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A Unified Experiment Design Framework for Detection and Identification in Closed-loop Performance Diagnosis

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Abstract— **This paper presents a least-costly experiment design framework for closed-loop performance diagnosis using prediction error identification. The performance diagnosis methodology consists in verifying whether an identified model of the true system lies in a performance-related region of interest. The experiment design framework minimizes the overall excitation cost incurred for detecting the cause of the performance drop and re-identifying the system dynamics when the degraded performance is due to control-relevant system changes. The optimal design of excitation signals is performed for a desired detection rate and a pre-specified level of accuracy required for the re-identified model.**

I. INTRODUCTION

Performance monitoring and diagnosis is a crucial step in maintenance of control systems. In real practice, most systems are inherently time-variant as the system and/or disturbance dynamics can change over time. The changes may result from new operating conditions that differ from those at the commissioning stage of the controller, revamp or malfunctioning of system components, variations in disturbance characteristics of system inputs and outputs, etc. These changes often lead to performance degradation of the existing controller, which performed satisfactorily at commissioning, unless the closed-loop performance of the control system is regularly monitored and updated.

A key issue in maintenance of controllers is how to distinguish system changes from variations in disturbance dynamics and amplitudes. In the event of an observed performance drop, diagnostic tools should allow us to verify whether the unsatisfactory closed-loop operation is due to changes in system dynamics or changes in disturbance characteristics. Over the last three decades, research on the detection of abrupt system changes has received increasing attention in the control community with the aim to address issues such as fault diagnosis, data segmentation, gain updating for adaptive algorithms, etc; see, e.g., [1], [5], and [7]. In [9], we presented a closed-loop performance diagnosis methodology based on prediction error identification. The methodology can be used when a performance drop is observed. Hypothesis testing is exploited to distinguish control-relevant system changes in closed-loop operation from variations in disturbance dynamics. The hypothesis test consists in verifying whether an identified model $G(z, \hat{\theta}_N)$ of the system $G_0(z)$ lies in a set $\mathcal{D}_{adm}$. We define $\mathcal{D}_{adm}$ as the set of all systems resulting in a satisfactory closed-loop performance under the original disturbance levels. It is self-evident that when $G(z, \hat{\theta}_N)$ lies in the set $\mathcal{D}_{adm}$ the performance drop is due to variations in disturbance dynamics, whereas in the case of $G(z, \hat{\theta}_N) \notin \mathcal{D}_{adm}$ the performance degradation arises from control-relevant system changes.

When the performance drop results from variations in disturbance dynamics, the existing controller can be retained unchanged as the disturbance variations may be temporary. On the contrary, if the degraded performance is due to changes in system dynamics with respect to commissioning, we should often redesign the existing controller in order to restore the nominal performance level. However, the model $G(z, \hat{\theta}_N)$ identified in the detection step is not typically accurate enough for reconstructing the controller. Therefore, the detection step should often be followed by re-identification of system dynamics when the performance drop is due to system changes.

In this work, the system information obtained in detection is exploited to perform the re-identification of system dynamics using the Bayesian framework. This allows us to increase the accuracy of the re-identified model. The merit of utilizing the Bayesian framework in the re-identification step arises from the fact that the diagnosis methodology is based on prediction error identification. We extend the experiment design framework presented in [10] to combine detection with the subsequent re-identification step if a system model with a high accuracy is required for performance restoration. The framework enables us to minimize the overall excitation cost incurred for detection and re-identification through optimal design of the excitation signals. In the design of the excitation signal used for detection, we consider not only the desired detection rates, but also the required level of accuracy of the re-identified model to be used for controller redesign. In addition, we will be able to limit the incurred excitation cost when the performance drop is due to changes in disturbance dynamics. To alleviate the standard problem of unknown true system in experiment design [4], we present a so-called two scenario approach to obtain initial system estimates using routine closed-loop data for two hypothetical performance drop scenarios.

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II. PERFORMANCE DIAGNOSIS

Closed-loop performance degradation typically leads to increased variance of system inputs and outputs. The performance diagnosis methodology proposed in [9] aims to verify whether the observed performance drop results from control-relevant system changes. We investigate the closed-loop performance of the system shown in Fig. 1. Our attention is restricted to a stable linear time-invariant single input single output system described as:

$$y(t) = \underbrace{G(z,\theta_0)}_{G_0(z)} u(t) + \underbrace{H(z,\theta_0)e(t)}_{v(t)}, \tag{1}$$

where $\theta_0 \in \mathbf{R}^k$ is the unknown true parameter vector; $e(t)$ is a white noise signal with variance σ_e^2; $G(z,\theta_0)$ and $H(z,\theta_0)$ are stable discrete-time transfer functions. $H(z,\theta_0)$ is assumed to be monic and minimum-phase [8]. In Fig. 1, $r(t)$ denotes the excitation signal used for system identification.

