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Identification of Experimental Models for Control Design

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Abstract *– The identification of linear models that are particularly suitable for serving as a basis for (robust) model-based control design has recently attracted considerable attention. Both the system identification community and the control community have spent considerable efforts in developing a coherent approach to the problem. Typical problems that have to be dealt with consider questions of optimal experiment design, feedback-relevant system approximations and control-relevant model uncertainty specifications.*
Research into these problems has delivered several attempts for bridging the gap between identification and control theory. In this lecture these developments will be highlighted, directing particular attention to the identification of control-relevant approximate models, the use of closed-loop experimental data for identification, the quantification of model uncertainty, and the use of identification criteria that are motivated by control performance cost functions.

Keywords *– System identification, control design, closed-loop identification, uncertainty modelling, robustness properties.*

I. INTRODUCTION

The identification of dynamic models out of experimental data has very often been motivated and supported by the presumed ability to use the resulting models as a basis for model-based control design. As such, control design is considered an important intended-application area for identified models. On the other hand, model-based control design is built upon the assumption that a reliable model of the plant under consideration is available. In situations where a first-principles-based model alone does not provide a sufficiently accurate description of reality, the analysis of experimental data is an important tool for providing the necessary accurate description of the system dynamics.

The question how to identify dynamic models that are most suitable for a model-based control design could have the following straightforward answer:

1 Identify from experimental data that dynamical model that is the most accurate description of reality;
2 Use that model as a basis for control design;
3 Implement the designed controller on the real plant.

Actually this answer is based on two related simplifying assumptions; from an identification point of view it is assumed that the real underlying system dynamics can be identified exactly from experimental data. This assumption is partly covered by the expression that the system is supposed to belong to the considered model set ($\mathcal{S} \in \mathcal{M}$) [1]. In a control design context the considered situation is reflected by the *certainty equivalence principle*, meaning that a control system is designed on the basis of a model that is supposed to reflect the exact dynamical behaviour of the underlying plant. This assumption then justifies the implementation of the designed controller on this plant.
For arriving at a methodology that is feasible for reliable application in high-performance and advanced control problems, a more solid approach to the problem is required, and also possible. Such a more solid approach requires the incorporation of robustness properties in both the identification and control problems. With the incorporation of robustness properties it should be avoided that the end result (the performance of the implemented control system) is critically dependent on model inaccuracies that might occur.

In the last two decades the development of robust control theory (see e.g. [2]) has provided tools for designing control systems that take account of model inaccuracies. This is done by evaluating and designing control systems that guarantee a particular level of performance for all models in a suitable model uncertainty set, rather than just for a single nominal model.
In an identification context the construction of model uncertainty bounds (in the format of parameter confidence intervals) has been part of the classical parameter estimation methods ([3], [1]). However, the validity of these confidence bounds is limited to the situation of consistent estimates, i.e. the situation that the considered set of candidate models contains the real data generating system ($\mathcal{S} \in \mathcal{M}$). This restriction can be rather severe when dealing with real-life situations. Therefore attention has been given to the development of uncertainty bounds for experimental models, also in situations of model approximations.

Another aspect of the model building is the question which dynamical aspects of the underlying system are most relevant to take into account in view of the performance of the resulting model-based controller. There are many examples from (industrial) practice that show that very complex plants can effectively be controlled by rather simple (linear) controllers. This directly implies that only simple (low order) models should be sufficient as a basis for appropriate control design. The question then occurs: which dynamical phenomena of the system have to be modeled accurately, and which aspects are less relevant. A quantification of these phenomena could lead to particular guidance in the experiment design: only simple experiments can be performed that are directed towards retrieving that model information that is required for designing a reliable control system, see. e.g. [4] for an example of this.

In this paper, the several aspects mentioned above will be discussed separately and the results and contributions that are achieved will be highlighted. The overall goal that will be pursued is:

> The design of appropriate identification experiments and identification methods that support the design of a high-performance model-based control system.

Attention will be limited to the identification of black box parametric models. The results also have their impact on the situation of first-principles-based model structures, but that will not be pursued any further here.

First attention will be given to control-relevant nominal models; then identification of these models will be considered on the basis of open-loop and closed-loop experiments. After a discussion on model uncertainty estimation, attention will be given to the use of identified models (and their uncertainty bounds) in robust control design.

