Instrumental-variable methods for continuous-time model identification in closed-loop

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Abstract— **System identification in closed-loop has been of considerable interest in the last two decades. Most of the existing methods have been developed for discrete-time models. In this paper, various instrumental variable-based methods are proposed for identifying continuous-time models of systems operating in closed-loop. The accuracy of these methods is also investigated leading to the definition of the optimal IV estimator which gives minimum variance. As this needs the exact knowledge of the noise model, it cannot be used directly in practice. Several alternatives are therefore proposed to cope with this drawback and illustrated with a simulation example.**

I. Introduction

System identification is an established field in the area of systems and control. It is aimed at determining useful models for dynamical systems based on observed inputs and outputs. Although dynamical systems in the physical world are actually in continuous-time (CT) domain, most system identification schemes have been based in the past on discrete-time (DT) models without concern for the merits of the native CT models. The development of CT model based system identification techniques began in the middle of the last century but was overshadowed by the overwhelming developments of DT methods. This was mainly due to the 'go completely digital' trend that was spurred by parallel developments in digital computers. Interest in CT approaches to system identification has however been growing in the last decade (see e.g. [1], [2], [3]).
Recent investigations [4], [5] have drawn attention to difficulties that can be encountered when utilizing standard DT estimation algorithms under conditions, such as rapidly sampled data and dominant system modes with widely different natural frequencies. However, there are many issues which have not received adequate attention so far in the case of CT model identification. One such issue is the identification of closed-loop systems.
Many results were established during the last decade in this area [6], [7]. When looking at methods that can consistently identify plant models of systems operating in closed-loop while relying on simple linear (regression) algorithms, instrumental variable (IV) techniques seem to be attractive, but not often applied. On the other hand, when dealing with highly complex processes that involve models of high dimension in terms of inputs and outputs, it can be attractive to rely on methods that do not require non-convex optimization algorithms.
For closed-loop identification a basic IV estimator has been proposed [8], and more recently the so-called tailor-made IV algorithm [9], where the closed-loop plant is parametrized using (open-loop) plant parameters has been suggested. The class of algorithms denoted by BELS (for Bias-Eliminated Least-Squares), e.g. [10], is also directed towards the use of linear regression algorithms only. It has recently been shown that these algorithms are also particular forms of IV estimation schemes [11], [9]. Then, when comparing the several available IV algorithms, the principal question to address should be: how to achieve the smallest variance of the estimate? An optimality result has been recently developed for the closed-loop DT model identification problem, showing consequences for the optimal choice of the design parameters [12]. The same reasoning can be applied to the closed-loop CT model identification case.
In this paper, several instrumental variable methods are proposed to handle the identification problem of CT models of linear dynamic systems operating in closed-loop, and the influence for the several design variables (related to optimal variance) is considered. Since for optimal variance, the noise model has to be known exactly, several bootstrap methods are also proposed for approximating this required information from measurement data. These proposed methods are compared by means of a simulation example, showing that a near optimal estimator can be obtained by an appropriate choice of the design parameters.

II. Preliminaries

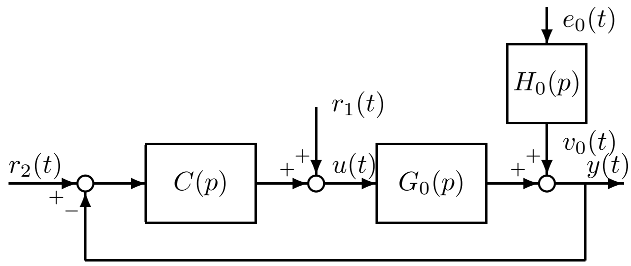

Fig. 1. Closed-loop configuration.

