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PERFORMANCE ENHANCEMENT ON THE BASIS OF IDENTIFIED MODEL UNCERTAINTY SETS WITH APPLICATION TO A CD MECHANISM

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Abstract. We address the design of robust feedback controllers based on nominal models and model error bounds that are obtained from measurement data. In order to keep controller complexity limited an approach is adopted where controller synthesis is performed on the basis of a low order (approximate) nominal model; prior to controller implementation the robust performance of the controller is evaluated based on a (highly accurate) system uncertainty set employing a dual Youla uncertainty structure. The procedure is based on an iterative scheme of identification and control design and is experimentally verified on the radial tracking control of a Compact Disc mechanism.

Keywords. Identification, robust control, modelling errors, uncertainty, mechanical systems.

1. INTRODUCTION

Nowadays servo systems in consumer electronic products must satisfy harsher requirements regarding tracking performance and disturbance rejection at decreasing cost. Application of advanced feedback control offers an opportunity to comply with these requirements. A procedure for design of feedback controllers, directed towards an *enhanced* tracking performance of the servo system, requires the availability of mathematical models that describe the dynamical properties of the system. A model however inherently constitutes an approximate system description due to the fact that knowledge of system dynamics is inaccurate or the dynamics is too complex to describe exactly. Hence, a model based control design inherently confronts us with the issue how well a controller that is designed on the basis of an approximate model, will do for the physical system. The paradigm of robust control design has offered viable tools to take model inaccuracies into account in the design in view of producing a robust controller (Zhou *et al.*, 1996). Instrumental in robust control is the availability of uncertainty models that describe the mismatch between a nominal model and the physical plant.

In this work a model-based robust control design procedure is proposed and experimentally verified on the radial actuator of a Compact Disc mechanism. A full procedure will be presented for the following problem:

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given a plant with unknown dynamics that is controlled by a given controller and that is available for taking experimental data; design a controller on the basis of identified dynamical model information, that leads to an improved performance of the controlled plant.
A driving argument is that a controller should be of limited complexity, yet establishing a high robust performance. We adopt an approach where *low complexity* (hence approximate) nominal models are utilized for controller synthesis. This is motivated by two arguments. As for many model-based control design approaches the complexity of the resulting controller is directly determined by the complexity of the nominal model (and corresponding weighting functions), this approach directly limits the complexity of the controller. Secondly, it can be observed that for many applications a reduced order nominal model can provide a high performance model-based controller, as long as the nominal model accurately reflects the control-relevant part of the system dynamics.
To ensure robust performance of a designed controller prior to its implementation, the performance that is achieved for the plant is evaluated utilizing an accurate system uncertainty set. The ingredients of the control design procedure hence are:

- Identification of a nominal model that is designed to capture the control-relevant dynamics of the plant;
- Quantification of a model uncertainty set that comprises the mismatch between a nominal model and the physical plant;
- Controller synthesis using nominal model information on the basis of a loop-shaped $\mathcal{H}_\infty$ procedure and quantified model error bounds in order to evaluate robustness properties.

The several steps of the procedure are indicated in Figure .1. As an approximate nominal model puts restrictions on a maximum achievable robust performance enhancement, we can not achieve our design objective in one synthesis step. We have to explore the limitations of the underlying approximate model by conducting synthesis and performance evaluation for a number of prespecified, gradual performance enhancement objectives. The identification of approximate models for design of high robust performance controllers has received extensive attention in literature, in particular in the format of iterative identification and control (Gevers, 1993; Van den Hof and Schrama, 1995). The approach presented here is one example of an iterative scheme.

The particular design of a robust controller is the topic of section 2. In section 3 the identification of nominal models and model error bounds is addressed. The effectiveness of the proposed procedure will be illustrated in section 4 by an application to the optical radial servo system in an industrial compact disc player.