II. NOMINAL MODELS FOR MODEL-BASED CONTROL DESIGN

When discussing control systems the configuration will be considered as shown in figure 1.

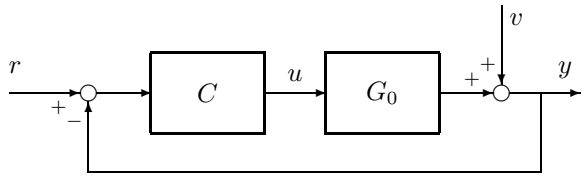

Fig. 1. Closed-loop configuration.

In this configuration the signal r is a tracking or a setpoint signal being uncorrelated with v which is a (stochastic) noise disturbance; G_0 and C are the scalar plant and controller, q is the forward shift operator: $qu(t) = u(t+1)$. The closed-loop system equations are:

$$y(t) = G_0CS_0r(t) + S_0v(t) \quad (1)$$
$$u(t) = S_0Cr(t) - CS_0v(t) \quad (2)$$

with the sensitivity function $S_0 = (1+CG_0)^{-1}$.
It will be assumed that the closed-loop system is internally stable, meaning that the four transfer functions in (1)-(2) are stable, i.e. analytic in $|z| \geq 1$. In order to avoid technicalities, it will also be assumed that the product G_0C is strictly proper, which means that either C or G_0 contains a time delay.
The feedback controller C is designed to achieve particular properties of the closed-loop system, such as:

- stability,
- tracking properties, i.e. the complementary sensitivity function $T_0 = G_0CS_0$ (the transfer from r to y) should have a high bandwidth to guarantee a fast response of the closed-loop system
- disturbance attenuation, i.e. the sensitivity function S_0 (the transfer from v to y) should be sufficiently small in those frequency areas where v has significant contributions, and possibly
- limited power (or amplitude) of the control signal u, through a limitation on CS_0.

Additionally these properties should be maintained under slight variations on plant (and model) dynamics. Typical curves for T_0 and S_0 are sketched in Figure 2.

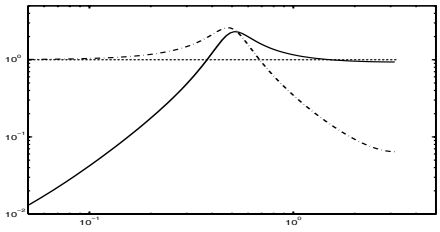

Fig. 2. Typical curve for Bode magnitude plot of sensitivity function $S_0(e^{i\omega})$ (solid) and related complementary sensitivity $T_0(e^{i\omega})$ (dashed).

The next example shows that not all frequency ranges are equally important when it comes down to constructing a control-relevant model.

Example 1: Suppose that our control design procedure involves the incorporation of an integrating (I) action in the controller to avoid static errors. Then $|C(e^{i\omega})| >> 1$ for ω small. As a result for low frequencies:

$$|S_0| \approx |CG_0|^{-1} << 1 \qquad |T_0| \approx 1$$

as long as $|CG_0| >> 1$. If the controller C would be designed on the basis of a model $\hat{G}$, e.g. to achieve an appropriate bandwidth, the low frequency behaviour of this model would hardly influence the control system. As a result a high level of accuracy in $\hat{G}$ in the low frequency range does not essentially contribute to the performance of the control system. □

It is a well know rule of thumb that for designing a model-based controller the accuracy of the model should be high in particular in the frequency range around the bandwidth of the closed-loop system. However this qualitative information one would like to incorporate in a structural way when designing identification experiments, and selecting/developing identification methods.
One attempt to formalize this situation is to rewrite the control performance cost function into a related modelling (or identification) criterion:
Consider a control performance cost function $\|J(G_0, C_{\hat{G}})\|$, related to a closed-loop system with plant G_0 and controller $C_{\hat{G}}$ being designed on the basis of a plant model $\hat{G}$. One can think of J as e.g. a weighted sensitivity function:

$$J(G_0, C_{\hat{G}}) = \frac{V}{1 + C_{\hat{G}} G_0} \tag{3}$$

as a cost function aiming at a control system that satisfies: $|S_0(e^{i\omega})| < |V(e^{i\omega})|^{-1}$; alternative choices for J include LQ/LQG criteria, model reference control, and robustness optimization procedures (see e.g. [5], [6]). It is the aim of the control system to achieve a minimum value of $\|J(G_0, C_{\hat{G}})\|$, through an appropriate choice of $\hat{G}$ and $C_{\hat{G}}$. As a global optimization problem this will generally be intractable as it requires both a modelling and a control design variable. However employing the triangle inequality:

$$\begin{array}{c} \Big| \|J(\hat{G}, C_{\hat{G}})\| - \|J(G_0, C_{\hat{G}}) - J(\hat{G}, C_{\hat{G}})\| \Big| \leq \\ \leq \|J(G_0, C_{\hat{G}})\| \leq \\ \leq \|J(\hat{G}, C_{\hat{G}})\| + \|J(G_0, C_{\hat{G}}) - J(\hat{G}, C_{\hat{G}})\|, \end{array} \tag{4}$$

it appears that the *achieved performance cost* $\|J(G_0, C_{\hat{G}})\|$ will be close to the designed performance cost $\|J(\hat{G}, C_{\hat{G}})\|$ provided that the *performance degradation term* $\|J(G_0, C) - J(G, C)\|$ is sufficiently small. This exactly shows the main issue that a control design engineer is aiming at: designing a model-based controller that -after implementation on the real plant- shows a performance cost that is similar to the performance of the controlled model. In this way this degradation term can be given the interpretation of a control-performance induced modelling criterion:

$$\hat{G} = \arg\min_G \|J(G_0, C) - J(G, C)\|.$$

For the choice of J as given in (3) this takes the form:

$$\begin{aligned} \hat{G} &= \arg\min_G \left\| \frac{V}{1 + CG_0} - \frac{V}{1 + CG} \right\| && (5) \\ &= \arg\min_G \left\| \frac{V(G_0 - G)C}{(1 + CG_0)(1 + CG)} \right\|. && (6) \end{aligned}$$

Note however that the C on the right hand side of these expressions actually is again dependent on G. By fixing the controller on the right hand side, a feasible optimization problem results that might be embedded in an iterative scheme of solving for $\hat{G}$ and C in an alternating way. The modeling criterion derived above shows that for model-based control design two models should be considered "close" to each other if their corresponding weighted sensitivity functions are close.
In the next section the possibilities will be evaluated to design identification methods based on the modelling criterion (6).

III. IDENTIFICATION OF NOMINAL MODELS

A. The open-loop case

In the open-loop situation ($C \equiv 0$) the standard way of handling the identification is to parametrize the model $(G(q, \theta), H(q, \theta))$ using a finite- dimensional parameter vector $\theta \in \Theta$. A parameter estimate is constructed by minimizing a least-squares criterion:

$$\hat{\theta}_N = \arg\min_\theta \frac{1}{N} \sum_{t=1}^{N} \varepsilon_f^2(t, \theta) \tag{7}$$

where the filtered prediction error

$$\varepsilon_f(t, \theta) = L(q) H(q, \theta)^{-1} [y(t) - G(q, \theta) u(t)]$$

with L a stable (user-chosen) prefilter that is applied to the input/output data. If the input signal is persistently exciting, the plant G_0 can be consistently identified provided that (a) G_0 and H_0 can be modelled exactly within the model set chosen, or (b) G_0 can be modelled exactly, and $G(q, \theta)$ and $H(q, \theta)$ have no common parameters ([1]).

Under fairly general conditions the parameter estimate $\hat{\theta}_N$ converges to $\theta^\star$ with probability 1 for $N \to \infty$ where

$$\theta^\star = \arg\min_\theta \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_\varepsilon(\omega, \theta) d\omega. \tag{8}$$

and $\Phi_\varepsilon(\omega, \theta)$ the power spectral density of $\varepsilon_f(t, \theta)$. For an output error model structure ($H(q, \theta) = 1$), it follows that

$$\begin{aligned} \theta^\star &= \arg\min_\theta \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_0 - G(\theta)|^2 \Phi_u |L|^2 d\omega && (9) \\ &= \arg\min_\theta \|[G_0 - G(\theta)] H_u L\|_2 && (10) \end{aligned}$$

with H_u a stable spectral factor of Φ_u. Note that in (9), as in the sequel, the arguments $e^{i\omega}$ are suppressed for brevity. Reduced-order identification now involves an intrinsic model reduction step with a (frequency weighted) norm that can be directly tuned by the user through designing Φ_u and/or L.
When comparing the expression (10) with the criterion that was desired from a control-relevant modelling point of view (6) it appears that it will not be simply possible to match the two criteria by appropriate choices of H_u and L. This problem will appear to be quite different when considering closed-loop experimental data.

