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Optimal Experiment Design in Closed Loop with Unknown, Nonlinear and Implicit Controllers using Stealth Identification

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***Abstract*— We present a novel optimal experiment design method that is applicable to linear time-invariant systems regulated by unknown, nonlinear and implicitly-defined controllers. Current methods require an explicit expression for the controller in order to construct the optimal excitation spectrum. Consequently, these are limited to linearly-controlled systems. The identification scheme suggested in this paper circumvents the aforementioned requirement by ensuring that the excitation signal remains unnoticed by the controller, i.e., the identification data is gathered in an open-loop fashion.**
Our theoretical analysis is complemented with a numerical study on a six-parameter, single-input single-output linear system controlled by an MPC. We find that our method generates least-costly excitation signals which deliver identified models that lie close to the true system whilst honoring quality constraints, validating the novel optimal experiment design framework.

I. Introduction

In optimal identification experiment design, the excitation signal used during an identification experiment is designed to either maximize the accuracy of the to-be-identified model under some constraints on the cost of the identification [1], [2] or, conversely, to obtain a given accuracy at minimal cost [3]. Generally, the accuracy of the identified model is measured in terms of the inverse of the covariance matrix, P_θ^{-1}, of the identified parameter vector. This matrix is a function of the spectrum Φ_r of the excitation signal r.

We consider here least-costly experiment design [3]. Hence, the optimal experiment design problem is formulated as determining the excitation spectrum minimizing the cost of the identification experiment whilst guaranteeing that $P_\theta^{-1} \succeq R_{adm}$, with R_{adm} a matrix corresponding to the minimal accuracy required for the use of the model (e.g. for control purpose). There exist different techniques to construct the matrix R_{adm} (see e.g. [4], [5]).

We restrict to optimal experiment design for direct closed-loop identification. Direct closed-loop identification [6] is an identification technique that allows to determine a model of a real-life system G_0 (the so-called true system) when this true system is operated in closed loop. The procedure is as follows: an excitation signal r is added e.g. at the output of the controller for a given amount of time, during which the corresponding input and output of the true system are collected. The model of the true system is subsequently determined by minimizing the power of the prediction error computed with this data. The procedure delivers a consistent model of the true system provided some conditions are met (the strongest condition being that a full-order model structure is used). An advantage of this identification technique is that the procedure does not require the knowledge of the controller present in the loop during the identification experiment, and can thus be applied to systems with any type of stabilizing controllers (e.g. implicit, piece-wise linear, and nonlinear).

Unfortunately, this property does not hold for the optimal design of a direct closed-loop identification experiment [3]. In order to solve the experiment design problem using the convex optimization techniques of [3], an affine relation must exist between the decision variable Φ_r and P_θ^{-1}. Such a relation only exists if the controller is linear and explicitly expressed as a transfer function $C(z)$. Moreover, the transfer function must be known. In this paper, we extend the framework presented in [3] so that the optimal excitation spectrum Φ_r can also be determined when the controller in the loop is unknown, implicit, or nonlinear.

To this end, we modify the feedback mechanism during the identification experiment in such a way that the (possibly unknown, implicit or nonlinear) controller does not see the excitation signal. If this is the case, the signal r is applied in an open-loop fashion to the to-be-identified system G_0 and the dependence of P_θ^{-1} on Φ_r becomes independent of the expression of the controller. It enables to determine the optimal spectrum Φ_r using convex optimization techniques even though the true system is operated by an unknown, implicit or nonlinear controller. For the above property to hold, we will show that the feedback signal to the controller should be changed to $y - G_0 r$. Evidently, this scheme requires the knowledge of the unknown system G_0. We thus propose to replace G_0 by an initial estimate G_{id} of G_0 such as is generally done in optimal experiment design to circumvent the so-called chicken and egg problem (i.e. the fact that P_θ is dependent on the unknown system G_0).

The paper is organized as follows. In Section II we

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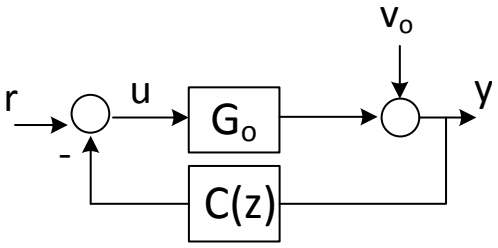

Fig. 1. Schematic overview of the true closed-loop system.

introduce the direct method. We restrict to SISO notation for ease of notation, but note that generalization to multiple-input multiple-output (MIMO) systems is straightforward. Section III explains the current limitation of optimal experiment design and introduces our novel identification method. Our method is further elaborated in Section IV, where we show its applicability to nonlinear controllers. The framework is then tested on a SISO example in Section V. Finally, conclusions are drawn in Section VI.

