

Identifiability of diffusively coupled linear networks with partial instrumentation

JULY 2023

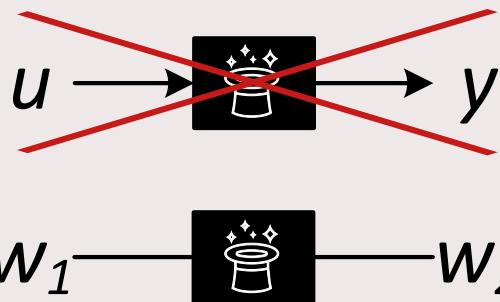


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Diffusive coupling

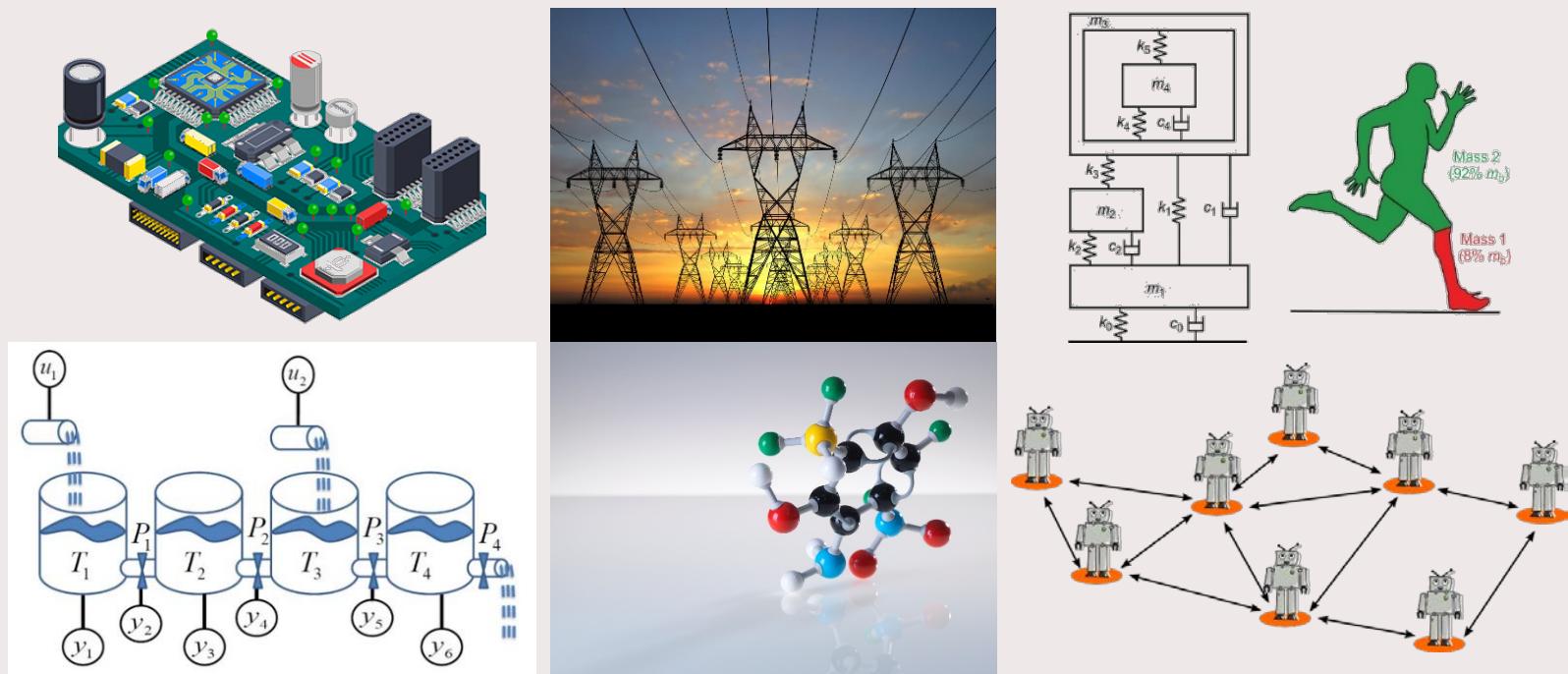


$$v_1 - v_2 = Ri$$



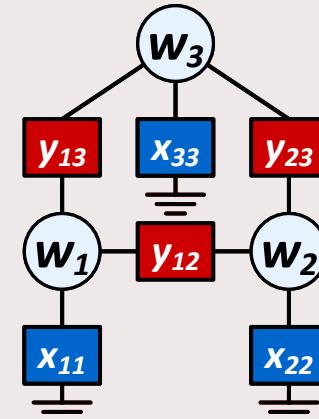
$$x_1 - x_2 = -\frac{1}{K}f$$

Diffusively coupled linear network



Network model

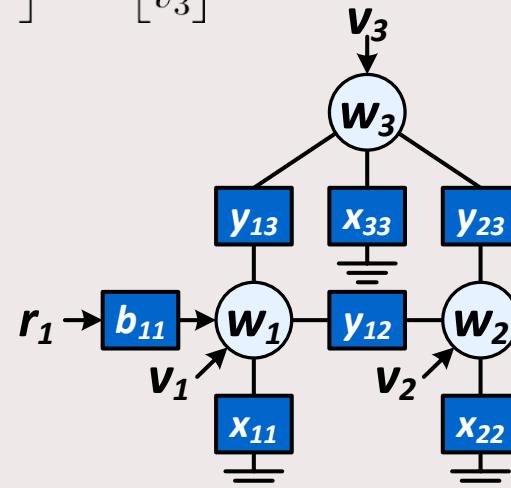
$$\begin{bmatrix} x_{11} & & \\ & x_{22} & \\ & & x_{33} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$



Network model

$$\begin{bmatrix} x_{11} & & \\ & x_{22} & \\ & & x_{33} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} \star & -y_{12} & -y_{13} \\ -y_{12} & \star & -y_{23} \\ -y_{13} & -y_{23} & \star \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} b_{11} \\ \\ \end{bmatrix} r_1 + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

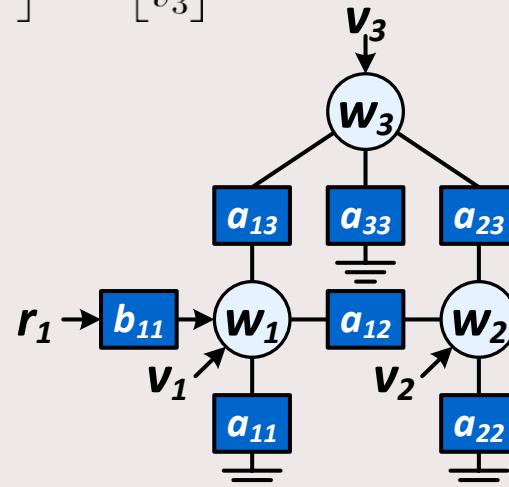
$$X(q^{-1})w(t) + Y(q^{-1})w(t) = B(q^{-1})r(t) + v(t)$$



