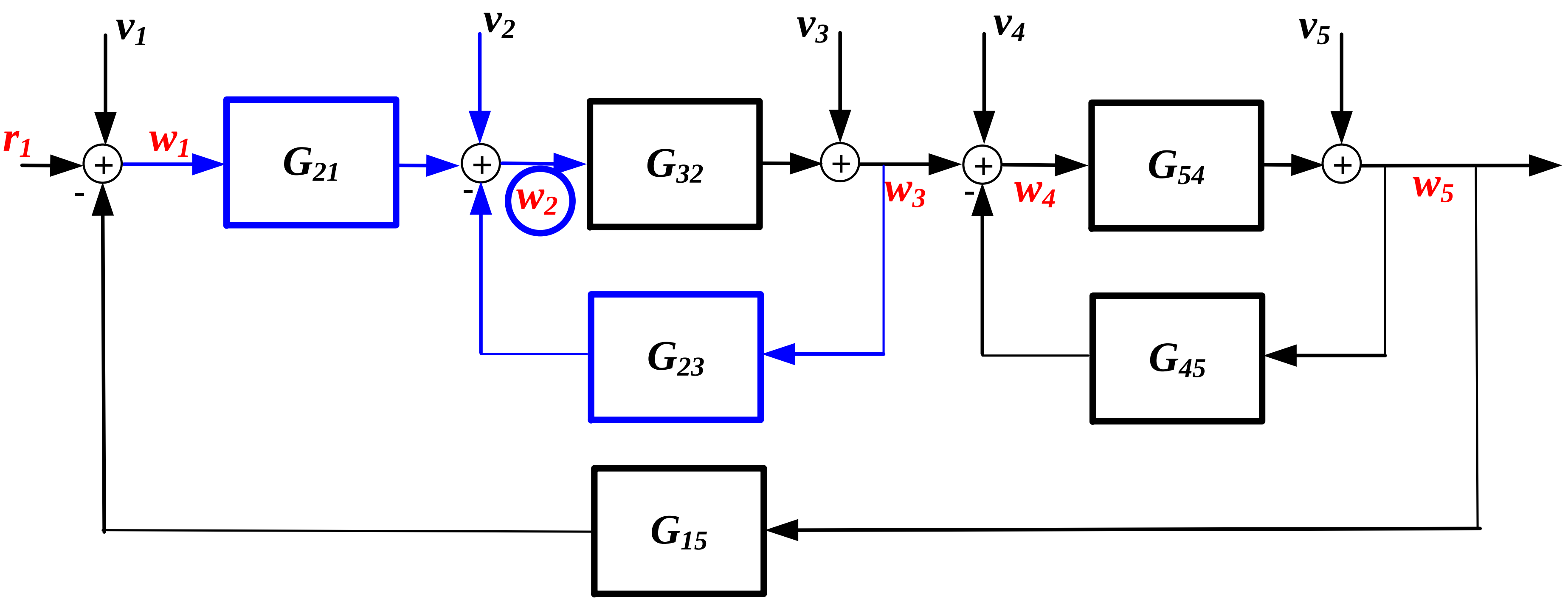

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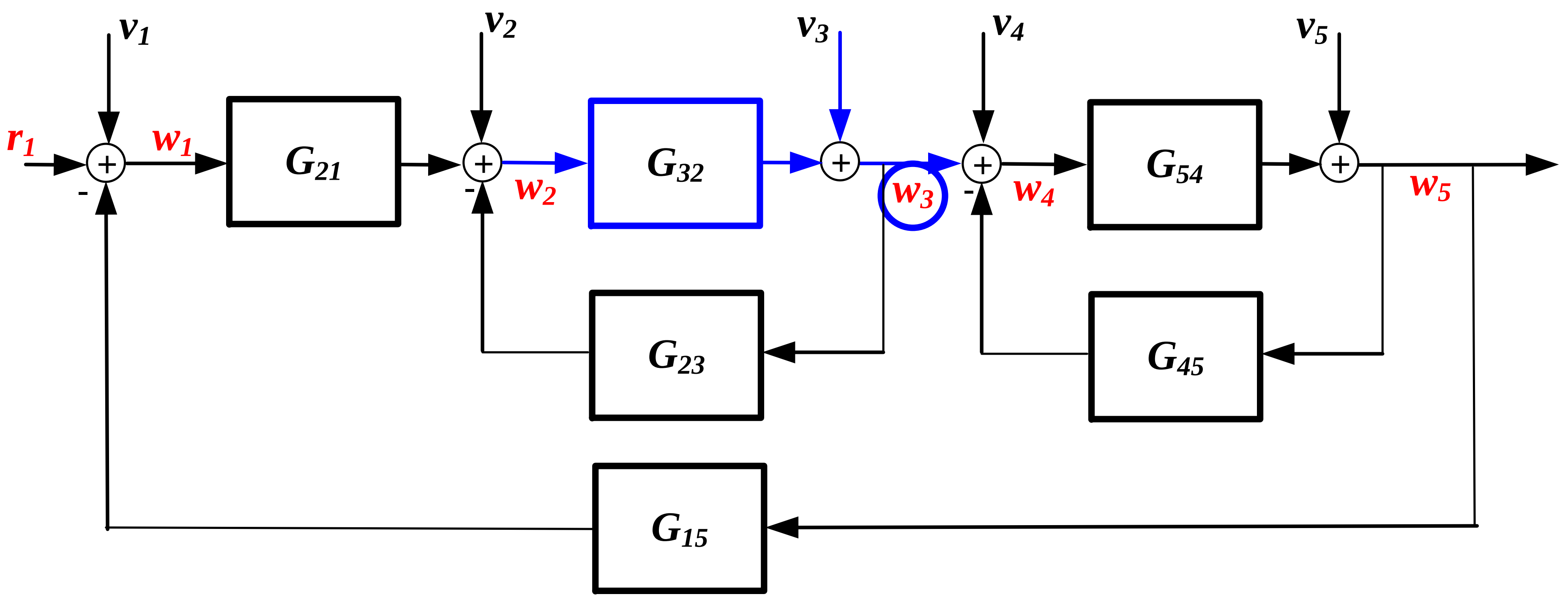