II. Direct closed-loop identification with a known linear controller

We will first consider the case of a linear controller. We then consider a linear time-invariant true system $\mathcal{S}$, operated in closed loop with a linear controller $C(z)$, consisting of output $y(t) \in \mathbb{R}$ and input $u(t) \in \mathbb{R}$,

$$y(t) = G(z,\theta_0)u(t) + H(z,\theta_0)e(t), \quad (1)$$
$$u(t) = r(t) - C(z)y(t). \quad (2)$$

Here, $\theta_0 \in \mathbb{R}^n$ is the unknown true parameter vector with $\dim(\theta_0) = n$, $e(t) \in \mathbb{R}$ is white noise with variance σ_e^2; $G(z,\theta_0) = G_0$ and $H(z,\theta_0) = H_0$ are stable, discrete-time transfer matrices. Furthermore, $H(z,\theta_0)$ is monic and minimum-phase. Lastly, we assume that G_0 or $C(z)$ has one delay.

As can be seen in (2) (see also Fig. 1), the closed-loop system is excited via an external signal $r(t) \in \mathbb{R}$ that is added to the output $u_C(t) = -Cy$ of the controller. Applying the excitation signal $r(t)$ for $t = \{0,\dots,N-1\}$ to the system and measuring the signals $Z_N = \{u(t), y(t) \mid t = 0,\dots,N-1\}$, a model $\left\{G(z,\hat{\theta}_N), H(z,\hat{\theta}_N)\right\}$ of the true system can be identified. The parameter vector $\hat{\theta}_N$ is defined as $\hat{\theta}_N = \arg\min_\theta V_{id}(\theta)$, with

$$V_{id}(\theta) = \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon^2(t,\theta), \quad (3)$$

where $\varepsilon(t,\theta) = H^{-1}(z,\theta)\left[y(t) - G(z,\theta)u(t)\right]$ is the prediction error. Throughout this paper, we assume that r is sufficiently exciting and that the model has been identified in a model structure $\mathcal{M}$ containing the true system (1)-(2), i.e., $\mathcal{S} \in \mathcal{M}$. In this way, the parameter vector $\hat{\theta}_N$ identified with (3) will be asymptotically normally distributed around the true parameter vector θ_0. Hence, we have for $N \to \infty$ that $\hat{\theta}_N \to \mathcal{N}(\theta_0, P_\theta)$, with P_θ a strictly positive definite matrix given by

$$P_\theta = \frac{\sigma_e^2}{N}\left(\mathbb{E}\left[\left(\frac{\partial\varepsilon(t,\theta)}{\partial\theta}\mid_{\theta_0}\right)\left(\frac{\partial\varepsilon(t,\theta)}{\partial\theta}\mid_{\theta_0}\right)^T\right]\right)^{-1} \quad (4)$$

that can be estimated from $\hat{\theta}_N$ and Z_N [6].

Alternatively, in the frequency domain, the inverse of P_θ is of the form:

$$P_\theta^{-1} = \frac{N}{2\pi\sigma_e^2}\int_{-\pi}^{\pi}\big[F_r(e^{i\omega},\theta_0)F_r(e^{i\omega},\theta_0)^*\Phi_r(\omega) + F_v(e^{i\omega},\theta_0)F_v(e^{i\omega},\theta_0)^*\sigma_e^2\big]\,d\omega, \quad (5)$$

with $F_r(z,\theta_0) = \frac{S}{H_0}\Lambda_G(z,\theta_0)$, $F_v(z,\theta_0) = \frac{\Lambda_H(z,\theta_0)}{H_0} - C(z)S\Lambda_G(z,\theta_0)$, $S = (1+C(z)G_0)^{-1}$ the sensitivity function of the closed-loop system $(G_0, C(z))$, $\Lambda_G = \frac{\partial G}{\partial\theta}\mid_{\theta_0}$, $\Lambda_H = \frac{\partial H}{\partial\theta}\mid_{\theta_0}$, and $\Phi_r(\omega)$ is the spectrum of the excitation signal.

