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Refined Instrumental Variable methods for closed-loop system identification

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Abstract: This paper describes optimal instrumental variable methods for identifying discrete-time transfer function models when the system operates in closed-loop. Several noise models required for the design of optimal prefilters and instruments are analyzed and different approaches are developed according to whether the controller is known or not. Moreover, a new optimal refined instrumental variable technique is developed to handle the identification of a linear (ARX) predictor combined with an ARMA noise model in a closed-loop framework. The proposed refined instrumental variable algorithm achieves minimum variance estimation of the process model parameters. The performance of the proposed approaches is evaluated by Monte-Carlo analysis in comparison with other alternative closed-loop estimation methods.

Keywords: System identification; closed-loop identification; optimal instrumental variable.

1. INTRODUCTION

Feedback is present in a variety of practical situations due to safety and/or economic restrictions. In the last two decades, various attempts have been made to handle linear system identification in the presence of feedback. Indeed, closed-loop system identification leads to several difficulties due to the correlation between the disturbances and the control signal induced by the loop. Several methods have therefore been developed to cope with this problem either for discrete-time transfer function model identification (see *e.g.* Söderström and Stoica [1989], Van den Hof [1998], Forssell and Chou [1998], Forssell and Ljung [1999], Zheng [2003], Gevers [2005]) or for continuous-time TF model identification (see *e.g.* Gilson et al. [2008]). This paper focusses more specifically on instrumental variable (IV) techniques which present the major advantage of being able to consistently identify plant models in closed-loop while relying on simple linear-like ("pseudo-linear regression") algorithms and not relying on linearity nor knowledge of the controller. For closed-loop identification, a basic IV estimator was first suggested by Young [1970]; the topic was later discussed in more detail by Söderström et al. [1987]. More recently a so-called "tailor-made IV algorithm" was proposed Gilson and Van den Hof [2001], where the closed-loop plant is parameterized using the (open-loop) plant parameters. Then, an optimal (minimal) variance result was developed in the closed-loop extended IV identification case, revealing consequences for the choice of weights, filters and instruments Gilson and Van den Hof [2005].

This paper aims at presenting several solutions according to the chosen model structure. Therefore, after a quick review of two available solutions (for ARX and ARARX models), a new optimal IV-based technique is presented which is based on the identification of a more realistic Box-Jenkins (BJ) model, where the plant and the noise models are not constrained to have common polynomials. The identification problem is rewritten to make use of a linear-in-the-parameters predictor next to an additional noise model identification required for determining the optimal prefilter and instrument. Moreover, two situations are studied depending on the controller knowledge.

The paper is organized as follows. After the preliminaries, the lower bound of the covariance matrix is recalled in Section 3. Section 4 shows how the refined IV method can be used to provide consistent and efficient estimates to the closed-loop identification problem. The different algorithms are then presented in Section 5. Finally, in Section 6, the comparison between different methods is illustrated with the help of Monte Carlo simulation examples.

2. PROBLEM FORMULATION

Preliminaries.

Consider a stable, linear, Single Input Single Output, closed-loop system of the form shown in Figure 1. The data generating system is assumed to be given by the relations

$$\mathcal{S} : \begin{cases} y(t) = G_0(q)u(t) + H_0(q)e_0(t) \\ u(t) = r(t) - C_c(q)y(t). \end{cases} \tag{1}$$

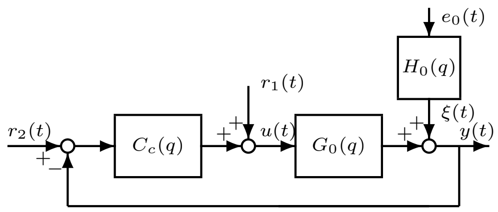

Fig. 1. Closed-loop configuration

The process is denoted by $G_0(q) = B_0(q^{-1})/F_0(q^{-1})$ with the numerator and denominator degree equals to n_0, the controller is denoted by $C_c(q)$ and q^{-1} is the delay operator with $q^{-i}x(t) = x(t-i)$. $u(t)$ describes the process input signal, $y(t)$ the process output signal. For ease of notation we introduce an external signal

$$r(t) = r_1(t) + C_c(q)r_2(t). \quad (2)$$

A coloured disturbance $\xi_0(t) = H_0(q)e_0(t)$ is assumed to affect the closed-loop, where $e_0(t)$ is a white noise, with zero mean and variance $\sigma^2_{e_0}$.