Consider a stable linear SISO closed-loop system shown in figure 1. The data generating system is assumed to be given by the relations

$$\mathcal{S}: \begin{cases} y(t) = G_0(p)u(t) + H_0(p)e_0(t) \\ u(t) = r(t) - C(p)y(t) \end{cases} \quad (1)$$

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The process is denoted by $G_0(p) = B_0(p)/A_0(p)$ and the controller by $C(p)$ where p is the differential operator ($p = d/dt$). $u(t)$ describes the process input signal, $y(t)$ the process output signal. For ease of notation we also introduce the reference signal $r(t) = r_1(t) + C(p)r_2(t)$. Moreover, it is also assumed that the CT signals $u(t)$ and $y(t)$ are uniformly sampled at T_s. A colored disturbance is acting on the loop. This noise term represented by $v_0(t) = H_0(p)e_0(t)$ has in the light of the spectral factorisation theorem been modelled as filtered white noise. The external signal $r(t)$ is assumed to be uncorrelated with the noise disturbance $v_0(t)$.

In order to avoid the mathematical difficulties due to the manipulation of a CT noise, it has been chosen to use a particular modelling for the noise contribution (by following the same reasoning as in [13]). Let T_s denotes the sampling period and assume that $\{e_{0k}\}$ is a zero-mean normally distributed DT noise sequence with covariance

$$\mathbb{E}\{e_{0i}e_{0j}^T\} = \lambda_0\delta_{ij} \quad (2)$$

In addition, we assume that $e_0(t) = e_0(kT_s)$ for $kT_s \leq t < (k+1)T_s$, i.e. $e_0(t)$ is assumed constant during the sampling interval. According to standard time series analysis, it follows that the spectral density of $\{e_{0k}\}$ is constant in the frequency range $[-\pi/T_s \;\; \pi/T_s]$. As the Fourier transform of the sampled signal is periodic with a period equal to the sampling frequency $\omega_s = 2\pi/T_s$, we conclude that the noise sequence $\{e_{0k}\}$ has constant spectral density for all frequencies smaller than ω_s. This noise model can then describe any rational spectral density for the sampled noise process in the relevant frequency range $[-\pi/T_s \;\; \pi/T_s]$ [13], [14].

The CT model identification problem is to find estimates of $G_0(p)$ from finite sequences $\{r_k\}_{k=1}^N$, $\{u_k\}_{k=1}^N$, $\{y_k\}_{k=1}^N$ of external, input and output DT data. The model to be identified is then described by the following equation

$$\mathcal{M}: \; y(t_k) = G(p,\theta)u(t_k) + H(p,\theta)e(t_k), \quad (3)$$

where $e(t_k)$ is a discrete-time white noise, with zero mean and variance λ. A parametrized process model is considered

$$\mathcal{G}: G(p,\theta) = \frac{B(p,\theta)}{A(p,\theta)} = \frac{b_0 + b_1 p + \cdots + b_{n_b}p^{n_b}}{a_0 + a_1 p + \cdots + p^{n_a}} \quad (4)$$

where n_b, n_a denote the orders of the numerator and denominator of the process respectively and with the pair (A, B) assumed to be coprime. The process model parameters are stacked columnwise in the parameter vector

$$\theta = [a_{n_a-1} \;\cdots\; a_0 \; b_{n_b} \;\cdots\; b_0]^T \in \mathbb{R}^{n_a+n_b+1}. \quad (5)$$

The numerator and denominator orders n_b and n_a are supposed to be known. The controller $C(p)$ is also assumed to be known and given by

$$C(p) = \frac{Q(p)}{P(p)} = \frac{q_0 + q_1 p + \cdots + q_{n_q}p^{n_q}}{p_0 + p_1 p + \cdots + p^{n_p}} \quad (6)$$

with the pair (P, Q) assumed to be coprime. In the following, the closed-loop system is assumed to be asymptotically stable and $r(t)$ is a signal that is persistently exciting of sufficient high order.

