

# PROCESS CONTROL-RELEVANT AND CLOSED-LOOP IDENTIFICATION

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**Abstract.** The identification of dynamical systems on the basis of data, measured under closed-loop experimental conditions, is a problem which is highly relevant in many (industrial) applications. When using models as a basis for model-based robust control design both nominal models and model uncertainty bounds are required. In this paper it is shown how -in particular- model uncertainty bounds can be obtained from closed-loop experimental data in the classical prediction error identification framework. The considered uncertainty structure is adjusted so as to allow direct evaluation of the performance robustness of both the actual and a to-be-designed controller.

**Keywords.** Closed-loop identification; system identification; robust control; robust performance; uncertainty.

## 1. INTRODUCTION

Many industrial processes operate under feedback control. Due to unstable behaviour of the plant, required safety and/or efficiency of operation, experimental data can only be obtained under so-called closed-loop conditions. Identification methods for dealing with closed-loop experimental data have been developed in the seventies and eighties, see Söderström and Stoica (1989) for an overview. These "classical" methods are typically directed towards solving the consistency problem, considering the situation that plant and disturbance model can be modeled exactly (system is in the model set).

Initiated by an emerging interest in the identification of models that are particularly suitable for model-based (robust) control design, renewed attention has been given lately to the problem of closed-loop identification. There is a number of arguments to prefer closed-loop experi-

ments over open-loop ones, in case one is interested in model-based control design. These arguments comprise aspects of bias and variance, input shaping, and the fact that a controller can linearize the (possibly nonlinear) plant behaviour in a relevant working point, thus enabling approximate linear modelling.

Unlike the classical situation, attention is now also given to properties of identified approximate models, handling the -more realistic - situation that plant and noise dynamics are not exactly present in the model set considered (Van den Hof, 1997; Forssell and Ljung, 1998).

For model-based control design, models are required that accurately describe those plant dynamics that are most essential for the subsequent control design. The question how to determine those dynamics, and how to extract them from experimental data is handled in the area "identification for control" of which accounts are given in the survey papers Gevers (1993) and Van den Hof and Schrama (1995). Whereas in these references main attention is given to the construction of appropriate nominal models, the area of "robust identification" has been

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directed more specifically towards the construction of model uncertainty sets for use in robust control design, see e.g. Ninness and Goodwin (1995).

In this paper particular attention will be given to the role of closed-loop experiments in the identification of uncertainty models to be used in subsequent (robust) control design. The experimental setup to be considered is depicted in Figure 1.

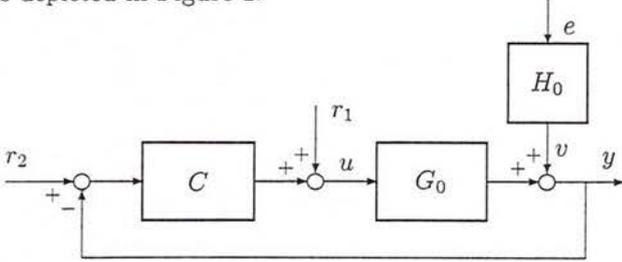


Fig. 1. Closed-loop configuration.

In this configuration  $r_1$  and  $r_2$  are external excitation signals, uncorrelated to the filtered white noise disturbance signal  $v = H_0 e$ . The sensitivity function of the closed-loop is denoted by  $S_0 = (1 + CG_0)^{-1}$ .

## 2. CLOSED-LOOP IDENTIFICATION

In the classical "direct" identification method one simply applies a standard (prediction error) identification procedure without taking account of the presence of a feedback controller. If the system  $\mathcal{S} := (G_0, H_0)$  is present in the model set  $\mathcal{M} := \{(G(q, \theta), H(q, \theta)), \theta \in \Theta\}$ , then a consistent estimate is obtained under mild conditions on the excitation of the closed-loop.

In recent years attention is given to generalizations of other classical schemes, aiming at the consistent estimation of  $G_0$  also in the case that  $H_0$  is not modelled exactly. This concerns e.g. the two-stage method (being of the joint i/o type) and the method based on a dual Youla/Kucera parametrization (being of the indirect type). For details the reader is referred to e.g. Van den Hof (1997).

