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Structural Identifiability of Grid Block and Geological Parameters in Reservoir Simulation Models

J. van Doren* (Delft University of Technology), J.D. Jansen (Delft University of Technology), P.M.J. Van den Hof (Delft University of Technology) & O.H. Bosgra (Delft University of Technology)

SUMMARY

It is well-known that history matching of reservoir models with production measurements is an ill-posed problem, e.g. different choices for the history matching parameters may lead to equally good history matches. We analyzed this problem using the system-theoretical concept of structural identifiability. This allows us to analytically calculate a so-called information matrix. From the information matrix we can determine an identifiable parameterization with a significantly reduced number of parameters.
We apply structural identifiability analysis to single-phase reservoir simulation models and obtain identifiable parameterizations. Next, we use the parameterization in minimizing an objective function that is defined as the mismatch between pressure measurements and model outputs. We also apply the structural identifiability analysis to an object-based parameterization describing channels and barriers in the reservoir.
We use the iterative procedure to determine for reservoir models with 2025 grid block permeability values a structurally identifiable parameterization of only 13 identifiable parameters. Next, we demonstrate that the parameterization leads to perfect history matches without the use of a prior model in the objective function. We also demonstrate the use of the identifiable object-based parameterization, leading to geologically more realistic history matches.

Introduction

It is well known that parameter estimation of reservoir simulation models using measured production data (i.e. 'history matching') is generally an ill-posed problem, see e.g. Gavalas et al. (1976) and Tavassoli et al. (2004). This is particularly true if it is attempted to estimate individual grid block parameters such as permeability or porosity values, which may lead to a very large number (10^5 to 10^6) of unknown parameters which can only be estimated with a large variance. Another challenging aspect in history matching is the need to retain geological realism while updating the parameter values. One way to overcome the ill-posedness of the parameter estimation problem is by constraining the solution space for the model parameters through the addition of regularization terms to the objective function. Another way is to reparameterize the parameter space, where the number of parameters is strongly reduced, while at the same time it may be possible to better maintain geological realism. Reparameterization techniques previously applied in reservoir engineering include zonation (e.g. Jacquard and Jain (1965), Grimstad et al. (2003)), grad zones (Bissell et al. (1994), Brun et al. (2004)), spectral decomposition and subspace methods (Shah et al. (1978), Reynolds et al. (1996)), principle component analysis (Sarma et al. (2007)), and discrete cosine transform (Jafarpour and McLaughlin (2007)). In this paper we will obtain a parameterization from structural identifiability analysis. The notion of structural identifiability was first stated by Bellman and Åström (1970). State-space model parameterizations have been analyzed by Glover and Willems (1974), Grewal and Glover (1976) and Walter (1987). A test for local structural identifiability of high-order state-space models has been proposed in Dötsch and Van den Hof (1996). In Van Doren et al. (2008) this test has been adapted and used to determine an identifiable parameterization of the permeability field of a reservoir simulation model. In this paper we will determine a identifiable parameterization and subsequently use it to estimate the grid block permeability in a single-phase reservoir model. We will also apply the structural identifiability analysis to an object-based parameterization describing channels and barriers in the reservoir.

First we will briefly describe structural identifiability and how we can use the notion of structural identifiability to determine an identifiable parameterization in terms of model parameters. In section 3 we introduce a parameterization that is capable of modeling channels and barriers in a reservoir grid, where the number of parameters is strongly reduced and geological realism is preserved after updating. In section 4 we use an iterative procedure to estimate the grid block permeability parameters with the identifiable parameterization resulting from the structural identifiability analysis. We also estimate the geological parameters of the channel parameterization.

Structural identifiability

Consider a linear, time-invariant, discrete time, state-space model structure, parameterized in $\boldsymbol{\theta}$:

$$\mathbf{x}(k+1) = \mathbf{A}(\boldsymbol{\theta})\mathbf{x}(k) + \mathbf{B}(\boldsymbol{\theta})\mathbf{u}(k) \quad (1)$$
$$\mathbf{y}(k) = \mathbf{C}(\boldsymbol{\theta})\mathbf{x}(k), \quad (2)$$

where $\mathbf{x}(k) \in \mathbb{R}^n$, $\mathbf{u}(k) \in \mathbb{R}^m$, $\mathbf{y}(k) \in \mathbb{R}^p$, and $\boldsymbol{\theta} \in \mathbb{R}^q$. In this paper we use the local structural identifiability formulation of Glover and Willems (1974):

