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Adjoint-Based History Matching of Structural Models Using Production and Time-Lapse Seismic Data

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Abstract

In spite of large uncertainties in the actual reservoir structure, structural parameters of a reservoir model are usually fixed during history matching and only the flow properties of the model are allowed to vary. This often leads to unlikely or even unfeasible property updates and possibly to a poor predictive capability of the model. In those cases it may be expected that updating of the structural parameters will improve the quality of the history match. Preferably such structural updates should be implemented in the static (geological) model, and not just in the dynamic (flow) model. In this paper we use a gradient-based history matching method to update structural properties of the static model. We use an adjoint method to efficiently compute the derivatives of the data mismatch with respect to grid block porosities in the dynamic model and convert the corresponding volume changes to structural updates (layer thicknesses) in the static model. This method is suitable for structural updating of large scale reservoir models using production data and/or time-lapse seismics or other spatially distributed data. The method is tested on a 3D synthetic model, where time-lapse as well as production data have been used to update depth of the reservoir's bottom horizon. We obtained significant improvements in the history match quality and the predictive capability of the model.

1. Introduction

The objective of history matching is to improve the predictive capacity of a reservoir simulation model through adjusting the model parameters until the simulated data match the historical data as closely as possible. In spite of large uncertainties in the reservoir structure, in many cases the structural model parameters are fixed during the history matching process and only the flow-related properties (e.g. permeability, porosity and net-to-gross ratio) of the model are allowed to vary. This often leads to either a poor history match or unlikely (or even unfeasible) parameter updates. Structural uncertainties can significantly affect various aspects of the reservoir model such as reservoir bulk volume and well positions and subsequently affect the predictive capability of the model (Thore et al., 2002; Seiler et al., 2010). Consequently, updating of the structural reservoir parameters by assimilation of production data and time-lapse seismic data has the potential to improve the quality of history matched models considerably.

In typical computerized modeling workflows, the static (geological) model is built based on seismic interpretations, logs, cores, outcrop data, and geological insight. Subsequently, because the static model contains millions of grid cells, it is upscaled to a coarser grid during export to the dynamic (reservoir flow) model for flow simulations. Thereafter, a set of flow-related uncertain reservoir model parameters is identified and then updated using historical data. There are different disadvantages associated with this workflow. Most importantly, because of the sequential nature of the approach, the history matched reservoir models are often inconsistent with static data and display geologically unrealistic features. Furthermore, in this workflow, modeling is typically based on 'low, medium and high cases' of static reservoir parameters related to stock tank oil initially in place (STOIIP), which are generated before quantifying the dynamic outcomes, such that the flow-related uncertainties are not necessary adequately captured. A related issue is that the geological models and the upscaled flow models often contain many details that are irrelevant to the flow response.

In contrast, a 'Big Loop' approach can be used to avoid the disadvantages of the traditional modeling workflow. The Big Loop approach is an integrated reservoir modeling workflow in which parameter updates are performed in the static model. Not only does this ensure more realistic updates, it also facilitates incorporating knowledge from different subsurface

disciplines. Moreover, systematic criteria can be derived that help to determine which level of geologic model detail is flow-relevant or business-decision-relevant. Several authors have proposed such an integrated workflow, (Chierici, 1992; Caers, 2003; Hamman et al., 2003; Gross et al., 2004; Hoffman et al., 2005; Suzuki and Caers, 2006; Elrafiie et al., 2009; Seiler et al., 2009; Kaleta et al., 2012). In this paper we do not employ a full Big Loop workflow, but, instead, we focus on updating some of the structural aspects of the static model. Such an approach to update structural parameters has been addressed in several studies before (Rivenæs et al., 2005; Suzuki et al., 2008; Schaaf et al., 2009; Seiler et al., 2010; Skjervheim et al., 2012). Rivenæs et al. (2005) generated various fault patterns and ran streamline simulations for the entire set of realizations. Next they performed model selection (rather than history matching) by ranking of the models based on the mismatch between simulated and measured production data. Suzuki et al. (2008) built a large set of models that covered a wide range of possible structural interpretations, and subsequently used stochastic search methods to find those realizations that matched the historical production data. Schaaf et al. (2009) presented a workflow that updates both geological and simulation models at the same time using two different optimization methods. Synthetic historical production data were assimilated in their workflow. The results showed a reduction in the objective function value (i.e. the averaged mismatch between historic and simulated data) but the data match was relatively poor. Seiler et al. (2010) proposed a method to handle structural uncertainties in the reservoir model and presented a history matching workflow for updating structural model parameters with the ensemble Kalman filter (EnKF) through assimilating production data. To represent structural uncertainty an ensemble of top and bottom reservoir horizons were generated around a base-case representing the most-likely interpretation. The vertical positions of points at the top and bottom horizons were considered as the uncertain parameters which were included in an augmented state vector and updated using the EnKF method. Skjervheim et al. (2012) introduced an integrated workflow in the form of a consistent modeling chain from depth conversion to flow simulation. They also represented the uncertainty with an ensemble of realizations and used various ensemble-based assisted history matching methods such as the ensemble smoother and the EnKF. Leeuwenburgh et al. (2011) also demonstrated the feasibility of an integrated work flow for structural parameter updating using the EnKF.

