From closed-loop identification to dynamic networks: generalization of the direct method

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Abstract— **Identification methods for identifying (modules in) dynamic cyclic networks, are typically based on the standard methods that are available for identification of dynamic systems in closed-loop. The commonly used direct method for closed-loop prediction error identification is one of the available tools. In this paper we are going to show the consequences when the direct method is used under conditions that are more general than the classical closed-loop case. We will do so by focusing on a simple two-node (feedback) network where we add additional disturbances, excitation signals and sensor noise. The direct method loses consistency when correlated disturbances are present on node signals, or when sensor noises are present. A generalization of the direct method, the joint-direct method, is explored, that is based on a vector predictor and includes a conditioning on external excitation signals. It is shown to be able to cope with the above situations, and to retain consistency of the module estimates.**

I. INTRODUCTION

Identification methods for modelling dynamic (cyclic) networks are receiving considerable attention. Among different non-parametric and parametric approaches [1], [2], [3], [4], [5], a framework for the extension of prediction error approaches to the case of dynamic networks has been presented in [6]. Focussing on the problem of identifying a single module in a dynamic network, conditions have been formulated for prediction error methods to arrive at consistent module estimates, see e.g. [6], [7], while identifiability of the network has been addressed in [8], [9], [10]. The identification methods typically considered, are based on classical closed-loop identification methods, referred to as "direct method" or "two-stage / projection" method ([11]). In a closed-loop configuration the direct method is very attractive as it leads to consistent model estimates and asymptotically reaches the Cramer-Rao lower bound for the variance of model estimates. While basic consistency results for direct methods applied in a dynamic network situation have been derived in [6], [7], the analysis has been limited to the estimation of a single module and for a particular set-up. When considering a module as a basic building block for a dynamic network, we need to generalize this set-up to include disturbances on inputs and outputs that can be mutually correlated, external excitation signals that can be present on different locations, and sensor

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noise that can affect the measurements. We will illustrate the consequences of this generalized set-up on a very simple two-node feedback configuration. Starting from a classical two-node closed-loop system we are going to consider the options of adding (correlated) disturbance signals, external excitation signals, and sensor noise, to show what the effects are on consistency of the identification results. We will introduce and analyse a generalization of the direct method, by considering a (symmetric) vector signal predictor as well as adding external excitation signals as predictor inputs, as introduced in [12]. The resulting so-called joint-direct method will be shown to be able to provide consistency in the situation of correlated disturbances and sensor noise, and therefore can serve as a prime identification method for modules in dynamic networks.

After introducing the system setup that we will be considering, the classical direct method of closed-loop identification will be briefly reviewed in Section III, and its results will be evaluated for our more general network setup in Section IV. A new joint-direct method will then be presented and analyzed, while the particular issue of dealing with sensor noise is addressed in Section VI.

II. SYSTEM SETUP

We consider a two-node network system, as depicted in Figure 1, where G_{12}^0 and G_{21}^0 are linear time-invariant systems, the circles are summation points, with the node signals w_1 and w_2 being the results of the summations respectively, r_1, r_2 are external excitation signals that are available to the user, and v_1, v_2 are non-measured disturbances, modelled as stationary stochastic processes with rational spectral density ([11]).
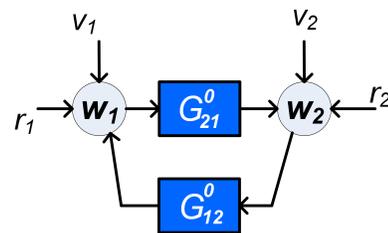

Fig. 1. Two-node dynamic network system.