In the disturbance rejection control problem at hand, we intend to study the ability of the loop $[C\ G_0]$ in coping with system disturbances, i.e. $v(t)$. The performance of a stable closed-loop system made up of a system $G(z,\theta)$ and an existing controller $C(z)$ is therefore defined as:

$$J(G,C,W_l,W_r) = \sup_{\omega} \bar{J}(\omega,G,C,W_l,W_r) \tag{2}$$

with

$$\bar{J}(\omega,G,C,W_l,W_r) = \bar{\sigma}\Big(W_l(e^{j\omega})F\big(G(e^{j\omega}),C(e^{j\omega})\big)W_r(e^{j\omega})\Big)$$

$$F(G,C) \triangleq \begin{pmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} \\ \frac{C}{1+GC} & \frac{1}{1+GC} \end{pmatrix}, \tag{3}$$

where $\bar{\sigma}(A)$ denotes the largest singular value of A; $W_l(z)$ and $W_r(z)$ are pre-specified diagonal performance weighting filters. A controller $C(z)$ will be deemed satisfactory for the system $G_0(z)$ if $[C\ G_0]$ is stable and $J(G_0,C,W_l,W_r) \leq 1$.

To formulate the hypothesis test, we define the sets $\mathcal{D}_{adm}$ and $\mathcal{V}_J$ as follows.

Definition 1: Given the existing controller $C(z)$, the region $\mathcal{D}_{adm}$ is the set of all systems $G(z,\theta)$ that are stabilized by $C(z)$ and achieve the nominal performance $J(G,C,W_l,W_r) \leq 1$.

Definition 2: The set $\mathcal{V}_J$ contains the power spectra $\Phi_v(\omega)$ of all disturbances $v(t)$ which are sufficiently rejected by loops $[C\ G]$ satisfying $J(G,C,W_l,W_r) \leq 1$. The disturbance is sufficiently rejected if the input and output signals have a reasonably small variance in accordance with the pre-specified requirements.

It is evident that the performance of the closed-loop system shown in Fig. 1 will be adequate if $G_0(z) \in \mathcal{D}_{adm}$ and $\Phi_v(\omega) \in \mathcal{V}_J$. This is ensured at the commissioning stage of the controller where the performance weighting filters $W_l(z)$ and $W_r(z)$ are selected in relation to $\Phi_v(\omega)$. Situations may however arise over time that the system dynamics $G_0(z)$ and/or the disturbance spectrum $\Phi_v(\omega)$ change. In case that $G_0(z) \in \mathcal{D}_{adm}$ after the performance drop, $\Phi_v(\omega)$ can no longer lie in $\mathcal{V}_J$; suggesting that the performance degradation is due to variations in disturbance characteristics. On the contrary, the performance drop is a result of system changes when $G_0(z) \notin \mathcal{D}_{adm}$. In the event of an observed performance degradation, the detection problem therefore consists in deciding between the following hypotheses:

$$\begin{aligned} \mathcal{H}_0 &: G_0(z) \in \mathcal{D}_{adm} \\ \mathcal{H}_1 &: G_0(z) \notin \mathcal{D}_{adm}. \end{aligned} \tag{4}$$

Remark 1: In this work, we consider the scenarios pertaining to the two extreme cases, namely the performance drop is either due to changes in system dynamics or due to changes in disturbances characteristics. The proposed methodology can be easily extended to include more realistic scenarios such as when the system dynamics and disturbance characteristics vary simultaneously.

Since $G_0(z)$ is unknown in practice, we perform closed-loop identification with the existing controller to obtain a model of the true system. The signal $r(t)$ for $(t = 0,\ldots,N-1)$ is exploited to excite the closed-loop system in order to measure the input and output signals $\{u(t),y(t)\,|\,t = 0,\ldots,N-1\}_{det}$ based on which the model $\{G(z,\hat{\theta}_N^{det}),H(z,\hat{\theta}_N^{det})\}$ is identified. We shall assume throughout this paper that a full order model structure can be constructed such that θ_0 is the only value of the parameter vector for which $\{G(z,\theta),H(z,\theta)\}$ represents the true system. The identified parameter vector is defined as:

$$\hat{\theta}_N^{det} = \arg\min_{\theta} \frac{1}{N} \sum_{t=0}^{N-1} \epsilon^2(t,\theta), \tag{5}$$

where $\hat{\theta}_N^{det}$ denotes the parameter vector obtained using the set $\{u(t),y(t)\}_{det}$ collected for detection. The residuals $\epsilon(t,\theta) = H(z,\theta)^{-1}\big(y(t) - G(z,\theta)u(t)\big)$ in Eq. (5) relate to $r(t)$ via the measured signals $\{u(t),y(t)\}_{det}$, i.e.