III. Continuous-time model identification in closed-loop

Let us first consider the data generating system. From (1), it can also be written as

$$y^{(n_a)}(t_k) = \phi^T(t_k)\theta_0 + v_0(t_k) \quad (7)$$

where

$$\phi^T(t_k) = [-y^{(n_a-1)}(t_k) \cdots - y(t_k) u^{(n_b)}(t_k) \cdots u(t_k)] \quad (8)$$

and $v_0(t_k) = A_0(p)H_0(p)e_0(t_k)$. $x^{(i)}(t_k)$ denotes the ith time-derivative of the CT signal $x(t)$ at time-instant t_k. There are two main time-domain approaches to estimate a CT model. The first is to estimate from the sampled data an initial DT model and then convert it into a CT model. The second approach consists in identifying directly a CT model from the DT data. In comparison with the DT counterpart, CT model identification raises several technical issues. The first point is related to implementation. Unlike the difference equation model, the differential equation model is not a linear combination of samples of only the measurable process input and output signals. It also contains input and output time-derivatives which are not available as measurement data in most practical cases. Various types of CT filters have been devised to circumvent the need to reconstruct these time-derivatives [3]. The CONtinuous-Time System IDentification (CONTSID) toolbox has been developed on the basis of these methods [15].

Suppose that a causal stable analog filter with Laplace transfer function $F(s)$ of minimal order n_a is selected. By passing both input and output measurements $u(t)$ and $y(t)$ through this filter, the time-derivatives of the filtered signals may be obtained. This operation is applied to model (3) at the time instant $t = t_k$ leading to the expression of the equation error

$$\varepsilon_f(t_k) = y_f^{(n_a)}(t_k) - \phi_f^T(t_k)\theta \quad (9)$$

with

$$\phi_f^T(t_k) = [-y_f^{(n_a-1)}(t_k) \cdots - y_f(t_k) \; u_f^{(n_b)}(t_k) \;\; \cdots \;\; u_f(t_k)]$$
$$y_f(t_k) = F(p)y(t_k), \quad u_f(t_k) = F(p)u(t_k) \quad (10)$$

or by using inverse Laplace transform $\mathcal{L}^{-1}$

$$y_f^{(i)}(t) = \mathcal{L}^{-1}[s^i F(s)Y(s)], \;\; u_f^{(i)}(t) = \mathcal{L}^{-1}[s^i F(s)U(s)]$$

For simplicity, it has been assumed that the differential equation model (3) is initially at rest. Note however that in the general case the initial condition terms do not vanish in equation (9). Whether they require estimation or they can be neglected depends upon the selected signal pre-processing method. There is much choice for the pre-filter. Four typical filters are as follows [3]:

$$F_1(s) = \left(\frac{\beta}{s+\lambda}\right)^{n_\alpha} \qquad F_2(s) = \left(\frac{\beta}{s+\lambda}\right)^{n_\alpha+1} \quad (11)$$

$$F_3(s) = \left(\frac{1}{s}\right)^{n_\alpha} \qquad F_4(s) = \left(\frac{1-e^{-lT_s s}}{s}\right)^{n_\alpha} \quad (12)$$

where $F_1(s)$ and $F_2(s)$ represent the filters used in the case of the minimal order multiple filter method and generalised Poisson Moment Functional (*gpmf*) approach respectively; $F_3(s)$ denotes the usual multiple integral operation while $F_4(s)$ is referred to a linear integral filter (*lif*).

IV. Basic instrumental variable estimators

The open-loop process transfer function parameters θ can be estimated using a basic instrumental variable (IV) estimator. The CT version of the basic IV estimate of θ is given by

$$\hat{\theta}_{iv}=sol\left\{\frac{1}{N}\sum_{k=1}^{N}F(p)z(t_k)F(p)[y(t_k)^{n_a}-\phi^T(t_k)\theta]=0\right\} \quad (13)$$

where N denotes the number of data and $z(t_k)$ gathered the instruments. In contrast with the DT model identification scheme, a filter already appears in the basic IV in order to handle the derivatives of the input/ouput signals. For this particular reason, the instruments $z(t_k)$ have also to be filtered by $F(p)$ to get the unknown derivatives of the instrumental variable signal.