The purpose of least-costly experiment design is to develop an excitation signal (or spectrum) that disturbs the process in a minimal way, whilst guaranteeing user-imposed constraints on the quality of the to-be-identified model.

Mathematically, we wish to solve the problem

$$\min_{\Phi_r(\omega)} \text{Identification cost} \quad \text{subject to } P_\theta^{-1} \succeq R_{adm}, \quad (6)$$

where P_θ^{-1} is as defined in (5) and R_{adm} a matrix corresponding to the minimal accuracy required for the use of the to-be-identified model. This matrix is computed numerically, see [4] for details. The above optimization problem can be reformulated into a Linear Matrix Inequality (LMI) problem, linear in the decision variable Φ_r, which can be numerically solved.

A few comments are in order. First of all, we do not exactly know σ_e^2 and θ_0, so these should be replaced by some estimates. Secondly, the decision variable $\Phi_r(\omega)$ has infinite dimension (since ω is continuous). We therefore parameterize the spectrum such that the LMI problem becomes tractable by writing the the excitation spectrum as [3]

$$\Phi_r(\omega) = \sum_{k=-m}^{m} c_r e^{i\omega r} \geq 0 \ \ \forall\omega, \quad (7)$$

where $c_r = c_{-r}$, $r = 0\dots,m$ are now the decision variables.

Observe that in order to solve the LMI problem, the expression of $C(z)$ needs to be known and be explicit. Indeed, the terms $F_r(e^{i\omega},\theta_0)$ and $F_v(e^{i\omega},\theta_0)$ require a closed-form expression of the controller $C(z)$. Therefore, closed-loop least-costly experiment design is only possible in this way when the controller is linear, time-invariant, and known.

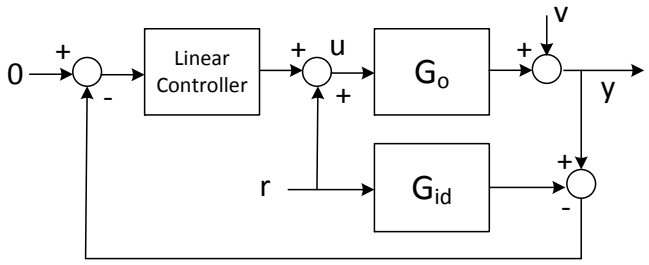

Fig. 2. Schematic overview of the stealth identification set up for a closed-loop system with a linear (possibly implicit or unknown) controller.

III. Experiment design with unknown and implicit linear controllers

We now present a novel least-costly identification method to allow for unknown and implicit linear controllers, see Fig. 2. The idea is to ensure that the excitation signal $r(t)$ is not noticed by the controller, i.e., $r(t)$ affects the system in an open-loop fashion. Notice that the feedback term to the controller is adjusted from $y(t)$ to $y(t) - G_{id}r(t)$, where G_{id} is an initial estimate of G_0. In the case $G_{id} = G_0$ the controller does not sense the excitation signal at all.

The input and output signals are, for any G_{id}, given by

$$\begin{aligned} u(t) &= u_r + u_e = \frac{1+CG_{id}}{1+CG_0}r(t) - \frac{CH_0}{1+CG_0}e(t), \\ y(t) &= y_r + y_e \\ &= G_0\frac{1+CG_{id}}{1+CG_0}r(t) + \frac{H_0}{1+CG_0}e(t), \end{aligned} \tag{8}$$

where $y_e(t)$ and $u_e(t)$ are the output and input signals that are generated by the closed loop $(G_0, H_0, C(z))$ without the presence of the excitation signal $r(t)$ (if $G_{id} = 0$, r remains in the equations).

Let us now use these equations to find the expression of the inverse of the covariance matrix P_θ^{-1} of a parameter vector identified using (3) with the data set $Z_N = \{y, u\}$ given by (8). The prediction error becomes

$$\begin{aligned} \varepsilon(t,\theta) &= H^{-1}(z,\theta)(y(t) - G(z,\theta)u(t)) \\ &= H^{-1}(z,\theta)(y_r(t) - G(z,\theta)r(t)) \\ &+ H^{-1}(z,\theta)(y_e(t) - G(z,\theta)u_e(t)) \\ &= \varepsilon_r(t,\theta) + \varepsilon_e(t,\theta), \end{aligned} \tag{9}$$

where $\varepsilon_e(t,\theta)$ is the prediction error that would be obtained if $\forall t, r(t) = 0$. We will give explicit expressions of both terms below.