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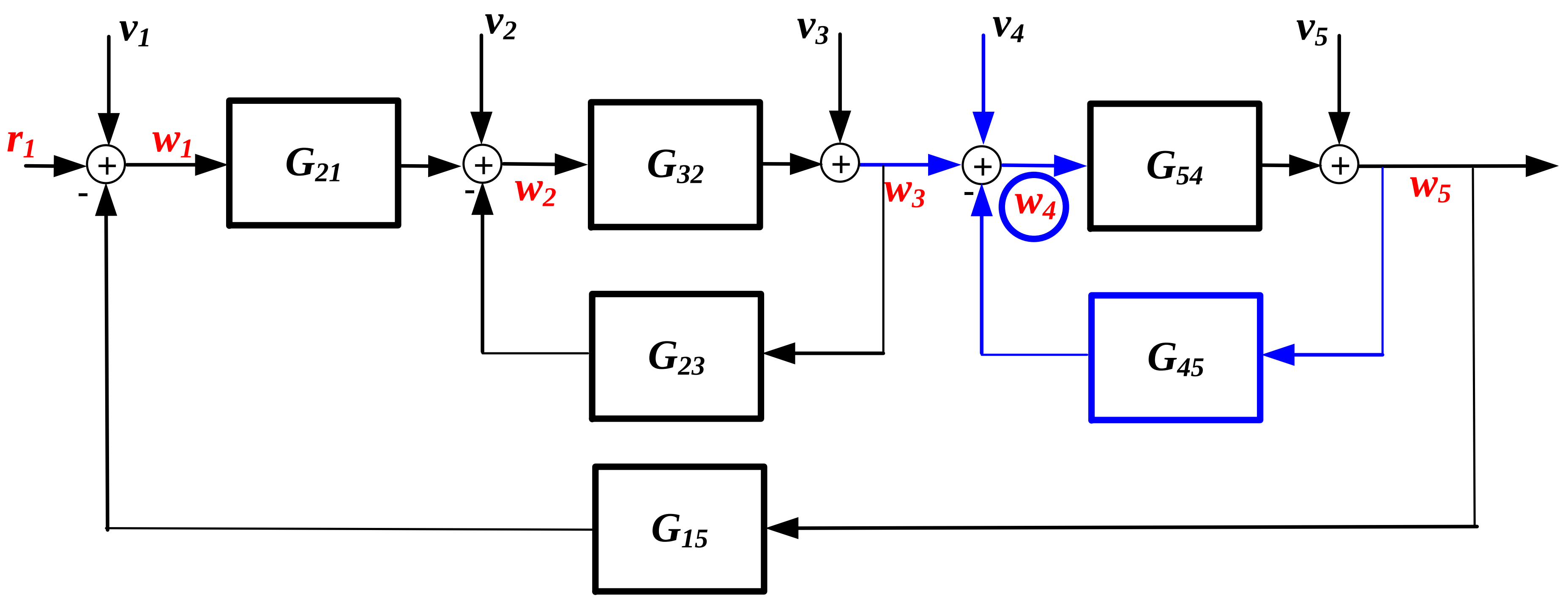