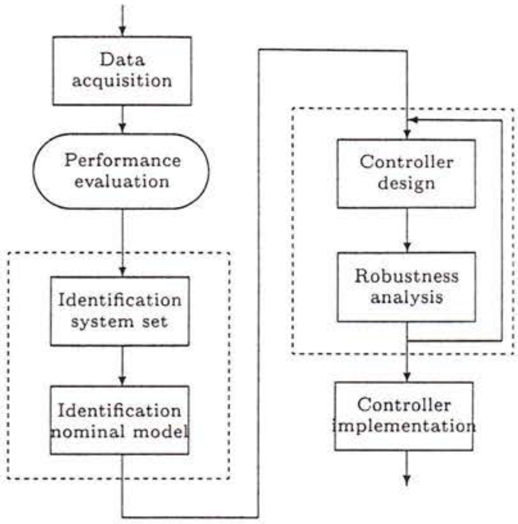

Fig. 1. Schematic diagram of the several steps in the procedure

2. CONTROL DESIGN

2.1 *Introduction*

We consider a feedback system configuration depicted in the block diagram of figure 2. A general feedback

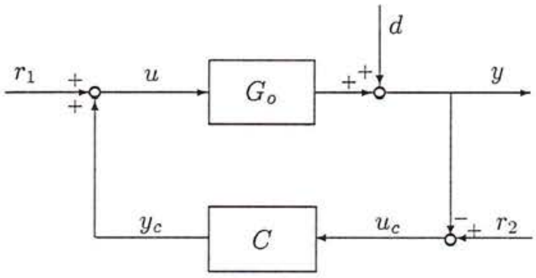

Fig. 2. Block diagram of a feedback system, consisting of a system G_o in configuration with a feedback controller C.

performance objective can be expressed in terms of the closed loop transfer function that maps $[r_2\ r_1]^T \rightarrow [y\ u]^T$

$$T(G_o, C) := \begin{bmatrix} G_o \\ 1 \end{bmatrix} [1 + CG_o]^{-1} [C \ \ 1],$$

which embodies all relevant feedback properties of the controlled plant, among which disturbance rejection, tracking properties and control effort.
As a measure of performance, we will require that a designed controller complies with performance specifications [2]:

$$\Gamma_{lo}(\omega) \leq |T(G_o, C)| \leq \Gamma_{up}(\omega). \tag{1}$$

[2] In this paper the notation $|A|, A \in \mathbb{C}^{p\times q}$ denotes a $p \times q$ matrix of the magnitudes of the elements of A.

2.2 *Controller synthesis*

Based on a nominal model $\hat{G}$ of the plant G_o, the controller synthesis is carried out by means of the following weighed optimization:

$$C_{\hat{G},\nu_b} =$$

$$\arg_{\tilde{C}} \min \left\| \begin{bmatrix} W(\nu_b)\hat{G} \\ 1 \end{bmatrix} [1+\tilde{C}\hat{G}]^{-1}[\tilde{C}/W(\nu_b) \;\; 1] \right\|_\infty \quad (2)$$

where W_{ν_b} is a filter, designed for a nominal design bandwidth of ν_b Hz. The controller synthesis is similar to a H_∞ loop shape design according to McFarlane and Glover (1990), which is known to have favourable robustness properties in view of model errors structured in a fractional (coprime factor) model framework. The weighting filter W_{ν_b} is designed to shape the loop transfer $W_{\nu_b}\hat{G}$ to achieve a prespecified bandwidth, and additional robustness properties are achieved by solving the optimization problem (2). However the *achieved* performance robustness will be the crucial argument in deciding which loop shape filter can be safely applied, so as to achieve a performance that is similar to the designed performance.

2.3 *Performance robustness analysis*

Although the synthesis is known to be optimally robust in view of normalized coprime factor perturbations, there is no guarantee that the synthesized controller actually is robustly stable or robustly performing. This has to be checked prior to controller implementation.