B. Closed-loop experiments

B.1 Direct identification

In the direct identification method one simply applies the standard (prediction error) identification procedure without taking account of the presence of a feedback controller. A parameter estimate is obtained similarly as in the open-loop case.
If the system $\mathcal{S} := (G_0, H_0)$ is present in the model set $\mathcal{M} := \{(G(q,\theta), H(q,\theta)), \theta \in \Theta\}$, then a consistent estimate is obtained under general persistency of excitation conditions. However in the situation that $\mathcal{S} \notin \mathcal{M}$ the properties of the direct method collapse, and even in the situation that the plant G_0 can be modeled exactly within $\mathcal{M}$, consistency of $G(q,\hat{\theta}_N)$ is lost if the noise model is misspecified. The asymptotic identification criterion in the frequency domain, as in (8), is governed by

$$\Phi_\varepsilon = \frac{|S_0|^2|G_0 - G(\theta)|^2|C|^2}{|H(\theta)|^2}\Phi_r + \frac{|H_0|^2|S_0|^2}{|H(\theta)|^2|S(\theta)|^2}\lambda_0$$

where $S(\theta) = (1 + CG(q,\theta))^{-1}$ is the sensitivity function of the parametrized model. Due to the fact that the two terms in the right hand side of the expression above are coupled through θ, it will not be possible to obtain an identificaiton criterion similar to (6). Note however that the appearance of the sensitivity function S_0 as a weighting function in Φ_ε already points to a phenomenon that is also present in (6).

B.2 Indirect approaches

As alternatives to the direct approach, several methods have been developed recently, being generalizations of the classical indirect and joint i/o-methods [7]. For an overview of the more recent developments see [8], [9].

- In the *two-stage method* first the transfer functon from r to u is identified, and with this estimate a noise-free input signal $\hat{u}_r$ is simulated. In the second step the transfer between $\hat{u}_r$ and y is identified.
- In the *coprime-factor method* a one input (r) two output (y, u) model is identified and a plant model is constructed by taking the quotient of the two estimated transfers.
- In the *indirect method* or *dual-Youla-Kucera method* the transfer function between r and y is identified, and with knowledge of the controller C a corresponding plant model is calculated.
- In the *tailor-made parametrization approach* the closed-loop transfer function from r to y is parametrized in terms of the model parameters of G and the known controller. Then the model parameters of G are identified on the basis of data r and y.

These identification methods have a number of properties in common. When using fixed (non-parametrized) noise models during identification, consistent plant models of G_0 can be obtained, and the limiting parameter estimate is given by[1]

$$\theta^\star = \arg\min_\theta \frac{1}{2\pi}\int_{-\pi}^{\pi} |S_0G_0C - S(\theta)G(\theta)C|^2 \frac{\Phi_r}{|K_*|^2} d\omega. \tag{11}$$

By designing the (fixed) noise model K_* (or the signal spectrum Φ_r), this bias expression can explicitly be tuned to the designer's needs. However the expression is different from the related open-loop expression (9). Instead of a weighted additive error on G_0, the integrand contains an additive error on the closed-loop transfer G_0S_0C. Straightforward calculations show that

$$G_0S_0 - G(\theta)S(\theta) = S_0[G_0 - G(\theta)]S(\theta),$$

so that the asymptotic bias distribution can be characterized by

$$\theta^\star = \arg\min \left\| \frac{[G_0 - G(\theta)]C\Phi_r^{\frac{1}{2}}}{(1 + CG_0)(1 + CG(\theta))K_*} \right\|_2. \tag{12}$$

This implies that the additive error on G_0 is always weighted with S_0, and thus emphasis will be given to an accurate model fit in the frequency region where S_0 is large, being typically the region that determines the bandwidth of the control system, as depicted in the typical situation of Figure 2. By setting K_* according to

$$\Phi_r(\omega) = |K_*|^2|V|^2$$

expression (12) will become equivalent to (6), and so a control-relevant identification criterion has been obtained.