The system equation reads:

$$w(t) = G^0(q)w(t) + v(t) + R^0 r(t) \qquad (1)$$

with

$$G^0(q) = \begin{bmatrix} 0 & G_{12}^0 \\ G_{21}^0 & 0 \end{bmatrix},$$

and $R^0 = I$ (the identity matrix) when both signals r_1, r_2 are present, and with q the shift operator, i.e. $qw_2(t) = w(t+1)$. Unless otherwise stated we will assume that (excitation) signals r are uncorrelated to (disturbance) signals v, and that the closed-loop system is stable. The noise process v is modelled through $v(t) = H^0(q)e(t)$, with H^0 monic, stable and minimum phase, and e a white noise process. In order to simplify technicalities, we will assume that the modules G_{12}^0 and G_{21}^0 are strictly proper. The node signals w are considered to be measured, possibly under the presence of sensor-noise. In that latter situation the measured variables become $\tilde{w}(t) := w(t) + s(t)$, with s sensor noise. Sensor noise is present only when it is particularly mentioned in the text.

III. THE CLASSICAL DIRECT METHOD

In the classical direct method, there is a noise process v_2 and possibly an external excitation signal r_1, while v_1 and r_2 are absent, and sensor noise is considered not to be present, see Figure 2.
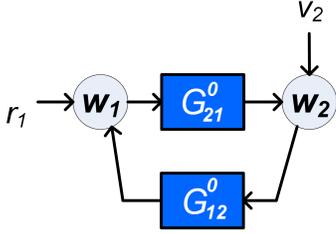

Fig. 2.   Classical closed loop system.

This implies that

$$H^0(q) = \begin{bmatrix} 0 & 0 \\ 0 & H_2^0 \end{bmatrix}, \text{ and } R^0(q) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The target of direct identification typically is to estimate one of the modules, i.e. the "plant" G_{21}^0, while the other module G_{12}^0 is considered to be a feedback controller that might or might not be known. In the direct method, a one-step-ahead predictor for node signal w_2 is considered,

$$\hat{w}_2(t|t-1) := \mathbb{E}\{w_2(t)|w^{t-1}\} \tag{2}$$

where $\mathbb{E}$ is expected value, and w^{t-1} denotes the past of the measured signal w up to time $t-1$. When applying the predictor to a parametrized model $(G_{21}(q,\theta), H_2(q,\theta))$, it can be expressed as

$$\hat{w}_2(t|t-1;\theta) := H_2(q,\theta)^{-1}G_{21}(q,\theta)w_1(t) + \\ + (1 - H_2(q,\theta)^{-1})w_2(t),$$

and a model estimate is obtained by applying a least squares identification criterion:

$$\hat{\theta}_N = \arg\min_\theta \frac{1}{N} \sum_{t=0}^{N-1} [w_2(t) - \hat{w}_2(t|t-1;\theta)]^2. \tag{3}$$

Under weak regularity conditions ([11]) consistent model estimates for G_{21}^0 and H_2^0 are obtained, provided that

- The data generating system is present in the model set;

- There are no algebraic loops in the network and in the parametrized model, i.e. $G_{12}(\infty,\theta)G_{21}(\infty,\theta) \neq 0$ for all θ, with $G(\infty,\theta) := \lim_{z\to\infty} G(z,\theta)$;
- There is sufficient excitation in the closed-loop ([11]), e.g. through presence of r_1;

Some characteristics of the direct method in this situation are:

- Asymptotically, the model estimates reach the Cramer-Rao lower bound for the variance, on the basis of the measured data w_1 and w_2;
- No explicit use is made of the external excitation signal r_1, although it serves its purpose of providing excitation in the loop;
- Possible prior knowledge of the controller G_{12}^0 does not help in improving the identification results. Note that G_{12}^0 can actually be estimated from w_1 and r_1 in a noise-free estimation setting.
- The consistency results hold true even if G_{12}^0 is nonlinear and/or time-varying.