Definition: An input/output model structure $G : \Theta \to \mathcal{G}$ with $\Theta \subset \mathbb{R}^q$ and $\mathcal{G} \subset \mathbb{R}(z)^{p\times m}$ is called locally structural identifiable in $\boldsymbol{\theta}^\star \in \Theta$ if for all $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$ in the neighborhood of $\boldsymbol{\theta}^\star$ it holds that

$$\{G(z, \boldsymbol{\theta}_1) = G(z, \boldsymbol{\theta}_2)\} \Rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2.$$

In words: in the neighborhood of $\boldsymbol{\theta}^\star$ there are no two models with distinct parameters which have the same input-output behavior. Note that $G(q, \boldsymbol{\theta})$ can be written as:

$$G(q, \boldsymbol{\theta}) = \sum_{k=1}^{\infty} \mathbf{M}(k, \boldsymbol{\theta}) q^{-k}, \quad (3)$$

where $\mathbf{M}(k,\boldsymbol{\theta}) = \mathbf{C}(\boldsymbol{\theta})\mathbf{A}^{k-1}(\boldsymbol{\theta})\mathbf{B}(\boldsymbol{\theta})$ are the Markov parameters. Based on (3) we can argue that equality of the models $G(q,\boldsymbol{\theta}_1)$ and $G(q,\boldsymbol{\theta}_2)$ is related to equality of the Markov parameters of $G(q,\boldsymbol{\theta}_1)$ and $G(q,\boldsymbol{\theta}_2)$. In this analysis the Markov parameters are organized row-wise $\overrightarrow{\mathbf{M}}(k,\theta) := [\mathbf{M}_{1*}(k,\boldsymbol{\theta}), ..., \mathbf{M}_{p*}(k,\boldsymbol{\theta})]$, where $\mathbf{M}_{j*}(k,\boldsymbol{\theta})$ denotes the j-th row of Markov parameter $\mathbf{M}(k,\boldsymbol{\theta})$, and are subsequently gathered in the map $\overrightarrow{\mathbf{S}}_r : \Theta \subset \mathbb{R}^q \to \mathbb{R}^{pmr}$ defined by:

$$\overrightarrow{\mathbf{S}}_r(\boldsymbol{\theta}) := \left[\ \overrightarrow{\mathbf{M}}(1,\boldsymbol{\theta}) \quad \overrightarrow{\mathbf{M}}(2,\boldsymbol{\theta}) \quad \ldots \quad \overrightarrow{\mathbf{M}}(r,\boldsymbol{\theta})\ \right] \in \mathbb{R}^{1\times pmr}. \tag{4}$$

As shown in Van Doren et al. (2008), the model structure is locally identifiable in $\boldsymbol{\theta}^\star$ if, for sufficiently large r, $\mathrm{rank}\left(\frac{\partial \overrightarrow{\mathbf{S}}_r(\theta)}{\partial \boldsymbol{\theta}}\right) = q$ in $\boldsymbol{\theta} = \boldsymbol{\theta}^\star$.

Using the notational conventions stated before, the information matrix $\mathcal{I}_r$ for a multi-input multi-output system is defined as

$$\mathcal{I}_r := \left.\frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}} \frac{\partial \overrightarrow{\mathbf{S}}_r^T}{\partial \boldsymbol{\theta}^T}\right|_{\boldsymbol{\theta}^\star} = \sum_{i=1}^{r}\sum_{j=1}^{p}\left.\left(\frac{\partial \mathbf{M}_{j*}(i)}{\partial \boldsymbol{\theta}} \frac{\partial \mathbf{M}_{j*}^T(i)}{\partial \boldsymbol{\theta}^T}\right)\right|_{\boldsymbol{\theta}^\star} \tag{5}$$

with dimensions $q \times q$, and where $\frac{\partial \mathbf{M}_{j*}(i)}{\partial \boldsymbol{\theta}}$ is given by

