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An instrumental variable method for closed-loop identification of coreless linear motors

Tuan T. Nguyen, Mircea Lazar, Hans Butler and Paul M.J. Van den Hof

Abstract— **This paper presents an instrumental variable (IV) method dedicated to identification of coreless linear motors (CLMs) operating in closed-loop. The dynamics of a CLM can be described as a linear dynamical system preceded by a static input gain that is nonlinearly dependent on the output. The nonlinear dependency on the output makes it challenging to find an appropriate predictor for identification. In this paper, we introduce a linear-in-the-parameter predictor for the CLM dynamics, which is a modification of the nonlinear autoregressive exogenous (NARX) model. It is proven that the IV method using the introduced predictor results in a consistent estimate. In addition, we show that in many applications, the simple NARX predictor, which does not require knowledge of the statistical properties of the output measurement noise, can provide an estimate that is very close to the true parameter. A numerical example is shown for demonstration.**

I. INTRODUCTION

Coreless linear motors (CLMs), also known as ironless linear motors, are widely used in industrial positioning systems. For high-precision control of CLMs, an accurate model is crucial. In many practical applications, first principle modeling is not accurate enough for high performance due to uncertainty of the physical parameters. Therefore, identification of the motor's model is of interest. The identification experiments must be performed in closed-loop for safety reasons.

In the literature, identification of linear motors is typically formulated as identification of the model of the force ripple, which basically is the sum of all force components other than the nominal force. The force ripple model is usually written as a sum of basis functions where the coefficients are to be estimated by fitting the model to experimental data. Some of the research works only consider position-dependent and velocity-dependent force ripple [1]–[3]. The current-dependent force ripple has been considered in [4]–[6]. However, the contribution of the current in each coil to the force ripple was not addressed. In [7], [8], a method to identify the force function of each coil was proposed, but was limited to linear motors with one set of three-phase coils. Furthermore, in the above-mentioned works, the effect of the output measurement noise on the parameters estimation is neglected.

In this paper, we aim to identify the force function of each coil separately, with no limit on the number of coils. It is assumed that the true model of the system is a nonlinear output-error (NOE) model with zero-mean white noise on the output measurement. We model a CLM as a multiple input-single output (MISO) nonlinear dynamical systems with electrical currents as inputs and the position of the translator as output. The model consists of a linear dynamical system preceded by a static input gain vector that is nonlinearly dependent on the output. The linear dynamical system captures the motion dynamics of the translator, while the static input gain vector contains the position-dependent force functions of the coils, which describe the relation between the currents in the coils and the resulting forces. It should be noted that this model structure is different from the well known Hammerstein model structure. The static nonlinear part of this model structure is nonlinear in the noise-free output, while the static nonlinear part of the Hammerstein model structure is nonlinear in the input [9].

The nonlinear dependency on the unknown noise-free output makes it difficult to find an appropriate predictor for identification. The simple nonlinear autoregressive exogenous (NARX) model structure suffers from unrealistic noise assumptions, which leads to a biased estimate in the presence of output measurement noise [10]. On the other hand, the more realistic NOE model structure can provide a consistent estimate, but it is nonlinear in the parameters and requires the global solution of a nonconvex optimization problem, which is usually difficult to find. In this paper, we employ the instrumental variable (IV) method and we introduce a new linear-in-the-parameter predictor, which is a modification of the NARX model. It is proven that the IV method using the introduced predictor provides a consistent estimate. The method only requires the analytical solution of a simple generalized linear least square problem. In addition, we provide an analysis of the bias obtained by using the NARX model. It is shown that in many applications where the output measurement noise is small compared to the magnet pole pitch of the CLM, the simple NARX predictor, which does not require knowledge of the statistical properties of the output measurement noise, can provide an estimate that is very close to the true parameter. A numerical example is presented for demonstration.

Regarding practical implementation, the new identification method is easy to implement compared to other identification methods for linear motors, e.g. [5], [7], [8]. It is able to identify the force functions of all the coils together with the motion dynamics of the translator from a single experiment. In addition, there is no need to apply a constant load on the motor.

The paper is organized as follows. Section II describes the identification problem. The IV framework is reviewed in

The authors are with the Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands. E-mails: {t.t.nguyen,m.lazar,h.butler,p.m.j.vandenhof}@tue.nl

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Section III. Section IV presents the bias-corrected predictor. A numerical example is shown in Section V for demonstration. The conclusions are summarized in Section VI.