At frequencies where S_0 will be small (low frequencies), poor process knowledge is obtained. This refers to the situation of Example 1 where it is motivated that in this frequency range accurate model information is not required.

[1] Details vary slightly over the methods.

Due to Bode's sensitivity integral, $\int_0^\pi log\ |S(e^{i\omega})|\ d\omega = c$, (constant), the attenuation of signal power in the low frequency range, will always be "compensated" for by an amplification of signal power in the higher frequency range.

Whereas direct identification needs consistent estimation of noise models in order to consistently identify G_0, the alternative methods can do without noise models. Incorporation of noise models here is very well possible, but this will result in bias distributions that become dependent on the identified noise models as well as on (the unknown) noise spectrum Φ_v.
The indirect methods for closed-loop identification are implemented in a public domain add-on to Matlab's System Identification toolbox, see [10].

C. Variance

The analysis above is directed towards the limiting parameter estimate $\theta^\star$, and therefore only considers structural (bias) errors in the models. For analyzing the asymptotic variance of the transfer function estimates the prediction error framework provides variance expressions that are asymptotic in both n (model order) and N (number of data) [1]. For the direct identification approach, and in the situation that $\mathcal{S} \in \mathcal{M}$ this delivers:

$$cov \begin{pmatrix} \hat{G}(e^{i\omega}) \\ \hat{H}(e^{i\omega}) \end{pmatrix} \sim \frac{n}{N}\Phi_v(\omega) \cdot \begin{bmatrix} \Phi_u(\omega) & \Phi_{eu}(\omega) \\ \Phi_{ue}(\omega) & \lambda_0 \end{bmatrix}^{-1}. \quad (13)$$

This reduces to ([11]):

$$cov \begin{pmatrix} \hat{G} \\ \hat{H} \end{pmatrix} \sim \frac{n}{N}\frac{\Phi_v}{\Phi_u^r} \cdot \begin{bmatrix} 1 & (CS_0H_0)^* \\ CS_0H_0 & \dfrac{\Phi_u}{\lambda_0} \end{bmatrix},$$

where Φ_u^r is the spectral density of u^r being that part of the input signal that is generated by the reference signal, i.e. $u^r = S_0Cr$. Consequently

$$cov(\hat{G}) \sim \frac{n}{N}\frac{\Phi_v}{\Phi_u^r} \qquad cov(\hat{H}) \sim \frac{n}{N}\frac{\Phi_v}{\lambda_0}\frac{\Phi_u}{\Phi_u^r}. \quad (14)$$

This shows that only the noise-free part u^r of the input signal u contributes to variance reduction of the transfer functions. Note that for $u^r = u$ the corresponding open-loop results appear. In [11] it is shown that for the alternative indirect methods presented before, these expressions are the same.
The asymptotic variance analysis tool gives an appealing indication of the mechanisms that contribute to variance reduction. It also illustrates one of the basic mechanisms in closed-loop identification, i.e. that noise in the feedback loop does not contribute to variance reduction. Particularly in the situation that the input power of the process is limited, it is relevant to note that only part of this input power can be used for variance reduction.

The results seem to indicate that from a variance point of view, open-loop identification always has to be preferred over closed-loop identification (since $\Phi_u^r \leq \Phi_u$). However when a controller is designed on the basis of both $\hat{G}$ and $\hat{H}$ also the nondiagonal terms of $cov(\hat{G}\ \hat{H})^T$ will play a role, leading to the situation that in those cases a closed-loop experiment leads to minimal variance of the resulting controller [12].

Because of the "doubly asymptotic" nature of the results $(N, n \to \infty)$, the asymptotic variance analysis tool is also quite crude. For finite model orders, the variance results will likely become different over the several methods, as discussed in [9].