Network model

$$\begin{bmatrix} x_{11} & & \\ & x_{22} & \\ & & x_{33} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} \star & -y_{12} & -y_{13} \\ -y_{12} & \star & -y_{23} \\ -y_{13} & -y_{23} & \star \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} b_{11} \\ \vdots \\ b_{33} \end{bmatrix} r_1 + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\boxed{\begin{aligned} A(q^{-1})w(t) &= B(q^{-1})r(t) + v(t) \\ y(t) &= C(q^{-1})w(t) \end{aligned}}$$



Contents

1. Diffusively coupled network
2. Problem statement
3. Identifiability
4. Conclusion

Contents

1. Diffusively coupled network
2. **Problem statement**
3. Identifiability
4. Conclusion

Identifiability

$$A(q^{-1})w(t) = B(q^{-1})r(t) + F(q)e(t)$$

$$y(t) = C(q^{-1})w(t)$$

Identifiability

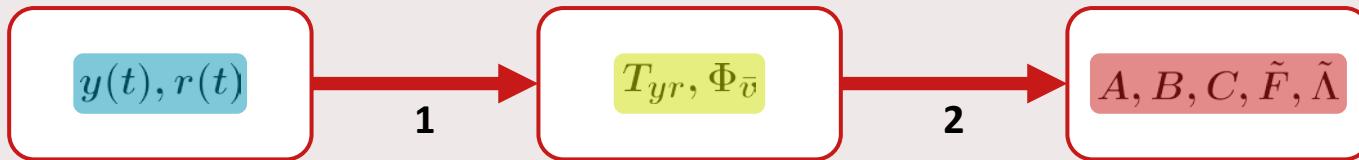
$$\begin{aligned} Aw(t) &= Br(t) + Fe(t) \\ y(t) &= Cw(t) \end{aligned}$$

$$y(t) = \underbrace{CA^{-1}B}_{T_{yr}} r(t) + \underbrace{CA^{-1}Fe(t)}_{\bar{v}(t)}$$