II. PROBLEM DESCRIPTION

A. Coreless linear motor

Fig. 1 shows a cross-sectional view of a CLM. A CLM contains a stationary part called the stator and a moving part called the translator. The stator consists of two permanent magnet arrays mounted on two iron plates. The translator contains one or multiple sets of three-phase coils placed in between the two magnet arrays. Each set of three-phase coils consists of three electrical coils A, B and C connected in star configuration.

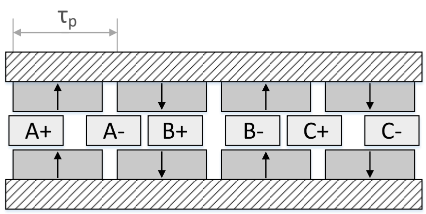

Fig. 1. Cross-sectional view of a CLM.

A CLM can be modeled as two separated parts, the electromagnetic part and the motion part, as shown in Fig. 2. This is a common approach in modeling of linear and planar electrical motors [11]–[13]. The electromagnetic part describes the relation between the force produced by the motor and the currents in the coils. It is modeled as a static gain $\Psi(x, \theta_0)$ which is nonlinearly dependent on the position of the translator. The motion part $G(q, \theta_0)$ is a linear time-invariant system that captures the motion dynamics of the translator. Here, q^{-1} is the delay operator with $q^{-i}x(t) = x(t-i)$, $\theta \in \mathbb{R}^{n_\theta}$ denotes the parameter vector and $\theta_0 \in \mathbb{R}^{n_\theta}$ denotes the true parameter vector. More details on the adopted model of a CLM can be found in [13].

B. Control loop

The standard control loop of a CLM is shown in Fig. 2. The controller consists of two parts: a feedback linearization block $\Psi^{-1}(\hat{x}, \hat{\theta})$, which aims to invert the nonlinearity, and a linear controller $C(q)$ which controls the linear dynamics.

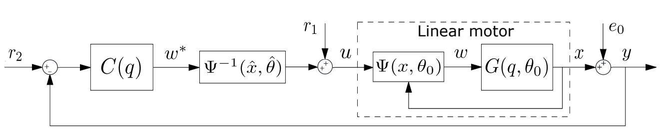

Fig. 2. A CLM control loop.

The description of the signals is listed below:

- $u(t) \in \mathbb{R}^{n_u}$: the input of the system, which is the vector of the currents in the coils. The number of independent coils in the translator is n_u.
- $x(t) \in \mathbb{R}$: the noise-free output of the system, which is the position of the translator.
- $y(t) \in \mathbb{R}$: the noise-corrupted output, which is the measurement obtained from the encoder.
- $e_0(t) \in \mathbb{R}$: the output measurement noise. It is assumed that $e_0(t)$ is a zero-mean white noise with a symmetric probability distribution.
- $w(t) \in \mathbb{R}$: an unmeasurable internal signal, which is the motor force.
- $r_2(t) \in \mathbb{R}$: the output reference.
- $r_1(t) \in \mathbb{R}^{n_u}$: the additional input excitation signal.

The input signal u and the output measurement y are known. The signals w and x cannot be measured.

The controller $C(q)$ is chosen such that there is no direct feedthrough from $y(t)$ to $u(t)$, which implies that $u(t)$ and $e(t)$ are uncorrelated. It follows that $x(t)$ and $e(t)$ are also uncorrelated.

C. System description

The data generating system is a NOE model of the following form

$$\mathcal{S} : \begin{cases} x(t) = G(q, \theta_0) w(t), \\ w(t) = \Psi(x(t), \theta_0) u(t), \\ y(t) = x(t) + e_0(t), \\ u(t) = r_1(t) + \Psi^{-1}(\hat{x}(t), \hat{\theta}) C(q)(r_2(t) - y(t)). \end{cases} \tag{1}$$

The linear dynamics is parameterized as follows

$$G(q, \theta) = \frac{B(q, \theta)}{A(q, \theta)} = \frac{\sum_{k=1}^{n_b} b_k q^{-k}}{1 + \sum_{j=1}^{n_a} a_j q^{-j}}. \tag{2}$$

The noise-free output can then be written as

$$x(t) = -\sum_{j=1}^{n_a} a_j^0 x(t-j) + \sum_{k=1}^{n_b} b_k^0 w(t-k). \tag{3}$$

The position-dependent static gain $\Psi(x, \theta)$ is a row vector of the force functions

$$\Psi(x, \theta) = \begin{bmatrix} \Psi_1(x, \theta) & \dots & \Psi_{n_u}(x, \theta) \end{bmatrix}. \tag{4}$$