Evaluation of robust performance using an identified model uncertainty set, in this work amounts to checking whether the corresponding feedback system magnitudes $|T(G_o,C)|$ comply with our specifications (1). As there is no exact knowledge available on the plant G_o, this test can not be performed directly, but it will be performed on the basis of a system uncertainty set $\mathcal{P}$ of which G_o is considered to be an element:

$$\Gamma_{lo}(\omega) \leq |T(G,C)| \leq \Gamma_{up}(\omega), \qquad \forall G \in \mathcal{P}.$$

For the common uncertainty sets based on e.g. additive, multiplicative or dual-Youla uncertainty, this test requires the evaluation of a linear fractional transformation (LFT) of the uncertainty Δ, which can be done without conservatism. For more details see section 3.2.

3. IDENTIFICATION

3.1 *Nominal models*

A nominal model is identified from closed-loop experimental data using a so-called coprime factor identification method (Van den Hof *et al.*, 1995). With this approach a possibly unstable plant can be identified while the model order can be controlled to be limited. Additionally, a model error expressed in terms of perturbed coprime factors have favourable continuity properties in view of robustness of the corresponding closed-loop system. Considering the following system description [3]:

$$y = G_oS_or + S_od \quad (3)$$

$$u = S_or - CS_od \quad (4)$$

with $r = r_1$, $S_o = 1/(1+CG_o)$, an appropriate filter F is constructed according to $F := 1/(D_x + CN_x)$, where (N_x, D_x) are normalized coprime factors of an auxiliary model G_x that is presumed to be an accurate system description. With this filter an auxiliary signal $x = Fr$ is obtained, leading to

$$y = N_ox + S_od \quad (5)$$

$$u = D_ox - CS_od \quad (6)$$

with $(N_o, D_o) = (G_oS_oF^{-1}, S_oF^{-1})$ being a normalized coprime factorization of G_o. The two factors N_o, D_o are then identified from experimental data by exciting the external excitation signal r, using a least squares prediction error approach (Ljung, 1987). The final plant model is obtained by taking the quotient of the two estimates, $\hat{G} = \hat{N}/\hat{D}$. Due to the normalization, the McMillan degree of the final model is equal to the McMillan degree of the separate factors.

Particular control-relevancy of the nominal model is achieved by applying the identification criterion:

$$\hat{\theta} = \arg\min_\theta \left\| V_{out} \begin{bmatrix} N_o - N(\theta) \\ D_o - D(\theta) \end{bmatrix} V_{in} \right\|_2 \quad (7)$$

where the weightings V_{out} and V_{in} are chosen to satisfy

$$V_{out} = \begin{bmatrix} W(\nu_b) & 0 \\ 0 & I \end{bmatrix}$$

$$V_{in} = (D_x + CN_x)^{-1}[C \;\; I] \begin{bmatrix} W^{-1}(\nu_b) & 0 \\ 0 & I \end{bmatrix}$$

such that the norm expression in (7) equals

$$\left\| \begin{bmatrix} W(\nu_b) & 0 \\ 0 & I \end{bmatrix} [T(G_o,C) - T(G(\theta,C))] \begin{bmatrix} W^{-1}(\nu_b) & 0 \\ 0 & I \end{bmatrix} \right\|_2$$

being in accordance with the control design performance criterion (2). In our approach the identification is performed in the frequency domain, employing periodic excitation signals.

[3] Without loss of generality we consider the situation $r_1 = r$, $r_2 = 0$.

3.2 *Model uncertainty*

Additional to the nominal model, a model uncertainty set is estimated to bound the uncertainty that is present in the estimated nominal model. The mixed probabilistic worst-case bounding procedure is adopted from de Vries and Van den Hof (1995). In this approach a probabilistic setting is taken for the effects of disturbance signals (variance aspects), while a worst-case approach is used for undermodelling issues. As (unstructured) model uncertainty structure, use is made of the dual-Youla parametrization, leading to an uncertainty set:

$$\mathcal{P} = \{G \mid G = \frac{N_x + D_c \Delta R}{D_x - N_c \Delta R}, \ \ |\Delta R(e^{i\omega})| \leq \gamma(\omega)\}$$