A. Consistency properties

By inserting (7) filtered by $F(p)$ into (13) and using the filtered signals, the following well-known equation is obtained

$$\hat{\theta}_{iv}=\theta_0+\left[\sum_{k=1}^{N}z_f(t_k)\phi_f^T(t_k)\right]^{-1}\left[\sum_{k=1}^{N}z_f(t_k)v_{0f}^T(t_k)\right] \quad (14)$$

where $v_{0f}(t_k) = F(p)v_0(t_k)$. It can be deduced from the previous equation that $\hat{\theta}_{iv}$ is a consistent estimate of θ if[1]

$$\begin{cases}\bar{E}[z_f(t_k)\phi_f^T(t_k)] \text{ is nonsingular} \\ \bar{E}[z_f(t_k)v_{0f}^T(t_k)] = 0\end{cases} \quad (15)$$

Different IV variants are obtained by different choices of the instruments $z(t_k)$, respecting the conditions given by (15).

B. Covariance properties

The asymptotic distribution of the parameter (13) estimated by an IV type of method has been extensively investigated in the open-loop DT context (e.g. [17], [16]). More recently, this work has also been carried out to the closed-loop DT model identification framework [12]. By considering equation (13), the former results can be applied to the case of the CT model estimate given by (3). As a result, under the assumptions formulated in section II and $G_0 \in \mathcal{G}$, $\hat{\theta}_{iv}$ is asymptotically Gaussian distributed

$$\sqrt{N}(\hat{\theta}_{iv}-\theta^*) \overset{dist}{\rightarrow} \mathcal{N}(0, P_{iv}) \quad (16)$$

with θ^* represents the limit of $\hat{\theta}_{iv}$ when $N \rightarrow \infty$ and where the covariance matrix is given by

$$P_{iv} = \lambda_0[\bar{E}z_f(t_k)\phi_f^T(t_k)]^{-1}[\bar{E}z_{fT}(t_k)z_{fT}^T(t_k)] \\ [(\bar{E}z_f(t_k)\phi_f^T(t_k))^{-1}]^T \quad (17)$$

with $z_{fT}(t_k)=H_0(p)A_0(p)z_f(t_k)$, λ_0 variance of $\{e_0(t_k)\}$.

V. Extended instrumental variable estimators

In CT model identification, the extended-IV estimate of θ is obtained by pre-filtering the input and output data appearing in (13) and by generalizing the so-called basic IV estimates of θ by using an augmented instrument $z(t_k) \in \mathbb{R}^{n_z}$ $(n_z \geq n_a + n_b + 1)$ so that an overdetermined system of equations is obtained

$$\hat{\theta}_{xiv} = \arg\min_{\theta}\left\|\left[\sum_{k=1}^{N}z_f(t_k)L(p)\phi_f(t_k)\right]\theta - \left[\sum_{k=1}^{N}z_f(t_k)L(p)y_f^{n_a}(t_k)\right]\right\|_Q^2 \quad (18)$$

where $L(p)$ is a stable pre-filter, and $\|x\|_Q^2 = x^TQx$, with Q a positive definite weighting matrix. The estimate is then obtained by solving a least-squares problem. This extended-IV gives a parameter estimator that requires more computations than the basic-IV. However, the enlargement of the IV vector could be used for improving the accuracy of the parameter estimates [17].

A. Consistency properties

The consistency conditions are easily obtained by generalizing (15) to the estimator (18): $\bar{E}[z_f(t_k)L(p)\phi_f^T(t_k)]$ is nonsingular and $\bar{E}[z_f(t_k)L(p)v_{0f}^T(t_k)] = 0$.