Analysis has shown that for these generalizations, the asymptotic bias expressions have the following form (Ljung, 1987):

$$\theta^* = \arg \min_{\theta \in \Theta} \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_0 - G(\theta)|^2 \frac{|S_0 S(\theta)|^2 \Phi_{r_1}}{|K_*|^2} d\omega,$$

where  $S(\theta) = (1 + CG(\theta))^{-1}$  and  $K_*$  is a fixed noise model used. This implies that the additive error on  $G_0$  is always weighted with the sensitivity function  $S_0$ , and thus emphasis will be given to an accurate model fit in the frequency region where  $S_0$  is large, being typically

the region that determines the bandwidth of the control system.

A similar weighting of the identification results with  $S_0$  is present in the asymptotic variance expressions of the estimated model  $\hat{G}$  that hold for all methods (Gevers et al., 1997; Ljung and Forsell, 1997):

$$\text{var}(\hat{G}(e^{i\omega})) \sim \frac{n}{N} \frac{\Phi_v(e^{i\omega})}{\Phi_{u_r}(e^{i\omega})}$$

where  $\Phi_{u_r} = |S_0|^2 \Phi_{r_1}$  and  $\Phi_v(e^{i\omega}) = |H_0|^2 \sigma_e^2$ . In the frequency range where the closed-loop configuration attains noise reduction (small  $S_0$ ), poor process knowledge is obtained. However, according to Bode's sensitivity integral,  $\int_0^\pi \log |S_0(e^{i\omega})| d\omega = c$ , (constant) and as a result the attenuation of signal power in a particular frequency range, will always be "compensated" for by an amplification of signal power in another frequency range.

As the dual-Youla/Kucera parametrization will be used in the construction of model uncertainty sets later on, brief attention will be given to this particular approach of closed-loop identification. It is based on the fact that the set of all linear plant models that are stabilized by a given  $C$  can be parametrized by

$$G = \frac{N_x + D_c R}{D_x - N_c R} \quad (1)$$

where  $(N_x, D_x)$  is a coprime factorization of just any model stabilized by  $C$  having a coprime factorization  $(N_c, D_c)$ , and  $R$  varies over the class of stable transfer functions. Coprimeness of the factorizations means that the (rational) factors are stable and do not have any cancelling unstable zeros. Applying the parametrization (1) to the plant  $G_0$  in Figure 1 results to the closed-loop configuration given in Figure 2. Analysis of

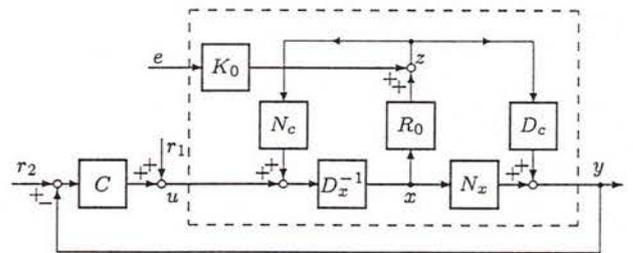


Fig. 2. Dual Youla/Kucera representation of closed-loop system

the corresponding configuration shows that  $z(t)$  and  $x(t)$  can simply be reconstructed from measured data using knowledge of the controller, and that  $x(t)$  and  $e(t)$  are uncorrelated. As a result,  $R_0$  (and possibly  $K_0$ ) can be

<sup>2</sup> Without loss of generality, the situation  $r_2 \equiv 0$  is considered.

identified by standard open-loop techniques, on the basis of available reconstructed signals  $x$  and  $z$ .

One of the particular advantages of this approach, is that every estimate  $\hat{R}$  that is stable, will provide a plant model  $\hat{G}$  that - by construction- is stabilized by the controller  $C$ . This is due to the particular Youla/Kucera parametrization.

### 3. IDENTIFICATION FOR CONTROL

#### 3.1 Introduction

In many situations models are identified for the purpose of using them as a basis for subsequent model-based (robust) control design. In that case the evaluation of models has to be undertaken in the scope of the control design. In other words: the best identified model (within a specific class) is that model that leads to a controller that controls the plant best. This application-dependent assessment of models is due to the fact that in practice identified models can only be an approximation of exact plant dynamics.

