# Adaptive Repetitive Control of a Compact Disc Mechanism


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**Abstract**

Radial track following of a compact disc player servo mechanism is severely exposed to periodic disturbances, induced by the eccentric rotation of the disc. The period of this disturbance is not available for measurement and varies slowly in time. Periodic disturbances can be adequately attenuated using the concept of repetitive control, provided the period is known. To deal with time varying periodic disturbances, a repetitive controller is tuned based on a simple though inaccurate physical model of the time varying character of the period. The model is tuned based on an estimate of the period, obtained through recursive identification. Experimental results show that the proposed scheme enables a significant improvement of the tracking accuracy of the radial servo mechanism.



**Keywords:** adaptive repetitive control, periodic disturbance signals, compact disc mechanism.


## 1 Introduction

In this paper the issue of track following is addressed as is encountered in a compact disc player. The compact disc servo mechanism considered here consists of two servo systems that enable track following, one for focus and one for radial tracking. In this paper attention is restricted to the radial part of the servo mechanism.

As the radial position of the track is not available for measurement, this signal may be interpreted as a disturbance acting on the servo system. Due to deficiencies of the track geometry and eccentric rotation of the disc, this disturbance mainly has a periodic character.

In view of present specifications, the required tracking accuracy is established using PID controllers, in spite of periodic components that are present in the tracking error. However, a much tighter bound on the tracking accuracy is desired due to desired higher information densities on the disc. Attenuation of periodic components of the tracking error therefore is imperative in order to comply with future specifications.


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In literature (a.o. [6, 3]) the concept of repetitive control has proven to be a useful control strategy for tracking reference signals or attenuating disturbances that are of periodic nature provided the period of the signal is known. However, as the period of the disturbance is not directly measured and varies in time, an estimate of the period of the error signal can be used to tune the repetitive controller. This problem has been elaborated based on simulation studies in [7].

In this paper an adaptive repetitive control scheme is proposed to meet the required attenuation of periodic disturbances having a time varying period. It is shown that fruitful use can be made of prior knowledge of the time varying character of the disturbance in order to provide better attenuation. As the prior knowledge is rather inaccurate, it has to be tuned based on an estimate of the period. As is always the case with adaptive systems, attenuation of time varying disturbances is a compromise between sufficient excitation of the estimation algorithm and sufficient attenuation of the disturbance.

The adaptive repetitive control scheme mainly consists of the following three parts: the repetitive controller (section 3), an identification algorithm for estimating the period of a periodic signal (section 4) and a tuning algorithm for adapting the repetitive controller according to the estimated period (section 5). The scheme is implemented on an experimental set up of a compact disc player. The experimental results show that the time varying periodic disturbances are adequately attenuated (section 6). The paper is concluded with some conclusions.

## 2 Problem description

For read-out of information in a compact disc player, a spirally shaped track on a rotating disc is scanned by an optical device. For adequate reading of the information, the optical device must be positioned with respect to the track within specified accuracy bounds. Positioning of the optical device is enabled by two actuators, one in radial direction of the track and one in focus direction. Both actuators are marginally stable and must therefore operate in closed loop with a stabilizing controller. An elaborate description of the compact disc system is found in [2].

Due to eccentric rotation of the disc and the fact that the track is not perfectly spirally shaped, the radial track position predominantly has a periodic character. The periodic components consist of a fundamental frequency corresponding to the rotation velocity of the disc and some higher harmonics.

Another feature of the disturbance is the time variance of its period. This results from the fact that the rotation frequency must be adjusted during play in order to keep the scanning velocity along the track at a constant value. This implies that the fundamental frequency decreases when reading the track from the centre to the edge of the disc. Moreover, in case the radial distance of the track would be known the fundamental frequency could be deduced from track geometry and the scanning velocity. Although the track geometry is not exactly known and differs per disc, some prior knowledge about the evolvement in time of the fundamental frequency is available, as will be shown in section 5.

As the track position is not available for measurement, it constitutes a disturbance acting on the closed loop system as shown in figure 1. The actuator is denoted as $P$ and $C$ is a stabilizing controller; $r$ is the track signal and $e$ the tracking error.

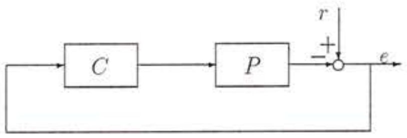


fig. 1: Block scheme of the compact disc servo mechanism

So, we are dealing with a disturbance having the character of a harmonic signal with time varying period.