Ideally, the force function of a coil is a perfect sinusoidal function of the position x. However, in reality, there are also other harmonic components. Therefore, it is common to model the force function $\Psi_l(x, \theta)$ as a Fourier series [4], [5], [7]

$$\Psi_l(x, \theta) = \sum_{n=1}^{n_F} \left(c_{l,n} \cos(\omega_n x) + d_{l,n} \sin(\omega_n x) \right). \tag{5}$$

Here, we denote $\omega_n = n\omega_1$, where ω_1 is the fundamental frequency of the Fourier series, n_F is the number of Fourier harmonics, c and d are the Fourier coefficients. The force $w(t)$ can be written as

$$\begin{aligned} w(t) = & \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} c_{l,n}^0 \cos(\omega_n x(t)) u_l(t) \\ & + \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} d_{l,n}^0 \sin(\omega_n x(t)) u_l(t). \end{aligned} \tag{6}$$

For brevity, let us denote

$$\mathfrak{c}_{l,n}^x(t) = \cos(\omega_n x(t)) u_l(t), \ \mathfrak{s}_{l,n}^x(t) = \sin(\omega_n x(t)) u_l(t),$$

and similarly

$$\mathfrak{c}^y_{l,n}(t) = \cos(\omega_n y(t))u_l(t),\ \ \mathfrak{s}^y_{l,n}(t) = \sin(\omega_n y(t))u_l(t).$$

Substituting (3) and (6) into the system model (1), the input-output relation of the system can be written in the following form

$$\begin{aligned} y(t) =& -\sum_{j=1}^{n_a} a^0_j x(t-j) + \sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c^0_{l,n}\mathfrak{c}^x_{l,n}(t-k) \\ &+ \sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d^0_{l,n}\mathfrak{s}^x_{l,n}(t-k) + e_0(t). \end{aligned} \tag{7}$$

D. Proposed approach

The model structure (1) is different from the well known Hammerstein model in that its static part is nonlinearly dependent on the output instead of the inputs. The identification methods available for Hammerstein systems are therefore not applicable. The nonlinear dependency of the model structure (1) on the unknown noise-free output makes it challenging to find an appropriate predictor for identification.

Using the simple NARX model structure, the noise-corrupted output enters the predictor nonlinearly, which can result in a biased estimate. On the other hand, the more realistic NOE model is nonlinear in the parameters and thus requires solving a nonconvex optimization problem, which is generally non-tractable.

In this paper, we employ the IV method, together with a new linear-in-the-parameter predictor, which is a modification of the NARX model. We will prove that the IV method together with the new predictor provide a consistent estimate of the parameters. The method only requires the analytical solution of a generalized linear least square problem, which is simple to compute.

III. INSTRUMENTAL VARIABLE METHOD FOR CLOSED-LOOP IDENTIFICATION

A. The IV framework

Consider a linear-in-the-parameter predictor of the form

$$\hat{y}(t,\theta) = \varphi^\top(t)\theta, \tag{8}$$

where $\theta \in \mathbb{R}^{n_\theta}$ is the parameter vector, $\varphi(t) \in \mathbb{R}^{n_\theta}$ is the regressor vector and $\hat{y}(t,\theta) \in \mathbb{R}$ is the predicted output. It should be noted that the NOE model (7) is nonlinear-in-the-parameter and thus cannot be written in the linear form (8). Therefore, we have to employ other predictor models which are linear in the parameters as described in Section IV.

The IV estimate is the generalized version of the least square estimate and is given by [14]

$$\hat{\theta}_{\mathrm{IV}} = \left(\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\varphi(t)^\top\right)^{-1}\left(\frac{1}{N}\sum_{t=1}^{N}\zeta(t)y(t)\right), \tag{9}$$

where $\zeta(t) \in \mathbb{R}^{n_\theta}$ is the instrumental vector. The selection of the instrumental vector $\zeta(t)$ is discussed in Section III-B. It can be seen that the IV estimate is the analytical solution of a generalized linear least square problem and is therefore attractive from computational perspective.