IV. QUANTIFICATION OF MODEL UNCERTAINTY FROM EXPERIMENTAL DATA

A. The classical approach

In a statistical (prediction error) framework the parameter estimate (7) will -under fairly general conditions- lead to a consistent estimate of the underlying plant G_0, H_0 provided that the input signal is sufficiently informative, and the model set is flexible enough to contain the real system, i.e. $\mathcal{S} \in \mathcal{M}$ which means that there exists a $\theta_0 \in \Theta$ such that $G(q,\theta_0) = G_0(q)$ and $H(q,\theta_0) = H_0(q)$.
Moreover, under the same conditions, the parameter estimate $\sqrt{N}(\hat{\theta}_N - \theta_0)$ will converge in distribution to a Gaussian distribution for $N \to \infty$ with asymptotic covariance matrix P, being dependent on noise level and on derivatives of the cost function with respect to the parameters. Important point here is that P can also be estimated from data, and as a result a (statistical) parameter confidence interval can be constructed of the form:

$$(\hat{\theta}_N - \theta_0)^T \frac{\hat{P}^{-1}}{N}(\hat{\theta}_N - \theta_0) \in \chi^2(d)$$

where $\chi^2(d)$ refers to a χ^2 distribution with $d = dim(\theta)$ degrees of freedom.

This (parametric) quantification of model uncertainty has two shortcomings when considered in the scope of the problem discussed in the previous section. The bound leads to a probabilistic expression for model uncertainty, which at the moment can not easily be translated into hard guarantees for control performance. The model uncertainty dealt with in a robust control configuration is generally of a hard-bounded (non-probabilistic) nature ([2]). The second point is that the assumption of exact modeling ($\mathcal{S} \in \mathcal{M}$) has been considered a rather severe restriction of its applicability.

B. Hard-bounded model error bounds

As an alternative, and in particular in order to avoid probabilistic bounds, the so-called bounded error models have

been extensively given attention, see e.g. [13]. In this approach the framework is

$$y(t) = G_0(q)u(t) + v(t)$$

where v is any disturbance signal acting on the measurement, reflecting all kinds of deviations between measurement data and real system. A model $\hat{G}(q)$ is said to be unfalsified by the data if the residual signal

$$v'(t) = y(t) - \hat{G}(q)u(t)$$

belongs to some hypothesized class $\mathcal{V}$ of disturbance signals v. The set of models that is now unfalsified by the data is dependent on the choice of the class $\mathcal{V}$. Several alternatives for $\mathcal{V}$ have been considered, as e.g.

- Bounded error disturbances $|v(t)| \leq c \in \mathbb{R}$
- Bounded power disturbances $\frac{1}{N}\sum_{t=1}^{N} v^2(t) \leq c \in \mathbb{R}$

while related approaches are also pursued in a frequency domain setting (for an overview see e.g. [14]). The *assumption* on the noise disturbance class $\mathcal{V}$ will have a high influence on the set of models that is unfalsified by the data. If the chosen set $\mathcal{V}$ is hard-bounded, then the resulting set of unfalsified models will necessarily also be hard-bounded, and so a non-probabilistic specification of model uncertainty results.
If the considered model $G(q,\theta)$ is linear in θ (e.g. FIR models or other basis function expansions) the resulting problems are computationally more attractive and solutions to the considered problem are relatively easily obtained.
The principal problem and challenge in the bounded error approach is the choice of disturbance class $\mathcal{V}$. If this class is chosen too large in comparison with the actual disturbances on the data, the results will be unnecessarily conservative, i.e. the uncertainty sets will be hard-bounded but unnecessarily large. The information that then is retrieved from the identified uncertainty set is very much limited. This happens in particular if the disturbance signals are allowed to be correlated with the input signal (which is the case when only considering amplitude bounds on v). The incorporation of constraints on the correlation between u and v can then be added to limit this conservatism. For a more extensive discussion the reader is referred to [14].

C. Statistical approaches for approximate models

As mentioned before, the classical -statistical- approach to the problem is restricted to the situation $\mathcal{S} \in \mathcal{M}$. In the last decade several approaches have been followed to generate results that go beyond this restriction. In the approach of stochastic embedding [15] the undermodelling error is embedded in a stochastic framework. In [16] and [17] solutions are constructed by a combined worst-case/probabilistic approach for bounding model uncertainty. The method of [17] can be summarized as follows:

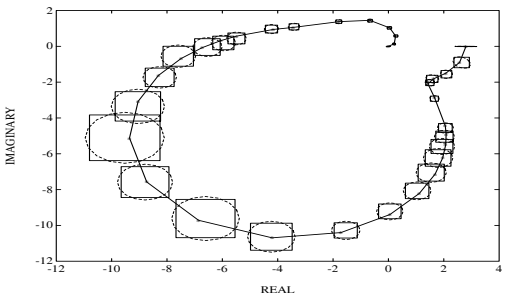

Fig. 3. Nyquist diagram with identified uncertainty bounds (rectangles, ellipsoids).

