Accelerating simulations of first principle models of complex industrial systems using quasi-LPV models

R. Bos, X. Bombois, P. M. J. Van den Hof
email: r.bos@dcsc.tudelft.nl

Abstract:

This poster presents a method to approximate computationally expensive first principles models (obtained for instance using finite differences modeling) with faster models having a quasi-Linear Parameter Varying (qLPV) model structure. To construct the qLPV models we introduce an identification algorithm that retains the physical interpretation of the state vector in the identified model. The CPU time associated with the resulting qLPV models is generally considerably less than the original first principles model.

Problem statement

Assume that the following state model of the process is given:

$$x(k+1) = f(x(k), u(k))$$

Assumptions about the known model:

- x(k) has a physical interpretation
- Evaluation of f(·) require lots of CPU time

Goal: Approximate f(·) with qLPV model structure s. t. the physical interpretation of x(k) is retained.

$$x(k+1) = A_0 x(k) + B_0 u(k) + x_{off,0} + \sum_{m=1}^{M} \phi_m(x(k), u(k))[A_m x(k) + B_m u(k) + x_{off,m}]$$

To construct the qLPV model we shall use simulation data generated with the original model!

Step 1: Determine A_0, B_0, $x_{off,0}$

Matrices A_0, B_0 and offset-vector $x_{off,0}$ are determined using least squares:

$$A_0, B_0, x_{off,0} = \arg \min_{A,B,x_{off}} \sum_k \| f(x(k), u(k)) - Ax(k) - Bu(k) - x_{off} \|^2$$

Step 2: Determine A_m, B_m, $x_{off,m}$

- Linearize given model f(x, u) at various points in intended working area. The locations should cover the entire working area.

$$\mathcal{A}_i = \left.\frac{\partial f(x,u)}{\partial x}\right|_{x=x_i, u=u_i} \qquad \mathcal{B}_i = \left.\frac{\partial f(x,u)}{\partial u}\right|_{x=x_i, u=u_i}$$

$$\mathcal{X}_{off,i} = f(x_i, u_i) - \mathcal{A}_i x_i - \mathcal{B}_i u_i$$

- Collect all linearizations in data matrix X

$$X = \begin{bmatrix} \text{vec}(\mathcal{A}_1 - A_0) & & \text{vec}(\mathcal{A}_{N_{lin}} - A_0) \\ \text{vec}(\mathcal{B}_1 - B_0) & \cdots & \text{vec}(\mathcal{B}_{N_{lin}} - B_0) \\ \mathcal{X}_{off,1} - x_{off,0} & & \mathcal{X}_{off,N_{lin}} - x_{off,0} \end{bmatrix}$$

- Compute A_m, B_m, $x_{off,m}$ for m=1, …, M by computing the SVD of X.The computed matrices A_m, B_m, $x_{off,m}$ correspond to the first left singular vectors corresponding to the largest singular values of X. The resulting matrices represent the directions in which the linearized behavior has the largest parameter variations.

Step 3: Determine $\phi_m(x,u)$

Since scheduling functions do not have physical interpretation we are free to choose a structure.

- Choose structure for $\phi_m(x,u,\theta_m)$, estimate parameters θ_m using prediction error identification.

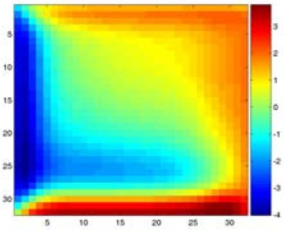

Figure 1: Realization of the heated plate model

Model	PE	CPU time
Orig.	0	406 s
Linear	$4.8 \cdot 10^{-3}$	4 s
qLPV	$0.48 \cdot 10^{-3}$	36 s

Table 1: Prediction errors and CPU time

Results

Using the described algorithm, an approximate model for a nonlinear finite element model of a 2-D heated plate (see Figure 1) was constructed. As indicated in Table 1, the identified qLPV model can be computed more than 10 times faster than the original model. Furthermore, the qLPV model is 10 times more accurate than the best possible linear model.