Since $\varepsilon_e(t,\theta)$ is independent of $r(t)$, the inverse of the covariance matrix can be expressed (via (4) and Parseval's theorem) as

$$P_\theta^{-1} = M_r + M_e, \tag{10}$$

with M_r a matrix that is affine in the excitation spectrum Φ_r and M_e the matrix corresponding to the term $\varepsilon_e(t,\theta)$. Matrix M_e is easily evaluated by simulating the system with $r(t) = 0\,\forall\, t$, gathering the data $Z_N = \{y, u\}$ and identifying the system. The resulting covariance matrix is M_e. We stress that (8) - (10) hold for any G_{id}.

If we now restrict to $G_{id} = G_0$, the matrix M_r is computed as

$$\frac{N}{\sigma_e^2}\frac{1}{2\pi}\int_{-\pi}^{\pi} \bar{F}_r(e^{i\omega})\bar{F}_r(e^{i\omega})^*\Phi_r(\omega)\,d\omega, \tag{11}$$

with $\bar{F}_r(z) = \frac{\Lambda_G(z,\theta_0)}{H_0}$ (which can be obtained by using (8) with $G_{id} = G_0$ and the expression (4)). This is the contribution of the excitation signal r to the quality of the parameter vector of the to-be identified model. Notice that this expression is not a function of the controller anymore. To perform optimal experiment design of Φ_r, we will suppose that $G_{id} = G_0$ and solve (6) with the expressions (9) - (11) for P_θ^{-1}.

The actual identification experiment can now be started. We use the direct method as explained in the previous section. We excite the true system represented in Fig. 2 (with $G_{id} = G_0$) with the signal $r(t)$ that was computed from (6), and gather the data $Z_N = \{y, u\}$. We then identify $\hat{\theta}_N$ with (3).

A. Convergence and Consistency

The stealth identification scheme delivers consistent estimates of the true system, even when $G_{id} \neq G_0$. We summarize the result in the following theorem. We will use the notations in [6] and refer to theorems therein.

Theorem 1: Assume that the true system $\mathcal{S}$ is operated as in Fig. 2 and that the considered closed loop is stable (condition D1 in [6]). Consider a model set $\mathcal{M} := \{G(z,\theta), H(z,\theta)\}$ that contains the true system, i.e., $\mathcal{S} \in \mathcal{M}$. Furthermore, we assume that our dataset Z_N is sufficiently informative w.r.t. $\mathcal{M}$, that there is a delay in either the controller or in both $G_0(z)$ and $G(z,\theta)$, and that the system is globally identifiable at θ_0.

Then the stealth identification scheme yields a consistent estimate of the true parameter vector θ_0, even when $G_{id} \neq G_0$.

Proof: Under the above assumptions, we have (cf. Theorem 8.2 in [6]) that $\hat{\theta}_N \to D_c$ w.p. 1 as $N \to \infty$, where

$$D_c = \arg\min_\theta \bar{V}(\theta) = \left\{\theta \mid \bar{V}(\theta) = \min_{\theta'} \bar{V}(\theta')\right\},$$

with

$$\bar{V}(\theta) = \bar{E}\frac{1}{2}\varepsilon^2(t,\theta).$$

The set D_c contains all parameter vectors θ^* that minimize the quadratic criterion. For consistency, we now need to prove that the set D_c only contains the true parameter θ_0, i.e. $D_c = \{\theta_0\}$. In other words, that θ_0 is the unique minimizer of the quadratic criterion (cf. Theorem 8.3 in [6]).

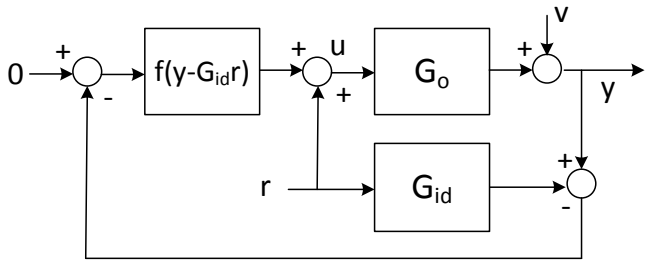

Fig. 3. Schematic overview of the stealth identification scheme for nonlinear controllers.