The following general model structure is chosen to model the system (1)

$$\mathcal{M}: \; y(t) = G(q,\rho)u(t) + H(q,\eta)\varepsilon(t,\theta), \quad (3)$$

where the parameter vector is given as $\theta^T = (\rho^T \; \eta^T)$. The parameterized process model takes then the form,

$$\mathcal{G}: G(q,\rho) = \frac{B(q^{-1},\rho)}{A(q^{-1},\rho)} = \frac{b_1q^{-1} + \cdots + b_nq^{-n}}{1 + a_1q^{-1} + \cdots + a_nq^{-n}}, \quad (4)$$

where n denotes the process model order and with the pair (A, B) assumed to be coprime. The process model parameters are stacked columnwise in the parameter vector

$$\rho = [a_1 \; \cdots \; a_n \; b_1 \; \cdots \; b_n]^T \in \mathbb{R}^{2n}. \quad (5)$$

The process model order n is assumed known or identified from the data and the parameterized noise model is assumed to be in the form of the following ARMA process,

$$\mathcal{H}: H(q,\eta) = \frac{C(q^{-1},\eta)}{D(q^{-1},\eta)} = \frac{1 + c_1q^{-1} + \cdots + c_mq^{-m}}{1 + d_1q^{-1} + \cdots + d_mq^{-m}}, \quad (6)$$

where the associated noise model parameters are stacked columnwise in the parameter vector,

$$\eta = [d_1 \; \cdots \; d_m \; c_1 \; \cdots \; c_m]^T \in \mathbb{R}^{2m}. \quad (7)$$

Note that this paper deals with IV-based methods which are known to give consistent plant model parameter estimates of G_0 irrespective of the structure of H_0.

Additionally, the controller $C_c(q)$ is given by

$$C_c(q) = \frac{Q(q^{-1})}{P(q^{-1})} = \frac{q_0 + q_1q^{-1} + \cdots + q_{n_c}q^{-n_c}}{p_0 + p_1q^{-1} + \cdots + p_{n_c}q^{-n_c}}, \quad (8)$$

with the pair (P, Q) assumed to be coprime. In the following, the closed-loop system is assumed to be asymptotically stable and $r(t)$ is an external signal that is persistently exciting of sufficient high order.

With these notations, the closed-loop system can be described as

$$y(t) = \frac{G_0(q)}{1 + C_c(q)G_0(q)}r(t) + \frac{1}{1 + C_c(q)G_0(q)}\xi_o(t)$$
$$u(t) = \frac{1}{1 + C_c(q)G_0(q)}r(t) - \frac{C_c(q)}{1 + C_c(q)G_0(q)}\xi_o(t). \quad (9)$$

In the following instrumental variable algorithms, use is made of the noise-free input/output signals deduced from (2) and denoted from hereon as

$$\mathring{u}(t) = \frac{1}{1 + C_c(q)G_0(q)}r(t), \; \mathring{y}(t) = \frac{G_0(q)}{1 + C_c(q)G_0(q)}r(t). \quad (10)$$

The noise-free regressor is then defined as follows

$$\mathring{\varphi}^T(t) = [-\mathring{y}(t-1) \cdots -\mathring{y}(t-n) \; \mathring{u}(t-1) \cdots \mathring{u}(t-n)]. \quad (11)$$

Now consider the relationship between the process input and output signals in (1),

$$y(t) = G_0(q)u(t) + H_0(q)e_0(t). \quad (12)$$

If the plant G_0 is included into the chosen model set $\mathcal{G}$ ($G_0 \in \mathcal{G}$), $y(t)$ can be written as

$$y(t) = \varphi^T(t)\rho_0 + v_0(t), \quad (13)$$

where ρ_0 denotes the true plant parameter vector,

$$\varphi^T(t) = [-y(t-1) \cdots -y(t-n) \; u(t-1) \cdots u(t-n)] \quad (14)$$

and $v_0(t) = F_0(q^{-1})H_0(q)e_0(t)$.

The objective is to estimate the parameter vector from the collected data $y(t)$, $u(t)$ and $r(t)$ with or without the knowledge of the controller.

Extended IV

The well-known extended-IV estimate is given by (see *e.g.* Söderström and Stoica [1983])

$$\hat{\rho}_{xiv}(N) = \arg\min_{\rho} \left\| \left[\frac{1}{N}\sum_{t=1}^{N} L(q)\zeta(t)L(q)\varphi^T(t) \right] \rho - \left[\frac{1}{N}\sum_{t=1}^{N} L(q)\zeta(t)L(q)y(t) \right] \right\|^2_W, \quad (15)$$

where $\zeta(t)$ is the instrument vector, $\|x\|^2_W = x^TWx$, with W a positive definite weighting matrix and $L(q)$ a stable prefilter.

