

## CONTROL-RELEVANT UNCERTAINTY MODELLING DIRECTED TOWARDS PERFORMANCE ROBUSTNESS

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**Abstract.** In line with recently introduced methods for iterative identification and control design, a method is proposed to identify uncertainty models from data with an uncertainty structure that is motivated by the (closed loop) control performance cost function. This allows the assessment of achieved plant performance on the basis of measured time series, as well as the evaluation of robust stability and robust performance for a newly designed controller prior to implementation. For the specific control performance measure considered, ( $\mathcal{H}_\infty$ -norm on a closed-loop transfer matrix), this naturally leads to the identification of upper bounds on either coprime factor model uncertainty or uncertainty on the (dual) Youla-parameter. Theoretical results are supported by experimental results of an application to the radial control loop in a compact disc servo mechanism.

**Keywords.** System identification, model uncertainty, coprime factorization, closed-loop identification, control-oriented models, robust performance.

### 1. INTRODUCTION

In the development of identification methods that provide models that are specifically suitable for model-based control design, one approach is in the area called iterative identification and control. Here (approximate) models are identified on the basis of closed-loop data while the plant is controlled by the latest controller, and next an improved controller is designed on the basis of the estimated model. One of the important phenomena in this

approach, is that the identification of an (approximate) nominal model is performed through an identification criterion that is induced by the control performance criterion. Examples of iterative schemes are provided by Zang et al. (1995), Lee et al. (1993) and Schrama (1992), see also the survey papers by Gevers (1993) and Van den Hof and Schrama (1995).

The basic line of reasoning is as follows. A control performance cost function is considered for a linear time-invariant plant  $P_o$  and a controller  $C$  to be determined by a function  $J(P_o, C)$  in a normed space; the control performance cost is then determined by its norm  $\|J(P_o, C)\|$ .

In the iterative procedures considered one distinguishes two steps:

- For a present controller  $C$ , identify a model  $\hat{P}$  that

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minimizes the *performance degradation* term  $\|J(P_o, C) - J(\hat{P}, C)\|$ ;

- For a given model  $\hat{P}$  design a controller  $C$  by minimizing the *designed performance cost*  $\|J(\hat{P}, C)\|$ .

In Van den Hof and Schrama (1995) it is illustrated how the several iterative schemes fit into this framework, by employing different performance cost functions. One of the important points in the iterative mechanism sketched is that when designing a new controller one has to be sure that the nominal design achieves sufficient performance robustness.

An intermediate question that plays a role in this respect, is the question, “What is the achieved plant performance for a given controller?” In other words: given a controller that is applied to the plant, can we monitor the achieved plant performance from general input/output time series? The answers to these questions require uncertainty modelling.

In this paper we will investigate the possibilities to use identified model uncertainty sets to robustify the control design in the iterative schemes, by choosing model uncertainty structures that are motivated by a general control performance cost function; this cost function is based on the closed-loop transfer function

$$T(P_o, C) = \begin{bmatrix} P_o \\ I \end{bmatrix} [I + CP_o]^{-1} \begin{bmatrix} C & I \end{bmatrix}, \quad (1)$$

which reflects the transfer function from the external signals  $(r_2, r_1)^T$  to the loop signals  $(y, u)^T$  as indicated in figure 1.

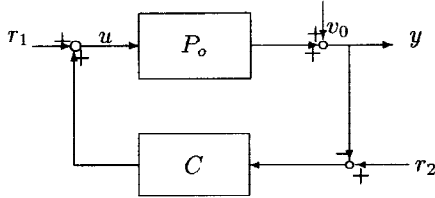


Fig. 1. Configuration of closed loop system  $T(P_o, C)$ .

The matrix  $T(P_o, C)$  incorporates all relevant feedback properties of the closed loop system composed of  $P_o$  and  $C$ . In this paper we will consider the control performance cost function

$$\|J(P_o, C)\| = \|V \cdot T(P_o, C) \cdot W\|_\infty \quad (2)$$

where  $V, W$  are user defined (stable) weighting functions, that for simplicity of notation and without loss of generality will be fixed to identity in this paper.