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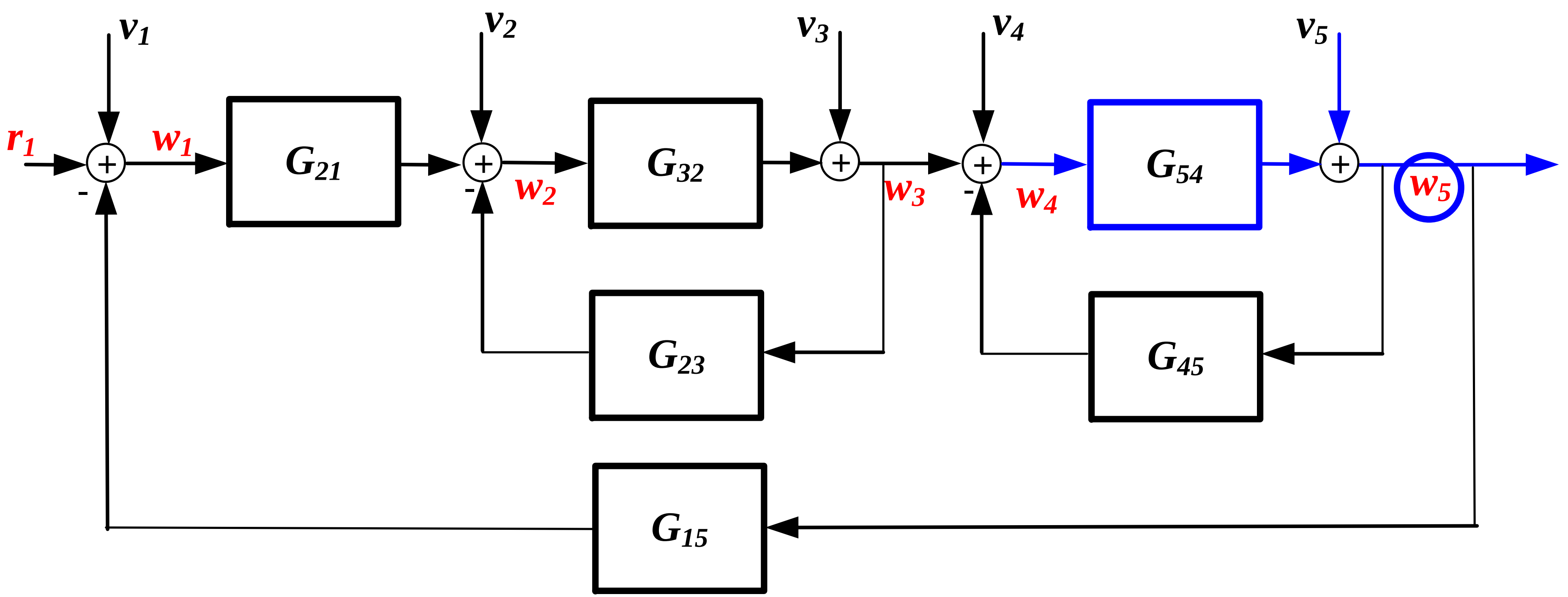
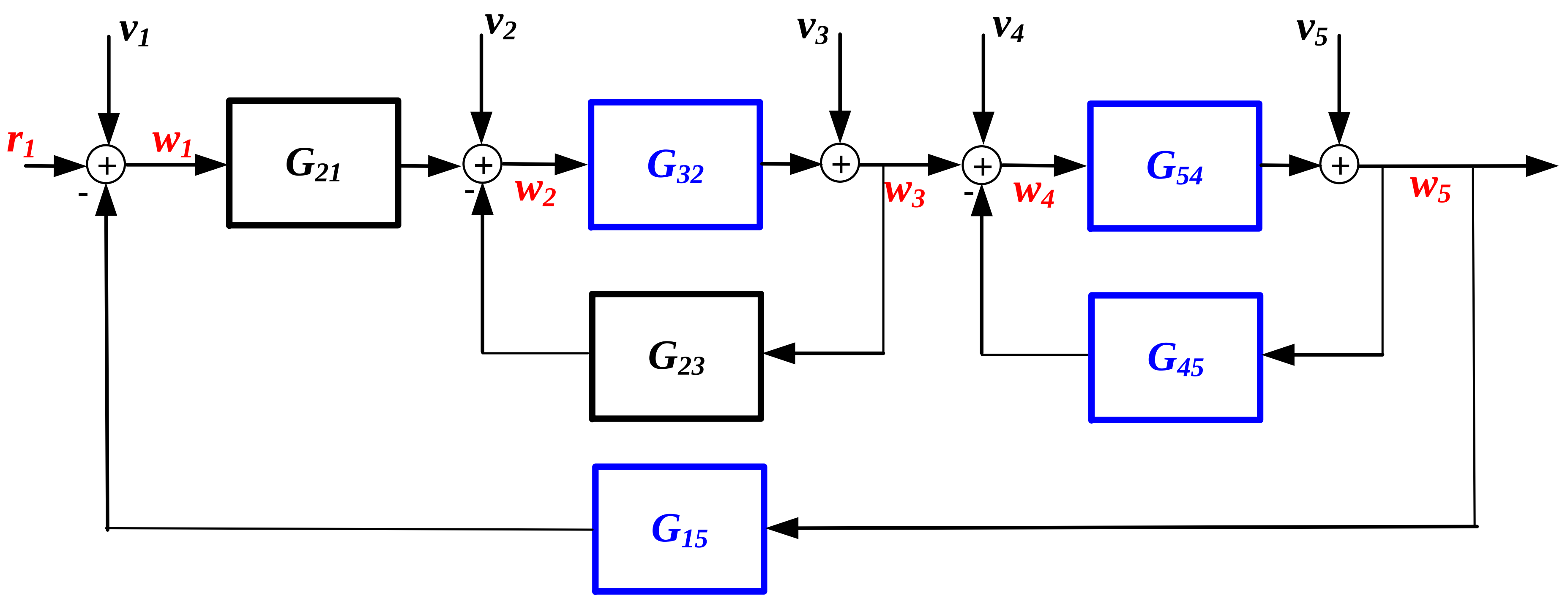