Identifiability

$$Aw(t) = Br(t) + \begin{bmatrix} 0 \\ \tilde{F} \end{bmatrix} \tilde{e}(t)$$
$$y(t) = Cw(t)$$

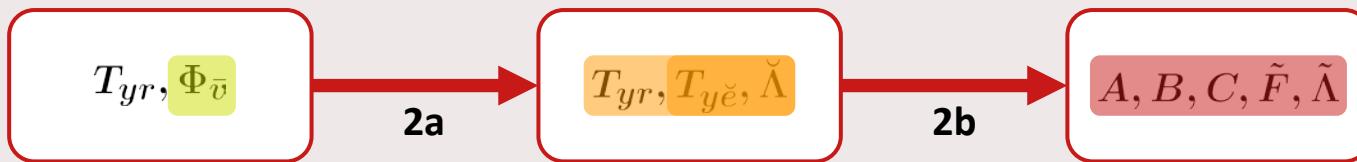
$$y(t) = \underbrace{CA^{-1}B}_{T_{yr}} r(t) + \underbrace{CA^{-1} \begin{bmatrix} 0 \\ \tilde{F} \end{bmatrix} \tilde{e}(t)}_{\bar{v}(t)}$$



Identifiability

$$y(t) = T_{yr}r(t) + \underbrace{T_{y\check{e}}\check{e}(t)}_{\bar{v}(t)}$$

$$y(t) = \underbrace{CA^{-1}B}_{T_{yr}} r(t) + \underbrace{CA^{-1} \begin{bmatrix} 0 \\ \tilde{F} \end{bmatrix} \tilde{e}(t)}_{\bar{v}(t)}$$

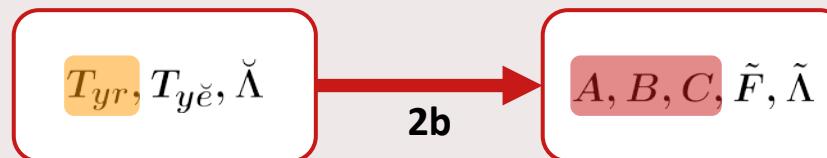


Problem statement

A diffusively coupled linear network with partial instrumentation
is identifiable from data if ...

$$Aw(t) = Br(t) + \begin{bmatrix} 0 \\ \tilde{F} \end{bmatrix} \tilde{e}(t)$$
$$y(t) = Cw(t)$$

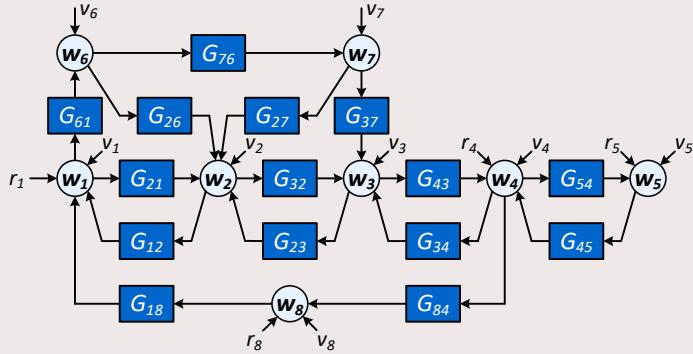
$$y(t) = \underbrace{CA^{-1}B}_{T_{yr}} r(t) + \bar{v}(t)$$