$$y(t) = S_0 v(t) + \underbrace{G_0 S_0 r(t)}_{y_r(t)} \tag{6}$$

$$u(t) = -CS_0 v(t) + \underbrace{S_0 r(t)}_{u_r(t)}, \tag{7}$$

where $S_0 = \frac{1}{1+G_0 C}$ is the sensitivity function of the closed-loop system. Note that when the closed-loop identification experiment is sufficiently informative, any parameter vector $\hat{\theta}_N$ identified through Eq. (5) based on $\{u(t),y(t)\}$ will be asymptotically normally distributed around the true parameter vector θ_0. This suggests that $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$ with P_θ being a strictly positive definite matrix [8]:

$$P_\theta = \frac{\sigma_e^2}{N}\Bigg(\mathbb{E}\Big[\big(\frac{\partial \epsilon(t,\theta)}{\partial \theta}|_{\theta_0}\big)\big(\frac{\partial \epsilon(t,\theta)}{\partial \theta}|_{\theta_0}\big)^T\Big]\Bigg)^{-1} \tag{8}$$

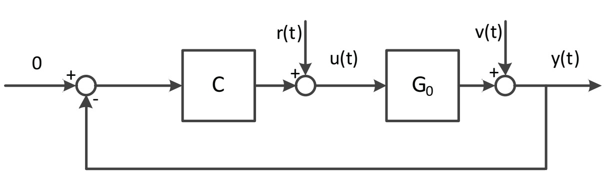

Fig. 1. The closed-loop system $[C\ G_0]$.

that can be estimated from $\hat{\theta}_N$ and $\{u(t), y(t)\}$, e.g. $\hat{\theta}_N^{det}$ and $\{u(t), y(t)\}_{det}$.

The identified model $G(z, \hat{\theta}_N^{det})$ can now be used in the following decision rule:

$$\begin{aligned} G(z, \hat{\theta}_N^{det}) \in \mathcal{D}_{adm} &\Rightarrow \text{choose} \quad \mathcal{H}_0 \\ G(z, \hat{\theta}_N^{det}) \notin \mathcal{D}_{adm} &\Rightarrow \text{choose} \quad \mathcal{H}_1. \end{aligned} \tag{9}$$

Since $G(z, \hat{\theta}_N^{det})$ is only an estimate of the true system, the decision rule may result in wrong conclusions regarding the cause of the performance drop. The null hypothesis $\mathcal{H}_0$ can be chosen erroneously when $G(z, \hat{\theta}_N^{det})$ has been generated by $G_0(z) \notin \mathcal{D}_{adm}$. This is in effect a wrong decision as the performance degradation is not due to variations in disturbance characteristics, but due to changes in system dynamics. On the other hand, the choice of the alternative hypothesis $\mathcal{H}_1$ is erroneous when $G_0(z) \in \mathcal{D}_{adm}$.

In case that the performance drop is due to variations in disturbance characteristics, i.e. $\mathcal{H}_0$, the existing controller can be kept intact as the changes in disturbance dynamics may be temporary. Alternatively, the disturbance model $H(z, \hat{\theta}_N^{det})$ identified along with $G(z, \hat{\theta}_N^{det})$ can be used to restore the disturbance rejection ability of the closed-loop system. On the contrary, when the performance diagnosis methodology detects $\mathcal{H}_1$ in the decision rule of Eq. (9), the system dynamics have changed with respect to $G_0(z)$ at the commissioning stage of the controller. The controller should therefore be redesigned to achieve the nominal performance level. However, the model $G(z, \hat{\theta}_N^{det})$ may not be sufficiently accurate for reconstructing the controller. This suggests that an additional re-identification step is often necessary to restore the control performance when $\mathcal{H}_1$ is selected.

To re-identify the system dynamics, we should yet again excite the closed-loop system using the excitation signal $r(t)$ to collect the input output signals $\{u(t), y(t) \,|\, t = 0, \ldots, N-1\}_{id}$. The measured signals can be used in a similar fashion as in the detection step to identify the model $\{G(z, \hat{\theta}_N^{id}), H(z, \hat{\theta}_N^{id})\}$ using prediction error identification, i.e. Eq. (5). We however exploit the Bayesian approach to identify the parameter vector $\hat{\theta}_N^{id}$ by utilizing the system information obtained through the detection signal [3]:

$$\hat{\theta}_N^{id} = \arg\min_{\theta} \frac{1}{N} \Big(\sum_{t=0}^{N-1} \epsilon^2(t, \theta) + \left\| \theta - \hat{\theta}_N^{det} \right\|^2_{P^{-1}_{\hat{\theta}_N^{det}}} \Big). \tag{10}$$

In fact, the term $\left\| \theta - \hat{\theta}_N^{det} \right\|^2_{P^{-1}_{\hat{\theta}_N^{det}}}$ allows us to use the knowledge of system dynamics obtained in the detection step to re-identify the parameter vector; $P^{-1}_{\hat{\theta}_N^{det}}$ is estimated from Eq. (8) using $\hat{\theta}_N^{det}$ and $\{u(t), y(t)\}_{det}$. The residuals $\epsilon(t, \theta)$ in Eq. (10) are calculated based on $\{u(t), y(t)\}_{id}$. Note that the measured input outputs sets $\{u(t), y(t)\}_{det}$ and $\{u(t), y(t)\}_{id}$ need not be of the same length N. Clearly, there is an excitation cost incurred to obtain each of these data sets.