In present applications the radial tracking error must not exceed 100 $nm$. This is established by controllers having high gains in the low frequent region (up to 300 Hz) and sufficient high frequent roll off in order to guarantee robust stability. Control design based on loop shaping provides low order controllers that enable the required track following behaviour. In figure 2 a fragment of a time domain measurement and in figure 3 an estimate of the power spectrum of the radial error signal is shown using a PID controller.

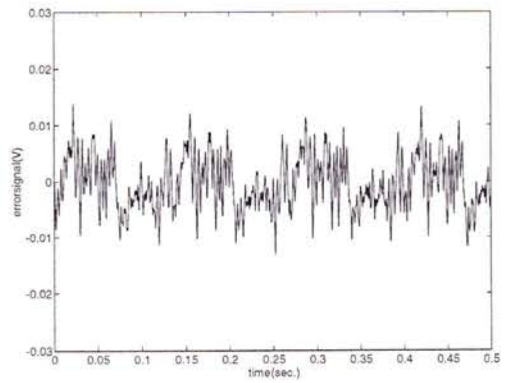


fig. 2: Radial error signal under PID control

The peak with the lowest frequency in figure 3 corresponds with the rotation frequency of the disc; the higher harmonics result from eccentric rotation. As is obvious from these figures, the periodic components have a considerable contribution in the magnitude of the error signal.

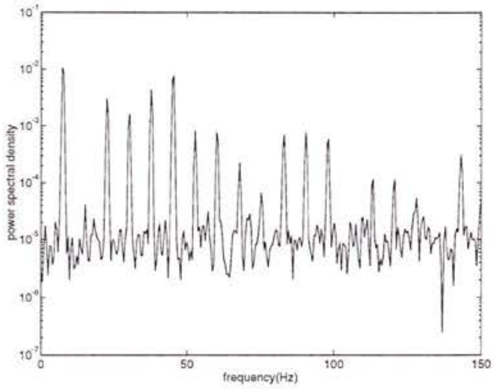


fig. 3: Power spectrum of the radial error signal

In future applications of optical data systems a desired feature is a higher information density on a disc. This is enabled by decreasing the radial distance of the track per revolution. An important consequence is that the maximum magnitude of the tracking error must be kept below a smaller value.

To establish a tighter specified bound on the maximum tracking error, the periodic components must be suppressed. One way to achieve this is increasing the controller gain in the low frequent region (f.i. by adding an integrator) resulting in a controller that also shows large disturbance attenuation at frequencies in between the frequencies of interest. Moreover, robustness is endangered since extra phase is added to the loop transfer function.

The periodic nature of the disturbance can be exploited for design of a controller for attenuation of specific frequencies. Since the period of the disturbance is not measured and *slowly* varies in time the problem of constructing an adaptive control scheme is considered where the time varying period is estimated from the error signal.

## 3 Repetitive control

In case the period of the disturbance is known and time invariant, repetitive control has proven to be an efficient control strategy. The concept of repetitive control is based on incorporating the repetitive nature of periodic signals in the control design (a.o. [6, 3]).

A repetitive controller is added to an existing controller in the closed loop as is schematically depicted in the block scheme of figure 4. Here the radial actuator is denoted as $P$, $C$ is a stabilizing controller and the repetitive controller is depicted in the dashed box.

The repetitive controller, also known as memory loop, is formalized in discrete time as follows. The signal buffer constitutes a delay of $N$ samples; the transfer from $e$ to $e'$ in figure 4 is expressed as

$$1 + \frac{q^{-N}}{1 - q^{-N}} = \frac{1}{1 - q^{-N}}$$

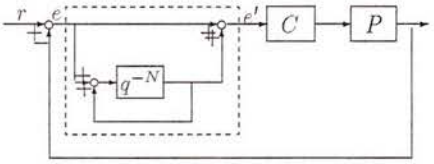


fig. 4: Configuration of repetitive control

where $q^{-1}e(t) = e(t-1)$. Adding the repetitive controller to the closed loop system results in an infinite gain of the loop transfer at frequencies $\omega = \frac{2\pi k}{N}$, $k = 1, 2, \ldots, N$. In case $r$ has the same period length of $N$ samples, the disturbance is suppressed asymptotically.