$$\begin{aligned} \frac{\partial \mathbf{M}_{j*}(k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} &= \frac{\partial \mathbf{C}_{j*}(\boldsymbol{\theta})\mathbf{A}^{k-1}(\boldsymbol{\theta})\mathbf{B}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \\ \frac{\partial \mathbf{C}_{j*}}{\partial \boldsymbol{\theta}}\mathbf{A}^{k-1}\mathbf{B} + \left(\mathbf{I}_q \otimes \mathbf{C}_{j*}\mathbf{A}^{k-1}\right)\frac{\partial \mathbf{B}}{\partial \boldsymbol{\theta}} &+ \sum_{l=1}^{k-1}\left(\mathbf{I}_q \otimes \mathbf{C}_{j*}\mathbf{A}^{l-1}\right)\frac{\partial \mathbf{A}}{\partial \boldsymbol{\theta}}\mathbf{A}^{k-1-l}\mathbf{B}. \end{aligned} \tag{6}$$

This expression can be calculated exactly given the state-space matrices and the analytical derivatives of the state-space matrices with respect to $\boldsymbol{\theta}$. At the moment this analysis has been applied to reservoir models up to $6,400$ grid blocks. Analysis of reservoir models with more grid blocks leads to memory problems on a laptop with 1Gb of RAM memory. After calculating $\mathcal{I}_r$ with (5) its rank is evaluated. The rank of $\mathcal{I}_r$ is denoted as $l := \mathrm{rank}\,(\mathcal{I}_r)$ and provides an estimate of the number of linearly independent rows or columns in $\mathcal{I}_r$. Here we use a singular value decomposition (SVD) to determine the numerical rank (Golub and Van Loan (1996)). Let

$$\mathcal{I}_r = \left[\ \mathbf{U}_1 \quad \mathbf{U}_2\ \right]\left[\begin{array}{cc} \boldsymbol{\Sigma}_1 & 0 \\ 0 & 0 \end{array}\right]\left[\begin{array}{c} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{array}\right], \tag{7}$$

be the SVD of $\mathcal{I}_r$, where $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ are unitary and $\boldsymbol{\Sigma}_1 = \mathrm{diag}(\sigma_1,\ldots,\sigma_l)$ with $\sigma_1 \geq \cdots \geq \sigma_l \gg \sigma_{l+1} \geq \ldots \geq \sigma_q$. The singular values $\sigma_{l+1},\ldots\sigma_q \geq 0$ are regarded as negligible. Numerical determination of the matrix rank l requires a criterion for deciding when a singular value σ_i should be treated as zero. In the example the choice is made that $\frac{\sigma_{l+1}}{\sigma_1} < 1 \times 10^{-5}$.

From (7) it can be seen that the columns of $\mathbf{U}_1$ provide an orthogonal basis of the column space of $\mathcal{I}_r$. The columns of $\mathbf{U}_1$ are regarded as directions in the parameter space that are structurally identifiable and serve as a mapping from high-dimensional parameter space $\boldsymbol{\theta}$ to a low-dimensional parameter space $\boldsymbol{\rho} = \mathbf{U}_1^T\boldsymbol{\theta}$. In case we choose the parameter to be estimated to be the permeability in each grid block, each column of $\mathbf{U}_1$ with length q can be projected on the N grid blocks of the reservoir model. This can be done because each parameter value in $\boldsymbol{\theta}$ corresponds to one grid block ($q = N$). Consequently, each column of $\mathbf{U}_1$ can also be interpreted as a spatial pattern, expressing the sensitivity of the Markov parameters w.r.t. the permeability vector. The columns of $\mathbf{U}_2$ provide an orthogonal basis of the null space of $\mathcal{I}_r$. The columns of $\mathbf{U}_2$ are regarded as directions in the parameter space that are structurally *not* identifiable. In other words, the information in the input-output data $\mathbf{u},\mathbf{y}$ does not hold any information about the parameter directions given by $\mathbf{U}_2$.

Geological parameterization

As described in section 1, desired features of history matching are that geological realism is preserved, and that the number of unknown parameters is not too large in order to avoid ill-posedness. A possible solution to realize these features is to choose a parameterization in terms of a limited number of geological objects (e.g. meandering channels) such that an update in the parameters results in a geologically realistic permeability field. Estimating channel parameters with production data has been considered by e.g. Rahon et al. (1998), Bi et al. (1999) and Phan and Horne (2002). The latter uses a deterministic method, where the mapping between the 14 channel parameters and the 3-dimensional permeability field is unique.