Illustrative example

Single transfers that can be identified by two-stage identification:



- **Input signal correlated with r_1**
- **Output does not have unmodelled components correlated with r_1**

Two-stage method – further results

Next questions:

- Can the conditions on r_m be checked in a structured way?
- Generalization of the approach towards:
  - MISO transfers
  - Variance reduction through multiple external signals

Two-stage method – further results

Check on topological structure of the network for two-stage identification of G_{ji}^0

Consider graph matrix: $A \in \{0,1\}^{L\times L}$ **that represents the presence of causal transfers:** $A_{ji} = 1$ if $G_{ji}^0 \neq 0$

In our example: $$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Then $[A^\ell]_{im}$ **equals the number of different (causal) path connections from** w_m **to** w_i.

Two-stage method – further results

Let $\mathcal{R}$ be set of node numbers *m* for which there is an r_m present

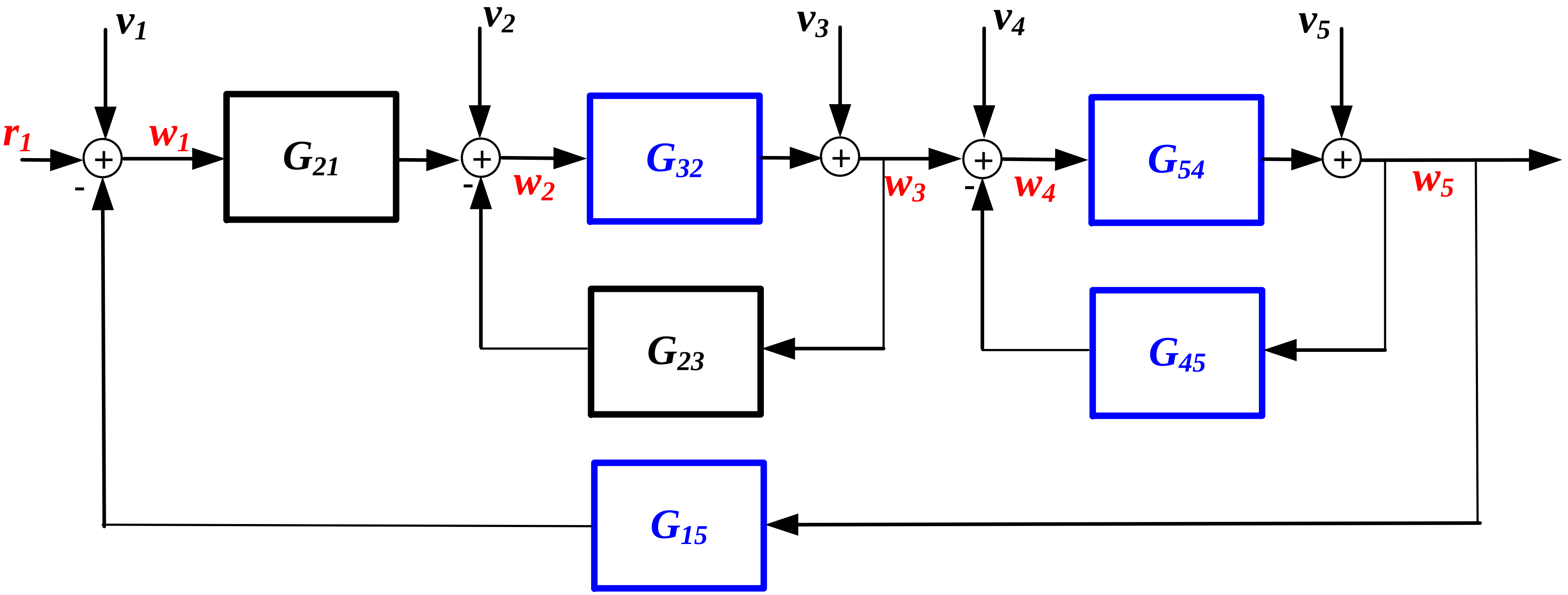
Topological conditions on two-stage identification of G_{ji}^0

- **There exists an $m \in \mathcal{R}$ and $\ell \in 1, 2, \cdots L$ such that**
  $$[A^\ell]_{im} \neq 0$$
- **For all $k \in \mathcal{U}_j^i$ it holds that $[A^\ell]_{km} = 0$ for all $\ell \in 1, 2, \cdots L$**

Simple manipulations on the graph matrix.

Two-stage method – further results

Extension to multiple transfers (MISO)



Two-stage method – further results

Extension to multiple transfers (MISO)