Adding infinite gains to the loop transfer endangers stability of the closed loop system since the actuator dynamics are not exactly known. Robust stability of a repetitive controller has been thoroughly analysed by Tomizuka ([6]) and Hillerström ([3]). To guarantee stability two filters $F_1$ and $F_2$ are added to the configuration of figure 4 as is shown in figure 5.

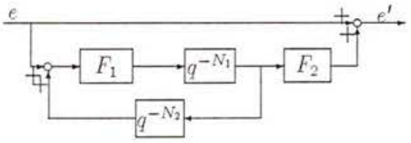


fig. 5: Modified repetitive controller

The filter $F_1$ modifies the infinite gain to a finite but high value and $F_2$ limits repetitive controller action to a frequency range where robustness is preserved. This leads to a modified repetitive controller expressed as

$$1 + F_2 \frac{F_1 q^{-N_1}}{1 - F_1 q^{-(N_1+N_2)}} = \frac{1 - F_1 q^{-N_1}(q^{-N_2} - F_2)}{1 - F_1 q^{-(N_1+N_2)}}.$$

To preserve high gains at the frequencies $\frac{2\pi k}{N}$ the transfer function $F_2 \frac{F_1 q^{-N_1}}{1-F_1 q^{-(N_1+N_2)}}$ is restricted to have a linear phase characteristic. This is achieved by choosing $F_1$ and $F_2$ to be low pass filters with linear phase characteristic.

## 4 Estimation of the fundamental frequency

In this section the estimation of the (time varying) period of a periodic signal is considered.

Consider the signal

$$e(t) = \sum_{k=1}^{M} \alpha_k \ \sin(k\theta_0 t) + v(t)$$

where $v(t)$ is some noise signal, $\alpha_k$ are the coefficients of the periodic components and $\theta_0$ is the fundamental frequency. We are concerned with estimating $\theta_0$ from $e(t)$.

The literature in the field of frequency estimation of a sum of sinusoids is quite extensive (a.o. [5, 8]). Important in this case is the fact that only one parameter is to be estimated (the fundamental frequency) and that the estimation has to be done on line.

A notch filtering technique as described by [4] is considered here for estimating the fundamental frequency. The identification procedure is based on minimizing the prediction error, expressed as:

$$\varepsilon(t, \theta) = H(q, \theta) \ e(t)$$

where

$$H(q, \theta) = \frac{A(q, \theta)}{A(\rho^{-1}q, \theta)} = \prod_{k=1}^{p} \frac{(1 - 2\cos k\theta \ q^{-1} + q^{-2})}{(1 - 2\cos k\theta \ \rho q^{-1} + \rho^2 q^{-2})}$$

where $\theta$ equals the fundamental frequency. The parametrized filter $H(q, \theta)$ has the structure of a notch filter having $p$ notches at $\omega = k\theta$, $k = \{1, \ldots, p\}$. The parameter $\rho$ determines the width of the notches. In case $\rho = 1$ the poles of $H(q, \theta)$ all lie on the unit circle and the notches have zero width, resulting in an unstable filter. By choosing $\rho < 1$ stability of the prediction error filter is obtained and the notches have nonzero width. The latter is imperative in order to increase the filters sensitivity with respect to the time variance of the periodic components. Specific feature of a notch filtering technique is that the fundamental frequency is the only parameter used in the estimation, due to a specific structure of the prediction error filter.

The parameter $\theta$ is estimated in a recursive setting as

$$\hat{\theta} = \arg_\theta \min \frac{1}{2} \sum_{k=1}^{M} \lambda^{M-k} \varepsilon^2(k, \theta)$$

where $\lambda$ is a forgetting factor. Since the periodic components are nonstationary, $\lambda$ must be chosen smaller than 1.

