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# CONTROLLER TUNING FREEDOM UNDER PLANT IDENTIFICATION UNCERTAINTY: DOUBLE YOULA BEATS GAP IN ROBUST STABILITY

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Abstract: In iterative schemes of identification and control one of the particular and important choices to make is the choice for a model uncertainty structure, capturing the uncertainty concerning the estimated plant model. This is typically done in some norm-bounded form, in order to guarantee robust stability and/or robust performance when redesigning the controller. Structures that are used in the recent literature encompass e.g. gap metric uncertainty, coprime factor uncertainty, and the Vinnicombe gap metric uncertainty. In this paper we study the effect of these choices when our aim is to maximize the (re)tuning freedom for a present controller (in terms of a norm-bounded perturbation) under conditions of robust stability. Particular attention will be given to the representation of plant uncertainty and controller tuning freedom in terms of Youla parameters. In the problem formulation considered here the so-called double Youla parametrization provides a norm-bounded set of robustly stabilizing controllers that is larger than corresponding sets that are achieved by using any of the other uncertainty structures.

Keywords: Identification for control; model uncertainty; robust stability; robust control; system identification; gap metric; Youla parametrization.

## 1. PROBLEM SET-UP

We consider linear time-invariant finite dimensional systems and controllers in $\mathbb{R}H_\infty$, in a feedback configuration depicted in figure 1, denoted by $H(G_0,C)$, where $G_0$ is the plant to be (modelled and) controlled, and $C$ a present and known controller to be redesigned.

The closed-loop dynamics of $H(G_0,C)$ are described by the transfer matrix

$$T(G_0,C) = \begin{bmatrix} G_0 \\ I \end{bmatrix} (I+CG_0)^{-1}[C \;\; I],$$

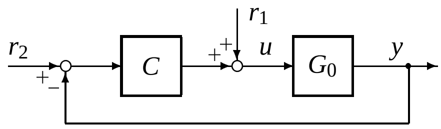


Fig. 1. Feedback interconnection $H(G_0,C)$.

which maps the vector of variables $col(r_1,r_2)$ into $col(y,u)$. The closed-loop system is stable if and only if $T(G_0,C) \in \mathbb{R}H_\infty$.
The problem field that we consider can be formulated as follows:

> Consider an (unknown) plant $G_0$ controlled by a known controller $C$, redesign the controller so as to achieve a better

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control performance for the controlled plant $G_0$.

There are several different aspects that can be distinguished in this problem, as e.g.

- One can construct an identified (uncertainty) model of the plant $G_0$ on the basis of experimental data, e.g. composed of a nominal model and some norm-bounded model or parameter uncertainty. See e.g. (Ninness and Goodwin, 1995; Hakvoort and Van den Hof, 1997; De Vries and Van den Hof, 1995; Gevers et. al., 1999).
- The redesigning of the controller can be performed on the basis of a single model (nominal design possibly extended with robustness verifications), a (norm-bounded) uncertainty model (robust design), or on no model at all (as e.g. in iterative feedback tuning (Hjalmarsson et. al., 1994)).

If in the controller redesign a (norm-bounded) uncertainty model is taken into account, then the worst-case performance of the newly designed control system can be optimized. This approach is e.g. followed in (de Callafon and Van den Hof, 1997) where the control design step is a robust control design optimizing the worst-case performance cost. If the identified uncertainty set contains the underlying real plant, guaranteed performance bounds will hold for the controlled real plant also. In this approach the control design utilizes all (uncertain) information on the plant that is available. The resulting control design algorithm becomes relatively complex ($\mu$-synthesis in the work of (de Callafon and Van den Hof, 1997)).

When in the controller redesign only a nominal model is taken into account for the design itself, and an uncertainty model for the plant is used a posteriori to verify the robustness of this design, there is a need for robustness tests concerning stability (and possibly performance).