Although, the EnKF is an efficient method for structural surface updating, it has a number of drawbacks. The ensemble and ensemble size in the EnKF need to be selected carefully such that uncertainty is sufficiently captured. In addition, to avoid high computational costs, a relatively small ensemble is generally chosen, i.e., the number of ensemble members (typically hundred) is small compared to the number of unknown parameters (typically in the order of the number of grid blocks). The assimilation of large amounts of data (e.g. as resulting from time-lapse seismic) with relatively small ensembles could lead to spurious correlations which may lead to unphysical updates of state variables and/or model parameters, see e.g. Aanonsen et al. (2009), or Oliver and Chen (2011). Gradient-based history matching is an alternative for structural updating, which does not suffer from these drawbacks.

In the studies cited above, only production measurements were assimilated in the history matching workflow. Production data provide localized spatial information about the area around the well locations and only very limited and averaged information about the regions in-between the wells. Consequently, the production data often contain insufficient information for history matching of large-scale structural parameters. On the other hand, time lapse seismic data can provide information on the areal distribution of pressure and saturation changes due to fluid production or injection. Hence, assimilation of time-lapse seismic data to estimate the structural parameters can result in more reliable results, see, e.g., Gosselin et al. (2001) or Van Essen et al. (2012). This paper proposes an assisted history matching workflow for structural parameter updating by assimilating time-lapse seismic data and/or production data with a gradient-based history matching method. The methodology is explained in the next section. Thereafter we present and discuss the results of three ‘twin experiments’ in which the method is tested with the aid of synthetic data.

2. Methodology

2.1. Adjoint Method for History Matching of Structural Models

Gradient-based history matching is an iterative procedure in which the update of the uncertain parameters is determined with the aid of the gradient vector (i.e. the vector of derivatives) of the mismatch objective function with respect to the uncertain parameters. Typically many iterations are required, and therefore it is essential to choose an efficient way to calculate the gradients. The gradients can be calculated either analytically or numerically. Methods that use numerical gradients are easy to implement but are computationally inefficient, especially when there is a large number of parameters. In the most time-consuming, deterministic, numerical variety one reservoir simulation is required for each uncertain parameter. In stochastic numerical approaches, this number can be somewhat reduced but still becomes prohibitively large for realistically sized reservoir models. On the other hand, methods that use analytical gradients, and in particular the adjoint method, are computationally much more efficient. The computational cost of the adjoint method depends on the number of objective functions and not on the number of variables, because this method provides the gradients of a given objective function with respect to all implemented variables by running a small number of simulations. Hence, among the existing methods for calculating gradients, the adjoint method is the most efficient one. The major disadvantage of the method is the significant amount of programming that is required to implement it in a reservoir simulation code. The adjoint method was first used for

history matching by Chen et al. (1974) and Chavalas et al. (1975), and thereafter refined by many authors. For detailed overviews, see the book of Oliver et al. (2008) or the review paper by Oliver and Chen (2011). In this study we propose to use the adjoint-method in a Big Loop approach for history matching of structural parameters.

2.2. Workflow

We use an in-house reservoir simulator with adjoint-functionality (Kraaijevanger et al., 2007). The simulator can provide the gradient of a mismatch function with respect to grid block parameters (e.g. permeabilities or porosities). However, in order to use gradient information for structural parameter updating we need to integrate static and dynamic modeling. To this end we use a Big Loop workflow in which the in-house dynamic simulator is coupled to commercial geological modeling software (Kaleta et al., 2012). In this workflow all the uncertain parameters are defined in the static domain. After constructing one or more static realizations these are exported to the dynamic domain for reservoir flow simulation, typically after an upscaling step (although the latter is not needed for the example described below). If there is a significant mismatch between the dynamic simulation results and the historical data, the gradients of the mismatch objective function with respect to dynamic grid block parameters are then used to update the static model parameters as will be discussed in detail below. The workflow is shown schematically in **Fig. 1**.

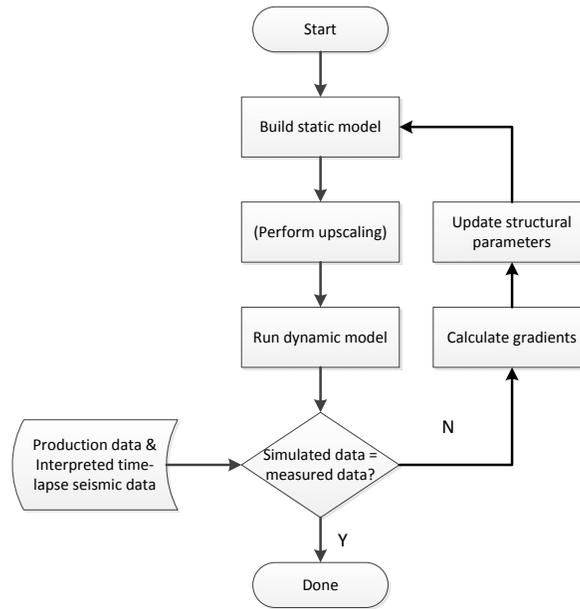


Fig. 1. Workflow for gradient-based history matching of structural parameters in the static model. Note: upscaling is not performed in our examples.