B. Covariance properties

The asymptotic distribution of parameter vector (18) is obtained by following the same reasoning as in section IV-B. Therefore, by considering the assumptions given in section II, equation (18), and under the assumption $G_0 \in \mathcal{G}$, $\hat{\theta}_{eiv}$ is asymptotically Gaussian distributed

$$\sqrt{N}(\hat{\theta}_{xiv}-\theta^*) \overset{dist}{\rightarrow} \mathcal{N}(0, P_{xiv}) \quad (19)$$

where the covariance matrix is given by

$$P_{xiv} = \lambda_0[R^TQR]^{-1}R^TQ[\bar{E}z_{fT}(t_k)z_{fT}^T(t_k)]QR[R^TQR]^{-1}$$

with $R = \bar{E}z_f(t_k)L(p)\tilde{\phi}_f^T(t_k)$, where $\tilde{\phi}_f^T(t_k)$ is the noise-free part of $\phi_f(t_k)$ and $z_{fT}(t) = L(p)H_0(p)A_0(p)z_f(t)$.

[1]The notation $\bar{E}[.] = \lim_{N\rightarrow\infty}\frac{1}{N}\sum_{k=1}^{N}E[.]$ is adopted from the prediction error framework of [16]

VI. Optimal Instrumental variable estimators

A. Theoretical results

The choice of the instruments $z(t)$, of n_z, of the weighting matrix Q and of the prefilter $L(q)$ may have a considerable effect on the covariance matrix P_{tiv}. In the open-loop DT situation the lower bound of P_{xiv} for any unbiased identification method is given by the Cramer-Rao bound [16], [18]. Optimal choices of the above mentioned design variables exist so that P_{xiv} reaches the Cramer Rao bound. For the closed-loop case, this type of reasoning is not viable for IV estimates, as the objective of reaching minimum variance conflicts with the restriction that instruments and noise should be uncorrelated. However it has been shown in [8] that there indeed exists a minimum value of the covariance matrix P_{xiv} as a function of the design variables $z(t)$, $L(q)$ and Q, under the restriction that $z(t)$ is a function of the external signal $r(t)$ only. All these works rely on the use of a DT model. However, even in the CT identification case, the covariance matrix can be optimized with respect to the design variables. By following the same reasoning as in the DT case, the covariance matrix optimal value is given by

$$P_{xiv} \geq P_{xiv}^{opt},\, P_{xiv}^{opt} = \lambda_0 \left\{ \bar{E}\left[(A_0(p)H_0(p))^{-1}\tilde{\phi}_f^T(t_k)\right]^T \left[(A_0(p)H_0(p))^{-1}\tilde{\phi}_f^T(t_k)\right]\right\}^{-1}, \tag{20}$$

P_{eiv}^{opt} is then obtained by taking e.g.

$$z_f(t_k) = \left[(A_0(p)H_0(p))^{-1}\tilde{\phi}_f^T(t_k)\right]^T,\ n_z = n_a + n_b + 1, \quad Q = I,\ L(p) = [A_0(p)H_0(p)]^{-1}. \tag{21}$$

Then, the optimal IV estimator can only be obtained if the true noise (and process) model $A_0(p)H_0(p)$ is exactly known and therefore optimality cannot be achieved exactly in practice. A solution to cope with this problem is to use bootstrapping methods, as it will be discussed in the next section.

Remark. In this framework, the filter $F(p)$ is supposed to be fixed *a priori* by the user and by the way, it is not included into the design variables of the method.

B. Approximate implementations

As the optimal IV method cannot be achieved in practice, approximate implementations of the optimal IV method will be considered. For this purpose one will need to take care that firstly a model of A_0H_0 is available in order to construct the prefilter $L(q)$ and the instruments $z(t)$, and secondly a first model of $G_0(q)$ is needed to compute the noise free part of the regressor $\tilde{\phi}(t)$.
The choice of the instruments and prefilter in the IV method affects the asymptotic variance, while consistency properties are generically secured. This suggests that minor deviations from the optimal value (which is not available in practice) will only cause second-order effects in the resulting accuracy. Therefore it is considered to be sufficient to use consistent, but not necessarily efficient estimates of the dynamics and of the noise when constituting the instrument and the prefilter [16].
Additionally for obtaining the necessary preliminary models a restriction is made to linear regression estimates in order to keep computational procedures simple and tractable.