III. THE EXPERIMENT DESIGN FRAMEWORK

A. The Unified Detection and Identification Framework

In this work, we aim to minimize the overall excitation cost through optimal design of the excitation signals used for detecting the cause of the performance drop and re-identifying a high-fidelity model when $\mathcal{H}_1$ is selected. The excitation signals should be designed such that some pre-specified requirements on detection probability as well as on precision of the re-identified model needed for performance restoration are fulfilled.

The overall excitation cost consists of two parts, namely the cost incurred to evaluate the decision rule of Eq. (9) and the cost that may be incurred to re-identify the system dynamics if the performance drop is due to changes in system dynamics. Like in [2], we define the excitation cost as:

$$\mathcal{J}_r = \beta_y \Big(\frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{y_r}(\omega) d\omega \Big) + \beta_u \Big(\frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_r}(\omega) d\omega \Big), \tag{11}$$

where $\Phi_{y_r}(\omega)$ and $\Phi_{u_r}(\omega)$ are the power spectra of the disturbance signals $y_r(t)$ and $u_r(t)$, respectively, with β_y and β_u being some user specified scalars. The cost $\mathcal{J}_r$ quantifies the effect of perturbations u_r and y_r caused by the excitation signal on the input and output signals, respectively.

As described in Section II, the decision rule of Eq. (9) may lead to erroneous decisions since it relies on an estimate $G(z, \hat{\theta}_N^{det})$ of the true system. The excitation signal used for detection should therefore be designed such that some predetermined detection probability is attained in Eq. (9). The detection probability corresponds to the probability that $G(z, \hat{\theta}_N^{det})$ lies in the performance-related region of interest, i.e. either inside or outside $\mathcal{D}_{adm}$, according to the selected hypothesis. To ensure that $G(z, \hat{\theta}_N^{det})$ lies in the designated performance-related region with a pre-specified probability, the following expressions should hold:

$$\begin{aligned} Pr\Big(G(z, \hat{\theta}_N^{det}) \in \mathcal{D}_{adm} | \mathcal{H}_0\Big) &\geq \alpha_1 \\ Pr\Big(G(z, \hat{\theta}_N^{det}) \notin \mathcal{D}_{adm} | \mathcal{H}_1\Big) &\geq \alpha_2, \end{aligned} \tag{12}$$

where α_1 and α_2 are the desired levels of detection probability corresponding to hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Eq. (12) implies that $G(z, \hat{\theta}_N^{det})$ lies in the region $\mathcal{D}_{adm}$ with the least probability α_1 if $\mathcal{H}_0$ is the true hypothesis, whereas $G(z, \hat{\theta}_N^{det}) \notin \mathcal{D}_{adm}$ with the least probability α_2 when $\mathcal{H}_1$ is the correct hypothesis.

In case that the alternative hypothesis $\mathcal{H}_1$ is selected in the decision rule, the detection step is followed by re-identification of system dynamics with a pre-specified accuracy. The re-identified system model $\{G(z, \hat{\theta}_N^{id}), H(z, \hat{\theta}_N^{id})\}$ will allow us to construct a new controller $C'(z)$ which ensures a satisfactory performance in closed-loop operation with the true system $G_0(z)$. We express the required level of accuracy of the re-identified model as:

$$Pr\Big(G(z, \hat{\theta}_N^{id}) \in \mathcal{D}'_{adm}\Big) \geq \alpha_3. \tag{13}$$

$\mathcal{D}'_{adm}$ is defined as the set of all systems $G(z, \theta)$ whose modeling error with respect to the true system, i.e. $G(z, \theta) -$

$G_0(z)$, is sufficiently small to allow the design of a controller $C'(z)$ that provides a satisfactory performance in closed-loop operation with the true system [6]. The user specified α_3 denotes the desired probability level at which the to be re-identified model $G(z, \hat{\theta}_N^{id})$ should lie in $\mathcal{D}'_{adm}$.