This leads to a nonlinear parametrized optimization problem and as the identification has to be done on line, only an approximate estimate is obtained according to an algorithm of a recursive prediction error type ([4]):

$$\begin{aligned}
\hat{\theta}(t) &= \hat{\theta}(t-1) + L(t) \ \varepsilon(t) \\
\varepsilon(t) &= H(q, \hat{\theta}(t-1)) \ e(t) \\
L(t) &= P(t)\psi(t) = \frac{P(t-1)\psi(t)}{\lambda(t)I + \psi^T(t)P(t-1)\psi(t)} \\
P(t) &= \frac{1}{\lambda(t)}\{P(t-1) - \frac{P(t-1)\psi(t)\psi^T(t)P(t-1)}{\lambda(t)I + \psi^T(t)P(t-1)\psi(t)}\}
\end{aligned}$$

Here $\lambda(t)$ is a time dependent forgetting factor. The algorithm requires calculation of

$$\psi(t) := \frac{\partial \varepsilon(t)}{\partial \hat{\theta}(t)} \approx \frac{\phi(t)}{A(\rho^{-1}q, \hat{\theta}(t-1))}$$

where $\phi(t)$ is a regression vector and of the prediction error $\varepsilon(t, \hat{\theta}(t-1))$.

## 5 Tuning of the controller

In case an accurate estimate of the fundamental frequency is obtained, the repetitive controller can be adjusted accordingly. Crucial issue is to determine whether adjustment based on an estimate available improves disturbance attenuation or not.

A sound strategy with respect to tuning of the repetitive controller hinges on a compromise between two requirements. On the one hand the memory loop is *not* to be tuned as long as periodic disturbances are sufficiently attenuated. On the other hand the memory loop *must* be tuned in case adjustment based on the estimated period provides better disturbance attenuation.

Using adaptive algorithms these requirements are closely related and somehow conflicting, as is stated in [1]. Once the periodic components are sufficiently suppressed in the error signal, excitation of the estimation algorithm is lost, resulting in a bad estimate of the period. Tuning of the memory loop based on this estimate is more likely to deteriorate disturbance attenuation. So, the memory loop must only be adjusted in case the periodic disturbance sufficiently excites the estimation algorithm (good estimate) without allowing the magnitude of the error signal to become too large (sufficient attenuation).

The problem boils down to finding a compromise between allowing some periodic excitation in order to prevent that larger periodic signals arise in the loop.

To formalize the compromise between sufficient attenuation and sufficient excitation, tuning takes place in case the magnitude of the measured error signal exceeds a threshold value. This tuning criterion heavily relies on the assumption that the magnitude of the error signal is mainly due to periodic components. As is obvious from figures 2 and 3 this holds to some extent since the error signal contains nonperiodic noise. It is noted that the tuning criterion is not very robust against nonperiodic disturbances like shocks. The threshold value is determined by trial and error.

Although the exact track position is not known, rough prior knowledge about the track geometry and the scanning velocity is available. From this prior knowledge a physical model of the rotation frequency as a function of time is derived as follows.

The radial distance between the track position at angle $\alpha$ and the center of a spirally shaped track can be expressed as

$$r(\alpha) = r_0 + \frac{dr}{d\alpha}\alpha, \qquad r_0 = r(0).$$

Now the length of the track at $\alpha = \phi$ equals:

$$x(\phi) = \int_0^{\phi} r(\alpha)\, d\alpha = r_0\, \phi + \frac{1}{2}\, \frac{dr}{d\phi}\, \phi^2 \tag{1}$$

and the length of the track scanned at time $t$ is expressed as

$$x(t) = \int_0^{t} v_T(\tau) d\tau = v_T \cdot t \tag{2}$$

with $v_T$ being the (constant) scanning velocity.

Equating the expressions for $x(t)$ and $x(\phi)$ gives

$$v_T \cdot t = r_0\, \phi + \frac{1}{2}\frac{dr}{d\phi}\, \phi^2. \tag{3}$$

Expressing $\phi$ explicitly as a function of time $t$ and differentiating with respect to time gives the following result:

$$\omega(t) := \frac{d\phi}{dt} = \frac{\pi v_T}{\sqrt{\pi^2 r_0^2 + \pi v_T d_T\, t}} \tag{4}$$

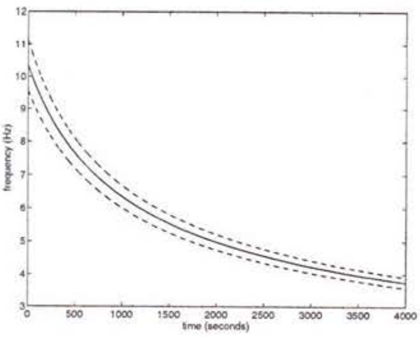


fig. 6: The rotation frequency evolving in time with $v_T = 1.3m/s$ (solid line), $1.4m/s$ (upper dashed line) and $1.2m/s$ (lower dashed line).$d_T = 1.6 \times 10^{-6}m, r_0 = 2 \times 10^{-2}m$.

where $d_T$ is the increase of the radius after $2\pi$ rad. revolution and $\frac{dr}{d\phi} := \frac{d_T}{2\pi}$.