The IV estimate is consistent, i.e. $\hat{\theta}_{\mathrm{IV}} \to \theta_0$ with probability 1 when $N \to \infty$, if the following two conditions are satisfied [14]

$$\bar{\mathbb{E}}[\zeta(t)\varphi(t)^\top] \text{ is nonsingular}, \tag{10}$$

$$\bar{\mathbb{E}}[\zeta(t)(y(t) - \hat{y}(t,\theta_0))] = 0. \tag{11}$$

Here, the notation $\bar{\mathbb{E}}[\cdot] = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\mathbb{E}[\cdot]$ is adopted from the prediction error framework [15]. Condition (10) is satisfied if the system is sufficiently excited and $\zeta(t)$ is well correlated with $\varphi(t)$. To satisfy condition (11), $\zeta(t)$ must be uncorrelated with the measurement noise and the predictor should be selected appropriately. The selection of the predictor will be discussed in Section IV.

B. Instrumental vector selection

The instrumental vector $\zeta(t)$ should be selected such that it is uncorrelated with the output measurement noise $e_0(t)$ and well correlated with $\varphi(t)$. In this paper, we choose the instrumental vector as the noise-free version of $\varphi(t)$, obtained by simulating the noise-free model

$$\mathring{\mathcal{S}} : \begin{cases} \mathring{y}(t) = G(q,\hat{\theta})\mathring{w}(t), \\ \mathring{w}(t) = \Psi(\mathring{x}(t),\hat{\theta})\mathring{u}(t), \\ \mathring{u}(t) = r_1(t) + \Psi^{-1}(\mathring{x}(t),\hat{\theta})C(q)(r_2(t) - \mathring{y}(t)). \end{cases} \tag{12}$$

Here, $\hat{\theta}$ is an estimate of θ_0, which can be obtained by first principle modeling using nominal physical parameters provided by the manufacturer. The signal $r_1(t)$ and $r_2(t)$ here are the same as those used in the real experiment, in order to make $\zeta(t)$ and $\varphi(t)$ well correlated. It is obvious that $\zeta(t)$ is uncorrelated with $e_0(t)$.

IV. PREDICTOR MODELS

In this section we discuss the selection of the predictor for the IV method. This is a challenging problem due to the nonlinear dependency of the system on the unknown noise-free output. We will show that the simple NARX model would result in a biased estimate, although the bias is negligible in many applications. A bias-corrected predictor is then introduced and consistency of the resulting IV estimate is proven.

A. NARX predictor

Let us consider the NARX predictor

$$\begin{aligned} \hat{y}_{\mathrm{NARX}}(t,\theta) =& -\sum_{j=1}^{n_a} a_j y(t-j) \\ &+ \sum_{k=1}^{n_b} b_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c_{l,n}\mathfrak{c}^y_{l,n}(t-k) \\ &+ \sum_{k=1}^{n_b} b_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d_{l,n}\mathfrak{s}^y_{l,n}(t-k) \\ =& \varphi^\top_{\mathrm{NARX}}(t)\theta, \end{aligned} \tag{13}$$

where θ is the parameter vector

$$\begin{aligned} \theta = [a_1 \ \ \dots \ \ a_{n_a} \ \ b_1 c_{1,1} \ \ \dots \ \ b_{n_b} c_{n_u,n_F} \\ b_1 d_{1,1} \ \ \dots \ \ b_{n_b} d_{n_u,n_F}]^\top, \end{aligned} \tag{14}$$

and $\varphi_{\mathrm{NARX}}(t)$ is the regressor vector

$$\begin{aligned}\varphi_{\mathrm{NARX}}(t) = [&-y(t-1) \quad \dots \quad -y(t-n_a)\\ &\mathfrak{c}^y_{1,1}(t-1) \quad \dots \quad \mathfrak{c}^y_{n_u,n_F}(t-n_b)\\ &\mathfrak{s}^y_{1,1}(t-1) \quad \dots \quad \mathfrak{s}^y_{n_u,n_F}(t-n_b)]^\top. \end{aligned} \quad (15)$$

We have $n_\theta = n_a + 2n_b n_u n_F$.

Note that the NARX model (13) is written in a linear-in-the-parameter form by using the overparameterization technique, which transforms a bilinear-in-the-parameter model to a linear-in-the-parameter model by replacing every crossproduct of parameters with new independent parameters [16]. When the new parameters have been identified, the original parameters can be obtained by performing singular value decomposition as explained in [16].

In addition, it is important to note that this parameterization is not unique. Let us define the parameter vectors $b = [b_1 \ \dots \ b_{n_b}]^\top$, $c = [c_1 \ \dots \ c_{n_F}]^\top$ and $d = [d_1 \ \dots \ d_{n_F}]^\top$. Then any set of parameter vectors $\tilde{b} = \beta b$, $\tilde{c} = \beta^{-1}c$ and $\tilde{d} = \beta^{-1}d$ provides identical input-output relation as the one in (13). To have a unique parameterization, a common approach is to fix the first element of b to a constant.