To minimize the overall cost of detection and re-identification, we present a unified experiment design framework that combines the detection step and the subsequent re-identification step when $\mathcal{H}_1$ is the correct hypothesis. The framework allows us to minimize the overall excitation cost of detection and re-identification pertaining to the case of $\mathcal{H}_1$ while ensuring an acceptable detection cost if $\mathcal{H}_0$ is the correct hypothesis. The latter is achieved by defining an upper bound on the detection cost $\mathcal{J}_{r,det}^{\mathcal{H}_0}$. The unified experiment design framework is formulated as:

$$\begin{aligned} \min_{\Phi_{r,det}(\omega),\Phi_{r,id}(\omega)} \quad & \mathcal{J}_{r,det}^{\mathcal{H}_1} + \mathcal{J}_{r,id}^{\mathcal{H}_1} \\ \text{s.t.:} \quad & Pr\Big(G(z,\hat{\theta}_N^{det}) \in \mathcal{D}_{adm} | \mathcal{H}_0\Big) \geq \alpha_1 \\ & Pr\Big(G(z,\hat{\theta}_N^{det}) \notin \mathcal{D}_{adm} | \mathcal{H}_1\Big) \geq \alpha_2 \\ & Pr\Big(G(z,\hat{\theta}_N^{id}) \in \mathcal{D}'_{adm}\Big) \geq \alpha_3 \\ & \mathcal{J}_{r,det}^{\mathcal{H}_0} < \beta, \end{aligned} \tag{14}$$

where $\Phi_{r,det}(\omega)$ and $\Phi_{r,id}(\omega)$ are the power spectra of the excitation signals designed for detection and re-identification, respectively; $\mathcal{J}_{r,det}^{\mathcal{H}_0}$ and $\mathcal{J}_{r,det}^{\mathcal{H}_1}$ are the excitation costs incurred for detection when $\mathcal{H}_0$ and $\mathcal{H}_1$ are the correct hypothesis, respectively; $\mathcal{J}_{r,id}^{\mathcal{H}_1}$ is the excitation cost incurred for re-identification; β is the maximum admissible excitation cost to achieve $Pr\Big(G(z,\hat{\theta}_N) \in \mathcal{D}_{adm} | \mathcal{H}_0\Big) \geq \alpha_1$.

It should be noted that only the spectrum $\Phi_{r,det}(\omega)$ will be applied to the closed-loop system to detect the cause of the performance drop. Depending on the outcome of the detection step, re-identification of system dynamics can be performed to obtain a model $G(z, \hat{\theta}_N^{id})$ in order to reconstruct the controller. The power spectra $\Phi_{r,det}(\omega)$ and $\Phi_{r,id}(\omega)$ can be designed for a fixed experiment duration N.

Remark 2: We restrict our attention to excitation signals with power spectrum:

$$\Phi_r(\omega) = R_r(0) + 2\sum_{i=1}^{m} R_r(i)cos(i\omega) \geq 0 \quad \forall\omega, \tag{15}$$

where m is a positive integer. This implies that $R_{r,det}(i)$ for $(i = 0, \ldots, m_1)$ and $R_{r,id}(i)$ for $(i = 0, \ldots, m_2)$ comprise the decision variables in Eq. (14); m_1 and m_2 denote the parameterization degree of $\Phi_{r,det}$ and $\Phi_{r,id}$, respectively. Such power spectrum ensures that the closed-loop identification experiment is sufficiently informative and, consequently, the approximation $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$ is valid. The parameters $R_r(i)$ can be considered as the auto-correlation sequence of a signal which has been generated by passing a white noise signal through an FIR filter of length $m+1$.

Remark 3: The probability α_2 may be chosen to be larger than α_1. This is to ensure that we can identify a system model $G(z, \hat{\theta}_N^{det})$ lying outside $\mathcal{D}_{adm}$ with a reasonably high accuracy in the case of $\mathcal{H}_1$. Note that Eq. (14) exploits the latter model in a Bayesian framework to identify $G(z, \hat{\theta}_N^{id})$.

B. The Two Scenario Approach

In practice, any experiment design exercise requires the knowledge of the true system $G_0(z)$ which is to be identified through optimal design of the excitation signal [4]. There is therefore a 'chicken and egg' problem inherent in experiment design. The fact that $G_0(z)$ is not known poses yet an additional difficulty in the experiment design problems which ensure certain detection probability in the decision rule, i.e. Eq. (9). As thoroughly described in [10], the formulation of the experiment design problem connected to the detection step relies on the cause of the performance drop. The knowledge of the true system is therefore required to select a hypothesis in Eq. (9) and accordingly design the spectrum of the excitation signal $r(t)$ in order to achieve some predetermined detection probability.

In this work, a so-called two scenario approach is adopted to circumvent the aforementioned 'chicken and egg' problem. We shall assume that the performance drop results only from one of the following scenarios:

1) variations in the disturbance characteristics, suggesting that the true system dynamics remain identical to those at commissioning;
2) changes in the true system dynamics, while the disturbance characteristics are the same as those at commissioning.