where (N_x, D_x) is a coprime factorization of an auxiliary model G_x, stabilized by the current controller C with coprime factorization (N_c, D_c). For each ω, the uncertainty bound $\gamma(\omega)$ is determined such that the dual-Youla factor R_o of the plant G_o satisfies the uncertainty bound with a prespecified probability α.
An attractive property of the particular uncertainty set, is that for all $G \in \mathcal{P}$

$$T(G,C) - T(G_x, C) = \begin{bmatrix} D_c \\ -N_c \end{bmatrix} \Delta R (D_x + C N_x)^{-1} [C \ \ 1] \tag{8}$$

which is an affine expression in ΔR. Consequently, a bound on the error ΔR enables a simple evaluation of magnitude bounds of $T(G,C)$ for C being the actual controller during experimentation. It also implies that when reducing the uncertainty in ΔR, the uncertainty in the closed-loop properties $T(G,C)$ is directly reduced.

For evaluation of the uncertainty in $T(G, C_{new})$ for a newly designed, not yet implemented controller, the corresponding expressions become a linear fractional transformation of ΔR, which can also be calculated without conservatism. This also holds for the calculation of equivalent (open-loop) plant uncertainty, corresponding to $C_{new} = 0$. For more details see Dötsch (1998).

4. APPLICATION TO A COMPACT DISC MECHANISM

4.1 *Problem and initial situation*

The model-based control design procedure presented is applied to the radial servo mechanism of a swing-arm type Compact Disc player. A schematic depiction is shown in figure 3. Characteristic for the dynamical properties of the radial actuator is that it contains a double integrator and mechanical flexibilities are present in the

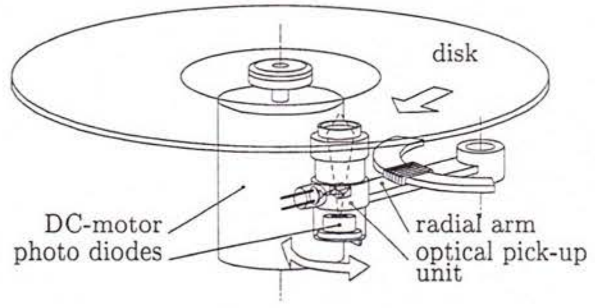

Fig. 3. Schematic depiction of a swing-arm type Compact Disk servo mechanism.

frequency range $\{400, 4000\}$ Hz which may prove disastrous for tracking performance when not adequately accounted for by the controller. An improved tracking performance, expressed in terms of the sensitivity transfer function, is phrased into the performance objective of an enhanced disturbance rejection at higher frequencies.
In the initial situation the plant is controlled by a 4th order controller, achieving a bandwidth of around 500 Hz.

4.2 *Identification*

Using the coprime factor identification procedure sketched in section 3.1, an auxiliary model is identified having order 48, and an approximate nominal model is obtained of order 8. The auxiliary model is used as a basis for the construction of a model uncertainty set, according to the procedure utilized in section 3.2. Figure 4 shows the Bode magnitude diagrams of the error bounds of $T(G,C)$ for the resulting uncertainty set. The magni-

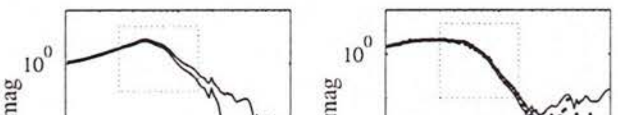

Fig. 4. Upper and lower magnitude bounds of $\{T(G,C), G \in \mathcal{P}\}$ (solid) and averaged ETFE (dot); frequency range [100, 10.000] Hz.

tude error bounds of $T(G, C)$ are obtained from expression (8). As the plant model $G = (N_x + D_c R)/(D_x - N_c R)$ is a linear fractional transformation of R, a dual Youla error bound can be transformed into an additive error bound on $\hat{G}$. For details we refer to Dötsch (1998). In figure 5 the Bode magnitude diagram is shown of (transformed) additive error bounds of the radial actuator.