- It is assumed that measurement data y, u satisfies the system equations:

  $$y(t) = G_0(q)u(t) + v(t)$$

  with v a stationary stochastic process, independent of u, and that G_0 allows a series expansion $G_0(z) = \sum_{k=0}^{\infty} g_0(k) f_k(z)$ with the expansion coefficients bounded by an a priori known bound $|g_0(k)| \leq \bar{g}(k)$, $k > 0$.
- A linearly parametrized model $G(q,\theta)$ is chosen, and a finite number of expansion coefficients is identified with a least squares (linear regression) algorithm.
- Then for a fixed frequency ω the estimated model $\hat{G}(e^{i\omega})$ can be shown to satisfy

  $$\Delta(e^{i\omega}) := \hat{G}(e^{i\omega}) - G_0(e^{i\omega}) = \beta_1(\omega) + \beta_2(\omega) + \beta_3(\omega)$$

  where
  - β_1 reflects the neglected tail of the expansion; this term can be worst-case bounded;
  - β_2 reflects a bias term on the estimated coefficients due to the neglected tail; this term can also be worst-case bounded;
  - β_3 reflects a variance contribution, which can be bounded in a probabilistic way by using the asymptotic theory according to [1].
- The uncertainty bounds can be computed in any user-chosen frequency grid, and lead to a Nyquist curve with uncertainty regions in user-specified frequencies, as illustrated in figure 3. In all cases the three different sources of uncertainty can be distinguished, which allows the user to determine which part is dominant, and to adjust the experimental setup so as to reduce the overall uncertainty bound.

The result is an uncertainty bound

$$\begin{bmatrix} \text{Re } \Delta(e^{i\omega}) \\ \text{Im } \Delta(e^{i\omega}) \end{bmatrix}^T \Lambda^{-1} \begin{bmatrix} \text{Re}\Delta(e^{i\omega}) \\ \text{Im}\Delta(e^{i\omega}) \end{bmatrix} \leq \gamma(\omega)$$

with probability $\geq \alpha$ in a user-defined frequency grid Ω, and for a user-specified choice of α. The main difference of this result with the result from the classical statistical approach, is that in the latter situation there is a parametric uncertainty set, leading to a global confidence interval. When considering unmodelled dynamics, the parametric structure is lost and a probabilistic expression results for a particular frequency ω that is frozen, and where relations between several different frequencies is not simply taken into account.

D. A generalized linear model structure

In almost all uncertainty bounding procedures in the literature use is made of linearly parametrized models. The most commonly known linearly parametrized model structure in identification and uncertainty bounding procedures is a finite impulse response (FIR) model:

$$G(q,\theta) = b_0 + b_1 q^{-1} + \cdots b_n q^{-n},$$

which (because of the linear way in which b_i enters G) has several attractive properties. However, for an accurate modelling of moderately damped systems it may require a large number of parameters. This can be overcome by using dynamical (orthogonal) basis functions $f_k(q)$, leading to a model structure:

$$G(q,\theta) = c_0 + c_1 f_1(q) + \cdots c_n f_n(q).$$

Generalized orthogonal basis functions can be constructed on the basis of an all-pass function G_b ($|G_b(e^{i\omega})| = 1$) with a balanced state-space realization (A,B,C,D). Then the scalar components of

$$V_k(z) = (zI - A)^{-1}B \cdot G_b^{k-1}(z)$$

form an orthonormal basis for the space of all stable systems, and a generalization is obtained for the FIR models (corresponding to $G_b = z^{-1}$), Laguerre models (first-order G_b) and Kautz models (second order G_b). An interesting phenomenon is that it can be shown that whenever the set of poles of a system G_0 and of the all-pass function G_b approach each other, the rate of convergence of the series expansion $G_0 = \sum c_k f_k(z)$ increases. This allows more accurate modelling with a limited number of parameters. For details on this parametrization and its use in identification one is referred to [18], [19].