In the 2-dimensional modeling applied in our paper we assume that the channels are straight and have a uniform permeability distribution. Also the permeability of the background is uniform. These assumptions are motivated by the fact that we wanted to reduce the number of parameters and only wanted to model those features of the reservoir that are relevant for prediction and control. Additionally, we reasoned that the flow behavior in a channel with a slight curvature would be approximately equal to the flow behavior in a straight channel with a slightly lower permeability.

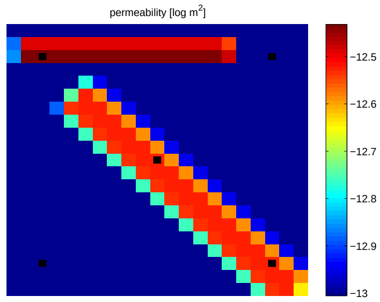

Figure 1: Example of permeability field generated with the channel and barrier modeling method. The parameters are given in Table 1.

The channels in our modeling method are modeled with the morphological structuring element *strel* as available in the Image Processing Toolbox of Matlab. Each channel is described by six parameters: orientation, position in x direction, position in y direction, length, width and channel permeability. An additional parameter describes the permeability of the background permeability of the reservoir model. This means that the permeability field with two channels in Figure 1 is described by 13 parameters. The values of the channel parameters are given in Table 1, where the longest channel is defined as channel 1. If channels intersect with each other, then the younger channel replaces the older channel. The channels are generated on a fine-scale grid, and then upscaled to the simulation grid size using the arithmetic mean. For the example depicted at Figure 1 we have chosen 420×420 grid blocks of $10 \times 10\text{m}^2$ each, and subsequently upscaled to 21×21 grid blocks of $200 \times 200\text{m}^2$.

With the structural identifiability analysis as described in the previous section it is possible to calculate an information matrix of the geological parameters

$$\mathcal{I}_{r,ch} = \frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}_{ch}} \frac{\partial \overrightarrow{\mathbf{S}}_r^T}{\partial \boldsymbol{\theta}_{ch}^T} \bigg|_{\boldsymbol{\theta}_{ch}^\star} . \tag{8}$$

In essence we simply apply the chain rule $\frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}_{ch}} = \frac{\partial \overrightarrow{\mathbf{S}}_r}{\partial \boldsymbol{\theta}_{gb}} \frac{\partial \boldsymbol{\theta}_{gb}}{\partial \boldsymbol{\theta}_{ch}}$, where $\boldsymbol{\theta}_{gb}$ is the grid block permeability. The term $\frac{\partial \boldsymbol{\theta}_{gb}}{\partial \boldsymbol{\theta}_{ch}}$ is simply calculated using finite differences where each geological parameter is perturbed in positive and negative direction on the fine-scale grid. The step size

Symbol	Meaning	Value	Unit	$\sum_{i=1}^{13} \lvert\mathbf{U}(i)\rvert \mathbf{\Sigma}(i,i)$
θ_{geo1}	orientation channel 1	45	[°]	79.3×10^{10}
θ_{geo2}	orientation channel 2	90	[°]	10.3×10^{10}
θ_{geo3}	position x channel 1	200	[m]	31.7×10^{10}
θ_{geo4}	position x channel 2	90	[m]	4.57×10^{10}
θ_{geo5}	position y channel 1	1000	[m]	2.62×10^{10}
θ_{geo6}	position y channel 2	150	[m]	8.24×10^{10}
θ_{geo7}	width channel 1	600	[m]	60.9×10^{10}
θ_{geo8}	width channel 2	200	[m]	24.7×10^{10}
θ_{geo9}	length channel 1	4400	[m]	14.9×10^{10}
θ_{geo10}	length channel 2	3000	[m]	2.34×10^{10}
θ_{geo11}	permeability channel 1	1000	[mD]	148×10^{10}
θ_{geo12}	permeability channel 2	1000	[mD]	36.4×10^{10}
θ_{geo13}	permeability background	100	[mD]	136×10^{10}

Table 1: Channel parameters of the permeability field in Figure 1.

of the perturbation is 10^{-5} times the absolute value of the parameter in question. Before this calculation the parameters are scaled so that the parameters are all roughly of the same magnitude. For the specific example with 13 geological parameters given in Figure 1, we obtain, after upscaling, the plot in Figure 2. When we calculate the singular vectors and singular values of (8) we see the correlation between the parameters. This greatly adds to the insight in the reservoir behavior and the history matching process, since it is now possible to see which parameters are correlated before one actually starts the updating process, and which parameters are structurally not identifiable.