2.3. Parameterization

Structural uncertainties may result from different sources such as migration, picking, and time-to-depth conversion errors (Thore et al., 2002). Here we assume that the depths of the top and bottom reservoir horizons are the major uncertainties in the static model, which results in an uncertainty in reservoir thickness. Moreover, in this study the top surface of the reservoir is fixed such that the uncertainty in reservoir thickness is due to the uncertainty in the reservoir bottom depth only. Hence, the reservoir bottom depth is used as history matching parameter. Unfortunately, gradients with respect to grid block heights are not available from our simulator, but gradients with respect to grid block porosity, summed over the height of the reservoir, can be interpreted as indications of where the reservoir height, and thus the bottom depth, needs to be adapted.

2.4. Mismatch Objective Function

We apply the following objective function to represent the averaged mismatch between between historical and simulated data:

$$J(\boldsymbol{\varphi}) = (\mathbf{d} - \mathbf{h}(\boldsymbol{\varphi}))^T \mathbf{P}^{-1} (\mathbf{d} - \mathbf{h}(\boldsymbol{\varphi})), \quad (1)$$

where \mathbf{d} is a vector of measured data, $\mathbf{h}(\boldsymbol{\varphi})$ is a vector of simulated data, $\boldsymbol{\varphi}$ is a vector of grid block porosities, and \mathbf{P} is a square positive semi-definite matrix of weight factors which is often chosen as the measurement error covariance matrix. Note that we do not use a prior term. We aim to assimilate production measurements as well as time-lapse seismic data, where we assume

that the time lapse seismic results are available in the form of interpreted saturations. Hence, we use two different objective functions: one defined as the mean squared difference between observed and simulated production data, and one defined as the mean squared difference between observed (i.e. interpreted) and simulated grid block saturations. The gradient of the objective function is defined as the column vector containing the partial derivatives of J with respect to the components of the uncertain grid block porosities:

$$\nabla J_{\boldsymbol{\varphi}} = \left(\frac{\partial J}{\partial \boldsymbol{\varphi}} \right)^T = \left[\frac{\partial J}{\partial \varphi_1} \quad \frac{\partial J}{\partial \varphi_2} \quad \frac{\partial J}{\partial \varphi_3} \quad \dots \quad \frac{\partial J}{\partial \varphi_n} \right]^T, \quad (2)$$

where n is the number of grid blocks in the dynamic reservoir model.

2.5. Structural Model Updating

The gradient of the objective function is calculated by the adjoint method in the dynamic simulator. Subsequently this gradient information is used to update the position of the reservoir bottom horizon by converting the gradient with respect to porosities to a gradient with respect to bottom depths defined as

$$\nabla J_{\mathbf{b}} = \left(\frac{\partial J}{\partial \mathbf{b}} \right)^T = \left[\frac{\partial J}{\partial b_1} \quad \frac{\partial J}{\partial b_2} \quad \frac{\partial J}{\partial b_3} \quad \dots \quad \frac{\partial J}{\partial b_m} \right]^T, \quad (3)$$

where \mathbf{b} is a vector of reservoir bottom depths, and m is the number of grid blocks in the bottom layer of the reservoir model. Using a simple volume balance the relation between the gradients is given by

$$\frac{\partial J}{\partial b_i} = \sum_{k=1}^K \left(\frac{\partial J}{\partial \varphi_k} \frac{1}{\Delta h_k} \right), \quad i = 1, \dots, m, \quad (4)$$

where Δh_k is the height of grid block k , and K is the number of gridblocks over the height of the reservoir. In our implementation we simply use only the porosity gradients for the bottom layer of grid blocks such that $K = 1$. We used a simple steepest descent method to update the bottom depth values, i.e. the vertical coordinates (with positive axis pointing downwards) at the centers of the grid blocks in the bottom layer of the reservoir model:

$$\mathbf{b}^{j+1} = \mathbf{b}^j + \alpha \left(\frac{\partial J}{\partial \mathbf{b}} \right)^T_j, \quad (5)$$

where the positive scalar α is a fixed step length, and j is an iteration counter. The reservoir thickness at the grid blocks corresponding to the well locations is assumed to be known, such that the gradients in those grid-blocks are zero.

3. Results and Discussions

3.1. Twin Experiments

We performed three ‘twin experiments’, using the same ‘truth model’ (used to generate synthetic data) but two different uncertain prior models. The first two experiments involved the assimilation of production data, while in the last one we assimilated time lapse seismics.

3.1.1. Truth Model

The ‘truth model’ represents a simple three dimensional reservoir with three layers: an impermeable shale layer in between permeable top and bottom zones; see **Fig. 2**. The average reservoir thickness is 65 m, the average depth is 4100 m, the water-oil-contact is located at 4085 m and the STOIP is 1.14×10^8 bbl. The initial water saturation is 0.1 and the initial reservoir pressure is 40 MPa at the top perforations. The top zone and the bottom zone have a constant permeability of 500 mD and 650 mD and a constant porosity of 0.15 and 0.2 respectively. Corey-type relative permeabilities are used for relperms. The fluid properties and Corey exponents are given in **Table 1**. Eight injectors and four producers, perforated over the entire height of the producing layers, are located in the field. Fig. 2 shows the permeability field of the ‘truth model’ together with injector and producer locations. The reservoir model contains three layers of grid blocks with a total number of grid blocks equal to 3888.