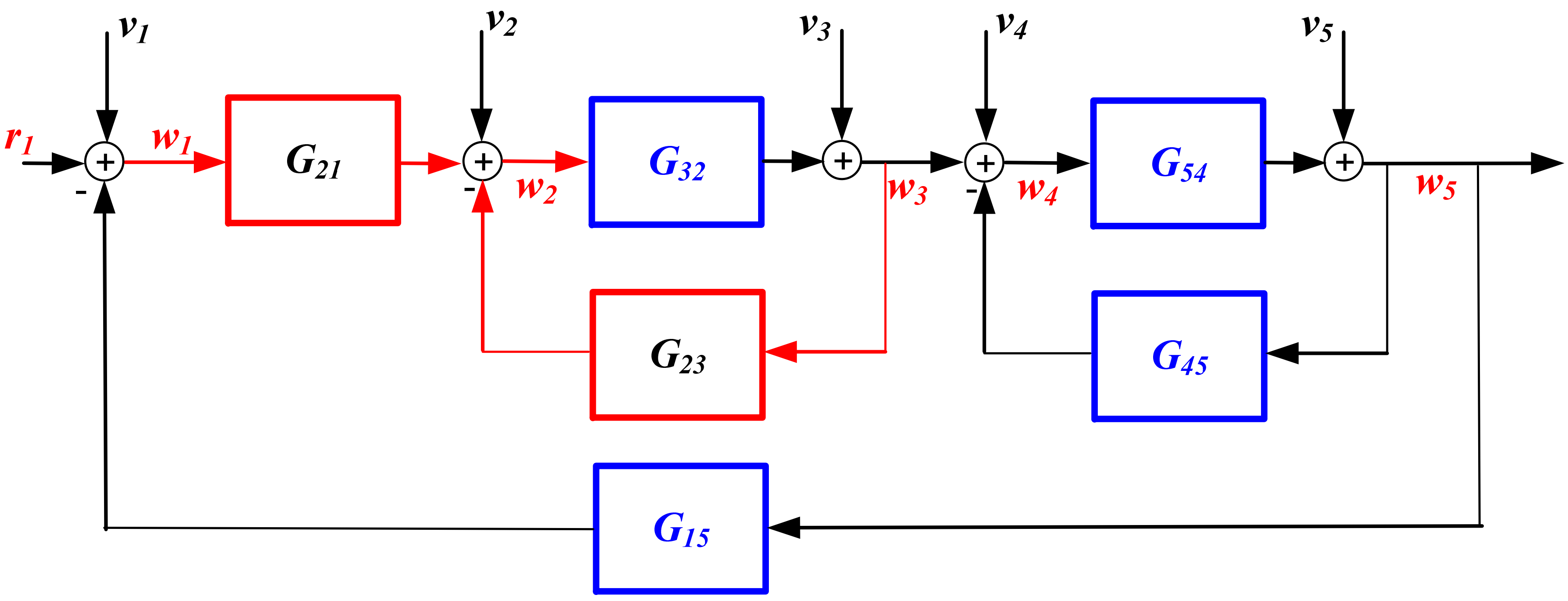
Idea:

Combine r_m-correlated inputs to output w_j into a MISO problem

- **Determine for r_m to which signals $w_k, k \in \mathcal{N}_j$ it is connected, $\Longrightarrow \mathcal{N}_j^m$**
- **Identify the transfers $G_{jk}, k \in \mathcal{N}_j^m$ in a MISO setting with the two-stage method**