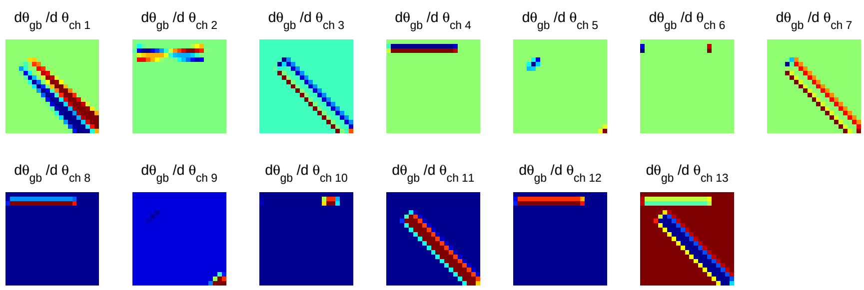

Figure 2: Each column of $\frac{\partial \boldsymbol{\theta}_{gb}}{\partial \boldsymbol{\theta}_{ch}}$ is projected onto the reservoir grid. Parameter $\theta_{ch1}, \ldots, \theta_{ch13}$ are denoted in Table 1.

Application

In this analysis we use a two-dimensional model, that contains only one fluid. The resulting model is linear in the states, but non-linear in the parameters. A five-point finite difference discretization in space and an implicit discretization in time yield the following state-space ordinary differential equation in discrete time

$$\mathbf{p}(k+1) = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{p}(k) + \mathbf{B}\mathbf{u}(k), \quad \mathbf{p}(0) = \mathbf{p}_0 \tag{9}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{p}(k), \tag{10}$$

where $k \in \mathbb{Z}$ denotes discrete time. The state variables $\mathbf{p} \in \mathbb{R}_+^n$ denote fluid pressures in each grid block. The number of states is equal to the number of grid blocks. The input vari-

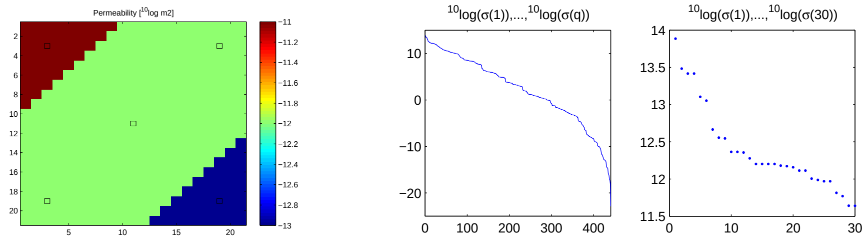

Figure 3: Permeability field used to generate measurements $\bar{\mathbf{y}}_k$ where the rectangles indicate the well positions (left), singular values of (5) (middle) and first 30 singular values of (5) (right).

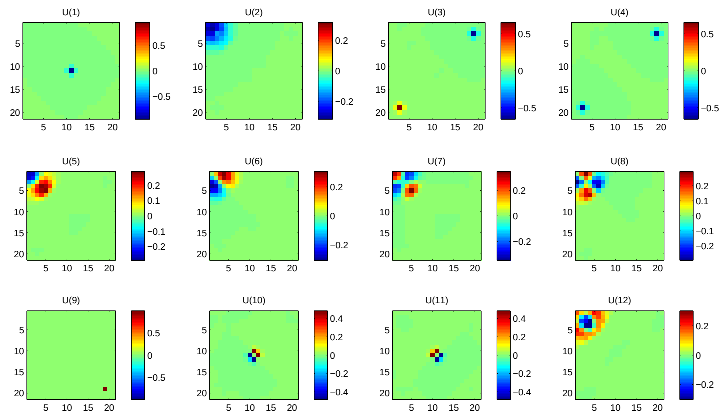

Figure 4: Projected singular vectors corresponding to first 12 singular values of (5) using the permeability field depicted in Figure 3.