To this end, we substitute Eqs. (8) into (9) and obtain

$$\begin{aligned}\varepsilon(t,\theta) = & \quad \frac{G_0(z)-G(z,\theta)}{H(z,\theta)}\left(\frac{1+CG_{id}}{1+CG_0}\right)r(t) \\ + & \quad \frac{H_0(z)}{H(z,\theta)}\left(\frac{1+CG(z,\theta)}{1+CG_0}\right)e(t), \end{aligned} \quad (12)$$

where the first and the second term on the r.h.s. are respectively $\varepsilon_r(t,\theta)$ and $\varepsilon_e(t,\theta)$ (cf. (9)).

Using the assumption on the presence of a delay in either $C(z)$ or in both $G(z,\theta)$ and G_0, the power of ε reaches its minimum at $\theta = \theta_0$. It is furthermore a unique minimizer due to the assumption of an informative data set. ■

We note that the proof also holds for other minimization criteria. Please see [6] for details. Notice that when $G_{id} = 0$ the prediction error (12) reduces to the expression one would have obtained from the direct method for a closed-loop system. Furthermore, when $G_{id} = G_0$, we see that the first term on the r.h.s. becomes $H^{-1}(z,\theta)(G_0 - G(z,\theta)r(t))$. This term is now equivalent to the one which would have been obtained with identification of an open-loop system.

IV. Experiment design with (implicit) nonlinear and MPC controllers

As the previous section showed, the stealth identification scheme enables least-costly experiment design. However, we only considered the concept for systems regulated by an LTI controller. In this section, we continue the theoretical analysis of the scheme by generalizing to (possibly implicit or unknown) nonlinear controllers, see Fig. 3.

To this end, we first adapt Eqs. (8) to incorporate various nonlinear controllers:

$$\begin{aligned} u(t) &= u_r + u_{r,e} = r(t) + f(y - G_{id}r), \\ y(t) &= y_r(t) + y_{r,e}(t) \\ &= G_0 r(t) + G_0 f(y - G_{id}r) + H_0 e(t), \end{aligned} \quad (13)$$

with $y_{r,e} = G_0 f(y - G_{id}r) + H_0 e(t)$, and where we suppose that $f(\cdot)$ completely describes the dynamics of the controller. Observe that it is a function of y, G_{id} and r. Furthermore, we define $y_e(t)$ and $u_e(t) = f(y - G_0 r) = f(y_e)$ as the output and input signals that are generated by the closed loop (G_0, H_0, f) shown in Fig. 3 without the presence of the excitation signal $r(t)$.

The expression for the prediction error in the nonlinear case becomes (cf. (9))

$$\begin{aligned} \varepsilon(t,\theta) &= H^{-1}(z,\theta)(y(t) - G(z,\theta)u(t)) \\ &= H^{-1}(z,\theta)(y_r + y_{r,e} - G(z,\theta)(u_r + u_{r,e})) \\ &= \varepsilon_r(t,\theta) + \varepsilon_{r,e}(t,\theta) \end{aligned} \quad (14)$$

with $\varepsilon_{r,e} = H^{-1}(y_{r,e} - G(z,\theta)u_{r,e})$. Indeed, substituting Eqs. (13) into (14) gives

$$\varepsilon(t,\theta) = \frac{G_0(z) - G(z,\theta)}{H(z,\theta)}(r + f(y - G_{id}r)) + \frac{H_0(z)}{H(z,\theta)}e(t),$$

showing that the first term on the r.h.s. is $\varepsilon_r(t,\theta)$ and the latter two define $\varepsilon_{r,e}(t,\theta)$.

In the case where $G_{id} = G_0$ the above equation reduces to

$$\begin{aligned} \varepsilon(t,\theta) &= \frac{G_0 - G(z,\theta)}{H(z,\theta)}(r + u_e) + \frac{H_0}{H(z,\theta)}e(t) \\ &= \varepsilon_r(t,\theta) + \varepsilon_e(t,\theta), \end{aligned} \quad (15)$$

since, by definition, $u_e = f(y - G_0 r) = f(y_e)$. We find that $\varepsilon_e(t,\theta)$ is now independent of r, so the covariance matrix can be written as an affine function of Φ_r (cf. (10)-(11)).