By definition, when $G_0 \in \mathcal{G}$, the extended-IV estimate provides a consistent estimate under the following two conditions [1]

- $\bar{\mathbb{E}}L(q)\zeta(t)L(q)\varphi^T(t)$ is full column rank,
- $\bar{\mathbb{E}}L(q)\zeta(t)L(q)v_0(t) = 0$.

3. LOWER BOUND FOR AN IV METHOD

The choice of the instrumental variable vector $\zeta(t)$, the number of instruments n_ζ, the weighting matrix W and the prefilter $L(q)$ may have a considerable effect on the covariance matrix P_{xiv} produced by the IV estimation algorithm. In the open-loop situation the lower bound of the covariance matrix for any unbiased identification method is given by the Cramer-Rao bound (see *e.g.* Söderström and Stoica [1983] and Ljung [1999]). The closed-loop situation has been investigated recently in Gilson and Van den Hof [2005], therefore only the main results to be used in the following are recalled here. It has been shown that a minimum value of the covariance matrix P_{xiv} as a function of the design variables $\zeta(t)$, $L(q)$ and W exists under the restriction that $\zeta(t)$ is a causal function of the external signal $r(t)$ only. In that case

$$P_{xiv} \geq P^{opt}_{xiv}$$

[1] The notation $\bar{\mathbb{E}}[.] = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \mathbb{E}[.]$ is adopted from the prediction error framework of Ljung [1999].

with

$$P_{xiv}^{opt} = \sigma_{e_0}^2 [\bar{\mathbb{E}} \mathring{\varphi}_f(t) \mathring{\varphi}_f^T(t)]^{-1}, \quad (16)$$

$$\mathring{\varphi}_f(t) = L^{opt}(q) \mathring{\varphi}(t), \quad (17)$$

$$L^{opt}(q) = \frac{1}{F_0(q^{-1}) H_0(q)}, \quad (18)$$

and where $\mathring{\varphi}(t)$ is the noise-free part of $\varphi(t)$ (see equation (11)). It has been shown that the minimum variance result can be achieved by the following optimal choice of design variables

$$W = I \text{ and } n_\zeta = 2n, \quad (19)$$

$$L(q) = L^{opt}(q) \quad (20)$$

$$\zeta(t) = \mathring{\varphi}(t). \quad (21)$$

Using equations (15) and (19)-(21), the IV estimate using these optimal design variables is given by

$$\hat{\rho}^{opt}(N) = \hat{R}_{\zeta_f \varphi_f}^{-1}(N) \hat{R}_{\zeta_f y_f}(N) \quad (22)$$

with

$$\hat{R}_{\zeta_f \varphi_f}(N) = \frac{1}{N} \sum_{t=1}^{N} \zeta_f(t) \varphi_f^T(t), \quad (23)$$

$$\hat{R}_{\zeta_f y_f}(N) = \frac{1}{N} \sum_{t=1}^{N} \zeta_f(t) y_f(t) \quad (24)$$

and where the regressor $\varphi_f(t) = L^{opt}(q)\varphi(t)$, the output $y_f(t) = L^{opt}(q)y(t)$ and the instrument vector $\zeta_f(t) = L^{opt}(q)\zeta(t)$ are filtered by $L^{opt}(q)$ (18).

As a result, note that in the considered IV estimator (15), the optimal choice of instruments and prefilter is dependent on unknown system properties, *i.e.* the plant as well as the noise dynamics. Whereas dependency of plant dynamics could be taken care of by an iterative procedure where the instruments and prefilter are constructed on the basis of a previous process model $\hat{\rho}^{i-1}$ in order to provide an improved process model $\hat{\rho}^i$ by applying (15); knowledge of the noise dynamics $H_0(q)$ is generally missing in an IV estimator like (15) as it is not particularly estimated.

Therefore the next step in an optimal IV method should be to extend the estimator (15) with a procedure to estimate an appropriate noise model, to be used as a basis for constructing the optimal prefilter $L^{opt}(q)$ given in (18).