In this contribution we focus on the latter situation, assuming that the controller $C$ has to be redesigned (retuned) into $C_{new}$ by some (not specified) design method, and that the aim is to construct a norm-bounded area around $C$ that characterizes the tuning freedom for $C_{new}$ under conditions of robust stability, i.e. under the condition that $C_{new}$ stabilizes all models in the identified uncertainty set. The size of the norm-bounded set of controllers is typically dependent on the uncertainty structure that is chosen to represent the plant identification uncertainty. In this paper different structures will be analysed and compared. In particular a gap metric uncertainty structure will be applied and will be shown to lead to results that are more conservative than the results that are obtained when employing a so-called double Youla representation of plant uncertainty and controller retuning freedom. In a second stage similar results will be derived for uncertainty sets in terms of the Vinnicombe gap metric and the so-called $\Lambda$-gap.

## 2. PRELIMINARIES

A coprime factor framework will be used to represent plants and controllers, employing both right coprime and left coprime factorizations:

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s)$$
$$C(s) = N_c(s)D_c^{-1}(s) = \tilde{D}_c^{-1}(s)\tilde{N}_c(s) \qquad (1)$$

where $(N,D)$ and $(N_c,D_c)$ are right coprime factorizations *(rcf)* and $(\tilde{N},\tilde{D})$ and $(\tilde{N}_c,\tilde{D}_c)$ are left coprime factorizations *(lcf)* over $\mathbb{R}H_\infty$ (Vidyasagar, 1985). The coprime factorizations are normalized $(nrcf)$, $(nlcf)$ if they additionally satisfy $\bar{N}^*\bar{N}+\bar{D}^*\bar{D}=I$ and $\tilde{N}\tilde{N}^*+\tilde{D}\tilde{D}^*=I$, where $(\cdot)^*$ denotes complex conjugate transpose. The notation $\overline{(\cdot)}$ will be used to denote normalized factorizations. Let $G$ and $C$ have coprime factorizations as in (1) and let $\Lambda,\tilde{\Lambda}\in\mathbb{R}H_\infty$ be defined as

$$\Lambda = \bar{\tilde{N}}_c\bar{N}+\bar{\tilde{D}}_c\bar{D} \quad \tilde{\Lambda} = \bar{\tilde{N}}\bar{N}_c+\bar{\tilde{D}}\bar{D}_c, \qquad (2)$$

then $H(G,C)$ is stable iff $\Lambda^{-1}\in\mathbb{R}H_\infty$ which is equivalent to the condition $\tilde{\Lambda}^{-1}\in\mathbb{R}H_\infty$ (Vidyasagar, 1985).

## 3. ROBUST STABILITY RESULTS FOR DOUBLE-YOULA REPRESENTATIONS

Uncertainty on a plant $G_0$ can be described in very many different ways. In a norm-bounded formulation, there are options for additive, multiplicative, coprime-factor, gap-metric uncertainties, all having their particular robust stability tests. See e.g. (Callafon et. al., 1996) for an overview in a rather uniform (coprime factor) framework.

When considering robust performance tests on norm-bounded uncertainty sets, it has been motivated in (de Callafon and Van den Hof, 1997) that for general classes of performance measures, norm-bounded uncertainty in a dual Youla parametrization framework has particular advantages. In this parametrization, a norm-bounded plant uncertainty set is considered of the form:

$$\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G) := \left\{G_\Delta = (\bar{N}_x+\bar{D}_c\Delta_R)(\bar{D}_x-\bar{N}_c\Delta_R)^{-1} \mid \left\|Q_c^{-1}\Delta_R Q\right\|_\infty \leq \gamma_G\right\}$$

with $G_x$ a nominal model, $C$ the present controller stabilizing $G_x$, and $Q,Q_c$ stable and stably invertible weighting functions. In terms of stability, the dual-Youla parametrization has the basic property that an element in $\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G)$ is stabilized by $C$ if and only if the corresponding $\Delta_R$ is stable.

Similar to characterizing plant uncertainty, a retuning or adaptation of the controller can be represented as a Youla-type "perturbation" on the present controller $C$. This results in the so-called double Youla parametrization, indicated in Figure 2, where

$$C_{new} := C_\Delta = (\bar{N}_c+\bar{D}_x\Delta_C)(\bar{D}_c-\bar{N}_x\Delta_C)^{-1}.$$

The following stability results apply to this situation (Tay et. al., 1989; Schrama et. al., 1992).