TABLE 1 - FLUID PROPERTIES AND COREY EXPONENTS		
Property	Value	Unit
ρ_w	1009	Kg/m ³
ρ_o	880	Kg/m ³
μ_w	1×10^{-3}	Pa.s
μ_o	4×10^{-3}	Pa.s
S_{wc}	0.2	-
S_{or}	0.1	-
k_{rw}^0	0.9	-
k_{ro}^0	0.8	-
n_w	3	-
n_o	4.75	-

At the start of production the injectors operate at a constant flow rate of 300 m³/day and the producers at a bottom hole pressure of 39 MPa at the top perforations, i.e. 1 MPa below the reservoir pressure. This ‘truth model’ is used to create synthetic production data over a period of 12 years. The measurements (oil rate, water rate per well) are taken monthly. After eight years of production one seismic survey is conducted. Interpreted time-lapse seismic data is represented as saturation changes per grid block. **Fig. 3** shows the oil saturation in the bottom layer of the truth model after eight years of production. No measurement errors are added to the data.

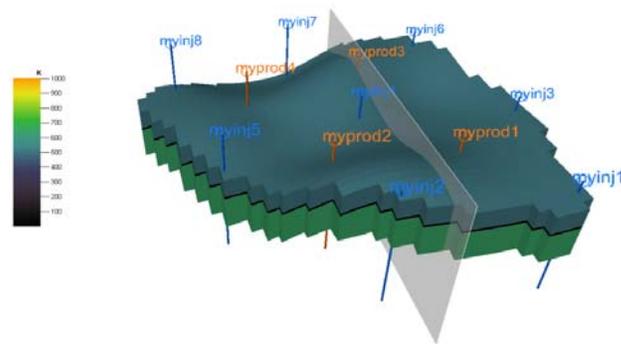


Fig. 2. The ‘truth model’ permeabilities. Seven injectors are placed around the field and one in the center. Four producers are located in central part of the field. The transparent plane indicates the cross section corresponding to Figs. 4 and 5.

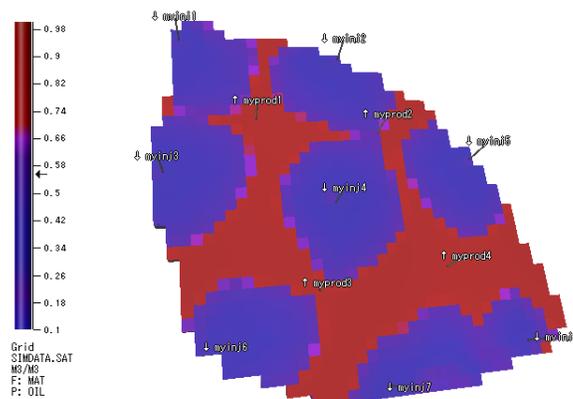


Fig. 3. Oil saturation in the bottom layer of the ‘truth model’ after 8 years of production.

3.1.2. Prior Models

In this study, the true bottom horizon is assumed to be unknown. The bottom horizon of the prior models is obtained by stochastic manipulation of the bottom horizon of the ‘truth model’. Two different prior models are chosen: prior #1 represents a reservoir structure that is relatively close to the truth model, and prior #2 a structure that displays significant differences compared to the truth. All remaining parameters in the prior models are chosen identical to those of the ‘truth model’ and

assumed to be known. **Fig. 4** and **Fig. 5** show cross sections of the prior models and the ‘truth’ for prior #1 and prior #2 respectively. The blue line represents the bottom of the ‘truth model’, the red line represents the bottom of the prior model, and the black line represents the top of the bottom layer (i.e. the bottom of the shale layer).

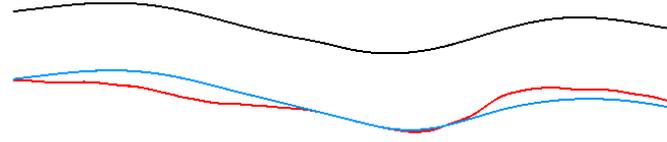


Fig. 4. A cross section of the ‘truth model’ and prior #1. This prior is close to the truth. Red represents the bottom of the prior model, blue represents the bottom of the truth, and the black represents the bottom of the shale layer.

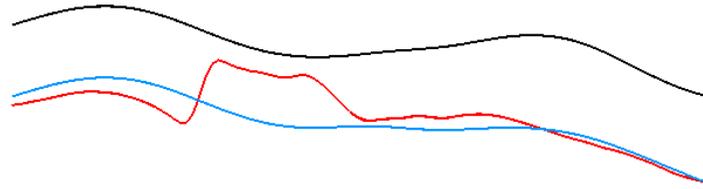


Fig. 5. A cross section of the truth model and the prior #2. This prior is significantly different from the truth. Red represents the bottom of the prior model, blue represents the bottom of the truth, and the black represents the bottom of the shale layer.