In what follows we will analyze the consistency of the IV estimate obtained by using the NARX predictor. For that purpose we will calculate the term in condition (11). By substituting $y(t) = x(t) + e_0(t)$ and using the following trigonometric identities

$$\begin{aligned}\cos(\chi_1+\chi_2)&=\cos(\chi_1)\cos(\chi_2)-\sin(\chi_1)\sin(\chi_2),\\ \sin(\chi_1+\chi_2)&=\sin(\chi_1)\cos(\chi_2)+\cos(\chi_1)\sin(\chi_2),\end{aligned}$$

we can rewrite $\mathfrak{c}^y_{l,n}(t-k)$ and $\mathfrak{c}^y_{l,n}(t-k)$ as follows

$$\begin{aligned}\mathfrak{c}^y_{l,n}(t-k)=&\mathfrak{c}^x_{l,n}(t-k)\cos(\omega_n e_0(t-k))\\ &-\mathfrak{s}^x_{l,n}(t-k)\sin(\omega_n e_0(t-k)), \end{aligned}\quad (16)$$

$$\begin{aligned}\mathfrak{s}^y_{l,n}(t-k)=&\mathfrak{s}^x_{l,n}(t-k)\cos(\omega_n e_0(t-k))\\ &+\mathfrak{c}^x_{l,n}(t-k)\sin(\omega_n e_0(t-k)). \end{aligned}\quad (17)$$

Consequently, subtracting $\hat{y}_{\mathrm{NARX}}(t,\theta_0)$ from $y(t)$ results in

$$\begin{aligned}&y(t)-\hat{y}_{\mathrm{NARX}}(t,\theta_0)\\ =&\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c^0_{l,n}\mathfrak{c}^x_{l,n}(t-k)\left[1-\cos(\omega_n e_0(t-k))\right]\\ &+\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d^0_{l,n}\mathfrak{s}^x_{l,n}(t-k)\left[1-\cos(\omega_n e_0(t-k))\right]\\ &+\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c^0_{l,n}\mathfrak{s}^x_{l,n}(t-k)\sin(\omega_n e_0(t-k))\\ &-\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d^0_{l,n}\mathfrak{c}^x_{l,n}(t-k)\sin(\omega_n e_0(t-k))\\ &+\sum_{j=1}^{n_a} a^0_j e_0(t-j)+e_0(t). \end{aligned}\quad (18)$$

In order to analyze the term in condition (11), let us calculate the expected values of $\cos(\omega_n e_0(t-k))$ and $\sin(\omega_n e_0(t-k))$. The characteristic function of $e_0(t)$ is defined as

$$\phi_{e_0}(\alpha) = \mathbb{E}[e^{i\alpha e_0(t)}], \text{ where } \alpha \in \mathbb{R}. \quad (19)$$

We have the following proposition.

Proposition IV.1 *If $e_0(t)$ is a zero-mean white noise with a symmetric probability distribution then*

$$\mathbb{E}[\cos(\omega_n e_0(t))] = \phi_{e_0}(\omega_n), \ \ \mathbb{E}[\sin(\omega_n e_0(t))] = 0. \quad (20)$$

Proof: Using Euler's formula we have

$$\phi_{e_0}(\omega_n) = \mathbb{E}[\cos(\omega_n e_0(t))] + i\mathbb{E}[\sin(\omega_n e_0(t))]. \quad (21)$$

Since $e_0(t)$ is a zero-mean white noise with a symmetric probability distribution, the characteristic function $\phi_{e_0}(\omega_n)$ is real-valued [17]. By equating the real part and the imaginary part of the two sides of equation (21), the proposition is proven. ∎

Now let us substitute (18) into the term in condition (11). We note that $e_0(t-k)$ is uncorrelated with $x(t-k)$, $u(t-k)$ and $\zeta(t)$. As a result, using the fact that

$$\mathbb{E}[\chi_1\chi_2] = \mathbb{E}[\chi_1]\mathbb{E}[\chi_2] \quad (22)$$

if χ_1 and χ_2 are independent variables, and using the result of Proposition IV.1, it follows that

$$\begin{aligned}&\bar{\mathbb{E}}[\zeta(t)(y(t)-\hat{y}_{\mathrm{NARX}}(t,\theta_0))]\\ =&\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c^0_{l,n}\left(\bar{\mathbb{E}}\left[\zeta(t)\mathfrak{c}^x_{l,n}(t-k)\right](1-\phi_{e_0}(\omega_n))\right)\\ &+\sum_{k=1}^{n_b} b^0_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d^0_{l,n}\left(\bar{\mathbb{E}}\left[\zeta(t)\mathfrak{s}^x_{l,n}(t-k)\right](1-\phi_{e_0}(\omega_n))\right)\\ \neq& 0. \end{aligned}\quad (23)$$

Therefore, condition (11) is not satisfied. We conclude that the IV method using the NARX predictor results in a biased estimate.