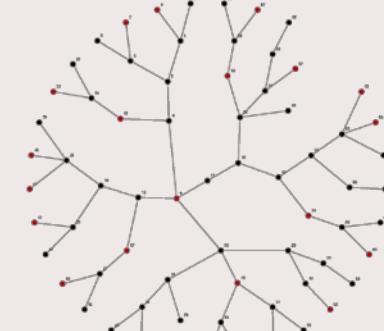
Contents

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Literature



$$y(t) = G(q)y(t) + R(q)u(t) + H(q)e(t)$$



$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

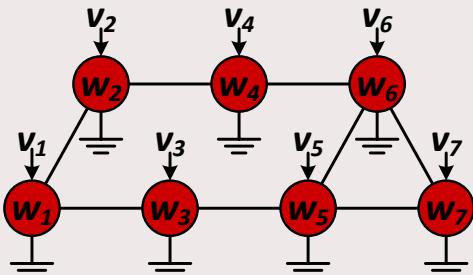
$$A(q^{-1})y(t) = B(q^{-1})u(t) + F(q^{-1})e(t)$$

Instrumentation

Full Measurement

L Measurement

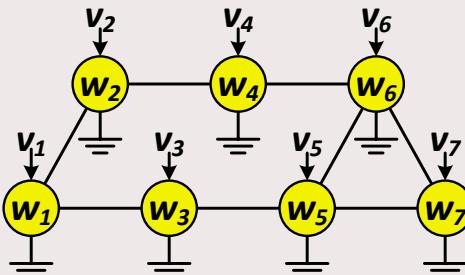
? Excitation



Full Excitation

? Measurement

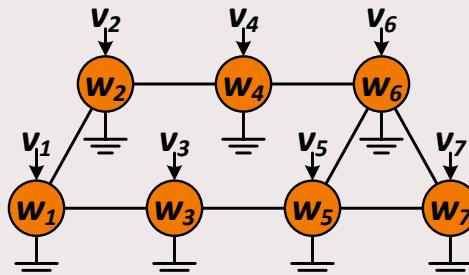
L Excitation



Partial instrumentation

? Measurement

? Excitation



Full measurement^[1]

$$T_{yr} = [\textcolor{red}{T} A]^{-1} \textcolor{red}{T} B$$

Full measurement^[1]

$$T_{yr} = [\textcolor{red}{U} A]^{-1} \textcolor{red}{U} B$$

- 1 Excitation
- Left coprime A, B

Full measurement^[1]

$$T_{yr} = [\mathcal{D}A]^{-1} \mathcal{D}B$$

- 1 Excitation
- Left coprime A, B
- 1 Diagonality constraint^[1,9]

Full measurement^[1]

$$T_{yr} = [\alpha \mathbf{I} A]^{-1} \alpha \mathbf{I} B$$

- 1 Excitation
- Left coprime A, B
- 1 Diagonality constraint^[1,9]

Full measurement^[1]

$$T_{yr} = A^{-1}B$$

- 1 Excitation
- Left coprime A, B
- 1 Diagonality constraint^[1,9]
- 1 Parameter constraint

Full measurement^[1]

$$T_{yr} = A^{-1}B$$

- 1 Excitation
- Left coprime A, B
- 1 Diagonality constraint^[1,9]
- 1 Parameter constraint

Full excitation^[10]

$$T_{yr} = CA^{-1}$$

- 1 Measurement
- Right coprime C, A
- 1 Diagonality constraint
- 1 Parameter constraint

[1] E.M.M. Kivits and P.M.J. Van den Hof, TAC, June 2023.

13 [9] E.M.M. Kivits and P.M.J. Van den Hof, CDC, 2022.

[10] E.M.M. Kivits and P.M.J. Van den Hof, IFAC WC, 2023.

Partial instrumentation^[10]

$$T_{yr} = C \mathbf{T}_C [\mathbf{T}_B A \mathbf{T}_C]^{-1} \mathbf{T}_B B$$

Partial instrumentation^[10]

$$T_{yr} = C \color{red}{U_C [U_B A U_C]^{-1} U_B B}$$

$$K + c \geq L + 1$$

- $\color{red}{K}$ Excitation & $\color{red}{c}$ Measurements
- Left coprime A, B & Right coprime C, A

Partial instrumentation^[10]

$$T_{yr} = C \mathbf{D}_C [D_B A \mathbf{D}_C]^{-1} D_B B$$

$$\mathbf{D}_C = \alpha D_B$$

- \mathbf{K} Excitation & c Measurements
- Left coprime A, B & Right coprime C, A
- 1 Diagonality constraint A, B & 1 Diagonality constraint C, A