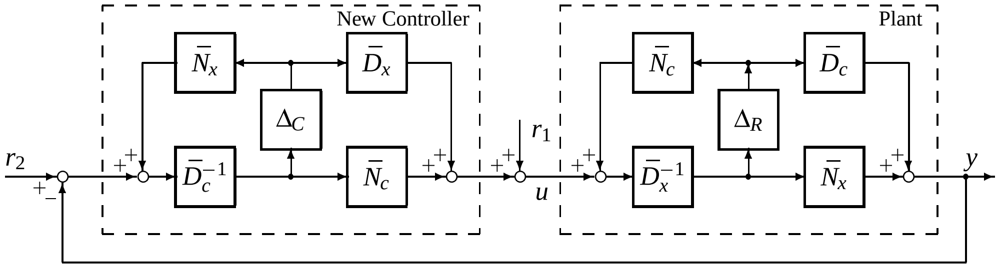


Fig. 2. Double Youla parametrization.

*Proposition 1.* Let $G_x$ and $C$ have normalized coprime factorizations as described above, and let $H(G_x,C)$ be stable. Denote

$$G_\Delta = (\bar{N}_x + \bar{D}_c\Delta_R)(\bar{D}_x - \bar{N}_c\Delta_R)^{-1} \qquad (3)$$

$$C_\Delta = (\bar{N}_c + \bar{D}_x\Delta_C)(\bar{D}_c - \bar{N}_x\Delta_C)^{-1}. \qquad (4)$$

Then for $\Delta_R, \Delta_C \in \mathbb{R}H_\infty$
(a) $H(G_\Delta, C_\Delta)$ is stable if and only if for some unimodular[2] $Q, Q_c \in \mathbb{R}H_\infty$, $H(Q_c^{-1}\Delta_R Q, Q^{-1}\Delta_C Q_c)$ is stable;
(b) $H(G_\Delta, C_\Delta)$ is stable if there exist some unimodular $Q, Q_c \in \mathbb{R}H_\infty$ such that

$$\left\|Q^{-1}\Delta_C Q_c\right\|_\infty \cdot \left\|Q_c^{-1}\Delta_R Q\right\|_\infty < 1. \qquad (5)$$

□

The unimodular matrices $Q$ and $Q_c$ can be interpreted to reflect the freedom in choosing the coprime factorizations of $G_x$ and $C$. Based on this result the next proposition can be formulated.

*Proposition 2.* Given a nominal model $G_x$ and a nominal controller $C$, with *nrcf's* as described before, such that $H(G_x,C) \in \mathbb{R}H_\infty$. Define a set of plants $\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G)$ and a set of controllers $\mathcal{C}_Y(G_x,C,Q,Q_c,\gamma_C)$ as

$$\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G) := \left\{G_\Delta = (\bar{N}_x + \bar{D}_c\Delta_R)(\bar{D}_x - \bar{N}_c\Delta_R)^{-1} \mid \left\|Q_c^{-1}\Delta_R Q\right\|_\infty \le \gamma_G\right\}$$

$$\mathcal{C}_Y(G_x,C,Q,Q_c,\gamma_C) := \left\{C_\Delta = (\bar{N}_c + \bar{D}_x\Delta_C)(\bar{D}_c - \bar{N}_x\Delta_C)^{-1} \mid \left\|Q^{-1}\Delta_C Q_c\right\|_\infty < \gamma_C\right\}.$$

Then all plants in $\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G)$ are stabilized by all controllers contained in the set $\mathcal{C}_Y(G_x,C,Q,Q_c,\gamma_C)$ if and only if $\gamma_G \cdot \gamma_C \le 1$. □

*Proof 1.* The result is direct by applying a small gain argument and employing Proposition 1.

This Proposition serves as a means to specify the allowable area for retuning the controller $C$ so as to guarantee robust stability with all models in the plant uncertainty set. Since the result is based on a small gain criterion, part a) of Proposition 1 can be used to show that the resulting set of controllers is equal to the exclusive set of all controllers stabilizing the entire set $\mathcal{G}_Y(G_x,C,Q,Q_c,\gamma_G)$.