In using identified models for control design, there are two paths to follow:

- Control-relevant identification of nominal models
- Identification of model uncertainty sets for robust control.

The first issue addresses the problem of identifying a reduced-order model to accurately fit those plant dynamics that are most essential for the nominal control design based on the plant model  $\hat{G}$  (and possibly noise model  $\hat{H}$ ). Based on a control performance cost function denoted by  $\|J(G_0, C)\|$ , this has led to the formulation of a performance-induced identification criterion

$$\hat{G} = \arg \min_G \|J(G_0, C) - J(G, C)\|$$

that depends on the controller  $C$  and the control performance cost function being chosen.

When restricting attention to the bias components of the modelling errors, this identification criterion can be shown to be equivalent to the identification criteria used in some indirect closed-loop identification methods (Gevens, 1993; Van den Hof and Schrama, 1995).

#### 3.2 Identification of model uncertainty sets for robust control

A starting point for constructing a model uncertainty set, would be to collect all models that are not invali-

dated by the data and the prior information on the system, sometimes called the *Feasible System Set* (FSS):

$$\mathcal{P}_{\text{fss}} := \{G \mid y(t) - G(q)u(t) = v(t), v \in \mathcal{V}\}$$

where  $\mathcal{V}$  is the hypothesized set of disturbance signals on the output. Dependent on the particular character of  $\mathcal{V}$  the feasible system set  $\mathcal{P}_{\text{fss}}$  will be of a deterministic or a probabilistic nature.

In general, the set  $\mathcal{P}_{\text{fss}}$  contains all unfalsified models and has a format that is unmanageable for a robust control design procedure. To make a set of models suitable for robust control design, a so-called uncertainty set of a prechosen nature need to be used where e.g. an additive or multiplicative norm-bounded uncertainty is formulated around a nominal model. A general characterization of such a set is:

$$\mathcal{P}_f(\hat{G}, \gamma) = \{G \mid G = f(\hat{G}, \Delta), |\Delta(e^{i\omega})| \leq \gamma(\omega), \forall \omega\}$$

with  $\gamma$  a positive real-valued function of  $\omega$  and  $f$  a linear fractional transformation (LFT):

$$f(\hat{G}, \Delta) = \hat{G} + P_{21}\Delta(1 - P_{11}\Delta)^{-1}P_{12}. \quad (2)$$

Although such an uncertainty set generally is denoted as “unstructured”, the choice of  $f$  does provide the set with a particular uncertainty structure. Note that an additive uncertainty results through the choice  $P_{21} = P_{12} = 1$ ,  $P_{11} = 0$ .

For a particular choice of  $f$  and  $\hat{G}$ , the “size”  $\gamma$  of the set (2) has to be chosen as small as possible so as to contain the feasible system set with the real plant  $G_0$ . This situation is depicted in an abstract way in Figure 3, where the shaded area reflects the set  $\mathcal{P}_{\text{fss}}$  which is encapsulated in the uncertainty set  $\mathcal{P}_f$ . Note that the former set is principally implied by the measurement data, whereas the latter set is partly just chosen by the user.

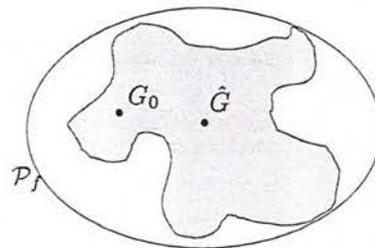


Fig. 3. Uncertainty set  $\mathcal{P}_f(\hat{G}, \gamma)$  (ellipsoid) and set of unfalsified models (shaded area).

It is clear that there are many options for choosing  $\mathcal{P}_f$  such that all unfalsified models are contained. However in choosing this  $\mathcal{P}_f$  it is apparent that one should take

account of the performance cost function  $J$ , by avoiding the incorporation of (falsified) models (in the white area of Figure 3) that lead to poor performance costs. Such incorporation would lead to a control design with considerable conservatism.