IV. THE DIRECT METHOD FOR A NETWORK SYSTEMS SETUP

A. Consistency results

First we extend the situation of the classical closed-loop to a situation with an additional disturbance v_1 in Figure 3.
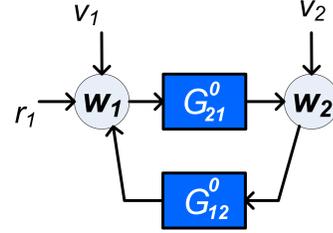

Fig. 3.   Classical closed loop system with input disturbance.

When analyzing the prediction error $\varepsilon(t,\theta) := w_2(t) - \hat{w}_2(t|t-1;\theta)$ for this situation, we arrive at (using shorthand notations and denoting $S^0 := (1 - G_{12}^0 G_{21}^0)^{-1}$):

$$\begin{aligned} \varepsilon &= H_2^{-1}[w_2 - G_{21}w_1] = H_2^{-1}[G_{21}^0 w_1 + v_2 - G_{21}w_1] \\ &= H_2^{-1}[G_{21}^0 - G_{21}]w_1 + H_2^{-1}v_2. \end{aligned}$$

When using $w_1 = S^0 r_1 + S^0 v_1 + G_{12}^0 S^0 v_2$, this leads to

$$\begin{aligned} \varepsilon = H_2(\theta)^{-1}[G_{21}^0 - G_{21}(\theta)]S^0 r_1 + \\ + H_2(\theta)^{-1}\left\{ \frac{S^0}{S(\theta)}v_2 + (G_{21}^0 - G_{21}(\theta))S^0 v_1 \right\} \end{aligned} \tag{4}$$

with $S(\theta) := (1 - G_{12}^0 G_{21}(\theta))^{-1}$.
We can now distinguish two situations:

1) If v_1 and v_2 are uncorrelated, then the expression for $\varepsilon(\theta)$ is composed of three independent terms. Minimization of the power of $\varepsilon(\theta)$, which is asymptotically achieved by (3), is then achieved by making the r_1- and v_1-dependent terms 0 which is implied by $G_{21}(\theta) = G_{21}^0$, and making the transfer from e_2

to $\varepsilon(\theta)$ unity, which is implied by $H_2(\theta) = H_2^0$. As a result consistency of the model estimates can be shown to hold under the same conditions as in the previous section. Addition of the disturbance term v_1 does not hurt the consistency results. Actually it adds to the power of the signal w_1 that excites G_{21}^0.

2) If v_1 and v_2 are correlated, then the above reasoning fails. It is not guaranteed anymore that $G_{21}(\theta) = G_{21}^0$ and $H_2(\theta) = H_2^0$ lead to a minimum of the power of $\varepsilon(\theta)$. As a result the consistency property is lost.

Quote 1: While the direct method is able to provide consistent model estimates in a classical closed loop setting, consistency is lost when the input signal is affected by an external disturbance that is correlated to the output disturbance of the system.

One might consider to add a second external excitation signal r_2 to the closed-loop in an attempt to retain the consistency property. However adding an r_2 signal will not change the contribution of the disturbance term in the expression (4) for $\varepsilon(\theta)$ and therefore will not solve the consistency issue.

Quote 2: Adding an additional external excitation signal r_2 to the closed-loop system has no effect on the consistency properties as meant in Quote 1.

When the node variables are measured under the influence of sensor noise, so the measured signals are $\tilde{w} = w_1 + s_1$ and $\tilde{w}_2 = w_2 + s_2$ with s_1, s_2 sensor noises, a more complicated situation occurs, which in the literature is referred to as an errors-in-variables problem. It is known that in this situation a direct method can not provide consistency results, without explicitly using the external excitation signal r_1, see [13].

Quote 3: Consistency properties of the direct method are lost when the node signals are measured under the influence of sensor noise.