## 4. GAP METRIC RESULTS

When considering the gap metric as a measure for bounding plant uncertainty a similar analysis can be given as presented in the previous section. The gap metric distance between two systems $G_x$, $G_\Delta$ is defined by

$$\delta(G_x,G_\Delta) = \max\{\vec{\delta}(G_x,G_\Delta),\ \vec{\delta}\,G_\Delta,G_x)\}$$

where the *directed gap* is:

$$\vec{\delta}(G_x,G_\Delta) = \inf_{Q_\delta,Q_\delta^{-1}\in H_\infty} \left\| \begin{bmatrix}\bar{N}_x\\ \bar{D}_x\end{bmatrix} - \begin{bmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{bmatrix} Q_\delta \right\|_\infty \qquad (6)$$

where $(\bar{N}_x, \bar{D}_x)$ and $(\bar{N}_\Delta, \bar{D}_\Delta)$ are *nrcf's* of $G_x$ and $G_\Delta$. The stability result that is applicable to our problem set up is the following.

*Proposition 3.* (Georgiou and Smith, 1990) Let $H(G_x,C)$ be stable. Then $H(G_\Delta,C_\Delta)$ is stable if

$$\delta(G_x,G_\Delta) + \delta(C,C_\Delta) < \|T(G_x,C)\|_\infty^{-1}. \qquad (7)$$

This sufficient condition for stability leads to the following formulation in terms of stabilizing sets of controllers.

*Corollary 1.* (Georgiou and Smith, 1990) Given a nominal model $G_x$ and a nominal controller $C$ such that $H(G_x,C) \in \mathbb{R}H_\infty$. The set $\mathcal{G}_\delta(G_x,\delta_G)$ defined as

$$\mathcal{G}_\delta(G_x,\delta_G) := \{G_\Delta \mid \quad \delta(G_x,G_\Delta) \le \delta_G\}$$

is stabilized by all controllers contained in the set $\mathcal{C}_\delta(C,\delta_C)$ defined as

$$\mathcal{C}_\delta(C,\delta_C) := \{C_\Delta \mid \quad \delta(C,C_\Delta) < \delta_C\}$$

if $\delta_C \le \|T(G_x,C)\|_\infty^{-1} - \delta_G$. □

In this proposition a sufficient condition for the retuning range (or the allowed "perturbation" from the present controller) is specified that is allowed under guarantee of robust stability. Unlike for proposition 2, $\mathcal{C}_\delta(C,\delta_C)$ with $\delta_C$ equal to the above mentioned upper bound, does not contain all controllers stabilizing the entire set $\mathcal{G}_\delta(G_x,\delta_G)$.

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[2] $A \in \mathbb{R}H_\infty$ is unimodular if $A^{-1} \in \mathbb{R}H_\infty$.

## 5. COMPARISON OF THE TWO UNCERTAINTY STRUCTURES

*Theorem 1.* Given a set of plants $\mathcal{G}_\delta(G_x,\delta_G)$ and a set of controllers $\mathcal{C}_\delta(C,\delta_C)$ satisfying the gap stability condition of Corollary 1. Then for the sets of Proposition 2 with $Q = Q_c = I$, it holds that

a) $\mathcal{G}_Y(G_x,C,I,I,\bar{\gamma}_G) \supseteq \mathcal{G}_\delta(G_x,\delta_G)$, with $\bar{\gamma}_G = \delta_G\|T(G_x,C)\|_\infty(1-\delta_G\|T(G_x,C)\|_\infty)^{-1}$
b) $\mathcal{C}_Y(G_x,C,I,I,\bar{\gamma}_C) \supseteq \mathcal{C}_\delta(C,\delta_C)$, with $\bar{\gamma}_C = \delta_C\|T(G_x,C)\|_\infty(1-\delta_C\|T(G_x,C)\|_\infty)^{-1}$
c) $\bar{\gamma}_G \cdot \bar{\gamma}_C \leq 1$, i.e. the two sets satisfy the stability condition of Proposition 2. □

*Proof.* A proof can be found in (Douma, Van den Hof and Bosgra, 2001).

The result of this theorem implies that even when embedding the gap uncertainty sets for plant and controller in (more conservative) sets in terms of Youla parametrizations, a simultaneous stabilization result remains valid. In other words: the related robust stability test for the Youla-structured uncertainty is less conservative than the test for the gap metric.