1) First solution: Several bootstrap IV methods have been proposed, in an attempt to approximate the optimal IV method, see e.g.[19], [18], [16] for the open-loop situation and [12] for the closed-loop one. A first solution consists in extending one of these algorithms to the CT situation. Here is developed the application of the IV4 method [16] to the CT closed-loop framework. The only difference between open-loop and closed-loop cases is that in the latter, also the input is correlated with the noise. Therefore, the instruments have to be uncorrelated with $u(t)$ but correlated with the noise-free part of $u(t)$. Moreover, according to section II, CT models are estimated to represent the transfers between the filtered output ($y_f(t_k)$) and the filtered excitation signal ($r_f(t_k)$) as well as for the transfer between the filtered input ($u_f(t_k)$) and the filtered excitation-signal ($r_f(t_k)$); but DT models are used to estimate the noise contribution.

Method clivc4.

1) Write the model structure as a linear regression

$$\hat{y}_f^{(n_a)}(t_k, \theta) = \phi_f^T(t_k)\theta. \tag{22}$$

Estimate θ by a least-squares method and get $\hat{\theta}_{ls}$ along with the corresponding CT transfer function $\hat{G}_{ls}(p)$.

2) Generate the instruments $z_{fls}(t_k)$ as

$$\tilde{y}_{fls}(t_k) = \frac{C(p)\hat{G}_{ls}(p)}{1 + C(p)\hat{G}_{ls}(p)} r_f(t_k) \tag{23}$$

$$\tilde{u}_{fls}(t_k) = \frac{1}{1 + C(p)\hat{G}_{ls}(p)} r_f(t_k) \tag{24}$$

$$z_{fls}(t_k) = [-\tilde{y}_{fls}(t_{k-1})\ \cdots\ -\tilde{y}_{fls}(t_{k-n_a})\ \ \tilde{u}_{fls}(t_{k-1})\ \cdots\ \tilde{u}_{fls}(t_{k-n_b})]^T \tag{25}$$

$z_{fls}(t_k)$ represents the noise-free part of the regressor $\phi_f(t_k)$. Determine the IV estimate of θ in (22) as

$$\hat{\theta}_{iv} = \hat{R}_{z_{fls}\phi_f}^{-1}\hat{R}_{z_{fls}y_f} \tag{26}$$

The corresponding estimated transfer function is given by $\hat{G}_{iv}(p) = \frac{\hat{B}_{iv}(p)}{\hat{A}_{iv}(p)}$.

3) Let $\hat{w}(t_k) = y_f^{(n_a)}(t_k) - \phi_f(t_k)\hat{\theta}_{iv}$. This equation holds for the CT case, and therefore it is still true at the sampling instants. By the way, an AR model of order $2n_a$ can be postulated for $\hat{w}(t_k)$: $L(q^{-1})\hat{w}(t_k) = e(t_k)$.
Estimate $L(q^{-1})$ using a least-squares method and denote the result by $\hat{L}(q^{-1})$.

4) Generate the instruments $z_{fiv}(t_k)$ as

$$z_{fiv}(t_k) = [-\tilde{y}_{fiv}(t_{k-1}) \ \cdots \ -\tilde{y}_{fiv}(t_{k-n}) \\ \tilde{u}_{fiv}(t_{k-1}) \ \cdots \ \tilde{u}_{fiv}(t_{k-n})]^T \quad (27)$$

with $\tilde{y}_{fiv}(t_k)$ and $\tilde{u}_{fiv}(t_k)$ computed as in equations (23)-(24) on the basis of $\hat{G}_{iv}(p)$. Using these instruments $z_{fiv}(t_k)$ and the prefilter $\hat{L}(q^{-1})$, determine the IV estimate of θ in (22) as

$$\hat{\theta}_{M_1} = \hat{R}^{-1}_{z_{fiv}\phi_{fT}} \hat{R}_{z_{fiv}y_{fT}}, \quad (28)$$

where $\phi_{fT}(t_k) = \hat{L}(q^{-1})\phi_f(t_k)$ and $y_{fT}(t_k) = \hat{L}(q^{-1})y_f(t_k)$.