Contrary to the linear case discussed in the previous section, the inverse of the covariance matrix is now no longer affine in the excitation spectrum $\Phi_r(\omega)$ for all G_{id}, due to the nonlinear nature of the controller. The underlying reason is that in the linear case we could make use of the superposition principle to obtain closed-form expressions for $u(t)$ and $y(t)$, which are not available in the nonlinear case.

An affine relation is, however, obtained when $G_{id} = G_0$. For this particular case it holds that $f(y - G_0 r) = f(y_e) = u_e$ as feedback term, and the expression for the prediction error then becomes $\varepsilon_r + \varepsilon_e$, as in the linear case.

We now give heuristic arguments that show that for nonlinear controllers a consistent estimate is also achieved with the stealth identification scheme. The main difference between the linear and nonlinear case is that no closed-form expressions for $u(t)$ and $y(t)$ exist. Fortunately, one is still able to prove consistence without much difficulty. The consistency result for the linear case can be extended to a nonlinear controller under the same assumptions (see Theorem 1). In particular, the loop shown in Fig. 4 should be exponentially stable, see [7] for details. The form of the prediction error (14) clearly shows that $\theta = \theta_0$ minimizes $\varepsilon(t,\theta)$ for all t.

The least-costly framework can thus also be applied to nonlinearly-controlled systems. The identification experiment that follows after construction the excitation signal $r(t)$ is equivalent to the one in the linear case. In practice, one assumes that $G_{id} = G_0$ and then computed the optimal excitation spectrum stipulated above.

A. Remarks for model predictive control

An MPC controller computes the optimal control strategy for future inputs $u(t), u(t+1), \ldots, u(t+N_u)$ by recursively solving a finite-time horizon control problem at each time instant. However, for each instant in time, only the input for the next time instance is applied to the system. This is called the receding horizon strategy [8]. An MPC controller is thus implicit and time-varying. Furthermore, constraints are imposed on the input and the output of the system.

By fooling the MPC with the stealth identification method, constraints will be respected for the signals u_e and y_e (i.e. input and output without excitation), but not necessarily for u and y (input and output during excitation). After having determined the excitation signal $r(t)$, the constraints for the MPC can be adapted so that we are sure that $u(t) = r(t) + u_e(t)$ satisfies the constraints. Another option is to saturate $r(t)$ as follows.

Suppose that the optimal spectrum can be realized by a signal $r_{opt}(t)$ (the time-domain solution of (6)) and that we have the following constraint on $u(t) : u(t) \leq |u_{max}|$. Once the MPC has computed its input $u_{MPC}(t)$ at time t, the component $r(t)$ added to time t is given by

$$r(t) = \begin{cases} r_{opt}(t) & \text{if } |u_{MPC}(t) + r_{opt}(t)| \leq u_{max}, \\ u_{max} - u_{MPC} & \text{if } u_{MPC}(t) + r_{opt}(t) > u_{max}, \\ -u_{max} - u_{MPC} & \text{if } u_{MPC}(t) + r_{opt}(t) < -u_{max}. \end{cases}$$

A similar approach can be used for the output. Enforcing this type of saturation decreases the information contents in the data. However, by extending the experiment time, one can compensate for this loss and ensure $P_\theta^{-1} \succeq R_{adm}$.

We now summarize some differences in stealth identification between linear and nonlinear controllers. Firstly, we recall that in the linear case an affine relation exists between Φ_r and P_θ^{-1} for any G_{id}, but in the nonlinear case only when $G_{id} = G_0$. This has the following implications.

For a linear, explicit and known controller, the novel least-costly framework can produce excitation signals that minimize the identification cost whilst honoring $P_\theta^{-1} \succeq R_{adm}$ for any G_{id} (in such cases, P_θ^{-1} will also be a function of G_{id}). The ability to find excitation signals in this way might prove beneficial for applications wherein the controller should not be disturbed (too much). This can, however, not be accomplished for known, explicit nonlinear controllers for $G_{id} \neq G_0$ as P_θ^{-1} is not affine in Φ_r.

Lastly, we consider the case when the (linear or nonlinear) controller is unknown or implicitly defined. In order to solve the optimization problem, one requires $G_{id} = G_0$, such that P_θ^{-1} becomes independent of the controller. In practice, we replace θ_0 by some estimate θ_{com} and assume $G_{id} = G_0$ in the optimization problem in order to solve it. The constraint $P_\theta^{-1} \succeq R_{adm}$ or the minimal identification cost could, however, not be met in this case. This should be investigated.