B. Confounding Variables

The lack of consistency in the presence of correlated disturbances, can also be explained using the notion of confounding variables. In [7] conditions have been formulated for consistency of the direct method in a general dynamic network setting. In the setting of the configuration of Figure 3, a confounding variable is a non-measured variable that has paths to w_2 as well as to w_1 that do not pass through measured variables, as e.g. the nonmeasured variable v_3 in Figure 4 (left). Confounding variables create a correlation between w_1 and w_2 that is not induced by the module G_{21}^0, and therefore can cause lack of consistency for the direct method.

In the situation of Figure 2, the variable v_2 has a path to w_2, but the path from v_2 to w_1 passes through the measured signal w_2 and therefore v_2 is not a confounding variable.

In the situation of Figure 3 the situation is more complex. If v_1 and v_2 are correlated, then the network configuration
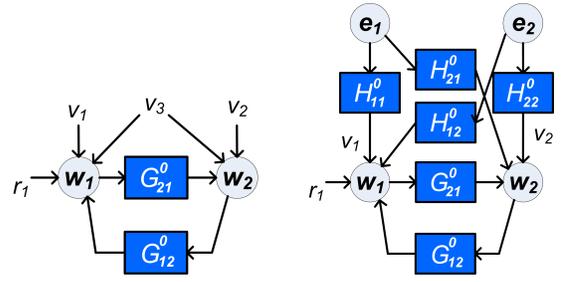

Fig. 4. Classical closed loop system with confounding variable v_3 (left) and correlated disturbances (right).

can actually be rewritten into Figure 4 (right), showing the presence of confounding variables e_1 and e_2.

As a result the lack of consistency in the considered situation can be explained through the presence of confounding variables. In Section VI it will be shown that also the presence of sensor noise can actually be phrased in terms of the occurrence of confounding variables.

V. THE JOINT-DIRECT METHOD

A. The two-node dynamic network situation

We will now consider the two-node dynamic network as depicted in Figure 1. In an attempt to overcome the lack of consistency of the direct method in the situation with correlated disturbances, we explore a network predictor that was introduced in [12], in particular for dealing with algebraic loops in networks. It is defined as:

$$\hat{w}(t|t-1) := \mathbb{E}\{w(t)|w^{t-1}, r^t\} \quad (5)$$

which, in comparison with (2), shows two differences:

1) Both signals w_2 and w_1 are predicted, and
2) External excitation signals r are included as predictor inputs.

As shown in [12], for the case of strictly proper modules, this network predictor is expressed by

$$\hat{w}(t|t-1) = \left(I - H(\theta)^{-1}(I - G(\theta))\right)w + H(\theta)^{-1}R^0 r \quad (6)$$

for a parametrized model $(G(q,\theta), H(q,\theta))$ while it is assumed that R^0 is known and fixed. A model is then identified by applying the identification criterion:

$$\hat{\theta}_N = \arg \min_\theta \frac{1}{N} \sum_{t=0}^{N-1} \varepsilon^T(t,\theta) W \varepsilon(t,\theta),$$

with $\varepsilon(t,\theta) := w(t) - \hat{w}(t|t-1;\theta)$, and W a 2×2 positive definite weighting matrix.

For analysing consistency properties of the estimated model $G(q, \hat{\theta}_N), H(q, \hat{\theta}_N)$, we analyse the prediction error:

$$\varepsilon(\theta) = H(\theta)^{-1}(I - G(\theta))(I - G^0)^{-1}[R^0 r + v] - H(\theta)^{-1}R^0 r$$
$$= H(\theta)^{-1}\left[(I - G(\theta))(I - G^0)^{-1} - I\right]R^0 r +$$
$$+ H(\theta)^{-1}(I - G(\theta))(I - G^0)^{-1}v. \quad (7)$$

On the basis of this expression for ε, and considering the situation $R^0 = I$ (both components of r present), consistency

of the model estimates is rather obvious, since the power of the r-dependent term is made 0 by $G(\theta) = G^0$, while additionally the power of the noise-induced term is minimized through the choice $H(\theta) = H^0$. There is no condition now on (absence of) correlations between the disturbance variables in v! This leads to the following quoted result:

> **Quote 4:** The joint-direct method, characterized by the use of a vector predictor (predicting both w_1 and w_2) and by including the two external excitation signals as predictor inputs, extends the consistency properties of the direct method to consistent estimates of the full network (and not only G_{21}^0), and to the case where v_1 and v_2 are correlated.