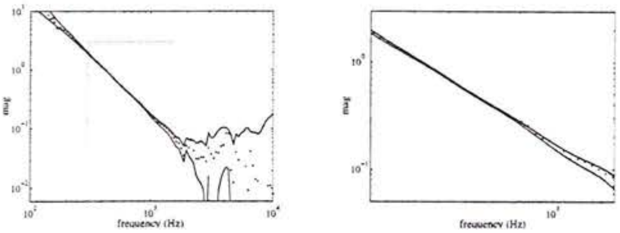

Fig. 5. Upper and lower magnitude error bounds of the plant G_o (solid) and a measured frequency response (dot); frequency range [100, 10.000] Hz (left) and [300, 1.500] Hz (right).

Note that the additive plant model error bounds are particularly tight in the frequency region $\{300, 700\}$ Hz while the corresponding feedback system error bounds are not specifically tighter in the specified region (where the sensitivity has large magnitude). This observation indicates that error bound identification employing a dual Youla model parametrization corresponds to additive plant model error bounds that are tight specifically in the frequency region relevant for closed loop dynamics.

4.3 *Control design*

Based on a crude model of the radial actuator of order 8, four controllers of order 14 are designed employing a third order weighting $W_3(\nu_b)$ that establishes a gradually enhanced disturbance rejection for a double integrator. The weighting function is parametrized as:

$$W_3(\nu_b) = K \frac{\tau_0 s + 1}{\tau_0 s} \frac{\omega_0^2}{s^2 + 2\beta\omega_0 s + \omega_0^2}$$
$$\tau_0 = \frac{\nu_b}{5}, \quad \omega_0 = 5\nu_b, \quad \beta = 0.4.$$

Bode diagrams of the designed controllers are shown in figure 6.

In order to assess robust performance of the synthesized controllers, in figure 7 error bounds are evaluated of the sensitivity function magnitudes on the basis of the identified model uncertainty set and the newly designed controllers. For the designs corresponding to $\nu_b = 650$ and

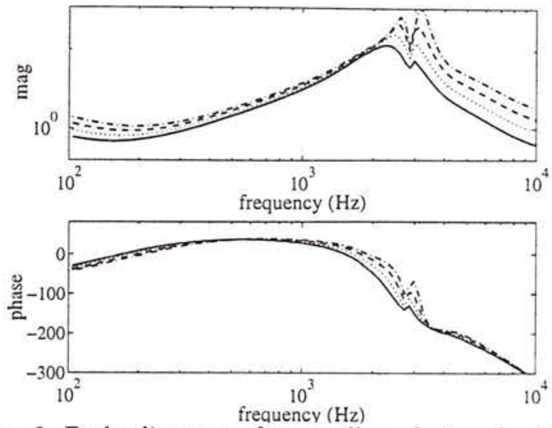

Fig. 6. Bode diagram of controllers designed with 3rd order weighting $W(\nu_b)$ for $\nu_b = 550$ Hz (solid), $\nu_b = 600$ Hz (dot), $\nu_b = 650$ Hz (dash) and $\nu_b = 700$ Hz (dash-dot).

700 (figure 7.c and d) the error bounds indicate a peaking of the sensitivity at 2 kHz and 4 kHz (due to resonance modes of the actuator) that is not accounted for by the approximate model (dashed line). Consequently, these resonance modes should be captured by the nominal model in case we strive for higher design bandwidths. Also it can be observed that the error bounds become larger the more the designed (future) bandwidth differs from the achieved (current) bandwidth.