3.2. Experiment #1: Assimilation of Production Data Starting From Prior #1

In the first twin experiment the history match is performed starting from prior #1 by assimilating production data. We did not use any scaling of the data (i.e. $\mathbf{P} = \mathbf{I}$). After some trial and error we selected a step size $\alpha = 7.5 \times 10^{-4} \text{ m}^2$, while a relative convergence of 0.005 was used as a stopping criterion. **Fig. 6** shows the convergence of the objective function after eleven iterations.

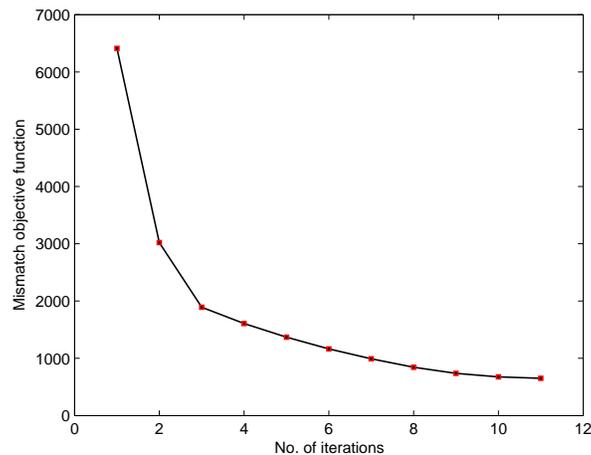


Fig. 6. Mismatch objective function for experiment #1.

Fig. 7 depicts a cross section of bottom horizons of the ‘truth model’ (blue), the prior model (red) and the updated model (green) after eleven iterations. **Fig. 8** depicts the prior and updated residual maps of the bottom horizon, where the residual is defined as the difference in depth between the ‘truth’ and the model. The colors represent the residuals (in m) and the contour lines indicate the shape of the true bottom horizon (without scale). The black line indicates the cross section corresponding to **Fig. 7**. In this experiment, which starts from a prior that is close to the truth, updating the reservoir bottom depth by assimilation of production data leads to an improved posterior.

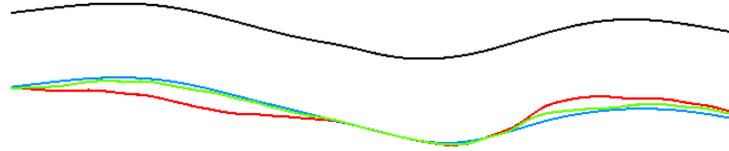


Fig. 7. Cross section through the bottom layer of the reservoir for experiment #1. Blue represents the truth, red represents prior #1, and green represents the updated model.

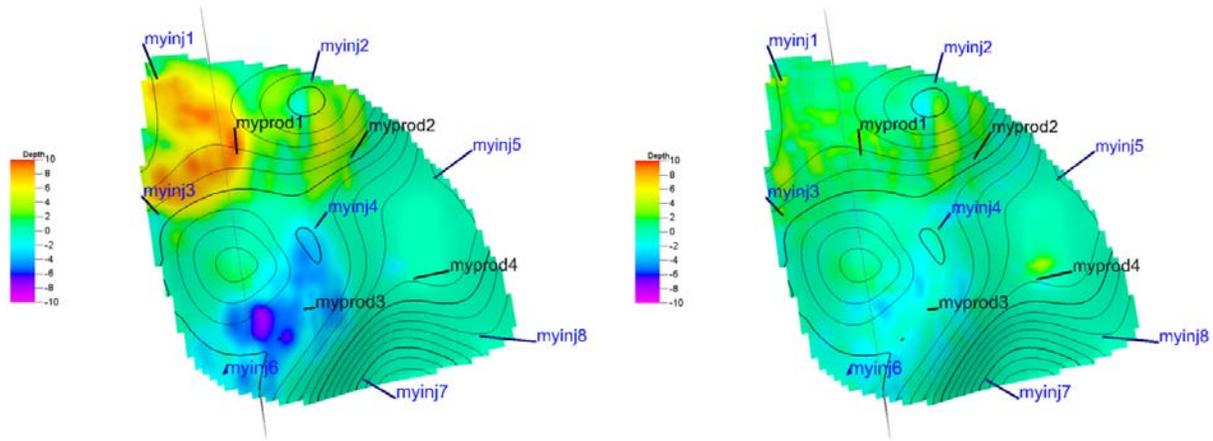


Fig. 8. Prior (left) and updated (right) residual maps for experiment #1. Colors represent the residuals in m. Contour lines indicate the true bottom depth (without scale).

3.3. Experiment #2: Assimilation of Production Data Starting From Prior #2

Experiment #2 involves the assimilation of production data starting from prior #2 which is further from the truth than prior #1. With a step length $\alpha = 7.5 \times 10^{-6}$, the objective function converged after 50 iterations but did not show a large drop in value; see Fig. 9. Note that the small increase in the objective function value in 21st iteration is due to the fact that we used a fixed step length without line search. The updated model does not show a good match with the truth; see Fig. 10. The same message is conveyed by Fig. 11 which depicts the prior and updated residual maps of the bottom horizon. Except for minor changes close to some of the well locations (myprod3, myprod4 and myinj4), the updated model has not changed significantly with respect to the prior model. This example illustrates that production data often do not contain enough information to reduce uncertainties in areas that are not in the immediate vicinity of the wells, and that the results of history matching using production data are dependent on the quality of the prior model.