For this performance function, we will consider how identified uncertainty sets can be used in three problems:

- performance assessment, i.e. the evaluation of control performance for the present controller;
- robust stability test, i.e. the prior assessment of closed loop stability for a future controller;
- robust performance test, i.e. the prediction of worst-case performance for a future controller.

In sections 2 and 3 we will present two uncertainty structures that we will consider, and it will be indicated how corresponding uncertainty sets can be identified from input/output data. The three tests mentioned above will be addressed in sections 4, 5 and 6, while experimental results on the radial loop of a compact disc servo mechanism are shown in section 7.

## 2. COPRIME FACTOR UNCERTAINTY SETS

An identification with the performance degradation criterion can be performed in the framework of coprime factor identification, where the possible unstable plant  $P_o$  is expressed as the ratio of two stable transfer functions  $N, D$  i.e.  $P_o = ND^{-1}$ . To gain access to specific coprime factorizations of the plant from closed loop measurements of  $r_1, r_2, u, y$  (figure 1) one can apply a filtering of the excitation signal  $r(t) := r_1(t) + C(q)r_2(t)$  according to:

$$x(t) = Fr(t) \quad (3)$$

with  $F$  a stable filter. Employing this (instrumental) signal  $x$  the system relations can now be formulated as:

$$y(t) = N_{o,F}x(t) + S_o v_o(t) \quad (4)$$

$$u(t) = D_{o,F}x(t) - CS_o v_o(t) \quad (5)$$

with  $S_o$  the plant sensitivity function  $S_o = [I + CP_o]^{-1}$ , and the (coprime) factors  $N_{o,F}, D_{o,F}$  given by

$$N_{o,F} = P_o(I + CP_o)^{-1}F^{-1} \quad (6)$$

$$D_{o,F} = (I + CP_o)^{-1}F^{-1}. \quad (7)$$

The freedom that is present in the choice of the stable filter  $F$  such that stable factors  $N_{o,F}, D_{o,F}$  result, is characterized in Van den Hof et al.(1995).

Note that the signal  $x(t)$  is uncorrelated with the noise  $v(t)$ , and can therefore be used as input signal in identification in an open loop way. The coprime factors  $N_{o,F}, D_{o,F}$  can be estimated by using  $x$  as input signal and  $[y \ u]^T$  as output signals. With the choice of a factorization (i.e. through the choice of  $F$ ), the coprime factorization to be estimated can be influenced. In Van den

Hof et al.(1995) this freedom is used to estimate possibly low order *normalized* factorizations of the plant. In de Callafon and Van den Hof (1995) the choice of  $F$  additionally is based on the demand for a model which is specifically suitable for application in the degradation criterion  $\|T(P_o, C) - T(P, C)\|_\infty$ . A related uncertainty set is defined by considering additive uncertainty on estimated coprime factors. To this end we denote the uncertainty set

$$\mathcal{P}_{CF}(\hat{N}, \hat{D}, \delta_N, \delta_D) := \left\{ P | P = (\hat{N} + \Delta_N)(\hat{D} + \Delta_D)^{-1}, \right. \\ \left. |\Delta_N(e^{i\omega})| \leq \delta_N(\omega), |\Delta_D(e^{i\omega})| \leq \delta_D(\omega) \right\} \quad (8)$$

where  $(\hat{N}, \hat{D})$  is a right coprime factorization (rcf) of the estimated model  $\hat{P}$ . Identification of such an uncertainty set can be performed by using one of the methods for coprime factor identification mentioned above, together with one of the available uncertainty identification procedures for open loop plants, either based on a worst-case deterministic approach or based on stochastic representations (Ninness and Goodwin, 1995). The considered uncertainty structure used in  $\mathcal{P}_{CF}$  will be shown to be specifically suitable for use in conjunction with the control performance function (2).