4. OPTIMAL IV IDENTIFICATION IN CLOSED-LOOP

4.1 Introduction

The optimal IV identification method is based on estimator (15). However, this latter one needs to be complemented with the estimation of a noise model. In this perspective, a general prediction error identification step is added to estimate a model for $H_0(q)$. This requires the choice of a particular parametrization for $H(q, \theta)$, possibly in relation to $G(q, \rho)$. Two options have been used so far in the literature Gilson and Van den Hof [2005]:

- ARX structure:

  $$A(q^{-1}, \rho)y(t) = B(q^{-1}, \rho)u(t) + \varepsilon(t, \rho).$$

  In this case the noise model

  $$H(q, \hat{\rho}) = 1/A(q^{-1}, \hat{\rho}) \quad (25)$$

  is already available from the process model estimate, and no additional noise model estimate is required. On the basis of the estimated process model $G(q, \hat{\rho})$ the corresponding filter is given by

  $$L(q, \hat{\rho}) = 1/[A(q^{-1}, \hat{\rho})H(q, \hat{\rho})] = 1 \quad (26)$$

  $\hat{\rho}$ is estimated during the model identification step.
- ARARX structure:

  $$A(q^{-1}, \rho)y(t) = B(q^{-1}, \rho)u(t) + \frac{1}{D(q^{-1}, \eta)}\varepsilon(t, \theta).$$

  In this case the corresponding noise model

  $$H(q, \eta) = 1/A(q^{-1}, \hat{\rho})D(q^{-1}, \eta) \quad (27)$$

  is not available from the process model $\hat{\rho}$ only as previously, but an additional noise model estimate is required. In Gilson and Van den Hof [2005], this is achieved by identifying $D(q^{-1}, \eta)$ as an autoregressive model $D(q^{-1}, \eta)w(t) = \varepsilon(t, \eta)$ with

  $$w(t) = A(q^{-1}, \hat{\rho})y(t) - B(q^{-1}, \hat{\rho})u(t), \quad (28)$$

  by using a first process model estimate ($\hat{\rho}$) and by applying a least square estimator

  $$\hat{\eta} = \arg\min_{\eta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t, \eta)^2.$$

  The optimal filter that results from this identification procedure is therefore

  $$L(q, \hat{\eta}) = D(q^{-1}, \hat{\eta}). \quad (29)$$

Note that these two methods rely on special structures of the estimated noise models, and therefore limit the possibility to reach consistent estimates of $H_0(q)$ in situations where these structures are not in accordance with the properties of the underlying systems. Therefore, in this paper, the options are extended to other more general model structures.

4.2 Extension to BJ and OE structure

The more general structure is obtained by choosing a *Box Jenkins* (BJ) model structure for identifying the noise dynamics, which takes the form

$$y(t) = \frac{B(q^{-1}, \rho)}{F(q^{-1}, \rho)}u(t) + \frac{C(q^{-1}, \eta)}{D(q^{-1}, \eta)}\varepsilon(t, \rho, \eta), \quad (30)$$

and a natural way to extend the IV estimator (15) with an identification of the noise model η is to write

$$v(t) = \frac{C(q^{-1}, \eta)}{D(q^{-1}, \eta)}\varepsilon(t, \hat{\rho}, \eta) \quad (31)$$

with $v(t) = y(t) - B(q^{-1}, \hat{\rho})/F(q^{-1}, \hat{\rho})u(t)$ being available as a measured/reconstructed signal once IV estimator (15) has delivered a process model $\hat{\rho}$. Estimation of η in the above equation is then undertaken by an ARMA estimation algorithm on the basis of $v(t)$.

The optimal prefilter that results from this identification procedure is then given by

$$L(q, \hat{\rho}, \hat{\eta}) = D(q^{-1}, \hat{\eta})/[F(q^{-1}, \hat{\rho})C(q^{-1}, \hat{\eta})]. \quad (32)$$

When choosing an *Output Error* (OE) model structure for the noise dynamics, we arrive at

$$y(t) = \frac{B(q^{-1}, \rho)}{F(q^{-1}, \rho)}u(t) + \varepsilon(t, \rho), \quad (33)$$

and of course no additional estimation of the noise dynamics is required. The optimal prefilter however changes, and now becomes given by

$$L(q, \hat{\rho}) = 1/F(q^{-1}, \hat{\rho}). \quad (34)$$

5. THE ITERATIVE IV ALGORITHM

The outline of the optimal IV algorithm in a unified way for the 4 considered model structures, within the closed-loop context is given below. Two versions are provided according to whether the controller is known or not. Moreover, the proposed algorithm is chosen to be iterative to refine the parameter estimates as it is achieved in the optimal refined IV estimation technique initially developed for open-loop system identification (see *e.g.* Young [1984]).