The question now is, whether there are performance-cost-relevant uncertainty structures. To address this question, consider the general situation of a control performance function  $J(G, C)$  that can be written in a LFT form involving  $G$  and  $C$ . With the LFT form of  $G$  in (2) expressed in  $\hat{G}$  and  $\Delta$ , the control performance function  $J(G, C)$  can be expanded in an LFT form:

$$J(G, C) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \quad (3)$$

where  $M_{ij}$  are functions of the nominal model  $\hat{G}$  and the controller  $C$ . This is due to the fact that an LFT connection of an LFT system remains an LFT system.

Considering the expression (3) for a fixed frequency  $\omega$ , it follows from the basic properties of an LFT that the circular area  $|\Delta(e^{i\omega})| \leq \gamma(\omega)$  in the complex plane is mapped to a circular area for  $J$ . However there is no guarantee that when  $\gamma(\omega)$  becomes smaller, the radius of the corresponding circle for  $J$  also becomes smaller. This latter situation is only guaranteed if  $J(G, C)$  becomes affine in  $\Delta$ , i.e.

$$J(G, C) = M_1 + M_2\Delta. \quad (4)$$

In this case there is a direct linear relationship between the "size"  $\gamma$  of the uncertainty set and the (frequency-dependent) worst-case performance cost  $|J(G, C)|$  over the set, in terms of

$$\sup_{G \in \mathcal{P}_f(\hat{G}, \gamma)} |J(G, C)| = |M_1(e^{i\omega})| + \gamma(\omega)|M_2(e^{i\omega})| \quad \forall \omega.$$

In this way the "shape" of the uncertainty set is directly tuned towards the performance criterion. Most combinations of uncertainty sets and performance functions will not deliver the above mentioned affine structure.

*Example 1.* (Additive uncertainty set). An additive uncertainty is characterized by  $f(\hat{G}, \Delta) = \hat{G} + \Delta$ . Substituting this expression for  $G$  in the performance function  $J(G, C) = V/(1+CG)$  delivers  $J(G, C) = \frac{V}{1 + C\hat{G} + C\Delta}$  which apparently is not affine in  $\Delta$ .

For a general class of closed-loop performance functions, an affine structure can be obtained, as indicated in the following proposition.

*Proposition 1.* (de Callafon and Van den Hof, 1997a). Consider the control performance function  $J(G, C)$ :

$$J(G, C) = U_2 \begin{bmatrix} G \\ 1 \end{bmatrix} [1 + CG]^{-1} [C \ 1] U_1$$

with  $U_1, U_2$  square weighting functions, and consider an uncertainty set

$$\mathcal{P}(\hat{G}, W) = \{G \mid G = \frac{\hat{N} + D_c\Delta_R}{\hat{D} - N_c\Delta_R}, |W^{-1}\Delta_R(e^{i\omega})| \leq 1\}$$

with  $C = N_c D_c^{-1}$  and  $\hat{G} = \hat{N}\hat{D}^{-1}$ . Then for all  $G \in \mathcal{P}(\hat{G}, W)$ ,  $J(G, C)$  satisfies

$$J(G, C) = M_1 + M_2\Delta_R$$

with

$$M_1 = U_2 \begin{bmatrix} \hat{G} \\ 1 \end{bmatrix} [1 + C\hat{G}]^{-1} [C \ 1] U_1 = J(\hat{G}, C)$$

$$M_2 = -U_2 \begin{bmatrix} -1 \\ C \end{bmatrix} D_c W (\hat{D} + C\hat{N})^{-1} [C \ 1] U_1. \quad \square$$

Note that the uncertainty set is written in the format of a dual Youla/Kucera parametrization, where the auxiliary model is given by the nominal plant model. This structure of  $G$  in the uncertainty set is depicted in figure 4. The performance cost function  $J$  is a very general (combination of) closed-loop transfer function(s), that can be tuned by specifying the weighting functions  $U_1$  and  $U_2$  and can be written in an LFT form.