### 3. ADDITIVE DUAL-YOULA PARAMETER UNCERTAINTY

An alternative to the coprime factor uncertainty set will also be shown to be directed towards the performance function (2). It is induced by the dual-Youla parametrization, which parametrizes all systems that are stabilized by a given controller.

**Proposition 3.1** *Let  $C$  have a rcf  $(N_c, D_c)$  and let  $P_x$  with rcf  $(N_x, D_x)$  be any system such that  $T(P_x, C)$  is stable. Then a plant  $P_o$  is stabilized by  $C$  if and only if there exists an  $R_p \in \mathbb{RH}_\infty$  such that*

$$P_o = (N_x + D_c R_p)(D_x - N_c R_p)^{-1}. \quad (9)$$

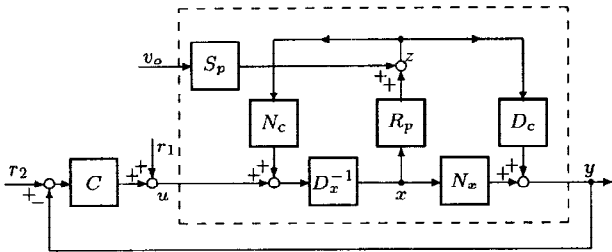


Fig. 2. Dual Youla-representation of the data generating system with noise.

The Dual-Youla parametrization is depicted in figure 2. Identification of the dual-Youla parameter  $R_p$  can be done in a way that closely relates to the coprime factor framework discussed previously. This is visualised by the fact that when choosing a filter  $F = (D_x + CN_x)^{-1}$ , the systems equations can be rewritten into the form

$$z(t) = R_p(q)x(t) + S_p(q)v_o(t) \quad (10)$$

$$\text{with } z(t) = (D_c + P_x N_c)^{-1}[y(t) - P_x u(t)] \quad (11)$$

$$S_p(q) = D_c^{-1}[I + P_x C]^{-1}W_o. \quad (12)$$

Based on expression (10) and using the fact that  $z(t)$  and  $x(t)$  can be obtained from filtering measured signals, while  $x$  is uncorrelated with  $v_o$ , we can now identify  $R_p$  in an open-loop way, using any uncertainty identification procedure mentioned in the previous section.

The dual-Youla parametrization leads to the following uncertainty set:

$$\mathcal{P}_R(N_x, D_x, N_c, D_c, \hat{R}_p, \gamma) := \\ \left\{ P | P = (N_x + D_c R_p)(D_x - N_c R_p)^{-1}, \right. \\ \left. R_p = \hat{R}_p + \Delta_R, |\Delta_R(e^{i\omega})| \leq \gamma(\omega) \right\}. \quad (13)$$

For given coprime factorizations of  $P_x$  and  $C$  the Youla-parameter uniquely relates to a plant  $P$ , according to:

$$R_p = (D_c + P N_c)^{-1}(P - P_x)D_x. \quad (14)$$

For evaluating robustness properties we have to consider the situation that a new controller  $C_{new}$  is going to be implemented on the plant. This new controller can be written in the form of a Youla-parametrization

$$C_{new} = [N_c + D_x R_c][D_c - N_c R_c]^{-1}. \quad (15)$$

In this way the stable Youla-parameter  $R_c$  uniquely relates to the new controller according to

$$R_c = (D_x + C_{new} N_x)^{-1}(C_{new} - C)D_c. \quad (16)$$

These expressions will be utilized when assessing robust stability and robust performance properties of the considered uncertainty sets.