These hypothetical scenarios in fact relate to the null hypothesis and the alternative hypothesis, respectively. The scenarios are determined based on routine closed-loop data, i.e. no excitation is required. In scenario 1, the underlying model of the controller synthesized at commissioning can describe the system dynamics, while the routine closed-loop data should be used to identify the disturbance characteristics. In scenario 2, on the contrary, the routine closed-loop data is utilized to identify only the model of the system dynamics.

We denote the model estimates obtained in scenarios 1 and 2 by $\{G(z,\theta_{\mathcal{H}_0}), H(z,\theta_{\mathcal{H}_0})\}$ and $\{G(z,\theta_{\mathcal{H}_1}), H(z,\theta_{\mathcal{H}_1})\}$, respectively. $G(z,\theta_{\mathcal{H}_0})$ is the model identified at the commissioning stage of the controller, whereas $G(z,\theta_{\mathcal{H}_1})$ is identified using the routine closed-loop data collected after the performance drop. The latter estimates of $G_0(z)$ and $H_0(z)$ can now be exploited to solve the unified experiment design problem. This is further discussed in the sequel.

C. Approximation of the Experiment Design Problem

Numerical solution of the unified experiment design problem is cumbersome. We can nevertheless construct a confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$ for the identified model $G(z, \hat{\theta}_N)$ based on $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$. The confidence set, centered at $G_0(z)$, contains the identified model with a pre-specified probability level α [4]:

$$\mathcal{D}(\theta_0, P_\theta, \mathcal{X}) = \Big\{ G(z,\hat{\theta}_N) \mid \hat{\theta}_N \in U, \tag{16}$$

$$U = \{\hat{\theta}_N \mid (\hat{\theta}_N - \theta_0)^T P_\theta^{-1}(\hat{\theta}_N - \theta_0) < \mathcal{X}\}\Big\}$$

where $\mathcal{X}$ is a real constant such that

$$Pr\Big(\chi^2(k) < \mathcal{X}\Big) = \alpha \tag{17}$$

with $\chi^2(k)$ being a chi-square distribution with k degrees of freedom.

We use the system models obtained from the two scenario approach as estimates for the unknown true parameter vector θ_0 in the confidence set $\mathcal{D}(\theta_0, P_\theta, \mathcal{X})$. The unified experiment design problem can therefore be recast as:

$$\begin{aligned} \min_{\Phi_{r,det},\Phi_{r,id}} \quad & \mathcal{J}_{r,det}(\Phi_{r,det}, \theta_{\mathcal{H}_1}) + \mathcal{J}_{r,id}(\Phi_{r,id}, \theta_{\mathcal{H}_1}) \\ \text{s.t.:} \quad & \mathcal{D}(\theta_{\mathcal{H}_0}, P_{\theta_1}, \mathcal{X}_1) \subseteq \mathcal{D}_{adm} \\ & \mathcal{D}(\theta_{\mathcal{H}_1}, P_{\theta_2}, \mathcal{X}_2) \subseteq \mathcal{C}_{\mathcal{D}_{adm}} \\ & \mathcal{D}(\theta_{\mathcal{H}_1}, (P_{\theta_2}^{-1} + P_{\theta_3}^{-1})^{-1}, \mathcal{X}_3) \subseteq \mathcal{D}'_{adm}(\theta_{\mathcal{H}_1}) \\ & \mathcal{J}_{r,det}(\Phi_{r,det}, \theta_{\mathcal{H}_0}) < \beta, \end{aligned} \tag{18}$$

where $\mathcal{C}_{\mathcal{D}_{adm}}$ is the complementary set of $\mathcal{D}_{adm}$; $\mathcal{J}_{r,det}(\Phi_{r,det}, \theta_{\mathcal{H}_1})$ and $\mathcal{J}_{r,id}(\Phi_{r,id}, \theta_{\mathcal{H}_1})$ are the excitation costs computed through the assumption that the model $\{G(z, \theta_{\mathcal{H}_1}), H(z, \theta_{\mathcal{H}_1})\}$ represents the true system dynamics, whereas $\mathcal{J}_{r,det}(\Phi_{r,det}, \theta_{\mathcal{H}_0})$ is determined based on $\{G(z, \theta_{\mathcal{H}_0}), H(z, \theta_{\mathcal{H}_0})\}$; P_{θ_i} for $\{i = 1, 2\}$ are the covariance matrices obtained with the detection spectrum $\Phi_{r,det}$ when the models $\{G(z, \theta_{\mathcal{H}_0}), H(z, \theta_{\mathcal{H}_0})\}$ and $\{G(z, \theta_{\mathcal{H}_1}), H(z, \theta_{\mathcal{H}_1})\}$ describe the system dynamics, respectively; P_{θ_3} is the covariance matrix calculated on the basis of $\Phi_{r,id}$ and $\{G(z, \theta_{\mathcal{H}_1}), H(z, \theta_{\mathcal{H}_1})\}$; $\mathcal{X}_1$, $\mathcal{X}_2$, and $\mathcal{X}_3$ are the quantiles related to the probabilities α_1, α_2, and α_3, respectively.