The rotation frequency $\omega(t) := \frac{d\phi}{dt}$ is expressed in terms of the scanning velocity $v_T$ along the track, the increase of the radial distance after one revolution $d_T$ and the radial distance of the track with respect to the center of the disc at time $t = 0$, $r_0$. The values of $v_T$ and $d_T$ are nominal values, standard for an audio compact disc ([2]). Figure 6 shows the rotation frequency as a function of time $t$ as expressed by (4) for 3 different values of $v_T$.

Based on this model a prediction of the rotation frequency at time $t$ can be made. If the model would be sufficiently accurate, the repetitive controller could be adjusted according solely to this model and estimation of the fundamental frequency would not be necessary. However, this is not the case since the description of the track geometry is rather rough and the parameter values of the model are not exact and vary for different discs. For instance, the scanning velocity $v_T$ is constant per disc but may vary between 1.2 m/s and 1.4 m/s.

An adaptive scheme is implemented where the repetitive controller is tuned based on the physical model described above and the model is adjusted in case the error exceeds its maximum value. The configuration is depicted in figure 7. The error signal is low pass filtered before it enters the estimation algorithm in order to prevent aliasing. The filtered error enters the notch filter and is simultaneously monitored to evaluate whether the magnitude does not exceed the threshold value. In case it does, the model is tuned based on the estimate of the period available at that time.

## 6 Experimental results

The adaptive scheme using prior knowledge of the rotation frequency is implemented on the radial servo mechanism of a compact disc player. The filter $F_1$ is a 10th order filter, $F_2$ an 8th order low pass filter, both having a linear phase characteristic. The buffer length of the repetitive controller is adjusted by changing the number of delays $N_1$. The repetitive controller is implemented with a sampling frequency of

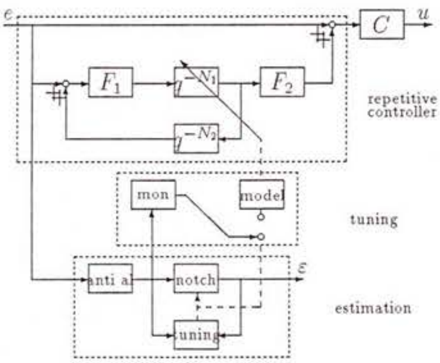


fig. 7: Scheme of the implemented adaptive repetitive controller

2 kHz and a stabilizing PD controller is implemented with 8 kHz using a digital signal processor. Note that the integrator is left out in order to obtain sufficient low frequent excitation of the estimation algorithm. Figure 8 shows the Bode plot of the repetitive controller.

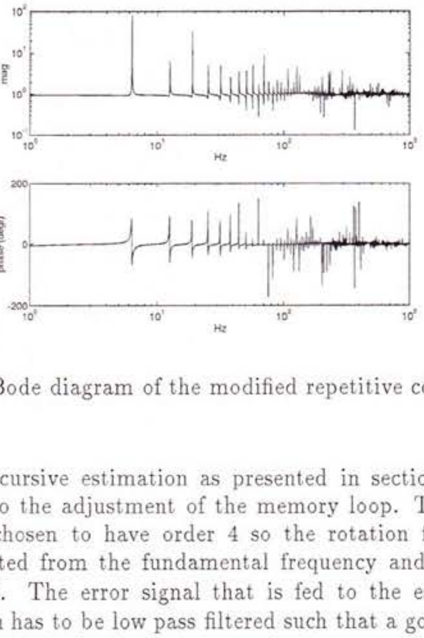


fig. 8: Bode diagram of the modified repetitive controller

The recursive estimation as presented in section 4 runs parallel to the adjustment of the memory loop. The notch filter is chosen to have order 4 so the rotation frequency is estimated from the fundamental frequency and the first harmonic. The error signal that is fed to the estimation algorithm has to be low pass filtered such that a good signal to noise ratio is obtained. A first order low pass filter having a cut off frequency at 20 Hz is implemented, indicated as "anti al." in figure 7.