B. Bias-corrected predictor

In this section we introduce a simple bias-correction for the NARX model. Let us define the bias-correction factors

$$\rho_n = \frac{1}{\mathbb{E}[\cos(\omega_n e_0(t))]} = \frac{1}{\phi_{e_0}(\omega_n)}, \quad n = 1,\dots,n_F. \quad (24)$$

Assume that $\phi_{e_0}(\omega_n)$ is known, we propose the following bias-corrected predictor

$$\begin{aligned}\hat{y}_{\mathrm{bc}}(t,\theta)=&-\sum_{j=1}^{n_a} a_j y(t-j)\\ &+\sum_{k=1}^{n_b} b_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} c_{l,n}\mathfrak{c}^y_{l,n}(t-k)\rho_n\\ &+\sum_{k=1}^{n_b} b_k \sum_{l=1}^{n_u}\sum_{n=1}^{n_F} d_{l,n}\mathfrak{s}^y_{l,n}(t-k)\rho_n\\ =&\varphi^\top_{\mathrm{bc}}(t)\theta, \end{aligned}\quad (25)$$

where

$$\begin{aligned}\varphi_{\mathrm{bc}}(t) = [&-y(t-1) \quad \dots \quad -y(t-n_a)\\ &\mathfrak{c}^y_{1,1}(t-1)\rho_1 \quad \dots \quad \mathfrak{c}^y_{n_u,n_F}(t-n_b)\rho_{n_F}\\ &\mathfrak{s}^y_{1,1}(t-1)\rho_1 \quad \dots \quad \mathfrak{s}^y_{n_u,n_F}(t-n_b)\rho_{n_F}]^\top. \end{aligned}\quad (26)$$

It can be seen that the proposed bias-corrected predictor preserves the linear-in-the-parameter property. We will now show that the IV method using the proposed predictor results in a consistent estimate.

Theorem IV.2 *Given that the condition* (10) *is satisfied, by using the predictor* (25) *and an instrumental vector which is uncorrelated with the output measurement noise, the IV estimate* (9) *is consistent.*

Proof: Let us consider condition (11). Due to (16) and (17), subtracting $\hat{y}_{\mathrm{bc}}(t,\theta_0)$ from $y(t)$ results in

$$\begin{aligned} & y(t) - \hat{y}_{\mathrm{bc}}(t,\theta_0) \\ =& \sum_{k=1}^{n_b} b_k^0 \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} c_{l,n}^0 \mathfrak{c}_{l,n}^x(t-k)\left[1-\cos(\omega_n e_0(t-k))\rho_n\right] \\ &+ \sum_{k=1}^{n_b} b_k^0 \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} d_{l,n}^0 \mathfrak{s}_{l,n}^x(t-k)\left[1-\cos(\omega_n e_0(t-k))\rho_n\right] \\ &+ \sum_{k=1}^{n_b} b_k^0 \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} c_{l,n}^0 \mathfrak{s}_{l,n}^x(t-k)\sin(\omega_n e_0(t-k))\rho_n \\ &- \sum_{k=1}^{n_b} b_k^0 \sum_{l=1}^{n_u} \sum_{n=1}^{n_F} d_{l,n}^0 \mathfrak{c}_{l,n}^x(t-k)\sin(\omega_n e_0(t-k))\rho_n \\ &+ \sum_{j=1}^{n_a} a_j^0 e_0(t-j) + e_0(t). \end{aligned} \tag{27}$$

Note that $e_0(t-k)$ is uncorrelated with $x(t-k)$, $u(t-k)$ and $\zeta(t)$. Substituting (27) into the term in condition (11) and using (22) and Proposition IV.1 we have

$$\bar{\mathbb{E}}[\zeta(t)(y(t)-\hat{y}_{\mathrm{bc}}(t,\theta_0))] = 0. \tag{28}$$

Therefore, condition (11) is satisfied. Given that condition (10) is also satisfied, the IV estimate using the bias-corrected predictor (25) is consistent. ■

Note that the bias-corrected predictor requires the knowledge of the probability distribution function of the output measurement noise. For example, the two most common probability distribution functions of the measurement noise are the normal (or Gaussian) distribution and the uniform distribution.