5.1 Case 1 - cliv *with known controller*

Step 1. ***Initialisation: estimate a first model***
Apply a basic IV method

$$\hat{\rho}^0 = \left[\sum_{t=1}^{N} \zeta(t)\varphi^T(t)\right]^{-1} \sum_{t=1}^{N} \zeta(t)y(t), \tag{35}$$

where $\varphi(t)$ is given by (14). Several IV methods may be chosen, as *e.g.* using the delayed version of the excitation signal $r(t)$ as instruments. This yields $B(q^{-1}, \hat{\rho}^0)$ and $F(q^{-1}, \hat{\rho}^0)$. Denote the corresponding transfer function by $G(q, \hat{\rho}^0) = B(q^{-1}, \hat{\rho}^0)/F(q^{-1}, \hat{\rho}^0)$. Set the initial noise model estimates $C(q^{-1}, \hat{\eta}^0) = D(q^{-1}, \hat{\eta}^0) = 1$ and $i = 1$.

Step 2. ***Estimate by IV***
Generate the filtered instruments [2] according to the model structure used as

$$\begin{cases} L(q, \hat{\theta}^{i-1}) \text{ computed using either (26), (29), (32), or (34)} \\ \hat{\mathring{y}}(t, \hat{\rho}^{i-1}) = \dfrac{G(q, \hat{\rho}^{i-1})}{1 + C_c(q)G(q, \hat{\rho}^{i-1})} r(t), \\ \hat{\mathring{u}}(t, \hat{\rho}^{i-1}) = \dfrac{1}{1 + C_c(q)G(q, \hat{\rho}^{i-1})} r(t), \\ \zeta_f(t, \hat{\theta}^{i-1}) = L(q, \hat{\theta}^{i-1}) \Big[-\hat{\mathring{y}}(t-1, \hat{\rho}^{i-1}) \ldots -\hat{\mathring{y}}(t-n, \hat{\rho}^{i-1}) \\ \qquad\qquad \hat{\mathring{u}}(t-1, \hat{\rho}^{i-1}) \ldots \hat{\mathring{u}}(t-n, \hat{\rho}^{i-1}) \Big] \end{cases}$$

$\zeta_f(t, \hat{\theta}^{i-1})$ can be seen as a filtered estimate of the noise-free part of the regressor vector $\varphi(t)$ (14) based on estimates $\hat{\mathring{y}}(t)$ and $\hat{\mathring{u}}(t)$ of the noise-free output and input to the plant, respectively. Determine the IV estimate using the prefilter and these instruments

$$\hat{\rho}^i = \left[\sum_{t=1}^{N} \zeta_f(t, \hat{\theta}^{i-1})\varphi_f^T(t, \hat{\theta}^{i-1})\right]^{-1} \sum_{t=1}^{N} \zeta_f(t, \hat{\theta}^{i-1}) y_f(t, \hat{\theta}^{i-1}) \tag{36}$$

$$\begin{cases} \varphi_f(t, \hat{\theta}^{i-1}) = L(q, \hat{\theta}^{i-1})\varphi(t) \\ y_f(t, \hat{\theta}^{i-1}) = L(q, \hat{\theta}^{i-1}) y(t) \end{cases} \tag{37}$$

This yields $B(q^{-1}, \hat{\rho}^i)$ and $F(q^{-1}, \hat{\rho}^i)$. Denote the corresponding transfer function by $G(q, \hat{\rho}^i) = B(q^{-1}, \hat{\rho}^i)/F(q^{-1}, \hat{\rho}^i)$.

Step 3. ***Obtain an optimal estimate of the noise model parameter vector η^i based on the estimated noise sequence:***
Use one of the noise model identification scheme described in Section 4 to estimate $\hat{\eta}^i$ and the associated transfer function $H(q, \hat{\eta}^i)$.

Step 4. ***For OE and BJ model strutures: repeat from step 2***. Stop when $F(q^{-1}, \hat{\rho})$, $B(q^{-1}, \hat{\rho})$, $H(q, \hat{\eta})$ and $L(q^{-1}, \hat{\theta})$ have converged.

[2] i stands for the ith iteration

Step 5. ***Compute the estimated parametric error covariance matrix $\hat{P}_\theta$ associated with the*** *cliv* ***parameter estimates, from***

$$\hat{P}_\theta = \hat{\sigma}^2 \left[\varphi_f^T(t, \hat{\theta})\varphi_f(t, \hat{\theta})\right]^{-1}, \tag{38}$$

where $\hat{\sigma}^2$ is the sample variance of the estimated residuals. Then, according to the model structure used, the resulting algorithm will be referred to as $cliv_{arx}$, $cliv_{ararx}$, $cliv_{bj}$ or $cliv_{oe}$.