In practice, the uncertainty set in terms of the Youla parametrization would be not be chosen as to enclose the set of the gap uncertainty but as to enclose the set of unfalsified plants. The direct consequence of the theorem for a controller-tuning problem is formulated in the next corollary.

*Corollary 2.* Given a set of (unfalsified) plants $\mathcal{G}$, a gap uncertainty set $\mathcal{G}_\delta(G_x,\breve{\delta}_G)$ and a Youla uncertainty set $\mathcal{G}_Y(G_x,C,I,I,\breve{\gamma}_G)$, where $\breve{\delta}_G,\breve{\gamma}_G$ are the smallest values of $\delta_G,\gamma_G$ such that $\mathcal{G} \subset \mathcal{G}_\delta(G_x,\delta_G)$ and $\mathcal{G} \subset \mathcal{G}_Y(G_x,C,I,I,\gamma_G)$. Then the largest stabilizing controller sets resulting from Proposition 2 and Corollary 1, satisfy and

$$\mathcal{C}_\delta(C,\|T(G_x,C)\|_\infty^{-1}-\breve{\delta}_G) \subset \mathcal{C}_Y(G_x,C,I,I,\breve{\gamma}_G^{-1}). \quad \square$$

Apparently, when describing plant uncertainty in either a gap metric bound or a norm bound in a dual-Youla representation, the latter format allows for a larger set of controllers that guarantee robust stability. The resulting set of controllers guaranteed to stabilize the set of unfalsified plants would still be larger when the freedom of applying weighting functions would be employed (cf. Proposition 1).

One of the principal differences in the two uncertainty structures is that a gap-metric distance between two plants is controller independent. A Youla formulation of the "distance" between two plants is taken under the presence of (and therefore dependent on) a particular controller. In the latter situation the closed-loop properties of the two plants can therefore be taken into account more particularly.

The formulation of the corollary technically allows that the sets are equal; in the next section a counter-example of this is shown.

## 6. EXAMPLE

An example, taken from (Schrama et. al., 1992), is considered in which robust stability is guaranteed by the condition of Proposition 2, but not by the gap-metric condition of Corollary 1. The systems of concern have the following transfer functions:

$$G_x = \frac{-s+1}{4s^3+0.4s^2+4s}$$
$$C = \frac{17s^2-2.3s+10}{s^2+3.3s+11}$$
$$G_\Delta = \frac{0.2s^7+3s^6+5.4s^5+7.8s^4-22s^3+5.2s^2-21s+3.2}{10s^7+31s^6+150s^5+123s^4+218s^3+87s^2+69s+7.1}$$
$$C_\Delta = \frac{30s^7+87s^6+131s^5+148s^4+130s^3+63s^2+41s+9.3}{s^7+8.3s^6+38s^5+83s^4+107s^3+97s^2+62s+13}$$

The Bode diagrams of these systems have been de-

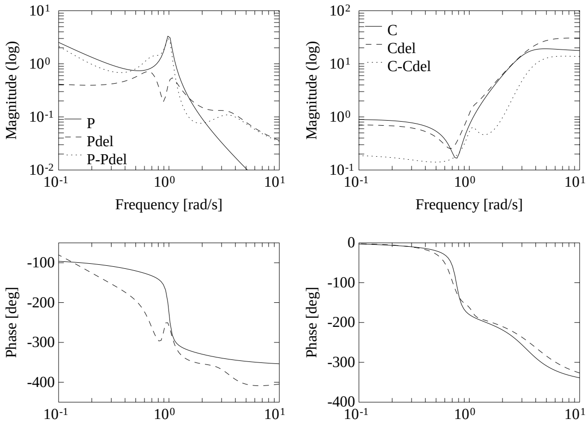


Fig. 3. Bode diagrams of the nominal and perturbed plants and controllers. a: Magnitudes of $G_x$ (solid), $G_\Delta$ (dashed) and $G_x - G_\Delta$ (dotted); b: Magnitudes of $C$ (solid), $C_\Delta$ (dashed) and $C - C_\Delta$ (dotted); c: Phases, see a; d: Phases, see b.