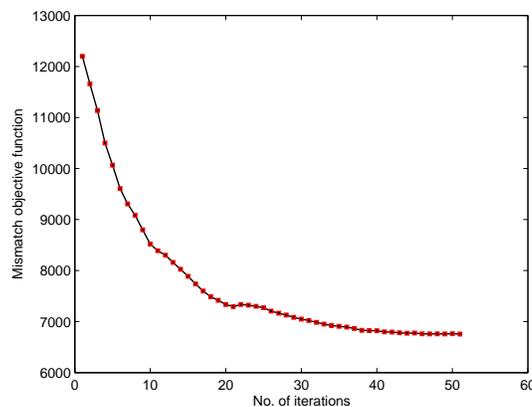


Fig. 9. Mismatch objective function for experiment #2.

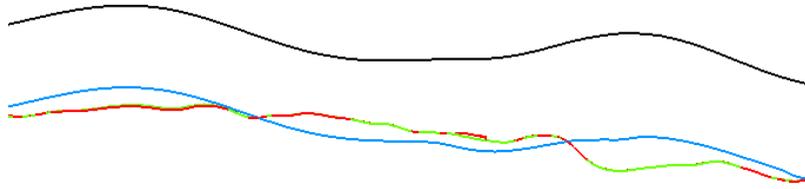


Fig. 10. Cross section through the bottom layer of the reservoir for experiment #2. Blue represents the truth, red represents prior #2, and green represents the updated model.

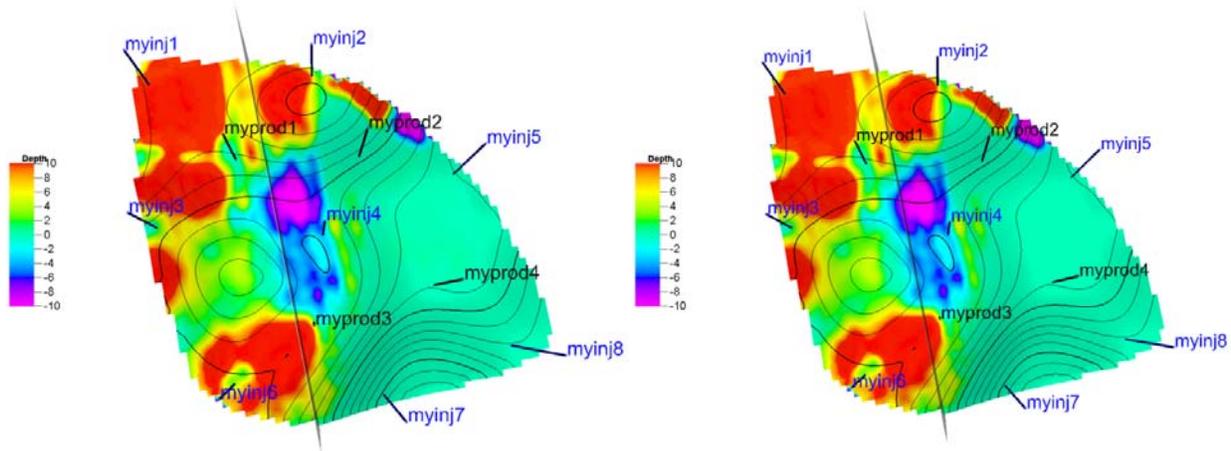


Fig. 11. Prior (left) and updated (right) residual maps for experiment #2. Colors represent the residuals in m. Contour lines indicate the true bottom depth (without scale).

3.4. Experiment #3: Assimilation of Time-Lapse Seismic Data Starting From Prior #2

Just like experiment #2, experiment #3 starts from the ‘poor’ prior #2, but now involves the assimilation of time-lapse seismic data instead of production data. The step length is chosen as $\alpha = 3.5$ and after 30 iterations the value of the objective function is reduced by a factor of ten; see Fig. 12.

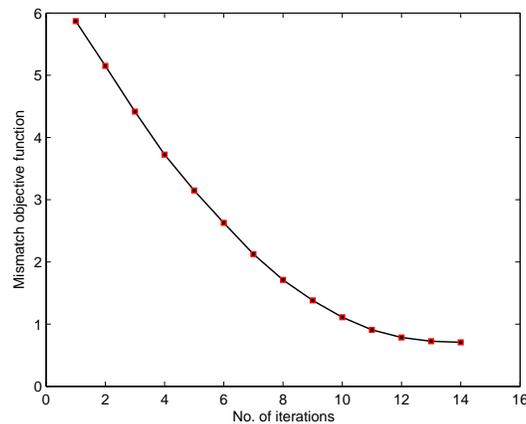


Fig. 12. Mismatch objective function for experiment #3.