So apparently by extending the predictor to (5), and identifying the two modules G_{21}^0 and G_{12}^0 simultaneously, we are able to handle the situation of correlated disturbances. The complexity that is added is that we need to solve a MIMO identification problem rather than a SISO problem.

Remark 1: There are other identification methods that can handle correlated disturbances, as e.g. the two-stage / IV type of methods analysed in [6], [7]. However the joint-direct method uses all available information in the node signals (rather than projecting them onto excitation signals r) and therefore can be is expected to achieve a smaller variance.

B. A network identifiability result

When considering the identification problem in the setting where only one of the two external excitation signals is present, a detailed analysis of the prediction error becomes more complicated. For that situation we can follow a different line of reasoning to arrive at a result.

For consistent identification of module dynamics and noise models in a dynamic network setting, the key condition is that the parametrized model set that is being used is network identifiable, [8], [10]. This implies that from the network transfers function $T_{wr} := (I - G(\theta))^{-1}R^0$ and spectral density $\Phi_v(\omega)$ there is a map to unique elements $(G(q, \hat{\theta}), H(q, \hat{\theta}))$ in the model set $\mathcal{M} := \{(G(q, \theta), H(q, \theta)), \theta \in \Theta\}$. Together with the condition that the data generating system is contained in the model set, and external excitation signals are sufficiently exciting, this will imply consistency of the model estimates.

In this paper we will use a condition for verifying network identifiability that has been introduced in [10]. For this purpose we need to denote the matrices

$$U(\theta) := \begin{bmatrix} R^0 & H(\theta) \end{bmatrix}, \quad \text{and} \quad T(\theta) := [I - G(\theta)]^{-1}U(\theta)$$

while $0 \leq K \leq 2$ is the number of external signals in r that is actually present in the network.

Proposition 1 ([10], Theorem 2.): Let $\mathcal{M}$ be a network model set that additionally satisfies the following properties:

a. Every parametrized entry in the model $\{(G(q, \theta), H(q, \theta)), \theta \in \Theta\}$ covers the set of all strictly proper rational transfer functions;

b. All parametrized transfer functions in the model $(G(q, \theta), H(q, \theta))$ are parametrized independently (i.e. there are no common parameters).

Then $\mathcal{M}$ is globally network identifiable if and only if

- each row i of the transfer function matrix $\begin{bmatrix} G(\theta) & U(\theta) \end{bmatrix}$ has at most $K + p$ parameterized entries, and
- for each i, $\check{T}_i(\theta)$ has full row rank for all $\theta \in \Theta$, where the $\alpha_i \times (K + p - \beta_i)$ matrix $\check{T}_i(q, \theta)$ is the submatrix of $T(q, \theta)$ that is constructed by taking the row numbers that correspond to the columns of $G(q, \theta)_{i\star}$ that are parametrized, and by taking the column numbers that correspond to the columns of $U(q, \theta)$ that are not parametrized.

It can be shown ([8]) that this identifiability result also holds row-wise for all elements in row i of the composed matrix $[G(q, \theta) \ H(q, \theta)]$ in the model set $\mathcal{M}$. The elements in row i are then identifiable if the above conditions are satisfied for the corresponding row i of $\begin{bmatrix} G(\theta) & U(\theta) \end{bmatrix}$ and for $\check{T}_i(\theta)$ respectively.