5.2 Case 2 - cliv *with unknown controller*

When the controller is known, it is worthwhile to use this information into the identification procedure, to provide with more accurate results. In the previous algorithm, the controller is used with the open-loop system model to construct the instruments and the filter. However, when it is unknown, another solution may be used to build up the instrumental vector which satisfies the optimal conditions (19)-(21). Indeed, the noise-free estimation of this instrumental vector can be achieved by using the two closed-loop transfers between $r(t), u(t)$ and between $r(t), y(t)$ instead of the open-loop one (between $u(t)$ and $y(t)$). The second step consists then in identifying the two closed-loop transfers $G_{yr}(q, \Theta_{yr})$ and $G_{ur}(q, \Theta_{ur})$ using two basic IV methods as

$$\hat{\Theta}_{yr} = \left[\sum_{t=1}^{N} \zeta_r(t)\varphi_{yr}^T(t)\right]^{-1} \sum_{t=1}^{N} \zeta_r(t)y(t) \tag{39}$$

$$\hat{\Theta}_{ur} = \left[\sum_{t=1}^{N} \zeta_r(t)\varphi_{ur}^T(t)\right]^{-1} \sum_{t=1}^{N} \zeta_r(t)u(t), \tag{40}$$

where the instruments are a delayed version of the excitation signal and the regressors make use of the input/ouput signals. These closed-loop estimates are then used to compute the instruments as

$$\hat{\mathring{y}}(t, \hat{\Theta}_{yr}) = G_{yr}(q, \hat{\Theta}_{yr})r(t), \;\; \hat{\mathring{u}}(t, \hat{\Theta}_{ur}) = G_{ur}(q, \hat{\Theta}_{ur})r(t).$$

The estimation of ρ is then achieved using the same way as in (36), only the instrument vector computation is changed.

5.3 Comments

- *Adaptive* optimal prefiltering of both I/O data signals is an inherent part of the *cliv* estimation.
- One of the advantages of the proposed algorithms is that it provides consistent plant estimates while still exploiting the pseudo-linear regression type of estimation. Indeed, the IV based pseudo-linear regression method recently suggested in Young [2006] can be used to estimate the ARMA process noise in the third step of the algorithms.
- The $cliv_{oe}$ algorithm obtained when $C(q^{-1}, \eta) = D(q^{-1}, \eta) = 1$ has some similarities with the algorithm of Steiglitz and McBride adapted to the closed-loop situation. However, the Steiglitz-McBride algorithm uses iterative least-squares rather than iterative optimal IV; as a result it is consistent only under restrictive conditions (see Stoica and Söderström [1981]) and therefore is less robust than $cliv_{oe}$.

6. SIMULATION EXAMPLES

The numerical examples presented in this section are used to illustrate the performance of the proposed methods. The plant to be identified is described by equation (1), where

$$G_0(q) = \frac{0.0997q^{-1} - 0.0902q^{-2}}{1 - 1.8858q^{-1} + 0.9048q^{-2}}, \quad n = 2$$

$$C_c(q) = \frac{10.75 - 9.25q^{-1}}{1 - q^{-1}},$$

$r(t)$ is a deterministic sequence: realization of a pseudo random binary signal of maximal length, with the number of stages of the shift register set to 9 and the clock period set to 8; $e_0(t)$ is a white noise uncorrelated with $r(t)$.

6.1 Example 1: white noise

Firstly, a white noise disturbance ($H_0(q) = 1$) is considered in order to evaluate the performance $cliv_{oe}$ algorithm in the case $\mathcal{S} \in \mathcal{M}$. The plant parameters are estimated on the basis of closed-loop data of length $N = 4088$. A Monte-Carlo simulation of 100 runs is used for a signal-to-noise (SNR) ratio equal to 35dB. The proposed method is compared to:

- *clivr*: the first basic IV method developed to handle the closed-loop case and which uses the delayed version of the reference signal as instruments (Söderström et al. [1987]);
- $cliv_{arx}$: the optimal IV method with an ARX model structure making use of the controller knowledge;
- *pem*: applied to the closed-loop data as described *e.g.* in Forssell and Ljung [1999] known to be theoretically efficient in the $\mathcal{S} \in \mathcal{M}$ case;
- $cliv_{oe1}$: making use of the controller knowledge;
- $cliv_{oe2}$: assuming the controller unknown.

The Monte Carlo simulation (MCS) results are presented in Table 1 where the mean and standard deviation of the estimated parameters are displayed. The average number of iterations (N_{iter}) for the *pem* and $cliv_{oe}$ algorithms are also given. It can be seen that all of the methods deliver accurate results, with smaller standard deviations for *pem* and for the proposed $cliv_{oe}$ type of methods. Indeed, these methods lead to better results thanks to the iterative estimation procedure, even though the number of iterations required for convergence is quite low. As expected, the *clivr* method provides the least accurate results since it is a simple basic (and not optimal) IV approach.