Fig. 11 depicts the prior and updated depths of the bottom horizon and Fig 14 the corresponding residual maps. In contrast with the previous example, history matching of the bottom horizon by assimilation of time-lapse seismic data results in an acceptable mismatch objective function value as well as a good match between the updated model and the truth.

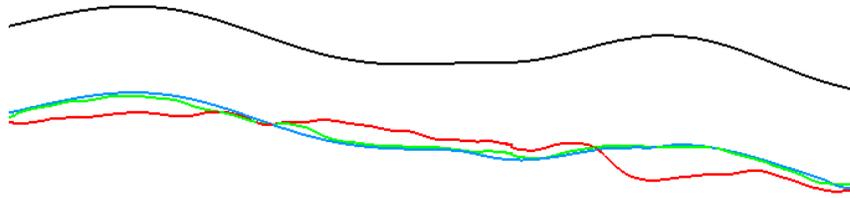


Fig. 13. Cross section through the bottom layer of the reservoir for experiment #3. Blue represents the truth, red represents prior #2, and green represents the updated model.

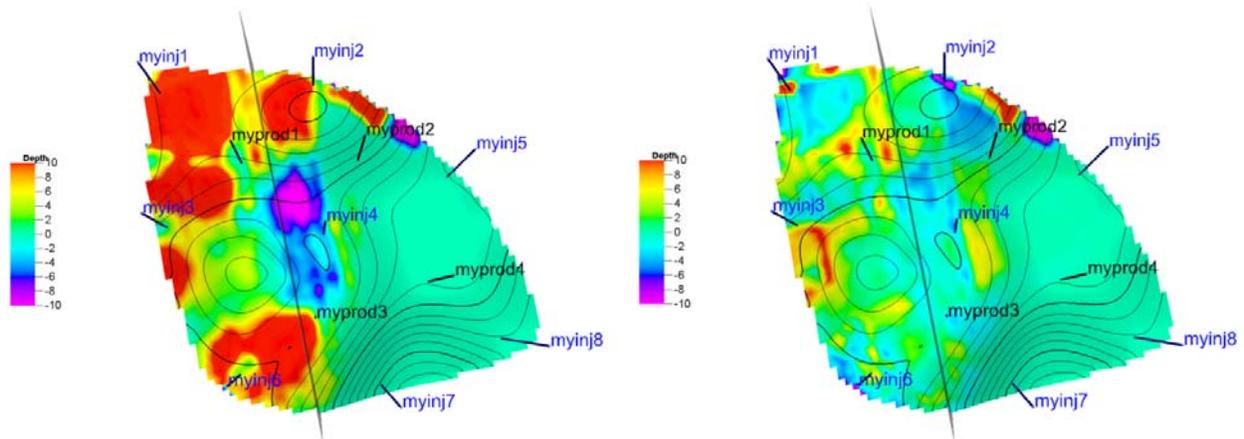


Fig. 14. Prior (left) and updated (right) residual maps for experiment #3. Colors represent the residuals in m. Contour lines indicate the true bottom depth (without scale).

Fig. 15 depicts the oil flow rates of each of the four production wells during 15 years of production for experiment #3. Production starts in 2004 and after eight years, in 2012, a time lapse seismic survey is conducted and the interpreted results are assimilated. The curves thereafter represent predictions of the future oil production. It can be observed that not only the simulated oil flow rates of the updated model are in a good agreement with the measured rates, but that also the predicted flow rates are much closer to the truth than those simulated with the prior model.

4. Discussion

In this study the uncertain parameters are updated without constraints. Moreover, the objective function does not contain a term that penalizes deviations from the prior parameter values, as is required in a Bayesian framework (see e.g. Oliver et al. 2008). Such a penalty term makes the problem well posed, restricts the parameter updates to values that keep the posterior values not too far from the prior values, and generally increases the smoothness of the results. For our examples, we expect that including a penalty term would lead to somewhat smoother updates but not to essentially different conclusions.

Assimilation of production data to update the bottom depth of prior #2 showed that production data do not provide sufficient information for reliable updates away from the wells whereas time lapse seismics provides much more spatially distributed information, leading to improved updates and improved predictions. Similar conclusions, but then for updating flow properties instead of structural properties have been reported before, see e.g. Walker and Lane (2007).