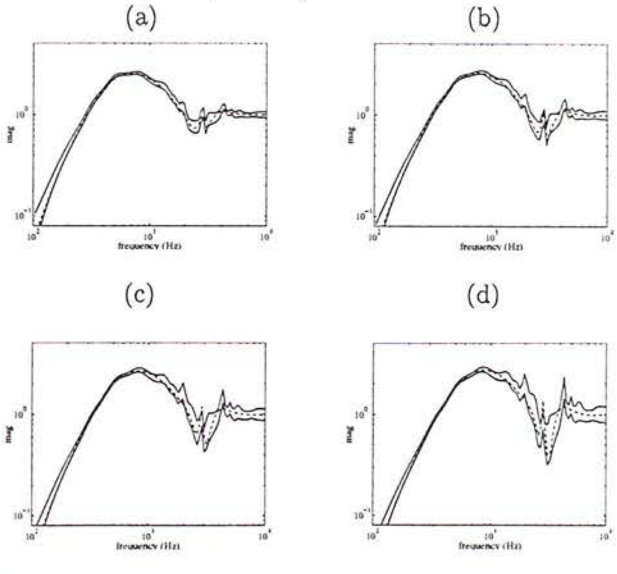

Fig. 7. Bode magnitude diagram of the sensitivity function: robust controllers designed with $W(\nu_b)$ for $\nu_b = 550$ (a), 600 (b), 650 (c), 700 (d) Hz in conjunction with the nominal model (dash) and the system uncertainty set bounds (solid).

To evaluate the quality of the error bounds, the predicted (a priori) performance of the controller $C_{\hat{G},750}$ designed with $\nu_b = 750$, is compared with frequency response estimates after implementation of this controller (a posteriori). This is done for both sensitivity and plant-times-sensitivity, and is shown in figure 8. For frequencies higher than 800 Hz the achieved response is within the predicted bounds. Up to 800 Hz the bounds do not capture the frequency response. From a range of possible causes of this phenomenon, such as e.g. nonlinear plant behaviour, it has appeared that -in particular- a mismatch between designed (and presumably known) controller and implemented controller is likely to be the cause of the mismatch that is found.

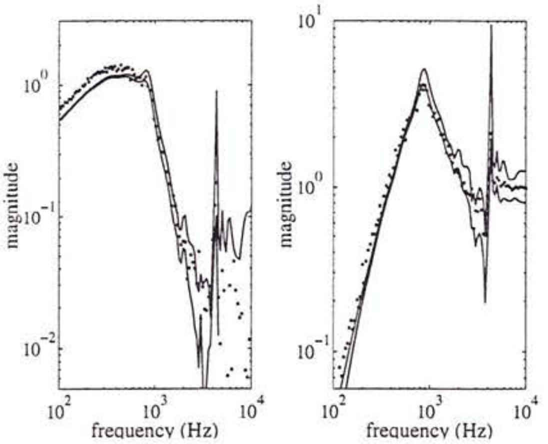

Fig. 8. $G/(1+CG)$ (left) and $1/(1+CG)$ (right) with controller $C_{\hat{G},750}$ in the loop: system uncertainty set (solid), identified with controller $C_{\hat{G},500}$, and frequency response estimate (dot).

New uncertainty sets are identified with the new controller $C_{\hat{G},750}$ implemented in the loop. The results for the sensitivity function are displayed in figure 9, showing that the error bounds now better capture the frequency response, although not 100% correctly.

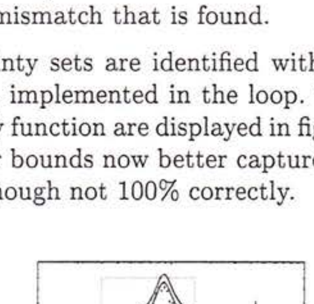

Fig. 9. Upper and lower magnitude bounds of sensitivity function, identified with controller $C_{\hat{G},750}$ (solid), and estimated frequency response (dot).

5. CONCLUSIONS

A procedure has been proposed for an enhanced performance design for an electro mechanical servo system by means of feedback control design, employing identified models and a quantified bound of the model error. Driving aspect is the need for a *low complexity* controller. Controller synthesis is based on a low order nominal model while robust performance is evaluated based on an accurate system uncertainty set, employing a dual Youla model structure. Conducting synthesis and analysis with a gradually increasing performance objective indicates the utility of the nominal model underlying the design. Critical issue in the procedure follows from experimental implementation on a Compact Disc mechanism: the procedure hinges on the availability of knowledge of the controller in the loop during experiment, which makes the method sensitive to inaccurate controller knowledge. A related approach relying on robust control design and application of μ-analysis procedures for performance robustness evaluation, is presented in De Callafon (1998).

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