6.2 Example 2: colored noise

A second example is used to analyse the performance of the proposed methods in the case of a colored noise, with

$$H_0(q) = \frac{1 + 0.5q^{-1}}{1 - 0.85q^{-1}}.$$

As previously, the proposed $cliv_{bj}$ methods are compared to *clivr*, $cliv_{ararx}$, *pem* algorithms. The plant parameters are estimated on the basis of closed-loop data of length $N = 4088$. A Monte Carlo simulation of 100 experiments is performed for SNR = 25dB. The mean and standard deviation of the estimated parameters for the 100 realizations are given in Table 2 along with the averaged number of iterations needed for the $cliv_{bj}$ methods. The Bode diagrams of the 100 models identified by the *clivr*, *pem*, $cliv_{ararx}$ and $cliv_{bj}$ algorithms are displayed in Figure 2.

These results show that all of the optimal-IV based methods give efficient results whereas the classical *clivr* and the *pem* algorithms fail to give accurate estimates. Indeed, the approximate optimal IV type of algorithms ($cliv_{ararx}$ and $cliv_{bj}$) give estimates with smaller standard deviation errors compared to the *clivr* algorithm, thanks to the optimal choices of instruments and filter. Furthermore, the $cliv_{bj}$ algorithms yields results with smaller standard deviation than the $cliv_{ararx}$ algorithm thanks to a more accurate noise model estimation.

Moreover, on the Bode diagrams, it can be noticed that the *pem* algorithm is not able to converge to the global minimum at each run and therefore leads to erroneous results.

It can also be noted that although the $cliv_{bj2}$ algorithm uses less information to identify the model (the controller is supposed to be unknown), the corresponding results remain really accurate. Therefore this method which aims at identifying a consistent BJ model without the knowledge of the controller should be very interesting in many practical situations.

7. CONCLUSION

This paper has considered the identification of several transfer function model structures within a closed-loop environment, using the instrumental variable technique modified to solve the closed-loop situation. The general Box-Jenkins model structure identification has been handled by making use of a linear-in-the-parameters predictor next to an additional noise model identification required for determining the optimal filter and the instruments. Two approaches have then been suggested according to whether the controller is known or not. It has been shown that a minimal value of the associated parametric error covariance matrix can be achieved by an appropriate choice of instruments and prefilters. The estimated Box-Jenkins model has the advantage of not constraining the plant and the noise models to have common polynomials.