### 4. PERFORMANCE ASSESSMENT

For the two uncertainty sets considered we can find expressions that are instrumental in assessing the performance of a given closed-loop configuration. With respect to the coprime factor uncertainty set, defined in section

2, simple manipulation of the equations shows that  $P_0 \in \mathcal{P}_{CF}(\hat{N}_{o,F}, \hat{D}_{o,F}, \delta_N, \delta_D)$  if and only if <sup>5</sup>

$$\left| T(P_o, C) - \begin{bmatrix} \hat{N}_{o,F} \\ \hat{D}_{o,F} \end{bmatrix} F [C \ I] \right| \leq \left| \begin{bmatrix} \delta_N(\omega) \\ \delta_D(\omega) \end{bmatrix} F [C \ I] \right| \quad (17)$$

for all  $\omega$ .

This non-conservative representation of  $\mathcal{P}_{CF}$  has direct connection with the control performance cost function considered. Using the fact that the  $\mathcal{H}_\infty$  norm of a matrix with rank 1 is equal to the supremum over frequency of its Frobenius norm, it follows that

$$\|T(P_o, C)\|_\infty \leq \sup_\omega \left\| \begin{bmatrix} |\hat{N}_{o,F}| + \delta_N(\omega) \\ |\hat{D}_{o,F}| + \delta_D(\omega) \end{bmatrix} |F| [|C| \ I] \right\|_F. \quad (18)$$

For the dual-Youla uncertainty set the results are quite similar. Considering a left coprime factorization (lcf)  $\tilde{D}_c, \tilde{N}_c$  of  $C$  such that  $\tilde{D}_c D_x + \tilde{N}_c N_x = I$ , then  $P_0 \in \mathcal{P}_R(N_x, D_x, N_c, D_c, \hat{R}_p, \gamma)$  if and only if

$$\left| T(P_o, C) - \begin{bmatrix} N_x + D_c \hat{R}_p \\ D_x - N_c \hat{R}_p \end{bmatrix} [\tilde{N}_c \ \tilde{D}_c] \right| \leq \begin{bmatrix} |D_c| \\ |N_c| \end{bmatrix} \gamma(\omega) [|\tilde{N}_c| \ |\tilde{D}_c|] \quad (19)$$

for all  $\omega$ , and consequently  $\|T(P_o, C)\|_\infty \leq$

$$\leq \sup_\omega \left\| \begin{bmatrix} |N_x + D_c \hat{R}_p| + |D_c| \gamma(\omega) \\ |D_x - N_c \hat{R}_p| + |N_c| \gamma(\omega) \end{bmatrix} [|\tilde{N}_c| \ |\tilde{D}_c|] \right\|_F. \quad (20)$$

Expressions (17) and (19) show that the two uncertainty sets considered can directly be characterized (non-conservatively) in a way that closely relates to the performance function considered. Equations (18) and (20) provide direct expressions for assessing achieved performance.

## 5. ROBUST STABILITY TEST

Using either of the two uncertainty sets  $\mathcal{P}_{CF}$  or  $\mathcal{P}_R$  a robust stability test can be performed for a designed (but not yet implemented) controller  $C_{new}$ .

**Proposition 5.1** *Consider the uncertainty set  $\mathcal{P}_{CF}$  (8). Then a stabilizing controller  $C_{new}$  for  $\hat{N}\hat{D}^{-1}$  stabilizes all plants  $P_o \in \mathcal{P}_{CF}$  if*

$$\left\| [\delta_D(\omega) + C_{new} \delta_N(\omega)] [\hat{D} + C_{new} \hat{N}]^{-1} \right\|_\infty < 1 \quad (21)$$

<sup>5</sup> The matrix inequality  $|A| \leq |B|$  will be used to indicate that the inequality holds for all scalar matrix elements separately.

**Proof:** See De Vries (1994).  $\square$

This result is a direct consequence of the small gain theorem. When the coprime factors of the plant are tuned to be normalized, a robust stability test based on the gap metric can also be used (see Van den Hof et al., 1995). For the dual-Youla uncertainty the following result holds.