C. Network identifiability for a reduced number of external excitation signals

For the situation when there is only one component of the r-signal present, we can analyse the identifiability conditions as formulated in Proposition 1. Without loss of generality we consider r_1 to be present and r_2 to be absent. Then

$$I - G(\theta) = \begin{bmatrix} 1 & -G_{12}(\theta) \\ -G_{21}(\theta) & 1 \end{bmatrix}$$

$$U(\theta) = \begin{bmatrix} 1 & H_{11}(\theta) & H_{12}(\theta) \\ 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

with $K + p = 1 + 2 = 3$, $\alpha_1 = \alpha_2 = 1$, and $\beta_1 = \beta_2 = 2$.

$$T(q, \theta) = \begin{bmatrix} 1 & -G_{12}(\theta) \\ -G_{21}(\theta) & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & H_{11}(\theta) & H_{12}(\theta) \\ 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

$$= (1 - G_{12}G_{21})^{-1} \cdot$$

$$\begin{bmatrix} 1 & H_{11} + G_{12}H_{21} & H_{12} + G_{12}H_{22} \\ G_{21} & G_{21}H_{11} + H_{21} & G_{21}H_{12} + H_{22} \end{bmatrix}$$

(where arguments (θ) have been dropped for brevity). The matrices $\check{T}_i$ are of dimension 1×1, and are given by

$$\check{T}_1 = (1 - G_{12}(\theta)G_{21}(\theta))^{-1}G_{21}(\theta)$$
$$\check{T}_2 = (1 - G_{12}(\theta)G_{21}(\theta))^{-1}.$$

The full rank property of $\check{T}_1$ and $\check{T}_2$ is guaranteed if $G_{21}(\theta)$ is not strictly equal to 0 for all θ. Under this condition the conditions of Proposition 1 are satisfied and the network is network identifiable. Under the common conditions of persistence of excitation of the appropriate signals, consistency of $G_{21}(\hat{\theta}_N)$ and $G_{12}(\hat{\theta}_N)$ then follows automatically. The converse situation of having only signals r_2 present, instead of r_1, follows directly by duality. As a result we can formulate the following statement:

> **Quote 5:** If in the situation of Quote 4 only 1 of the two reference signals is present, then the

consistency results remain valid for any correlation between the disturbances v_1 and v_2, provided that $G_{21}(\theta) \neq 0$ for all θ (if r_1 is present only), or provided that $G_{12}(\theta) \neq 0$ for all θ (if r_2 is present only).

In the situation that both excitation signals are absent, the situation becomes slightly different. Now we have

$$\begin{bmatrix} G(\theta) & U(\theta) \end{bmatrix} = \begin{bmatrix} 0 & G_{12}(\theta) & H_{11}(\theta) & H_{12}(\theta) \\ G_{21}(\theta) & 0 & H_{21}(\theta) & H_{22}(\theta) \end{bmatrix}$$

with $K + p = 0 + 2 = 2$, $\alpha_1 = \alpha_2 = 1$, and $\beta_1 = \beta_2 = 2$. Then according to the first condition of Proposition 1, we have too many parametrized terms in the rows of $[G(\theta\, U(\theta)]$. When evaluating the identifiabiltiy of $G_{21}(\theta)$ only, we can focus on the second row of $\begin{bmatrix} G(\theta) & U(\theta) \end{bmatrix}$. In the situation $H_{21}^0 = 0$ and $H_{21}(\theta) \equiv 0$ the number of parametrized terms in the second row of $[G(\theta)\ U(\theta)]$ satisfies the maximum number of 2 parametrized terms. The second condition of Proposition 1 then requires a full row rank of $\check{T}_2$. After some manipulations it can be shown that $\check{T}_2 = H_{11}(\theta) + G_{12}^0 H_{21}(\theta)$ which is a monic proper transfer function of full rank, and therefore the conditions for identifiability are satisfied. The converse situation of estimating G_{12}^0 under the condition that $H_{12}^0 = 0$ follows by duality.