ables $\mathbf{u} \in \mathbb{R}^m$ denote control settings such as injection or production rates or pressures in grid blocks containing wells. The output variables $\mathbf{y} \in \mathbb{R}^p$ denote measured pressures in grid blocks containing wells. $\mathbf{A}(\boldsymbol{\theta}) \in \mathbb{R}^{n\times n}$ is a penta-diagonal matrix with entries that are a function of grid block volume, fluid density, compressibility, fluid viscosity, porosity in each grid block, and permeability in each grid block. We choose not to use a well inflow model, and therefore $\mathbf{B} \in \mathbb{R}^{n\times m}$ is a sparse matrix containing ones in entries corresponding to a grid block containing a well. Matrix $\mathbf{C} \in \mathbb{R}^{p\times n}$ is also sparse containing ones in entries corresponding to a grid block containing a well. At the boundaries no-flow conditions are assumed.
We propose to analyze the structural identifiability of permeability and determine an identifiable parameterization as outlined in section 2. We re-parameterize the permeability as $\mathbf{k} = \mathbf{U}_1\boldsymbol{\rho}$ and estimate parameters $\boldsymbol{\rho} = \arg\min_{\boldsymbol{\rho}} V(\mathbf{U}_1\boldsymbol{\rho})$. The objective function $V(\mathbf{U}_1\boldsymbol{\rho})$ is defined as

$$V(\mathbf{U}_1\boldsymbol{\rho}) := \sum_{k=1}^{n} \left(\bar{\mathbf{y}}_k - \mathbf{y}_k(\mathbf{U}_1\boldsymbol{\rho})\right)^T \left(\bar{\mathbf{y}}_k - \mathbf{y}_k(\mathbf{U}_1\boldsymbol{\rho})\right), \tag{11}$$

where $\bar{\mathbf{y}}_k$ denotes the measurements at time step k. The advantage of this formulation is that only those (combinations of) grid block permeabilities are updated that are relevant for the input-output behavior. In Figure 3 the permeability field is depicted that is used to generate $\bar{\mathbf{y}}_k$. We define this permeability as the real field. It is a 21×21 reservoir grid penetrated by five wells, whose positions are indicated by the rectangles in Figure 3. We determine an identifiable parameterization using equations (5) and (7). The corresponding singular values are plotted in Figure 3. The singular vectors corresponding to the 12 largest singular values are projected onto

the reservoir grid and are depicted in Figure 4.
To estimate the permeability of the real field we start with an homogeneous initial guess $\theta_{init} = 5 \times 10^{-13}\mathrm{m}^2$ as depicted at the middle of Figure 5. The corresponding value of the objective function is $V(\boldsymbol{\theta}_{init}) = 6.76$. Based on $\boldsymbol{\theta}_{init}$ and using (5) and (7), we calculate $\mathbf{U}_1$ keeping only the first 12 columns. We add an extra column to $\mathbf{U}_1$ containing ones in every entry to account for an overall increase or decrease in permeability. Note that we have not used the true parameter value in our calculations. Subsequently, we use a gradient-based optimization procedure (e.g. the MATLAB function `lsqnonlin`) to minimize the objective function given in (11). Perfect measurements $\bar{\mathbf{y}}_1, \ldots, \bar{\mathbf{y}}_{200}$ are generated by simulating (9,10) with $\boldsymbol{\theta} = \boldsymbol{\theta}_{init}$, an initial state of $\mathbf{p}_0 = 100 \times 10^5\mathrm{Pa}$, and a manipulated input $u_0, \ldots, u_{199}$. The input is a pseudorandom, binary signal and is persistently exciting, i.e. contains enough frequencies to obtain informative measurements.
Since the identifiable parameterization partly depends on the permeability field we use an iterative procedure. If the value of the objective function given in (11) is not decreasing anymore (i.e. a local minimum is found) we determine a new identifiable parameterization. With the new set of basis functions we minimize V further. In this example the estimate has converged after 3 iterations to the permeability field depicted at the right of Figure 5. The value of the objective function has decreased to $V = 3.0 \times 10^{-4}$. The input-output behavior of this permeability field is similar to the behavior of the real permeability field. Apparently, the grid blocks with wells in the low and medium permeability field, together with the grid blocks in a slightly larger area around the well in the high permeable area, are structurally identifiable. The permeability values in the other grid blocks are in the structurally not identifiable directions and do not matter for the input-output behavior.
The estimated permeability field could be different when another set of initial parameter values were chosen to start the numerical search for a minimum of (11). There is always a risk that the numerical minimization gets stuck in a local minimum, and therefore it is advisable to try several different initial parameter values. Furthermore, although the objective function is small and the reservoir model is history matched, the primary purpose of the model is prediction and decision making. With the estimated permeability field the decision of e.g. deciding where a new well should be drilled can not be made adequately based on this estimated permeability field. Apparently, the information content in the measurements in not sufficient and the estimate is largely dominated by the initial parameter values.