- If the measurement noise is normally distributed with zero mean and variance σ^2, which is very common in practice, then the bias-correction factors are
$$\rho_n^{\text{normal}} = \frac{1}{\phi_{e_0}^{\text{normal}}(\omega_n)} = e^{\frac{\omega_n^2\sigma^2}{2}}. \tag{29}$$
- If the measurement noise is uniformly distributed on the interval $[-\eta,\eta]$ then the bias-correction factors are
$$\rho_n^{\text{uniform}} = \frac{1}{\phi_{e_0}^{\text{uniform}}(\omega_n)} = \frac{\omega_n\eta}{\sin(\omega_n\eta)}. \tag{30}$$

However, it will be shown in Section IV-D that in many applications, it is possible to obtain an estimate that is very close to the true parameter just by using the simple NARX predictor, which does not require the knowledge of the probability distribution function of the output measurement noise.

C. Relation between the NARX IV estimate and the bias-corrected estimate

Comparing the NARX regressor vector $\varphi_{\mathrm{NARX}}(t)$ in (15) and the bias-corrected regressor vector $\varphi_{\mathrm{bc}}(t)$ in (26) we have

$$\varphi_{\mathrm{bc}}(t) = \Omega\varphi_{\mathrm{NARX}}(t), \tag{31}$$

where $\Omega \in \mathbb{R}^{n_\theta \times n_\theta}$ is the diagonal bias-correction matrix

$$\Omega = \mathrm{diag}(1,\ldots,1,\rho_1,\ldots,\rho_{n_F},\rho_1,\ldots,\rho_{n_F}). \tag{32}$$

From (9) and (31), it follows that

$$\hat{\theta}_{\mathrm{IV}}^{\mathrm{bc}} = \Omega^{-1}\hat{\theta}_{\mathrm{IV}}^{\mathrm{NARX}}. \tag{33}$$

Therefore, the bias-corrected IV estimate can be obtained simply by multiplying the NARX IV estimate by Ω^{-1}.

D. Analysis of the bias in NARX IV

Let us calculate the bias of the IV estimate obtained using the NARX predictor. From Theorem IV.2, we know that $\hat{\theta}_{\mathrm{IV}}^{\mathrm{bc}} \to \theta_0$ with probability 1 as $N \to \infty$. Consequently, it follows from (33) that

$$\hat{\theta}_{\mathrm{IV}}^{\mathrm{NARX}} \to \Omega\theta_0 \text{ with probability 1 as } N \to \infty. \tag{34}$$

Therefore, as $N \to \infty$, the bias is

$$\hat{\theta}_{\mathrm{IV}}^{\mathrm{NARX}} - \theta_0 \to (\Omega - I)\theta_0 = \Gamma\theta_0, \tag{35}$$

where I is the identity matrix and

$$\Gamma = \mathrm{diag}(0,\ldots,0,\rho_1-1,\ldots,\rho_{n_F}-1,\rho_1-1,\ldots,\rho_{n_F}-1).$$

It is observed from (29) and (30) that if the variance of the output measurement noise is small compared to the Fourier period then the factors ρ_n are very close to 1. For example, if the measurement noise is white Gaussian noise with variance $\sigma = 0.01\tau_p$, where τ_p is the magnet pole pitch of the CLM, and the Fourier period is $2\tau_p$, then $\rho_1^{\text{normal}} - 1 = 5 \times 10^{-4}$. Consequently, the bias is negligible and we can thus safely use the NARX model as the predictor model. The knowledge of the probability distribution of the measurement noise is therefore not required in this case.

V. NUMERICAL EXAMPLE

In this section, a numerical example is presented to verify the performance of the proposed IV method. Assume that we have a CLM with three-phase coils A, B, C as shown in Fig. 1. The coils are connected in star configuration, which implies that the sum of the three currents is zero. Therefore, we actually only have two control inputs u_A and u_B.