Due to the Bayesian framework exploited in Eq. (10), the covariance matrix of the to be re-identified parameter vector $\hat{\theta}_N^{id}$ is $(P_{\theta_2}^{-1} + P_{\theta_3}^{-1})^{-1}$. The unified experiment design problem of Eq. (18) can be transformed into an LMI optimization problem; see [2] and [10] for further details.

IV. Numerical Illustrations

We apply the performance diagnosis methodology to a simulation case study in order to verify the adequacy of the unified experiment design framework. The true system has a Box-Jenkins structure: $y(t) = G_0(z)u(t) + H_0(z)e(t)$ where $G_0(z) = \theta_b z^{-1}/(1+\theta_f z^{-1})$ and $H_0(z) = 1+\theta_c z^{-1}$; $\theta_0 = (\theta_b \ \theta_c \ \theta_f)^T$ and $e(t)$ are the true parameter vector and a realization of a white noise process with variance $\sigma_{e,0}$, respectively. The performance weighting filters, i.e. $W_l(z) = \text{diag}\left(0, (0.52 - 0.46z^{-1})/(1 - 0.99z^{-1})\right)$ and

TABLE I

Settings of the unified experiment design problem

	Case I	Case II
N	500	500
β	0.01	0.1
$\alpha_1(\%)$	75.0	50.0
$\alpha_2(\%)$	80.0	95.0
$\alpha_3(\%)$	99.8	99.8

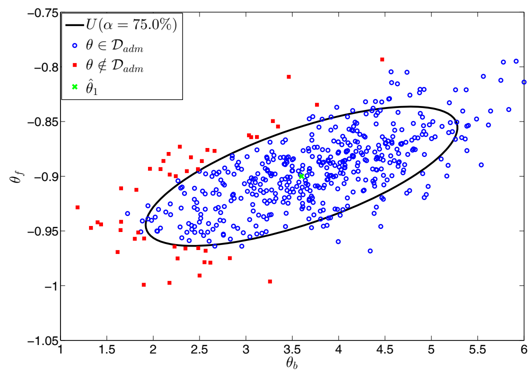

Fig. 2. Depiction of the probability of making an erroneous decision in Case I, i.e. $G_0(z) \in \mathcal{D}_{adm}$, when the detection signal is designed using the unified experiment design problem.

$W_r(z) = \text{diag}(0, 1)$, are selected such that the performance measure is stated in terms of the sensitivity functions. The true system $G_0(z)$ is in closed-loop operation with a H_∞-controller synthesized based on a Box-Jenkins model with $\theta_{com} = (3.6 \ \ -0.9 \ \ -0.9)^T$ and $\sigma_{e,com} = 1.0$. Note that the nominal performance level is attained at commissioning.

In the following, we consider two cases to compare the overall excitation cost of the proposed unified experiment design framework with that incurred when the excitation signals $\Phi_{r,det}$ and $\Phi_{r,id}$ are white noise signals. In both cases, the changes in system dynamics or disturbance characteristics lead to degradation of the closed-loop performance. This is observed through an increase in variances of the system input and output. The two scenario approach is used to obtain the initial system estimates required for experiment design. Note that the following simulation case studies are defined such that they pertain to the scenarios of the two scenario approach.

Case I: The disturbance characteristics are altered by varying the noise transfer function $H_0(z)$ and the variance of the white noise signal $e(t)$ with respect to the commissioning stage, i.e. $\theta_0 = (3.6 \ \ -0.1 \ \ -0.9)^T$ and $\sigma_{e,0} = 10.0$. It is evident that this case corresponds to $\mathcal{H}_0$ in the decision rule, i.e. Eq. (9).

We exploit the two scenario approach to get the system estimates $G(z, \theta_{\mathcal{H}_0})$ and $G(z, \theta_{\mathcal{H}_1})$ by using 1000 samples of the input and output signals $\{u(t), y(t)\}$ collected during the routine closed-loop operation. These system estimates are utilized to formulate the unified experiment design problem of Eq. (18) with the settings listed in Table I. To verify the results of experiment design, we perform a Monte Carlo

TABLE II

The actual closed-loop excitation costs incurred in the detection and re-identification steps

	Case I		Case II	
	White Noise	Signal Design	White Noise	Signal Design
$\mathcal{J}_{r,det}$	0.010	0.001	0.0064	0.0014
$\mathcal{J}_{r,id}$	NA	NA	0.0010	0.0039