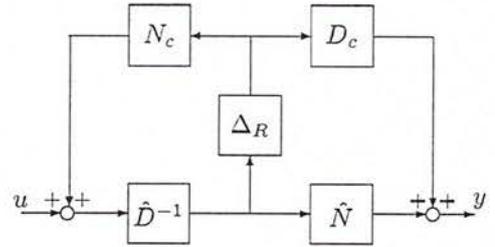


Fig. 4. Dual Youla/Kucera uncertainty representation.

For the particular choice of  $J$  as in Example 1 the resulting expression becomes

$$J(G, C) = \frac{V}{1 + CG} + M_2\Delta_R$$

with  $M_2 = \frac{-VN_cW}{\hat{D}(1 + C\hat{G})}$ , being a filter dependent on known quantities only.

A data-based uncertainty modelling procedure, should provide the smallest bound  $\gamma(\omega)$  (or equivalently  $|W(e^{i\omega})|$ ) that is required to guarantee that  $G_0$  is an element of the set  $\mathcal{P}_f(\hat{G}, W)$ . This minimization of  $\gamma(\omega)$  can be performed in a closed-loop experimental setup, by applying a model uncertainty estimation procedure to the

Youla/Kucera parameter. In terms of the mechanism discussed in section 2 this refers to choosing the auxiliary model  $G_x = \hat{G}$ , and identifying an upper bound for the transfer function between  $x$  and  $z$ . This motivates the use of closed-loop experimental data not only for nominal model identification, but also for control-relevant uncertainty bounding. The indicated mechanism also extends to more general performance cost functions, as shown in de Callafon and Van den Hof (1997a).

#### 4. UNCERTAINTY SET ESTIMATION

A dual Youla/Kucera uncertainty set, as discussed in the previous section, can be constructed on the basis of (closed-loop) measurement data, and prior assumptions on plant and noise. There is no restriction here on the uncertainty bounding approach that is chosen (hard-bounded or soft-bounded). Actually, any uncertainty bounding procedure designed to handle open-loop experiments can be used, as the required signals  $x$  and  $z$  for bounding  $\Delta R$  satisfy open-loop conditions (see section 2).

In almost all uncertainty bounding procedures in the literature use is made of linearly parametrized models as e.g. FIR models. However, for an accurate modelling of moderately damped systems it may require a large number of parameters. This can be overcome by using dynamical (orthogonal) basis functions  $f_k(q)$ , leading to a model structure:

$$G(q, \theta) = c_0 + c_1 f_1(q) + \dots + c_n f_n(q).$$

Generalized orthogonal basis functions can be constructed to contain a priori chosen dynamics. This allows more accurate modelling with a limited number of parameters. For details on this parametrization and its use in identification one is referred to Van den Hof *et al.* (1995) and Ninness and Gustafsson (1997).

A combined worst-case/probabilistic approach for bounding model uncertainty is presented in Hakvoort and Van den Hof (1997). It is based on the following main line of reasoning.

- It is assumed that measurement data  $y, u$  satisfies the system equations:

$$y(t) = G_0(q)u(t) + v(t)$$

with  $v$  a stationary stochastic process, independent of  $u$ , and that  $G_0$  allows a series expansion  $G_0(z) = \sum_{k=0}^{\infty} g_0(k) f_k(z)$  with the expansion coefficients bounded by an a priori known bound  $|g_0(k)| \leq \bar{g}(k)$ ,  $k > 0$ .

- A linearly parametrized model  $G(q, \theta)$  is chosen, and a finite number of expansion coefficients is iden-

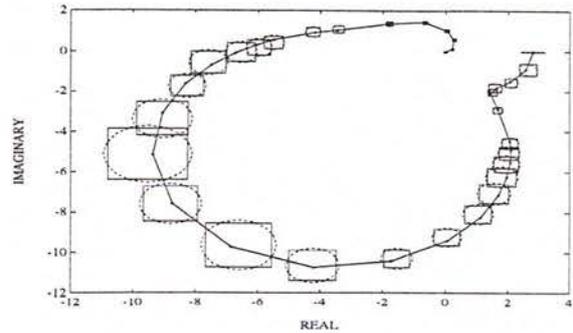


Fig. 5. Nyquist diagram with identified uncertainty bounds (rectangles, ellipsoids).

tified with a Least Squares (linear regression) algorithm.