picted in Fig. 3. The Figures 3.a and c display that $G_x$ (solid) and $G_\Delta$ (dashed) are strikingly different. The difference $G_x - G_\Delta$ (dotted) is quite large: its frequency response magnitude is at least 40% of $|G_x(i\omega)|$ over all frequencies, and it is even larger than 60% at those frequencies where $|G_x(i\omega)C(i\omega)| \approx 1$. The controller variation seems to be moderate, but $|C(i\omega) - C_\Delta(i\omega)|$ is larger than 15% of $|C(i\omega)|$ over all frequencies, and it is up to 70% at the frequencies where $|G_xC| \approx 1$.

$G_\Delta$ and $C_\Delta$ are modelled as perturbations of the normalized coprime factors of $G_x$, $C$. The corresponding plant and controller perturbations $\Delta_R$ and $\Delta_C$ are shown in Fig. 4. The $H_\infty$-norms of these perturbations are $\|\Delta_R\|_\infty = 0.968$ and $\|\Delta_C\|_\infty = 0.764$. The product of these norms is 0.734, so that even larger plant and controller perturbations are allowed in view of the

robust stability test of Proposition 2.
For the robust stability test based on the gap-metric condition of Corollary 1 we have the following numbers: $\delta(G_x,G_\Delta)=0.917$; $\delta(C,C_\Delta)=0.286$; $\|T(G_x,C)\|_\infty^{-1}=5.73\cdot 10^{-2}$.
Clearly $\delta(G_x,G_\Delta)+\delta(C,C_\Delta)$ is much larger than $\|T(G_x,C)\|_\infty^{-1}$. Hence from (7) it cannot be concluded that $H(G_\Delta,C_\Delta)$ is robustly stable. Moreover, as $\delta(G_x,G_\Delta)>\|T(G_x,C)\|_\infty^{-1}$ and $\delta(C,C_\Delta)>\|T(G_x,C)\|_\infty^{-1}$, the gap-metric condition fails even to guarantee stability of $H(G_\Delta,C)$ or of $H(G_x,C_\Delta)$. Finally, the small value of $\|T(G_x,C)\|_\infty^{-1}$ indicates that $H(G_x,C)$ has poor robustness properties in gap metric sense, while $H(G_x,C)$ is robustly stable against rather large perturbations as shown in Fig. 3.

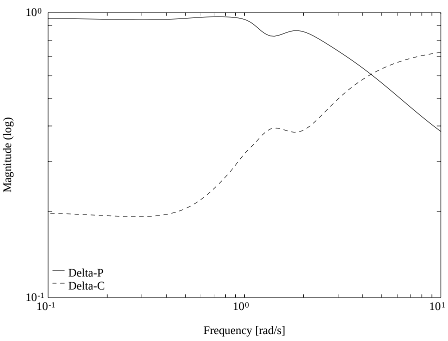


Fig. 4. Plant perturbation $\Delta_R$ (solid) and controller perturbation $\Delta_C$ (dashed).

## 7. EXTENSION TO ν-GAP AND Λ-GAP

The analysis as presented in this paper so far can readily be extended to other uncertainty structures as well, as e.g. the ν-gap and the Λ-gap. The Vinnicombe or ν-gap metric is defined as (Vinnicombe, 1993):

$$\delta_\nu(G_x,G_\Delta)=\begin{cases}\bullet\quad \left\|\begin{bmatrix}-\tilde{\bar{D}}_x & \tilde{\bar{N}}_x\end{bmatrix}\begin{bmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{bmatrix}\right\|_\infty, \\ \text{if } \det(\begin{bmatrix}\tilde{\bar{N}}_x^* & \tilde{\bar{D}}_x^*\end{bmatrix}\begin{bmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{bmatrix})\neq 0\ \forall\omega \\ \text{and} \\ W(\det\left(\begin{bmatrix}\tilde{\bar{N}}_x^* & \tilde{\bar{D}}_x^*\end{bmatrix}\begin{bmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{bmatrix}\right))=0 \\ \bullet\quad 1,\qquad \text{otherwise,}\end{cases} \tag{8}$$

where $W(g)$ denotes the winding number about the origin of $g(s)$ as $s$ follows the standard Nyquist $D$-contour.