2) Second solution: It has been shown previously that a first consistent estimation of the noise and process models has to be known in order to construct the instruments and the pre-filter. Since, the second order statistical property is not of crucial importance, a simple solution consists in estimating these models by using a least-squares estimator. The result will be obviously biased but the final estimate will not be affected by an inaccurate model obtained in this first step. Furthermore, in order to simplify the procedure and to avoid the noise modelling, it was chosen to restrict the second algorithm to the ARX models. In this particular case, the pre-filter $L(p)$ (eq (21)) is equal to 1.

Method clivc3.

1) Write the model structure as a linear regression (eq. (22)), and estimate θ by a least-squares method. Then, get $\hat{\theta}_{ls}$ along with the process and noise models $\hat{G}_{ls}(p) = \frac{\hat{B}_{ls}(p)}{\hat{A}_{ls}(p)}$, $\hat{H}_{ls}(p) = \frac{1}{\hat{A}_{ls}(p)}$ respectively.
2) Compute the noise-free part of the regressor

$$\tilde{\phi}_f(t_k) = [-\tilde{y}_{fls}(t_{k-1}) \ \cdots \ -\tilde{y}_{fls}(t_{k-n_a}) \\ \tilde{u}_{fls}(t_{k-1}) \ \cdots \ \tilde{u}_{fls}(t_{k-n_b})]^T$$

with $\tilde{y}_{fls}(t_k)$ and $\tilde{u}_{fls}(t_k)$ computed as in (23)-(24). Generate the instruments as $z_f(t_k) = \tilde{\phi}_f(t_k)$.
3) Using the instrument $z_f(t_k)$, determine the IV estimate in (22) as

$$\hat{\theta}_{M_2} = \hat{R}^{-1}_{z_f\phi_f} \hat{R}_{z_f y_f}. \quad (29)$$

VII. Example

The following numerical example is used to compare the performances of the proposed approaches. The process to be identified is described by equation (1), where

$$G_0(p) = \frac{p+1}{p^2+p-2}, \quad C(p) = \frac{10p+15}{p} \quad (30)$$

and $v_0(t_k) = H_0(p)e_0(t_k)$ is a white Gaussian noise. The excitation is chosen to be a pseudo-random binary signal of maximum length generated from a shift register of order 9 and a clock period equals to 8. The sampling period T_s is chosen equal to 1 ms.

From the comparative studies recently presented [3], the generalized Poisson moment functionals (*gpmf*) approach can be considered as one of the more efficient method to handle the time-derivative problem. This latter has been therefore associated with the two estimators (*clivc4*, *clivc3*) presented above. To illustrate the effectiveness of the algorithms and to investigate their performances, some Monte Carlo simulations of 200 runs with about 4000 data points have been performed for a signal to noise ratio

$$SNR = 10 \log \left(\frac{P_{y_d}}{\sigma_e^2} \right) = 0 \ \text{ dB}, \quad (31)$$

where σ denotes the standard deviation. P_{y_d} denotes the average power of the noise-free output fluctuation.

The *gpmf* transform of minimal order 3 has been applied and the Poisson filter coefficients have been set to $\lambda = \beta = 1$ (see $F_2(s)$ in equation (11)).