- Then for a fixed frequency  $\omega$  the estimated model  $\hat{G}(e^{i\omega})$  can be shown to satisfy

$$\hat{G}(e^{i\omega}) - G_0(e^{i\omega}) = \beta_1(\omega) + \beta_2(\omega) + \beta_3(\omega)$$

where

- $\beta_1$  reflects the neglected tail of the expansion; this term can be worst-case bounded;
- $\beta_2$  reflects a bias term on the estimated coefficients due to the neglected tail; this term can also be worst-case bounded;
- $\beta_3$  reflects a variance contribution, which can be bounded in a probabilistic way by using the asymptotic theory according to Ljung (1987).
- The uncertainty bounds can be computed in any user-chosen frequency grid, and lead to a Nyquist curve with uncertainty regions in user-specified frequencies, as illustrated in figure 5. In all cases the three different sources of uncertainty can be distinguished, which allows the user to determine which part is dominant, and to adjust the experimental setup so as to reduce the overall uncertainty bound.
- A similar procedure can be followed to quantify uncertainty bounds on step responses, pulse responses etcetera.

When applying this uncertainty bounding procedure to a closed-loop experimental situation as described in sections 2 and 3.2, this results in an uncertainty bound

$$|\Delta_R(e^{i\omega})| \leq \gamma(\omega) \quad \text{with probability } \geq \alpha$$

in a user-defined frequency grid  $\Omega$ , and for a user-specified choice of  $\alpha$ . On this basis a finite-dimensional weighting function  $W$  can be constructed to satisfy  $\|W \Delta_R\|_{\infty} \leq 1$ , being a format that can be handled in robustness issues in robust control.

## 5. IDENTIFICATION AND ROBUST CONTROL TOWARDS PERFORMANCE ENHANCEMENT

The mechanisms and results as discussed in the previous sections, can now be combined to construct an integrated procedure for performance enhancement of a controlled plant. Given the situation of a plant  $G_0$  being controlled by a controller  $C_i$ , the task is to design a new controller  $C_{i+1}$  on the basis of experimental data, such that the new controller achieves a better performance. Using the notation and concepts of this paper, this problem can be split in two parts:

- (a) Estimate  $\mathcal{P}_i$  and specify  $\gamma_i$  such that  $G_0 \in \mathcal{P}_i$  and

$$\|J(G, C_i)\| \leq \gamma_i \quad \forall G \in \mathcal{P}_i \quad (5)$$

- (b) Design  $C_{i+1}$  such that

$$\|J(G, C_{i+1})\| \leq \gamma_{i+1} < \gamma_i \quad \forall G \in \mathcal{P}_i \quad (6)$$

Part (a) is a problem of identifying an uncertainty set in such a way that the performance cost function is minimized over the set ( $\gamma_i$  is minimal). This can be obtained by choosing a control-relevant uncertainty set, as discussed in section 3.2, in combination with an appropriate uncertainty bounding procedure, as e.g. presented in section 4.

Part (b) of the problem is a robust control design problem. For the choices of  $J$  as discussed in this paper, this will generally require the solution of a  $\mu$ -synthesis problem.

A performance enhancement procedure along the lines sketched above is presented in De Callafon and Van den Hof (1997a), and a successful application to a multivariable wafer stepper motion control system is reported in De Callafon and Van den Hof (1997b).

## 6. CONCLUSIONS

Closed-loop experimental conditions are situations that are highly practical. Although some dynamical aspects of the plant (typically the low-frequent behaviour) may be harder to identify in closed-loop, other aspects will typically be amplified in the data. It is shown that it is possible to direct the closed-loop identification of models towards a control performance cost function, induced by a particularly chosen control objective. This implies that models can be identified that reflect the control-relevant aspects of the plant dynamics. It has been shown that this mechanism applies to identification of both nominal models, and model uncertainty bounds. This allows the formulation of an integrated approach to control-relevant uncertainty bounding and robust control design. An issue that deserves future attention in this research is the particular experiment design issues that

are involved in constructing model uncertainty sets that are sufficient to guarantee to specified level of robust performance.

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