Partial instrumentation^[10]

$$T_{yr} = C \mathbf{D}_C [D_B A \mathbf{D}_C]^{-1} D_B B$$

$$\underbrace{\begin{bmatrix} I \\ \star \end{bmatrix}}_{\mathbf{D}_C} = \alpha \underbrace{\begin{bmatrix} \star \\ I \end{bmatrix}}_{\mathbf{D}_B}$$

- \mathbf{K} Excitation & c Measurements
- Left coprime A, B & Right coprime C, A
- 1 Diagonality constraint A, B & 1 Diagonality constraint C, A
- L Parameter constraints

Partial instrumentation^[10]

$$T_{yr} = CA^{-1}B$$

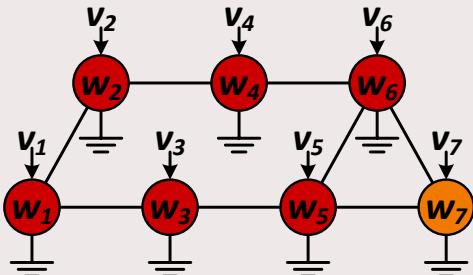
- $\textcolor{red}{K}$ Excitation
 - Left coprime A, B
 - 1 Diagonality constraint A, B
 - $\textcolor{red}{L}$ Parameter constraints
 - 1 Parameter constraint
- & $\textcolor{red}{c}$ Measurements
 - & Right coprime C, A
 - & 1 Diagonality constraint C, A

Instrumentation

Full Measurement

L Measurement

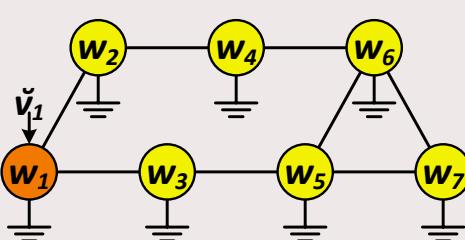
1 Excitation



Full Excitation

1 Measurement

L Excitation

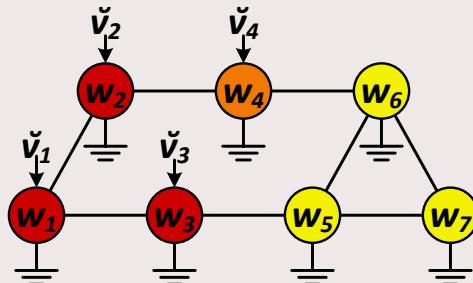


Partial instrumentation

K Measurement

c Excitation

$$K + c \geq L + 1$$



Transfer function

$$T_{yr} = CA^{-1}B$$

$$T_{yr} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1i} & \cdots & \alpha_{1,L-1} & \alpha_{1L} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2i} & \cdots & \alpha_{2,L-1} & \alpha_{2L} \\ \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ \alpha_{i1} & \alpha_{i2} & \cdots & \alpha_{ii} & \cdots & \alpha_{i,L-1} & \alpha_{iL} \\ \vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\ \alpha_{L-1,1} & \alpha_{L-1,2} & \cdots & \alpha_{L-1,i} & \cdots & \alpha_{L-1,L-1} & \alpha_{L-1,L} \\ \alpha_{L1} & \alpha_{L2} & \cdots & \alpha_{Li} & \cdots & \alpha_{L,L-1} & \alpha_{LL} \end{bmatrix}$$

Transfer function

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Transfer function

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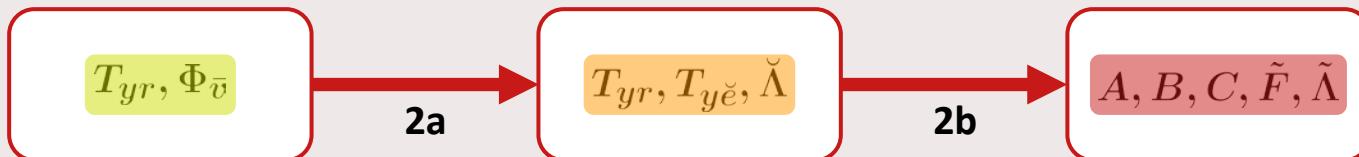
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Conclusion

A diffusively coupled linear network with partial instrumentation
is identifiable from data if ...