V. Numerical Study

In this section we test the stealth identification framework on a SISO system regulated by a model predictive controller. We verify whether the least-costly framework can indeed yield models that obey $P_\theta \succeq R_{adm}$ for $G_{id} = G_0$ as well as $G_{id} \neq G_0$.

A. Simulation set-up

We consider a data-generating system (1)-(2) embedded in a Box-Jenkins (BJ) model structure $\mathcal{M} = \{M(\theta), \theta \in \mathbb{R}^6\}$. The family of models $M(\theta)$ in this structure is given by $G(z,\theta) = \frac{\theta_1 z^{-1} + \theta_2 z^{-2}}{1 + \theta_5 z^{-1} + \theta_6 z^{-2}}$ and $H(z,\theta) = \frac{1 + \theta_3 z^{-1}}{1 + \theta_4 z^{-1}}$. A white-noise signal $e(t)$ with variance $\sigma_e^2 = 0.5$ is added through the filter $H(z,\theta_0)$, yielding $v = H(z,\theta_0)e(t)$. The noise realization $e(t)$ is fixed for all four experiments during a Monte Carlo step in order to compare results. At each new Monte Carlo step a new white-noise signal is randomly generated. The true system is defined as $S_0 = M(\theta_0)$, where $\theta_0 = [0.5,\ 0,\ 0,\ -0.6,\ -0.6,\ 0.8]^T$.

B. MPC algorithm

The data-generating system (1)-(2) is operated in closed loop with an MPC controller based on our commissioning model (G_{com}, H_{com}), which lies in the model structure $\mathcal{M}$, where $\theta_{com} = [0.15,\ 0.05,\ 0.92,\ 0.92,\ -0.30,\ 0.71]^T$. We denote this controller by $C(G_{com})$.

The MPC is tuned so that we get sufficient performance for the commissioning model (G_{com}, H_{com}). The MPC has a prediction horizon of $N_y = 40$ and a control horizon of $N_u = 20$. The output variable y has a weight of $Q = 1.0$, and the input a weight of $R = 0.1$. The constraint on the input u is $-3 \leq u \leq 3$, but are reduced to $-2.5 \leq u \leq 2.5$ to allow excitation. There are no constraints on the output.

C. Optimal Experiment Design parameters

As identification cost (6) we use

$$\mathcal{P}_{y_r} + \mathcal{P}_r = \frac{1}{2\pi} \int_{-\pi}^{\pi} (\Phi_{y_r} + \Phi_r)\, d\omega, \tag{16}$$

where $\mathcal{P}_{y_r}$ and $\mathcal{P}_r$ are the power of $y_r = G_0 r$ and of $r(t)$, respectively. The optimal cost is an output of the LMI algorithm. The matrix $R_{adm} = \frac{\gamma \mathcal{H}(\theta_0) \chi_\alpha^2(n)}{2}$, with $\alpha = 0.99$, $\gamma = 400$, and the Hessian $\mathcal{H}(\theta_0)$. We refer to [4], [5] for details on how to compute R_{adm}.

Following (6), we minimize the identification cost (16) subject to the constraint

$$P_\theta^{-1} = M_r + M_e \succeq R_{adm}, \tag{17}$$

The matrix M_e is estimated using (4) with $r(t) = 0$ during the entire simulation (in closed loop) with a length of $N = 10000$. Hence, the optimal excitation spectrum Φ_r is computed such that $M_r \succeq R_{adm} - M_e$.

D. Results

The optimal experiment design algorithm (6) for the identification cost (16) yields an excitation spectrum shown in Fig. 4. We computed all terms in (6) by substituting $\theta_0 = \theta_{com}$ since in reality, we don't know the true system beforehand.