Two-stage method – further results

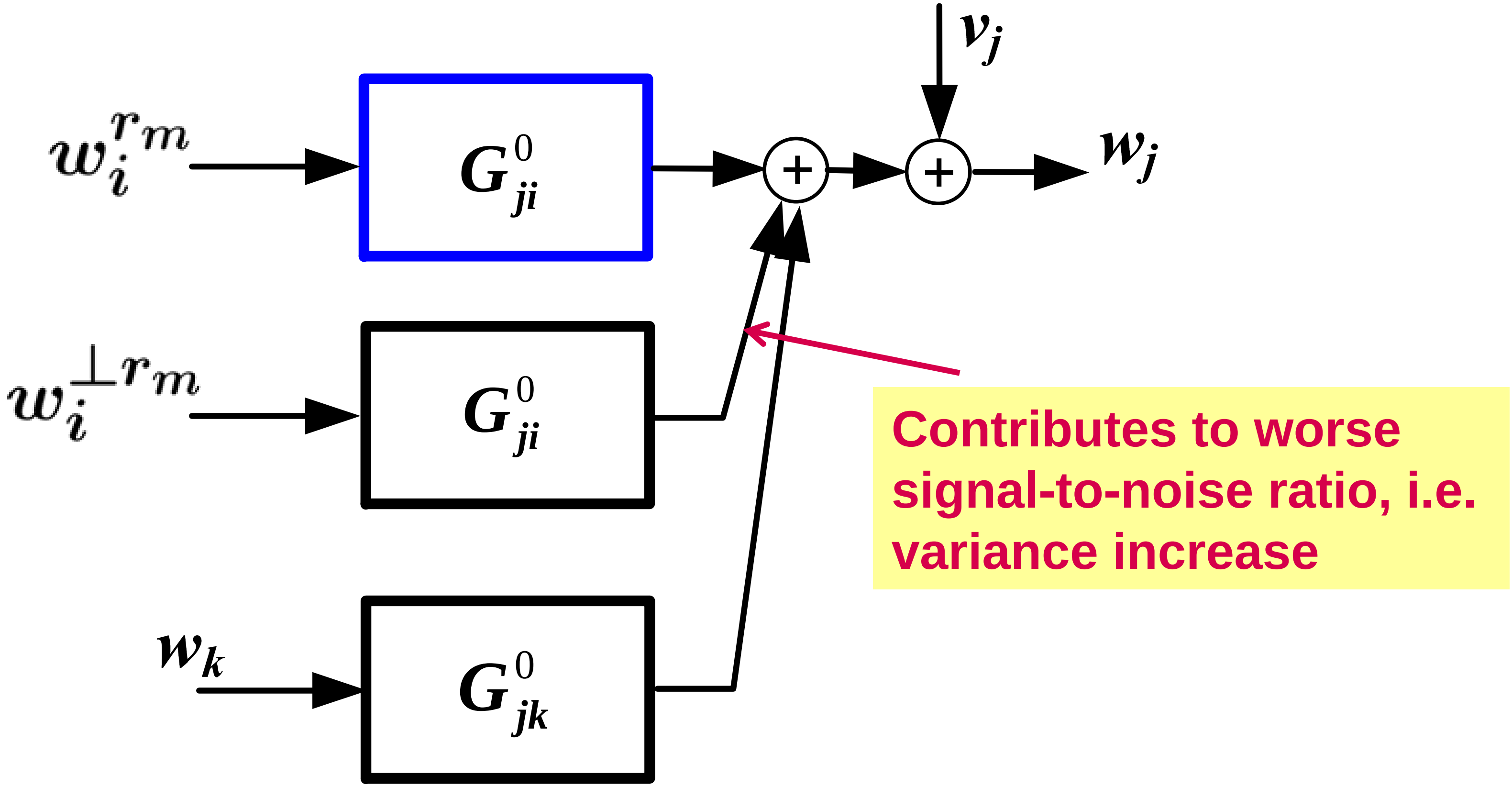
Extension to multiple transfers (MISO)



Identification of the transfers $\begin{pmatrix} w_1 \\ w_3 \end{pmatrix} \rightarrow w_2$ **can be done consistently, provided the excitation conditions are satisfied**

Two-stage method – further results

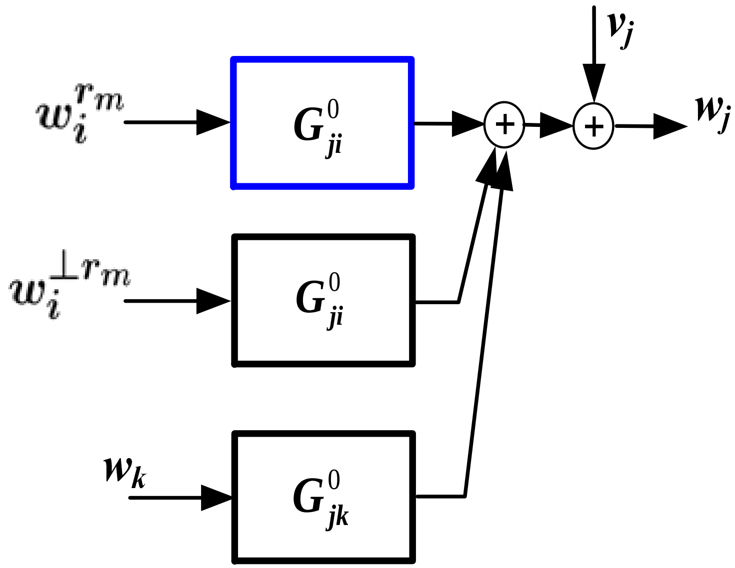
Extension to multiple excitation signals