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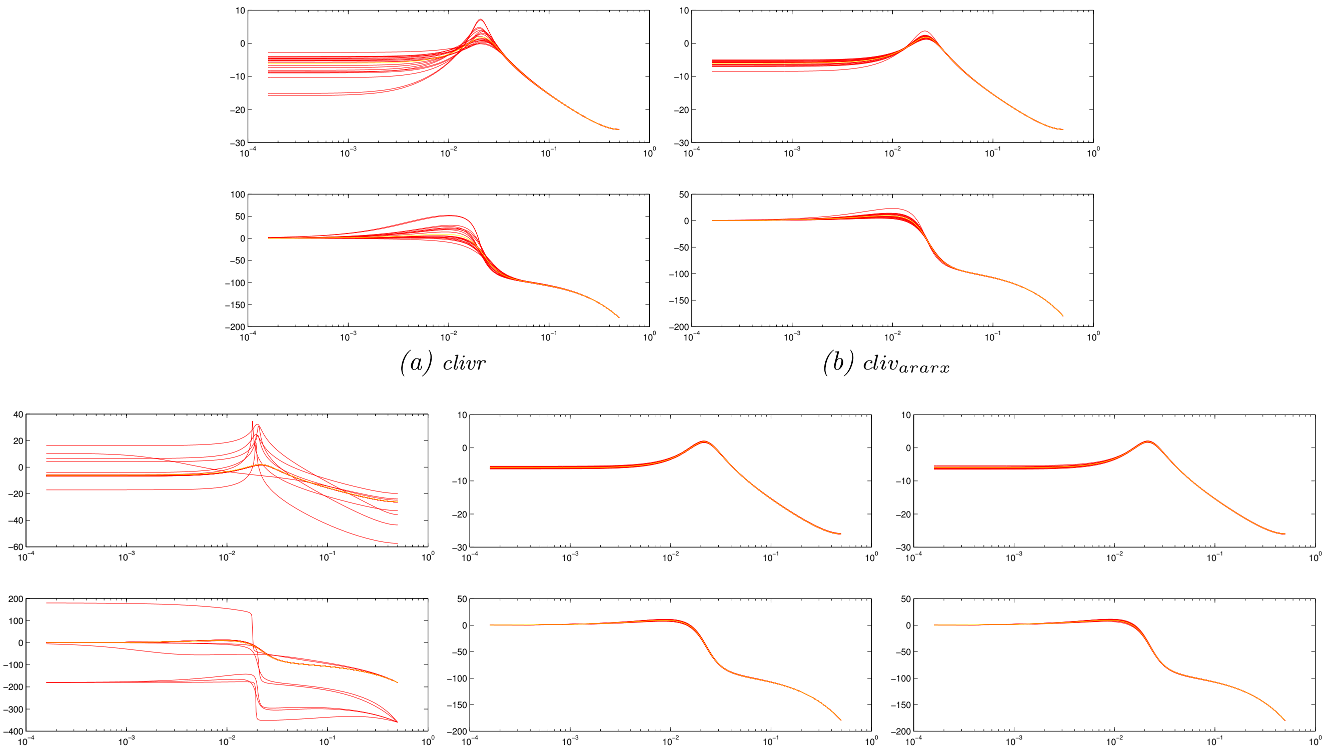
parameters true values	$\hat{b}_1$ 0.0997	$\hat{b}_2$ -0.0902	$\hat{a}_1$ -1.8838	$\hat{a}_2$ 0.9048	N_{iter}
$clivr$	$0.0997 \pm 0.3e{-}3$	$-0.0898 \pm 3.8e{-}3$	$-1.8821 \pm 35e{-}3$	$0.9014 \pm 32.4e{-}3$	
$cliv_{arx}$	$0.0997 \pm 0.3e{-}3$	$-0.0900 \pm 1.1e{-}3$	$-1.8772 \pm 9e{-}3$	$0.8977 \pm 8.5e{-}3$	
pem	$0.0996 \pm 0.05e{-}3$	$-0.0901 \pm 0.06e{-}3$	$-1.8858 \pm 0.12e{-}3$	$-0.9048 \pm 0.10e{-}3$	4.14
$cliv_{oe1}$	$0.0997 \pm 0.05e{-}3$	$-0.0902 \pm 0.06e{-}3$	$-1.8858 \pm 0.10e{-}3$	$0.9048 \pm 0.09e{-}3$	3.89
$cliv_{oe2}$	$0.0997 \pm 0.05e{-}3$	$-0.0902 \pm 0.06e{-}3$	$-1.8858 \pm 0.12e{-}3$	$-0.9048 \pm 0.10e{-}3$	3.53

Table 1. Mean and standard deviation of the 100 estimated models, white noise

parameters true values	$\hat{b}_1$ 0.0997	$\hat{b}_2$ -0.0902	$\hat{a}_1$ -1.8838	$\hat{a}_2$ 0.9048	N_{iter}
$clivr$	$0.0992 \pm 0.5e{-}3$	$-0.0900 \pm 4.9e{-}3$	$-1.8834 \pm 44.3e{-}3$	$0.9025 \pm 41.1e{-}3$	
$cliv_{ararx}$	$0.0998 \pm 0.5e{-}3$	$-0.0898 \pm 2.4e{-}3$	$-1.8823 \pm 21.1e{-}3$	$0.9014 \pm 19.6e{-}3$	
pem	$0.0793 \pm 64.8e{-}3$	$-0.0715 \pm 64e{-}3$	$-1.8993 \pm 34.7e{-}3$	$0.9181 \pm 33.8e{-}3$	4.22
$cliv_{bj1}$	$0.0997 \pm 0.7e{-}3$	$-0.0903 \pm 0.7e{-}3$	$-1.8856 \pm 3.6e{-}3$	$0.9048 \pm 3.2e{-}3$	5.88
$cliv_{bj2}$	$0.0997 \pm 0.6e{-}3$	$-0.0903 \pm 0.7e{-}3$	$-1.8860 \pm 3.7e{-}3$	$0.9050 \pm 3.4e{-}3$	5

Table 2. Mean and standard deviation of the 100 estimated models, colored noise



(a) $clivr$ (b) $cliv_{ararx}$

(c) pem (d) $cliv_{bj1}$ (e) $cliv_{bj2}$

Fig. 2. Estimated Bode diagrams (gain and phase (degree)) of the plant model $G(q, \eta)$ over the 100 MCS, colored noise

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