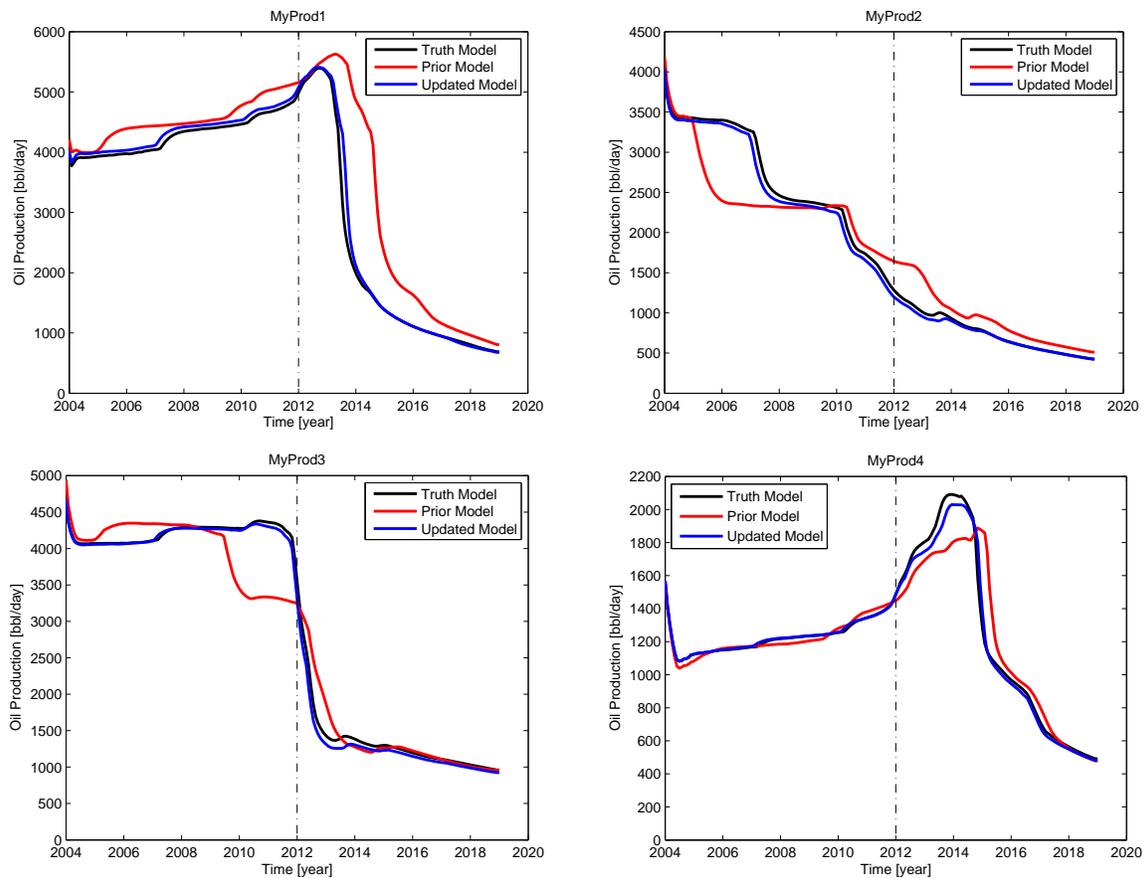


Fig. 15. Oil flow rates of each of the four production wells for experiment #3. Red curves represent the rates of the prior model simulation, blue curves represent the rates of the updated model, and black curves represent the true rates. The vertical dashed line indicates the moment that the history match is performed.

5. Conclusions

In this study we proposed a new method for updating uncertain structural reservoir parameters by combining static and dynamic reservoir models in a ‘big loop’ history matching workflow using gradient-based history matching. In particular we assumed the parameters defining the bottom horizon of the static reservoir model to be uncertain and updated them by assimilation of production data or time-lapse seismic data using the adjoint method. We tested the method on three simple 3D synthetic examples in which the bottom depth was the only uncertain parameter while the measurements were assumed to be error free. We conclude that for these examples

- The adjoint method is a computationally efficient method for history matching of the structural model parameters.
- Gradients of the mismatch objective function with respect to grid block porosities can serve as an acceptable approximation for gradients with respect to bottom depths and can thus be used to update the reservoir thickness.
- Production data contain mostly localized information from the near-well bore area and are therefore of limited value to update structural reservoir parameters in areas away from the wells. As a result the updated results are strongly dependent on the prior model.
- Time-lapse seismic data (in the form of interpreted saturations) contain much more spatially distributed information and are therefore a much better source of information to update structural reservoir parameters in areas away from the wells.
- Updating the structural parameters of the static reservoir model significantly improves the predictive capacity of the correspondingly updated dynamic reservoir model.

Further work is required to assess the wider validity of these conclusions, in particular for situations where more uncertain parameters are considered and where the data contain significant measurement errors.

6. Nomenclature

- b = grid block vertical position (depth) in the bottom layer of the reservoir model, L, m
 \mathbf{b} = vector of grid block vertical positions (depths) in the bottom layer of the reservoir model, L, m
 \mathbf{d} = vector of measured data, $L^{-1}m\ t^{-2}$, Pa (pressures), or $L^3\ t^{-1}$, m^3/s (flow rates), or dimensionless (saturations)
 Δh = grid block height, L, m
 \mathbf{h} = vector of simulated data (dimensions equal to those of \mathbf{d})
 J = mismatch objective function, dimensionless
 K = number of gridblocks over the height of the reservoir, dimensionless
 n = number of grid blocks in the reservoir model, dimensionless
 m = number of grid blocks in the bottom layer of the reservoir model, dimensionless
 \mathbf{P} = positive semi-definite matrix denoting the relative importance and correlation between the entries of \mathbf{d}
 (dimensions equal to those of \mathbf{d} squared)
 α = step length in steepest descent algorithm (dimensions equal to those of \mathbf{b} squared)
 φ = grid block porosity (dimensionless)
 $\boldsymbol{\varphi}$ = vector of grid block porosities (dimensionless)

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