**Proposition 5.2** *Consider the uncertainty set  $\mathcal{P}_R$  (13). Then  $C_{new}$  stabilizes all plants in the set  $\mathcal{P}_R$  if*

$$|R_c(e^{i\omega})| [|\hat{R}_p(e^{i\omega})| + \gamma(\omega)] < 1 \quad \text{for all } \omega. \quad (22)$$

**Proof:** Follows from Tay et al. (1989) and in Schrama (1992).  $\square$

This test can be applied using expression (16) for  $R_c$ . Note that when the new controller  $C_{new}$  is equal to the old controller, then  $R_c = 0$  and the latter stability test is non-conservative. In that case stability is guaranteed for *any* stable  $R_p$  because of the dual-Youla parametrization.

Both robust stability tests (21) and (22) can directly be performed based on one of the two uncertainty sets.

## 6. ROBUST PERFORMANCE TEST

For testing robust performance, we need to find an expression for  $\|T(P_o, C_{new})\|_\infty$  before having implemented the new controller on the plant. Using a coprime factor uncertainty set, this problem is quite hard to tackle for the general performance function that we consider here.

One option is to use the uncertainty that is estimated on the filtered sensitivity function  $S_o F^{-1} = D_{o,F}$  for deriving an additive uncertainty region around the open loop plant  $P_o$ . Once this is available, expressions for  $T(P_o, C_{new})$  can be generated; however these latter expressions will necessarily involve conservatism since we have to deal with the problem of bounding the uncertainty in expressions like  $P_o [I + C_{new} P_o]^{-1}$  where the uncertain element  $P_o$  appears in both the numerator and the denominator of the expression.

The procedure suggested is based on the following result that is valid for scalar plants only.

**Proposition 6.1** *Consider  $\mathcal{P}_D(\hat{D}_{o,F}, \delta_D) :=$*

$$\left\{ D | D = \hat{D}_{o,F} + \Delta_D, |\Delta_D(e^{i\omega})| \leq \delta_D(\omega) \right\},$$

*and denote  $\bar{S}(e^{i\omega}) := \hat{D}(e^{i\omega}) F(e^{i\omega})$  and  $\delta_S(\omega) := |F(e^{i\omega})| \delta_D(\omega)$ . Then  $D_{o,F} \in \mathcal{P}_D$  if and only if*

$$|P_o(e^{i\omega}) - \bar{P}(e^{i\omega})| \leq \delta_{\bar{P}}(\omega) \quad (23)$$

for all  $\omega$ , with

$$\bar{P}(e^{i\omega}) := \frac{1}{C} \left( \frac{\bar{S}^*}{|\bar{S}|^2 - \delta_S^2} - 1 \right) \quad (24)$$

$$\delta_{\bar{P}}(\omega) := \frac{1}{|C|} \frac{\delta_S}{|\bar{S}|^2 - \delta_S^2} \quad (25)$$

**Proof:** See Van Donkelaar et al.(1995).  $\square$

With the additive uncertainty description on the open-loop transfer function an uncertainty description can be derived for the sensitivity function of the plant in feedback with a newly designed controller, i.e.  $S_{o,new} = [I + C_{new}P_o]^{-1}$ . This can be done in a non-conservative way (Van Donkelaar et al., 1995), leading to the result that  $D_{o,F} \in \mathcal{P}_D$  if and only if

$$\left| S_{o,new} - \left( \frac{1 + C_{new}^* \bar{P}^*}{|1 + C_{new} \bar{P}|^2 - |C_{new}|^2 \delta_{\bar{P}}^2} \right) \right| \leq \frac{|C_{new}| \delta_{\bar{P}}}{|1 + C_{new} \bar{P}|^2 - |C_{new}|^2 \delta_{\bar{P}}^2} \quad (26)$$

for all  $\omega$ , and where  $C_{new}$  is any newly designed controller.

The above result shows that one can obtain non-conservative expressions for the uncertainty in the sensitivity function that is achieved for a newly designed controller. Similar expressions can also be obtained for the (2,1)-element of the matrix  $T(P_o, C_{new})$ . Problems occur when trying to handle the other elements in this matrix, as in those situations one has to deal with expressions for  $P_o$  in both numerator and denominator.