In the ideal case, the force function $\Psi_A(x)$ and $\Psi_B(x)$ only contains the first order harmonics $\omega_1 = \frac{\pi}{\tau_p}$, where $\tau_p = 0.04$m is the magnet pole pitch. In reality, however, there are also other harmonic components due to manufacturing tolerances. In this example, it is assumed that there is a higher-order harmonic component $\omega_2 = 2\omega_1$. The force produced by the motor can be written as

$$w = \begin{bmatrix} \sum_{n=1}^{2}\left(c_{A,n}\cos(\omega_n x) + d_{A,n}\sin(\omega_n x)\right) \\ \sum_{n=1}^{2}\left(c_{B,n}\cos(\omega_n x) + d_{B,n}\sin(\omega_n x)\right) \end{bmatrix}^\top \begin{bmatrix} u_A \\ u_B \end{bmatrix}.$$

The linear dynamical part is a mass-damper system which has the following discrete-time transfer function

$$G(q,\theta)=\frac{b_1q^{-1}+b_2q^{-2}}{1+a_1q^{-1}+a_2q^{-2}}. \tag{36}$$

The sampling frequency of the system is $F_s = 10\text{kHz}$. We fix $b_1 = 1\times10^{-7}$. The reference signal $r_2(t)$ is generated as a consecutive sequence of random forth-order motion profiles in the range $[0\text{m}, 0.08\text{m}]$. The input excitation signal is chosen as $r_1(t)=\sum_{n=1}^{100} p_n \sin(t+\psi_n)$, where p_n and ψ_n are random amplitudes and phase shifts. The output measurement noise is a zero-mean white Gaussian noise with standard deviation $\sigma = 5\times10^{-6}\text{m}$. The signal to noise ratio is 84dB.

The system parameters are estimated from closed-loop data of length $N = 2\times10^6$. A Monte-Carlo simulation of 150 runs is performed. The results are summarized in Table I. It is observed that there is no significant difference between the NARX IV method and the bias-corrected IV method, as the noise variance is small. Both methods give unbiased estimates. However, the standard deviations of the Fourier coefficients c and d are quite large. How to improve the statistical efficiency of the method needs further research.

TABLE I

MEAN AND STANDARD DEVIATION OF 150 ESTIMATED MODELS

Parameter	*True value*	*NARX IV*	*Bias-corrected IV*
a_1	-1.9950	-1.9950 ± 0.0001	-1.9950 ± 0.0001
a_2	0.9950	0.9950 ± 0.0001	0.9950 ± 0.0001
$\bar{b}_2$	0.9983	1.0230 ± 0.1881	1.0230 ± 0.1881
$c_{A,1}$	0	-0.0032 ± 0.2716	-0.0032 ± 0.2716
$c_{A,2}$	-0.6988	-0.6965 ± 0.1340	-0.6965 ± 0.1340
$c_{B,1}$	-9.0781	-9.0421 ± 0.8498	-9.0421 ± 0.8498
$c_{B,2}$	-0.2745	-0.2767 ± 0.1126	-0.2767 ± 0.1126
$d_{A,1}$	7.8619	7.8475 ± 0.7323	7.8475 ± 0.7323
$d_{A,2}$	-0.3694	-0.3503 ± 0.1082	-0.3503 ± 0.1082
$d_{B,1}$	-4.5391	-4.5189 ± 0.5322	-4.5189 ± 0.5322
$d_{B,2}$	0.4592	0.4598 ± 0.1288	0.4598 ± 0.1288

Note: $\bar{b}_2 = b_2\times10^7$.

Fig. 3 shows the force produced by the true system model and the averaged estimated model when three-phase sinusoidal current waveforms are applied to the coils:

$$u_A = i_p\cos\left(\omega_1 x+\frac{2\pi}{3}\right),\ u_B = i_p\cos(\omega_1 x), \tag{37}$$

where $i_p = 12.82\text{A}$ is the amplitude of the current. The maximum force error between the true system and the averaged estimated model is about 0.46%.

The normal linear least square method is also tested but the resulting estimate is very far from the true parameter and therefore is not shown here.

VI. CONCLUSIONS

This paper presented an IV method for closed-loop identification of CLMs. We introduced a linear-in-the-parameter predictor, which is a modification of the NARX model. It was proven that the IV method together with the introduced

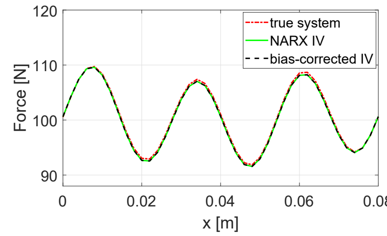

Fig. 3. Force produced by the system and the averaged estimated model.

predictor provide a consistent estimate. Besides, we analyzed the bias of the NARX IV method and showed that in many applications, the simple NARX predictor can provide an estimate that is very close to the true parameter. A numerical example was shown for demonstration.

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