The Λ-gap $\vec{\delta}_\Lambda(G_x,G_\Delta)$ between two plants $G_x$ and $G_\Delta$ is defined as ((Bongers, 1991; Bongers, 1994; Callafon et. al., 1996))

$$\vec{\delta}_\Lambda(G_x,G_\Delta)=\inf_{Q_\Lambda,Q_\Lambda^{-1}\in\mathbb{R}H_\infty}\left\|\begin{pmatrix}\bar{N}_x\\ \bar{D}_x\end{pmatrix}\Lambda^{-1}-\begin{pmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{pmatrix}Q_\Lambda\right\|_\infty \tag{9}$$

with $(\bar{N}_x,\bar{D}_x)$ and $(\bar{N}_\Delta,\bar{D}_\Delta)$ *nrcf's* of $G_x$ and $G_\Delta$, and $\Lambda$ as defined in (2).
The robust stability results -known from the literature- that can be exploited for our purpose of specifying a norm-bounded area around $C$ under robust stability guarantees read as follows (Vinnicombe, 1993; Bongers, 1994).

*Proposition 4.* Let $H(G_x,C)$ be stable. Then $H(G_\Delta,C_\Delta)$ is stable if
(a) $\delta_\nu(G,G_\Delta)+\delta_\nu(C,C_\Delta)<\|T(G_x,C)\|_\infty^{-1}$ (ν-gap condition) or
(b) $\vec{\delta}_\Lambda(G_x,G_\Delta)+\vec{\delta}_\Lambda(C,C_\Delta)<1$ (Λ-gap condition). □

These sufficient conditions for stability lead to the following formulation in terms of stabilizing sets of controllers.

*Corollary 3.* Given a nominal model $G_x$ and a nominal controller $C$ such that $H(G_x,C)\in\mathbb{R}H_\infty$. The set $\mathcal{G}_\nu(G_x,\delta_{\nu,G})$ defined as

$$\mathcal{G}_\nu(G_x,\delta_{\nu,G}):=\{G_\Delta|\quad \delta_\nu(G_x,G_\Delta)\leq\delta_{\nu,G}\}$$

is stabilized by all controllers contained in the set $\mathcal{C}_\nu(C,\delta_{\nu,C})$ defined as

$$\mathcal{C}_\nu(C,\delta_{\nu,C}):=\{C_\Delta|\quad \delta_\nu(C,C_\Delta)<\delta_{\nu,C}\}$$

if $\delta_{\nu,C}\leq\|T(G_x,C)\|_\infty^{-1}-\delta_{\nu,G}$. □

*Corollary 4.* Given a nominal model $G_x$ and a nominal controller $C$ such that $H(G_x,C)\in\mathbb{R}H_\infty$. The set $\mathcal{G}_\Lambda(G_x,\delta_{\Lambda,G})$ defined as

$$\mathcal{G}_\Lambda(G_x,\delta_{\Lambda,G}):=\left\{G_\Delta|\quad \vec{\delta}_\Lambda(G_x,G_\Delta)\leq\delta_{\Lambda,G}\right\}$$

is stabilized by all controllers contained in the set $\mathcal{C}_\Lambda(C,\delta_{\Lambda,C})$ defined as

$$\mathcal{C}_\Lambda(C,\delta_{\Lambda,C}):=\left\{C_\Delta|\quad \vec{\delta}_\Lambda(C,C_\Delta)<\delta_{\Lambda,C}\right\}$$

if $\delta_{\Lambda,C}\leq 1-\delta_{\Lambda,G}$. □

Note that $\mathcal{C}_\nu(C,\delta_{\nu,C})$ and $\mathcal{C}_\Lambda(C,\delta_{\Lambda,C})$ do not contain all controllers stabilizing the entire set $\mathcal{G}_\nu(G_x,\delta_{\nu,G})$ and $\mathcal{G}_\Lambda(G_x,\delta_{\Lambda,G})$, respectively. The use of the necessary and sufficient ν-gap condition $\delta_{\nu,G}\leq\|T(G_x,C_\Delta)\|_\infty^{-1}$ would result in a characterization of the exclusive set of all controllers stabilizing $\mathcal{G}_\nu(G_x,\delta_{\nu,G})$. This condition, however, does not allow for an explicit norm-bounded tuning range around a present controller which is sought for here.