For the dual-Youla uncertainty set tedious manipulations reveal (Schrama, 1992) that - using similar notation as in (19) - we can write

$$T(P_o, C_{new}) = \begin{bmatrix} N_x + D_c R_p \\ D_x - N_c R_p \end{bmatrix} [I + R_c R_p]^{-1} \begin{bmatrix} \tilde{N}_c + R_c \tilde{D}_x \\ \tilde{D}_c - R_c \tilde{N}_x \end{bmatrix}. \quad (27)$$

Note that in this expression all terms are known except for  $R_p$ . By bounding the amplitude of all transfers in this expression we can write:

$$\begin{aligned} & \left[ \begin{array}{l} |N_x + D_c \hat{R}_p| - |D_c| \gamma(\omega) \\ |D_x - N_c \hat{R}_p| - |N_c| \gamma(\omega) \end{array} \right] \left[ |I + R_c \hat{R}_p| + |R_c| \gamma(\omega) \right]^{-1} \\ & \cdot \left[ |\tilde{N}_c + R_c \tilde{D}_x| \quad |\tilde{D}_c - R_c \tilde{N}_x| \right] \leq |T(P_o, C_{new})| \leq \\ & \leq \left[ \begin{array}{l} |N_x + D_c \hat{R}_p| + |D_c| \gamma(\omega) \\ |D_x - N_c \hat{R}_p| + |N_c| \gamma(\omega) \end{array} \right] \left[ |I + R_c \hat{R}_p| - |R_c| \gamma(\omega) \right]^{-1} \\ & \left[ |\tilde{N}_c + R_c \tilde{D}_x| \quad |\tilde{D}_c - R_c \tilde{N}_x| \right]. \quad (28) \end{aligned}$$

leading directly to the result that  $\|T(P_o, C)\|_\infty$  is upper bounded by the supremum of the Frobenius norm of the right hand side matrix.

Note that for deriving expression (28) some conservatism is added by bounding the denominator term  $[I + R_c R_p]^{-1}$  in amplitude by substituting  $\gamma(\omega)$ . However note that the conservatism that is added here vanishes when  $R_c$  tends to 0, i.e. when the newly designed controller is close to the present controller.

## 7. EXPERIMENTAL RESULTS

The identification methods described in this paper have been applied to data of the (unstable) radial loop of a CD pick-up mechanism controlled with a PID-controller. Signal  $r(t)$  was chosen as a multi sinusoid with 8 periods of length  $N_0 = 1024$ . The frequencies are linearly spaced between 100 Hz and 10 kHz. For coprime factor identification the scheme is followed as proposed in de Callafon and Van den Hof (1995). The method developed by de Vries (1994) is applied for identification of model uncertainty bounds.

For the estimated  $\mathcal{P}_{CF}$  the performance assessment according to (17) is shown in figure 3. For the two transfer functions  $P_0 S_0$  and  $S_0$  that are directly accessible from data, the results of a spectral estimate are also given. The spectral estimates are well captured within the uncertainty bounds.  $\mathcal{P}_{CF}$  is also applied in the ro-

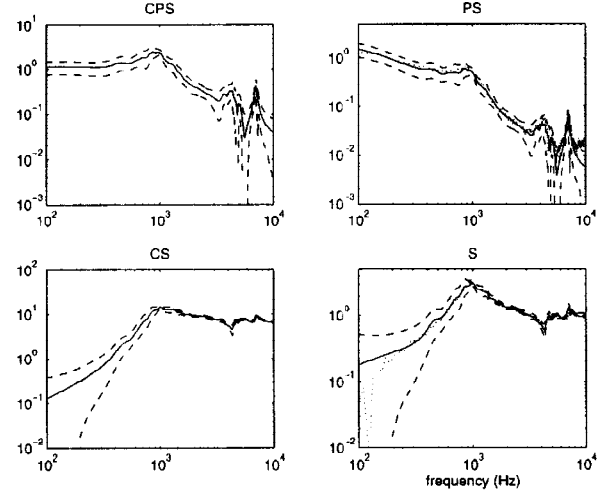


Fig. 3. Performance assessment; Bode amplitude plots of  $T(P_o, C)$  according to (17), with uncertainty bounds (---); spectral estimates of  $P_o S_0$  and  $S_0$  (..).