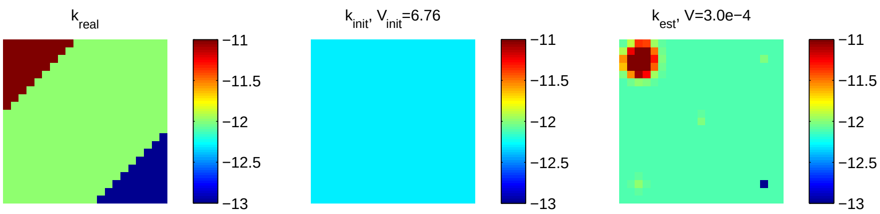

Figure 5: Real permeability field (left), initial permeability field (middle) and estimated permeability field (right) obtained with the identifiable parameterization.

In the second example we use the geological parameterization. We choose a real permeability field as depicted at the left in Figure 6, which is parameterized by 13 parameters. The well configuration is identical to the one in the previous example. We calculate for this permeability field with (8) and (7) the singular vectors $\mathbf{U} \in \mathbb{R}^{13\times 13}$ and singular values $\mathbf{\Sigma} \in \mathbb{R}^{13\times 13}$. Equally as in the previous example, the singular vectors can be interpreted as combinations of channel parameters. The values of $\sum_{i=1}^{13} |\mathbf{U}(i)|\mathbf{\Sigma}(i,i)$ denote a measure of how identifiable a specific

parameter is, and are denoted in Table 1. We conclude for this permeability field that the permeability of channel 1 and background permeability are best identifiable, and that the length of channel 2 and position y of channel 1 are least identifiable.
To estimate the channel parameters and resulting permeability field we start with initial parameter values resulting in a permeability field as depicted at the middle of Figure 6. The corresponding value of the objective function $V = 0.06$. With the same persistently exciting inputs we estimate the 13 channel parameters. This results after convergence in the permeability field depicted at the right of Figure 6. The value of the objective function has decreased to $V = 3.0 \times 10^{-4}$. However, the permeability field is different from the real permeability field.
The advantages of the channel parameterization is that it results after estimating the parameters in a geologically more realistic permeability field and that the number of parameters that need to be estimated is small. However, a disadvantage is that some channel parameters are difficult to identify, as is shown by the SVD of (8). Another difficulty with the channel parameterization is that it is not very flexible: if for example the permeability field does not contain channels, the procedure of estimating channel parameters will not converge or converge to a different estimate. Examples of more flexible parameterizations are e.g. the discrete cosine transform parameterization, that originates from the image compression community and has been used in history matching in the work of Jafarpour and McLaughlin (2007).

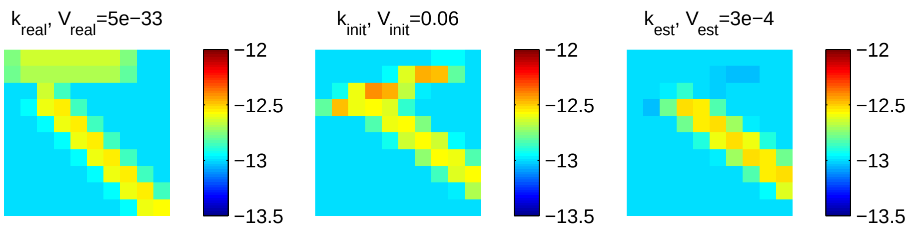

Figure 6: Real permeability field (left), initial permeability field (middle) and estimated permeability field (right) obtained with the channel parameterization.

Conclusions

In this paper a best identifiable, reduced-dimensional parameterization is constructed using structural identifiability analysis, which is applied to reservoir simulation models. In the original parameter space this leads to basis functions or spatial patterns, which have been used to estimate the permeability from production measurements. In addition. the structural identifiability of an object-based channel and barrier modeling method has been analyzed. The corresponding geological parameters have been estimated, where the parameters quickly converged to a geological realistically looking permeability field.

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