Two-stage method – further results

Extension to multiple excitation signals

Remedy:



- **Combine projections to as many external signals as possible**

$$w_i^{r_{m_1}} + w_i^{r_{m_2}} + \cdots$$

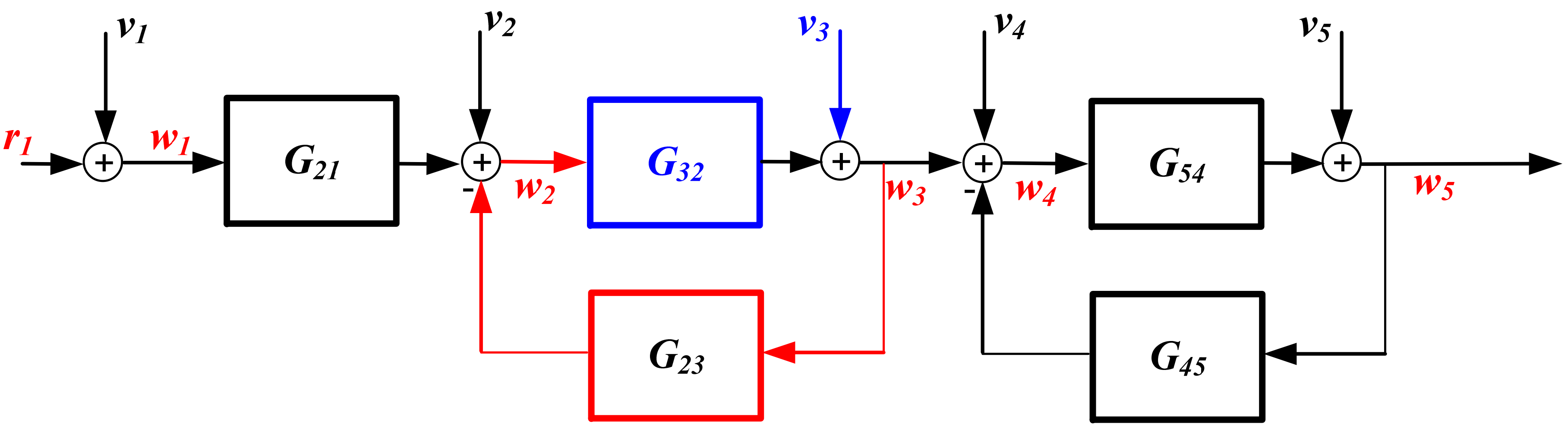
- **Even ``reconstructable'' noise signals can be used for this purpose**

$$w_i^{r_{m_1}} + w_i^{v_{m_2}} + \cdots$$

A noise signal v_j is reconstructable if all transfers into node j are known; then $v_j = w_j - r_j - \sum_{k \in \mathcal{N}_j} G_{jk}^0 w_k$ is known

Two-stage method – further results

Extension to multiple excitation signals



- If G_{32} is known
- noise signal v_3 can be reconstructed
- which excites w_3 to identify G_{23}
- and output w_2 is not disturbed by a v_3-dependent term

Identification of network structures

- So far considering the situation that the structure is known
- Identification of the structure (causality) is getting attention in the literature, in particular forms:
  - Tree-like structures (no loops)
  - Nonparametric methods (Wiener filter)
  - Mostly networks without external excitation

  see e.g. Materassi, Innocenti (TAC-2010)
- and in relation to compressive sensing methods for sparse identification
- Whenever loops are present and noises are non-white, the consistentcy results of closed-loop methods become important
  see e.g. Dankers et al., (IFAC SYSID 2012).

Summary

- **Data-driven modelling will only grow in its importance for high-performing engineering systems**
- **Current framework for open/closed-loop identification has to be extended to dynamic networks**
- **Methods for closed-loop identification extend to this case with some new properties**
- **They are expected to provide the basic tools for dealing with the structure identification problem also**
- **Many new questions pop up……**

Acknowledgements

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- **Sponsors:**

Thanks for your attention !

Challenges in System Identification

From closed-loop to dynamic network identification

Paul M.J. Van den Hof

24th Chinese Control and Decision Conference, 23-25 May 2012, Taiyuan, P.R. China

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