> **Quote 6:** In absence of external excitation signals r_1, r_2, the module G_{21}^0 can be estimated consistently when disturbance signals v_1, v_2 are correlated, under the condition that $H_{21}^0 = 0$. The dual result is that G_{12}^0 can be estimated consistently if $H_{12}^0 = 0$.

Note that the condition $H_{21}^0 = 0$ implies that v_1 does not causally affect v_2 in the sense of Granger ([4]). It does not imply that $R_{v_2 v_1}(\tau) = 0$, $\tau > 0$. This result is in agreement with the identifiability results in [14] where one-sided correlations between disturbance signals are considered for the situation of a joint-io identification method.

VI. A CLOSED-LOOP NETWORK WITH SENSOR NOISE

We will now focus on the situation that the node variables w are measured under the influence of additive sensor noise. To this end we write

$$\tilde{w} = w + s \tag{8}$$

where $\tilde{w}$ are the measured variables, and s is a (two-dimensional) stationary stochastic process, uncorrelated with signals r and v in the configuration, and with a diagonal spectral density.

First we will illustrate that sensor noise can be described in terms of the presence of confounding variables. For illustration purposes we will consider the open-loop situation ($G_{12}^0 = 0$) and $r_2 \equiv 0$. Then the system's equation $\tilde{w}_2 = G_{21}^0 w_1 + v_2 + s_2$ can be rewritten as $\tilde{w}_2 = G_{21}^0(\tilde{w}_1 - s_1) + v_2 + s_2$ leading to the equivalence of the two networks indicated in Figure 5, which shows in the right Figure that s_1 has become
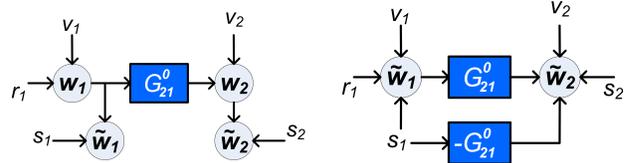

Fig. 5. Sensor noise equivalently described as confounding variable.

a confounding variable. When considering the general two-node network, and substituting the expression for $\tilde{w}$ into the system's equation $w = G^0 w + v + r$, (assuming $R^0 = I$), we obtain $\tilde{w} - s = G^0(\tilde{w} - s) + v + r$, leading to

$$\tilde{w} = G^0 \tilde{w} + \underbrace{v + (I - G^0)s}_{\tilde{v}} + r.$$

This system has the same structure as the original network, with the node variables w replaced by $\tilde{w}$ and the disturbance signal v by $\tilde{v}$, see Figure 6. The major difference is that the disturbance term now becomes

$$\tilde{v} = v + (I - G^0)s. \tag{9}$$

The consequence of this is that even if the disturbance signals v_1 and v_2 are uncorrelated, the new disturbance will have correlated components because of the term $(I - G^0)s$. So sensor noise can be considered as a special form of correlated disturbances. As a result, consistency properties for the modules in G^0 will remain invariant when sensor noise is present. In this situation the noise model will need to describe the disturbance $\tilde{v}$, i.e. a combined signal composed of disturbance signals and sensor noises.
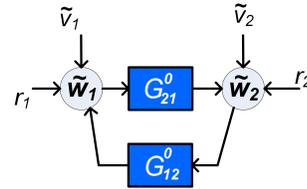

Fig. 6. Equivalent closed-loop network with sensor-noise measured variables as node signals.

> **Quote 7:** The consistency properties of the joint-direct method (Quotes 4-6) remain valid for the modules in G^0 when the measured node variables are subject to additive sensor noise.

It is a very strong property of the joint-direct method that it can deal with both correlated disturbances as well as with sensor noise. Note that these consistency properties can also be achieved by a two-stage / projection method as analyzed for the dynamic network situation in [6], [7], [15]. However in the latter situations not the full input signals can be used for estimation, leading to non-optimal variance results.