The process parameters are estimated by using methods *clivc4* and *clivc3*. Moreover, the results stemmed from the first closed-loop IV method developed by [8] and adapted to the CT model identification are also analyzed. This methods referenced as *clivc*, has been first presented in [20] and consists in using the reference signal time-derivatives as instruments; the estimate is thus given by

$$\hat{\theta}_{cliv} = \left[\sum_{i=1}^{N} \zeta_f(t_k)\phi_f^T(t_k) \right]^{-1} \sum_{i=1}^{N} \zeta_f(t_k) y_f^{(n_a)}(t_k) \quad (32)$$

$$\zeta_f^T(t_k) = \left[r_f^{n_a+n_b}(t_k) \quad \cdots \quad r_f(t_k) \right] \in \mathbb{R}^{n_a+n_b+1} \quad (33)$$

Furthermore, the results stemmed from the bias-eliminated least-squares method presented in [21] are also given for illustration purposes. This method is noted as *belsc*.

Bode diagrams of the 200 estimated CT models for the *clivc*, the *belsc*, *clivc4* and *clivc3* methods are plotted in figure 2. It can be seen that both near optimal IV techniques (*clivc4* and *clivc3*) are more accurate than the *bels*-based *gpmf* and the *clivc* algorithms. The proposed *clivc4* and *clivc3* methods give similar results and offer desirably accurate estimates, but in the particular case of an ARX model, the latter is numerically much cheaper to implement.

VIII. Conclusion

The identification problem of CT models of linear dynamic systems operating in closed-loop has been addressed in this paper by using closed-loop dedicated methods based on the instrumental variable techniques. Several closed-loop IV estimators have been studied along with their explicit expression for the covariance matrix of estimation errors. It is then shown that a minimal value of this covariance matrix can be achieved for a particular choice of instruments and prefilters. This minimal value requires the knowledge of the true system parameters and is therefore not reachable in practice. Several methods have thus been developed to determine the design parameters which allow to approximate the optimal closed-loop IV estimator. These methods have been compared to the recently suggested BELS methods which are known to lead to unbiased plant estimates in closed loop. However for arriving at estimates with attractive variance properties it is preferably to apply bootstrap IV methods as considered in this paper.

ACKNOWLEDGMENT

The authors gratefully acknowledge the useful comments by Professor G.P. Rao.

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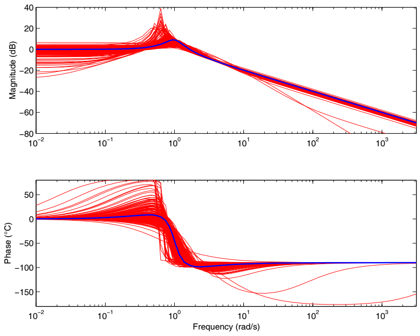
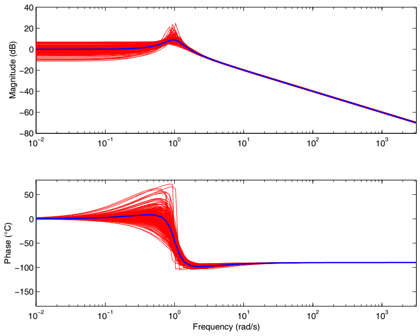
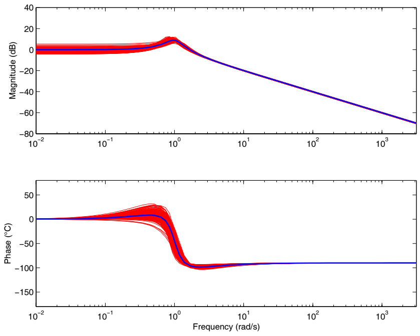


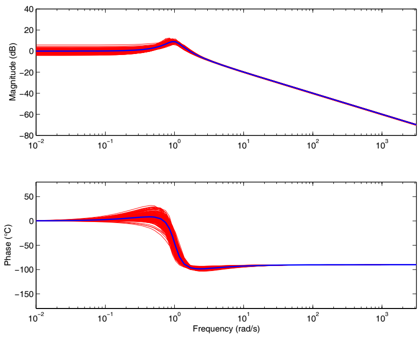
(a) *clivc* models



(b) *belsc* models



(c) *clivc4* models



(d) *clivc3* models

Fig. 2. Bode plots of the process (black) and of the estimates (grey)