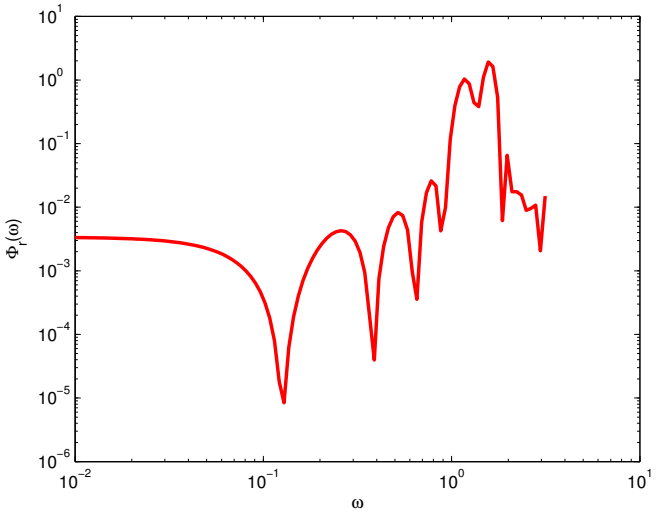

Fig. 4. Optimal excitation spectrum $\Phi_r(\omega)$ used in experiments 1 and 2.

Experiment	1	2	3	4
$10^4 \|\|\hat{\theta}_N - \theta_0\|\|^2$	2.45	8.28	4.15	2.95
$\lambda_{min}(P_\theta^{-1} - R_{adm})$	267	174	-3003	-3029
$10^3 \int(\Delta B)$	3.5	4.4	10.3	4.9

TABLE I
THE DIFFERENT MEASURES CALCULATED FOR FOUR TYPES OF EXPERIMENTS.

We compare different identified models with various measures, see Table I. The values in this table are averages over 250 Monte Carlo simulations. The first row in the table shows the average mean-square error of the identified parameter vector compared with the true vector θ_0. The second row shows the averaged minimal eigenvalue λ_{min} of the matrix $P_\theta^{-1} - R_{adm}$. When this value is negative, it means that the condition $P_\theta^{-1} \succeq R_{adm}$ is not met. The last row shows the integrated difference between the magnitude of the transfer function G_0 and the one obtained from the experiment, i.e., $20 \int_{-\pi}^{\pi} (\log|G(j\omega)| - \log|G_0(j\omega)|) d\omega$. This integral is indicated by $\int \Delta B$ in the table.

We now describe the experiments.

1) Experiment 1: The first identification was done with $G_{id} = G_0$. This is the situation in which the controller will not notice the excitation signal. The excitation spectrum is shown in Fig. 4. A good model is found that satisfies $P_\theta^{-1} \succeq R_{adm}$ with a low identification cost.

2) Experiment 2: The second experiment had $G_{id} = G_{com}$, i.e., the model available at commissioning was used. The excitation spectrum is equal to that of experiment 1. Again, a good model is found that satisfies $P_\theta^{-1} \succeq R_{adm}$ with a low identification cost.

3) Experiment 3: We use $G_{id} = G_{com}$ but now excite the system using a reference signal r having a white-noise spectrum with variance equal to the one of experiments 1 and 2. Note that a consistent model is found, but that the requirement $P_\theta^{-1} \succeq R_{adm}$ has not been met.

4) Experiment 4: Finally, we perform a white-noise excitation with a spectrum equal to that of experiment 3, but take $G_{id} = 0$. Again, we find a consistent model but we do not meet the accuracy requirements for the parameter vector that was imposed as a design constraint.

From the table, we conclude that the least-costly signals deliver precise models of the true system satisfying $P_\theta^{-1} \succeq R_{adm}$. The converse holds for the white-noise experiments. These models deliver a consistent parameter, but do not satisfy the model accuracy requirements.

VI. CONCLUSIONS

We have presented a novel experiment design method that is applicable to nonlinearly-controlled LTI systems. It generalizes current methods which are only applicable to LTI systems regulated by known, explicit linear controllers. This is achieved by modifying the feedback mechanism during the identification experiment in such a way that the (possibly unknown, implicit or nonlinear) controller does not see the excitation signal. We prove that this new identification scheme yields consistent estimates of the true system.

Our numerical results show that least-costly identification for nonlinear controllers result in identified models that obey user-defined criteria for parameter certainty (even when $G_{id} \neq G_0$). However, our method currently doesn't support an attractive way to deal with input constraints. This raises the question as to how one can compensate for the information loss that occurs when signals saturate (cf. Section IV)?

More theoretical and numerical analysis is required. From a theoretical point of view, a natural question to ask is: What conditions on $G_{id} \neq G_0$ are required such that we still minimize the identification error, and honor the constraint $P_\theta^{-1} \succeq R_{adm}$? Our findings seem to suggest that this is possible for G_{id} not very far from G_0.

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