V. USE OF UNCERTAINTY MODELS FOR ROBUST CONTROL ANALYSIS AND DESIGN

On the basis of the procedures presented in the previous sections several different types of model uncertainty bounds can be obtained. Continuing with the statistical approach for approximate models, a typical result could be

$$\mathcal{P}(\hat{G},\gamma) = \{G(e^{i\omega}) = \hat{G}(e^{i\omega}) + \Delta(e^{i\omega}),\ |\Delta(e^{i\omega})| \leq \gamma(e^{i\omega})\}$$

where the uncertainty set is structured as an additive error on the nominal model. This structure is retrieved from the frequency-dependent ellipsoidal uncertainty regions as discussed in section IV-C. The reformulation into a norm-bounded set, being equivalent to:

$$\|\gamma^{-1}\Delta\|_\infty \leq 1$$

is motivated by the use of these kind of uncertainty sets in robust control analysis and design. The choice for a particular structure however is still open. Note that when applying the procedure to closed-loop data, and using one of the indirect closed-loop identification methods, the related result would become e.g.

$$\begin{aligned} \mathcal{P}(\hat{G},\gamma) \quad = \quad & \{G(e^{i\omega}), \frac{CG}{1+CG} = \hat{T}(e^{i\omega}) + \Delta(e^{i\omega}) \\ & |\Delta(e^{i\omega})| \leq \gamma(e^{i\omega})\}. \end{aligned} \tag{15}$$

This latter uncertainty set is clearly controller dependent.

Does it make a difference which option is chosen? Yes, because just as the considerations around the choice of a control-relevant nominal model, the uncertainty area should be such that it is directed towards obtaining small uncertainty in the control-relevant area. Therefore a closed-loop setting for the uncertainty structure is again attractive. For a more extensive discussion on this see [20].

The corresponding uncertainty structures (varying from open-loop additive bounds to bounds on closed-loop objects) can be written in a linear fractional transformation as indicated in figure 4,

$$y = [M_{22} + M_{21}\Delta(1 - M_{11}\Delta)^{-1}M_{12}]u$$

with $M_{22} = \hat{G}$, the nominal model, and where Δ is norm-bounded: $\|\Delta\|_\infty < 1$. This formulation of the problem now matches the use of these uncertainty models in robustness analysis and robust control design. As a consequence they can now be used to

- Verify the worst-case performance cost achieved for the current controller:

$$\sup_{G\in\mathcal{P}} \|J(G,C)\|_\infty$$

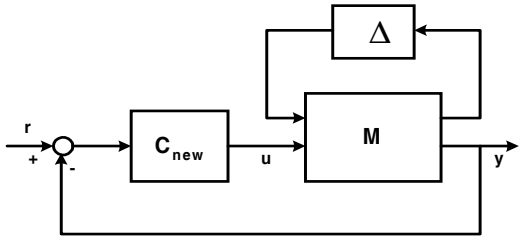

Fig. 4. New controller applied to a model uncertainty set with an LFT structured (norm-bounded) uncertainty.

- To verify whether a newly designed controller C_{new} will stabilize all models $G \in \mathcal{P}$, and
- To design a (robust) controller C_{new} that minimizes the worst-case performance cost:

$$\min_{C} \sup_{G \in \mathcal{P}} \|J(G, C)\|_\infty$$

These robustness analyses and synthesis problems are generally solved by so-called μ-analysis and synthesis tools, available from robust control theory, see e.g. [2]. For a generally formulated control performance cost function, these problems are worked out in [20]; applications to control designs in a wafer stepper and Compact Disc servo system are reported in [21], [22].
When employing the parametric uncertainty models as discussed in section IV-A robustness analysis can also be applied, as reported in [23], [24]. For application in control design of the bounded error type of modeling as discussed in section IV-B see e.g. [25].

VI. CONCLUSION

In this paper several aspects of the problem of identifying experimental models for use in model-based control design have been given attention. Interesting observations are that closed-loop experimental conditions should not necessarily be considered as a degenerate or unfavorable situation, but that they can play an important role in identifying accurate models for control. The challenging area of identification for control has delivered a couple of first solutions; there are still numerous open problems to be solved.

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