To demonstrate the merits of a physical model of the time variance of the period, first the adaptative repetitive controller is implemented without the model. A fragment of the measured error signal and the estimate of the rotation frequency are shown in figure 9.

The magnitude of the error signal gradually increases until the moment that the threshold value is exceeded. Adjustment of the memory loop leads to a decrease of the error magnitude, implying an improved attenuation of periodic components; so the estimate seems sufficiently accurate. The

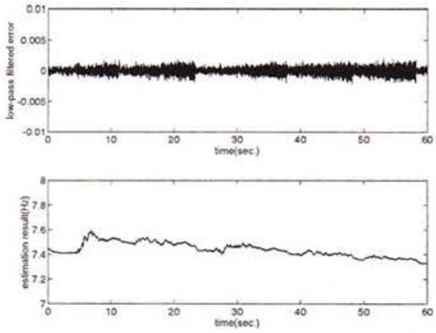


fig. 9: The measured radial error signal $e$ (upper) and the estimated frequency (lower) in case the memory loop is tuned directly by the estimated rotational frequency

compromise between attenuation and excitation is clearly illustrated by the gradual increase and abrupt decrease of the error magnitude.

Now figure 10 shows the same fragment of the error signal and the frequency estimate in case tuning is based on the physical model.

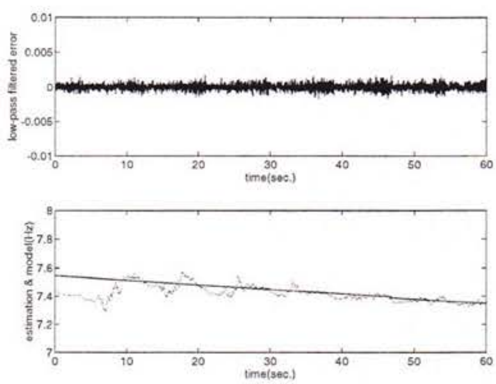


fig. 10: The measured error signal (upper) and estimated (dot) and modelled (solid) frequency (lower) in case the memory loop is tuned by the model of $\omega(t)$.

The maximum magnitude of the error signal is smaller than in case of direct tuning. This means that in the time span considered the physical model provides better suppression of time varying periodic disturbances.

Figure 11 shows what happens in case the model becomes less accurate in the course of time; the magnitude of the error gradually increases due to inadequate adjustment of the memory loop. In case the magnitude of the error signal exceeds a threshold value, the model is updated based on the current estimate of the rotation frequency. Due to loss of excitation of the estimation algorithm the frequency estimate deteriorates.

During the same time span, the use of a priori knowledge of the time varying character of the disturbance period pro-

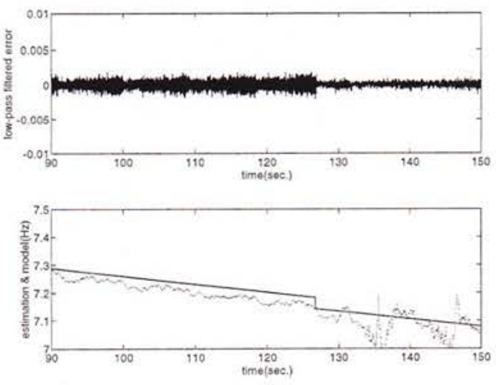


fig. 11: The measured error signal (upper) and estimated (dot) and modelled (solid) frequency (lower) in case the memory loop is tuned by the model of $\omega(t)$. The model is tuned after approximately 127 seconds.

vides better suppression of periodic components than direct tuning based on the estimate. Direct tuning of the memory loop occurs every 12 seconds; tuning of the model once in approximately 125 seconds.

Note that the maximum magnitude of the error is user defined in both schemes since it is determined by the tuning criterion. So in both schemes the error magnitude reaches the same maximum value.

## 7 Conclusions

In this paper the problem of attenuation of time varying periodic disturbances, acting on the radial servo mechanism of a compact disc player is addressed. An adaptive repetitive control scheme is proposed and implemented on an experimental set up of a compact disc player. A memory loop having adequate buffer length suppresses the periodic components in the error signal. Tuning of the memory loop is based on a physical model of the time variance of the rotation frequency. In case this model becomes inaccurate in the course of time, it is updated based on an estimate of the rotation frequency. The estimate is obtained through recursive prediction error identification, based on an adaptive notch filter technique. Use of a physical model provides an improved track following accuracy.

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