Based on these robust stability results one can now consider the same problem as is considered in the formulation of Theorem 1.

*Theorem 2.* Given a set of plants $\mathcal{G}_\nu(G_x,\delta_{\nu,G})$ and a set of controllers $\mathcal{C}_\nu(C,\delta_{\nu,C})$ satisfying the ν-gap stability condition of Corollary 3. Then for the sets of Proposition 2 with $Q=Q_c=I$, it holds that

a) $\mathcal{G}_Y(G_x,C,I,I,\bar{\gamma}_{\nu,G})\supseteq\mathcal{G}_\nu(G_x,\delta_{\nu,G})$, with $\bar{\gamma}_{\nu,G}=\delta_{\nu,G}\|T(G_x,C)\|_\infty(1-\delta_{\nu,G}\|T(G_x,C)\|_\infty)^{-1}$

b) $\mathcal{C}_Y(G_x, C, I, I, \overline{\gamma}_{\nu,C}) \supseteq \mathcal{C}_\nu(C, \delta_{\nu,C})$, with $\overline{\gamma}_{\nu,C} = \delta_{\nu,C} \|T(G_x,C)\|_\infty (1 - \delta_{\nu,C} \|T(G_x,C)\|_\infty)^{-1}$

c) $\overline{\gamma}_{\nu,G} \cdot \overline{\gamma}_{\nu,C} \leq 1$, i.e. the two sets satisfy the stability condition of Proposition 2. □

*Theorem 3.* Given a set of plants $\mathcal{G}_\Lambda(G_x, \delta_{\Lambda,G})$ and a set of controllers $\mathcal{C}_\Lambda(C, \delta_{\Lambda,C})$ satisfying the $\Lambda$-gap stability condition of Corollary 4. Then for the sets of Proposition 2 with $Q = \Lambda^{-1}$ and $Q_c = \tilde{\Lambda}^{-1}$, with $\Lambda, \tilde{\Lambda}$ as defined in (2), it holds that

a) $\mathcal{G}_Y(G_x, C, \Lambda^{-1}, \tilde{\Lambda}^{-1}, \overline{\gamma}_{\Lambda,G}) \supseteq \mathcal{G}_\Lambda(G_x, \delta_{\Lambda,G})$, with $\overline{\gamma}_{\Lambda,G} = \delta_{\Lambda,G} (1 - \delta_{\Lambda,G})^{-1}$

b) $\mathcal{C}_Y(G_x, C, \Lambda^{-1}, \tilde{\Lambda}^{-1}, \overline{\gamma}_{\Lambda,C}) \supseteq \mathcal{C}_\Lambda(C, \delta_{\Lambda,C})$, with $\overline{\gamma}_{\Lambda,C} = \delta_{\Lambda,C} (1 - \delta_{\Lambda,C})^{-1}$

c) $\overline{\gamma}_{\Lambda,G} \cdot \overline{\gamma}_{\Lambda,C} \leq 1$, i.e. the two sets satisfy the stability condition of Proposition 2. □

*Proof.* Proofs can be found in (Douma, Van den Hof and Bosgra, 2001).

These theorems show that, like the gap-metric uncertainty structure, also the $\nu$-gap and $\Lambda$-gap uncertainty structures lead to controller sets that in the considered problem formulation are more conservative than the sets that are obtained by a double Youla-parametrization.

## 8. CONCLUDING REMARKS

We have used the double Youla parametrization for purpose of specifying the maximum allowable tuning range for a new controller to deviate from the present controller while retaining robust stability. It is demonstrated that the result obtained when using this uncertainty structure is less conservative than when using the gap metric. An example has been provided to illustrate these results. The results imply that model uncertainty characterized in terms of a dual Youla-parametrization not only is advantageous from a performance point of view, but also for the situation where attention is restricted to robust stability aspects. Related results are provided for stability conditions in terms of the $\nu$-gap and the $\Lambda$-gap.

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