bust stability test (21), with respect to the original PID controller  $C$ , and a PID controller with enlarged gain of factor 1.6 ( $C_1$ ). For controller  $C_1$  the robust stability test is passed. This same holds for the test (22) applied to  $\mathcal{P}_R$ . Next the worst-case performance for the new controller  $C_1$  is assessed, based on identified sets  $\mathcal{P}_{CF}$  and

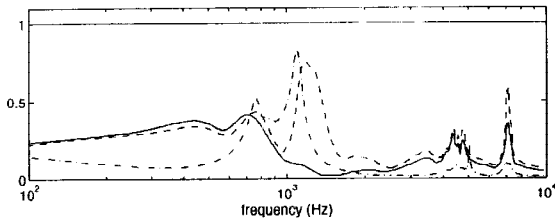


Fig. 4. Robust stability test (21) for  $C$  (—),  $C_1$  (---) and test (22) for  $C_1$  (-.-).

$\mathcal{P}_R$ . For  $\mathcal{P}_{CF}$  this test leads to the sensitivity bounds

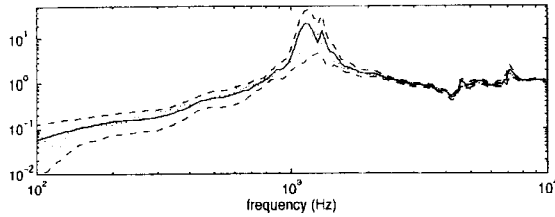


Fig. 5. Bode amplitude plot of  $S_{o,new} = (1 + C_1 P_o)^{-1}$  with uncertainty bounds based on  $\mathcal{P}_{CF}$ , and a spectral estimate obtained after implementation (···).

as shown in figure 5. For  $\mathcal{P}_R$  this test can be performed for all elements of the  $T(P_o, C_1)$  matrix. Results for the second column of  $T(P_o, C)$ , i.e. the elements  $S_{o,new}$  and  $P_o S_{o,new}$  are shown in figure 6.

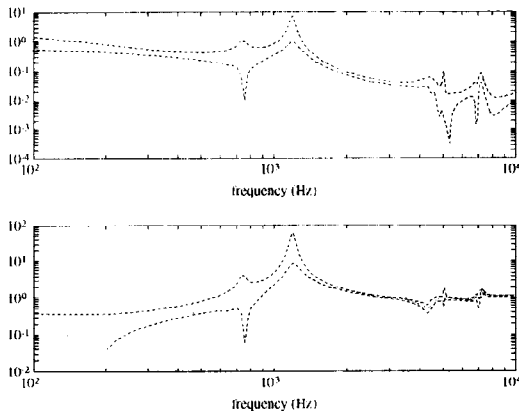


Fig. 6. Uncertainty bounds on Bode amplitude plots of  $P_o S_{o,new} = P_o(1 + C_1 P_o)^{-1}$  (upper) and  $S_{o,new}$  (lower) based on  $\mathcal{P}_R$ , with spectral estimate of  $S_{o,new}$  (dotted).

## CONCLUSIONS

It is shown that uncertainty sets based on additive coprime factor uncertainty and dual-Youla parameter uncertainty can be fruitfully used in either a performance

assessment test, a robust stability test and a robust performance test for a controlled plant for a general type of control performance criterion. The results can be used to robustify the control design in iterative schemes of identification and control design, where the control design is mostly restricted to a nominal design only. A further extension of the results presented here, and involving sub-optimal designs by employing  $\mu$ -tests, is provided in de Callafon and Van den Hof (1996).

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