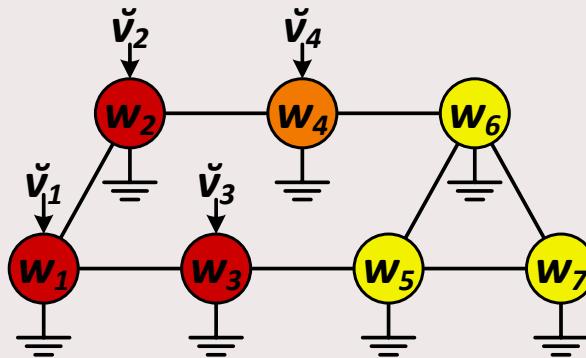
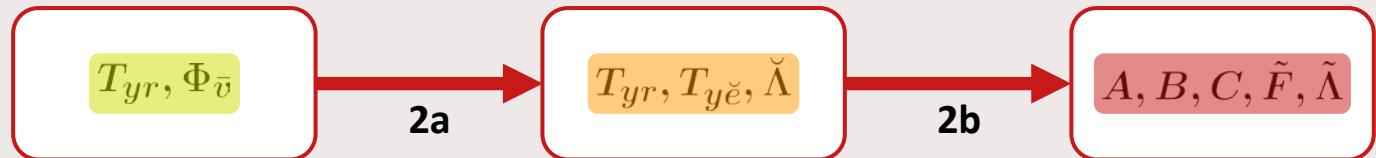
$$Aw(t) = Br(t) + \begin{bmatrix} 0 \\ \tilde{F} \end{bmatrix} \tilde{e}(t)$$
$$y(t) = Cw(t)$$

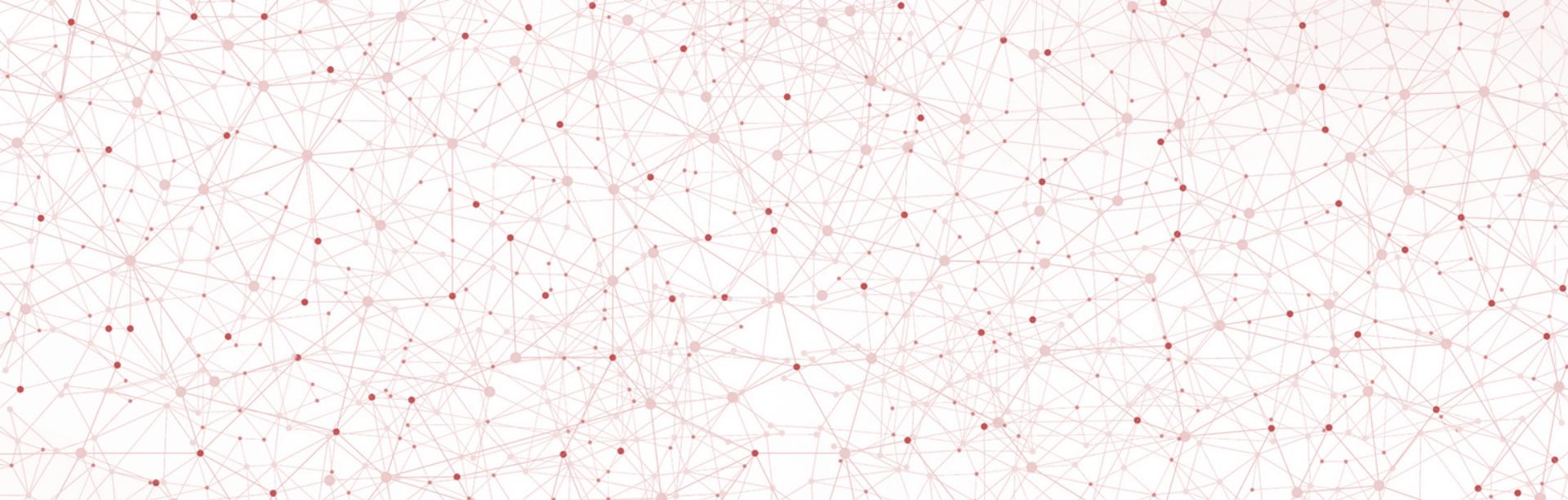
$$y(t) = \underbrace{CA^{-1}B}_{T_{yr}} r(t) + \bar{v}(t)$$



Conclusion

A diffusively coupled linear network with partial instrumentation
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Identifiability of diffusively coupled linear networks with partial instrumentation

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