In Table I an overview is listed of the consistency properties of the different cases that have been considered in this paper.

predictor outputs	predictor inputs	references present	disturbance present	disturbance correlation	sensor noise present	consistency	quote
w_2	w_1	r_1	v_2	n/a	no	yes	
w_2	w_1	r_1	v_1, v_2	none	no	yes	
w_2	w_1	none	v_1, v_2	none	no	yes	
w_2	w_1	r_1	v_1, v_2	H_{12} and/or H_{21}	no	no	1
w_2	w_1	r_1, r_2	v_1, v_2	none	no	yes	2
w_2	w_1	r_1, r_2	v_1, v_2	H_{12} and/or H_{21}	no	no	2
w_2	w_1	r_1, r_2	v_1, v_2	none	s_1, s_2	yes	3
w_2	w_1	r_1, r_2	v_1, v_2	H_{12} and/or H_{21}	s_1, s_2	no	3
w_1, w_2	w_1, w_2	r_1, r_2	v_1, v_2	none	no	yes	
w_1, w_2	w_1, w_2	r_1, r_2	v_1, v_2	H_{12} and/or H_{21}	no	no	
w_1, w_2	w_1, w_2, r_1, r_2	r_1, r_2	v_1, v_2	H_{12} and/or H_{21}	no	yes	4
w_1, w_2	w_1, w_2, r_1	r_1	v_1, v_2	H_{12} and/or H_{21}	no	yes	5
w_1, w_2	w_1, w_2, r_2	r_2	v_1, v_2	H_{12} and/or H_{21}	no	yes	5
w_1, w_2	w_1, w_2	none	v_1, v_2	H_{12}	no	yes (for G_{21}^0)	6
w_1, w_2	w_1, w_2	none	v_1, v_2	H_{21}	no	yes (for G_{12}^0)	6
w_1, w_2	w_1, w_2, r_1, r_2	r_1, r_2	v_1, v_2	H_{12} and/or H_{21}	s_1, s_2	yes (for G^0)	7
w_1, w_2	w_1, w_2, r_1	r_1	v_1, v_2	H_{12} and/or H_{21}	s_1, s_2	yes (for G^0)	7
w_1, w_2	w_1, w_2, r_2	r_2	v_1, v_2	H_{12} and/or H_{21}	s_1, s_2	yes (for G^0)	7
w_1, w_2	w_1, w_2	none	v_1, v_2	H_{12}	s_1, s_2	yes (for G_{21}^0)	7
w_1, w_2	w_1, w_2	none	v_1, v_2	H_{21}	s_1, s_2	yes (for G_{12}^0)	7

TABLE I

OVERVIEW OF CONSISTENCY RESULTS FOR THE DIFFERENT SITUATIONS CONSIDERED.

Remark 2: Since the presence of sensor noise can be recast into the situation of having correlated disturbance signals, as illustrated in Figure 6, we can conclude that sensor noises lead to the presence of confounding variables (see also Section IV-B). In contrast with the classical direct identification method, the joint-direct method is able to handle confounding variables for the simple 2-node feedback connection considered. Extending this claim to more complex dynamic network configurations is a topic for future research.

Remark 3: The joint-direct method has been introduced in [12], in particular for studying the problem of dealing with algebraic loops in closed-loop network systems. In the current paper we have assumed that all modules are strictly proper. The present results can be generalized to deal with (non-strictly) proper modules also.

VII. CONCLUSIONS

We have considered the identification of a single feedback system that acts as a basic building block in dynamic networks. The classical direct method of prediction error identification leads to consistent (and minimum variance) estimates of the plant model for a particular set of situations. However if we extend the situations to deal with correlated disturbances on the node signals as well as with sensor noise, then consistency is lost. The new joint-direct method is based on: (a) considering a vector predictor that predicts *all* node signals simultaneously, and (b) including external excitation signals as predictor inputs. Consistency properties have been shown in situations of correlated disturbances and of sensor noise. Both situations can also be qualified in terms of the presence of confounding variables.

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