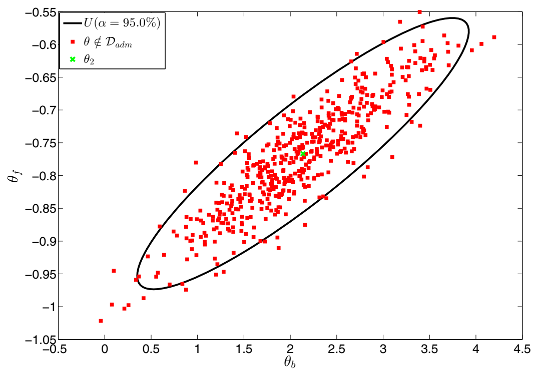

Fig. 3. Depiction of the probability of making an erroneous decision in Case II, i.e. $G_0(z) \notin \mathcal{D}_{adm}$, when the detection signal is designed using the unified experiment design problem.

simulation to identify the true system dynamics 500 times by applying the designed excitation signals. Fig. 2 depicts the parameters $\theta = (\theta_b\ \theta_f)^T$ of systems $G(z, \theta)$ identified through excitation with the detection signal, i.e. $\Phi_{r,det}$. The Monte Carlo simulation reveals that approximately 90.8% of the identified systems lie in the set $\mathcal{D}_{adm}$. Clearly, this is larger than the least detection probability $\alpha_1 = 75.0\%$ associated with the confidence region $\mathcal{D}(\theta_{\mathcal{H}_0}, P_{\theta_1}, \mathcal{X}_1)$ shown in Fig. 2. As can be seen, the ellipsoidal region is centered around $G(z, \theta_{\mathcal{H}_0})$ which is identical to $G(z, \theta_0) = G(z, \theta_{com})$ in this case.

The actual excitation cost incurred for detecting the cause of the performance degradation is given in Table II. The cost is evaluated by applying the excitation signal $\Phi_{r,det}$ to the true system $G_0(z)$. In this case, we often do not proceed with re-identification of the true system dynamics since the changes in disturbance characteristics may be temporary and, therefore, the controller can be kept intact. As shown in Table II, the detection cost incurred through applying the designed $\Phi_{r,det}$ is significantly lower than when a white noise signal with optimized variance is utilized for excitation. Note that the variance of the white noise is optimized by setting $m = 0$ in the power spectrum $\Phi_{r,det}$; see Eq. (15). It is evident that the detection cost pertaining to the white noise case hits the maximum admissible excitation cost β. This results from the choice of the user specified detection probability α_1 when $\mathcal{H}_0$ is the correct hypothesis. The latter probability correlates with the cost β to limit accordingly the unnecessary system perturbations if $\mathcal{H}_0$ is the true hypothesis.

Case II: We induce variations in the system dynamics by defining the parameter vector θ_0 as $(1.0\ \ -0.9\ \ -0.9)^T$. This implies that in Case II the performance degradation arises from the changes in $G_0(z)$, i.e. the alternative hypothesis.

Similarly as in Case I, we obtain two initial system estimates for experiment design through the two scenario approach. The settings of the experiment design problem is listed in Table I. The excitation signal $\Phi_{r,det}$ is used to identify the system $G(z, \theta)$ 500 times. As illustrated in Fig. 3, the Monte Carlo simulation indicates that 100% of the identified systems lie outside $\mathcal{D}_{adm}$, i.e. the actual detection probability is 100%. This is clearly in accordance with the cause of the performance drop, i.e. $\mathcal{H}_1$. Note that the pre-specified detection probability of the confidence region $\mathcal{D}(\theta_{\mathcal{H}_1}, P_{\theta_2}, \mathcal{X}_2)$ is 95%.

Since the performance drop is due to the changes in system dynamics, we should re-identify $G_0(z)$ with a reasonably high accuracy to be able to construct a new controller. We therefore re-excite the closed-loop system with the designed excitation signal $\Phi_{r,id}$. This allows us to attain a description of $G_0(z)$ that ensures an adequate closed-loop performance with the least probability $\alpha_3 = 99.8\%$. The actual excitation costs of the detection and re-identification steps are given in Table II. As compared to the case of excitation with white noise signals, the overall excitation cost, i.e. $\mathcal{J}_{r,det} + \mathcal{J}_{r,id}$, is approximately 26% lower when the detection and re-identification signals are designed through Eq. (18). Like in Case I, the optimal spectrum $\Phi_{r,det}$ results in an appreciably lower detection cost.

V. CONCLUSIONS

We have presented a unified framework for optimal design of excitation signals exploited for detection using the performance diagnosis methodology proposed in [9] and the subsequent re-identification when the performance drop is due to control-relevant system changes. It has been illustrated that through designing the power spectra of excitation signals used for detection and re-identification, we can significantly reduce the overall excitation cost as compared to excitation using white noise signals. In future, we will explore the use of experiment design for closed-loop performance diagnosis of model predictive controllers.

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