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Identifiability in Dynamic Acyclic Networks with Partial Excitation and Measurement

Xiaodong Cheng, Shengling Shi, and Paul M. J. Van den Hof

Abstract— **In this abstract, we study the identifiability problem of dynamic networks with acyclic topology, under a rather general setting where only a subset of its vertices are measured and a subset of the vertices are excited. Based on the concept of *tree/anti-tree covering*, we explore a condition under which all the modules are identifiable. This condition also induces a synthesis approach to allocate actuators and sensors in an acyclic network.**

I. Introduction

Network identifiability is an essential property when addressing data-driven modeling problems in dynamic networked systems, see e.g., [1], [2], [4]–[8], [10], [11] and the references therein. This property basically determines if we are able to distinguish a unique network model on the basis of measurement data.

In the literature, there are currently three notions of identifiability, namely, *global identifiability* [10], [11] that requires *all* the models in the set to be distinguishable, and *generic identifiability* [2], [5], [12], which means that *almost all* models in the model set can be distinguished. Recently, a local notion of generic identifiability is presented in [6], which is defined as identifiability on a neighborhood of network modules. Based on the different notions, identifiability analysis can be performed on a full dynamic network with various measurement and excitation schemes. In [2], [5], [10], all the vertices are assumed to be excited by sufficiently rich external signals, Depending on whether *generic identifiability* or *global identifiability* is considered, these conditions are interpreted in terms of the existence of *vertex disjoint paths* [2], [5] or *constrained vertex disjoint paths* [10] from the out-neighbors of each vertex to the measured vertices in a network. In contrast, the setting considered in [11] requires all the vertices to be measured, while only a subset of vertices are excited by external signals including noises and excitation signals manipulated by users, then network identifiability is characterized by the rank property of certain transfer matrix. Furthermore, if the generic rank of this transfer matrix is considered, a vertex-disjoint path condition for generic identifiability is obtained [12], which is dual to the ones in [2], [5]. In line with the setting in [11], [12], the work in [3], [4] provides a new characterization for generic identifiability using the concept of *disjoint pseudo-tree covering*, from which a graphical tool for synthesizing allocation of external excitation signals can be effectively addressed. While most existing works require all the vertices to be either excited or measured, [1], [6] study identifiability conditions in a more relaxed setting that allows for partial measurement and partial excitation. However, a graph-theoretical characterization for local identifiability is missing in [6]. The condition in [1] only provides primary results that either require a priori knowledge on network dynamics or deal with special networks with tree and cycle topologies.

In line with the network setting of [1], this abstract provides a more general graph-based condition for identifiably of dynamic acyclic networks, where only partial excitation and partial measurement signals are available. This condition is based on a graphical concept, called *disjoint tree/anti-tree covering*, and it can further lead to an algorithmic procedure that allocates actuators/sensors to admit generic identifiability of the model set of an acyclic network.

II. Preliminaries and Problem Setting

A. Notations

Denote $\mathbb{R}$ as the set of real numbers and $\mathbb{R}(q)$ as the rational function field over $\mathbb{R}$ with the variable q. The cardinality of a set $\mathcal{V}$ is represented by $|\mathcal{V}|$. A_{ij} denotes the (i,j) entry of a matrix A, and more generally, $[A]_{\mathcal{U},\mathcal{V}}$ denotes the submatrix of A that consists of the rows and columns of A indexed by two positive integer sets $\mathcal{U}$ and $\mathcal{V}$, respectively. e_i represents the ith column vector of the identity matrix.

B. Graph Theory

The topology of a dynamic network is characterized by a graph $\mathcal{G}$ that consists of a finite and nonempty vertex set $\mathcal{V} := \{1, 2, \cdots, L\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A directed graph is such that each element in $\mathcal{E}$ is an ordered pair of elements of $\mathcal{V}$. If $(i,j) \in \mathcal{E}$, we say that the edge is incident from vertex i to vertex j, and the vertex i is the *in-neighbor* of j, and j is the *out-neighbor* of i. Let $\mathcal{N}_j^-$ and $\mathcal{N}_j^+$ be the sets that collect all the in-neighbors and out-neighbors of vertex j, respectively.

A directed *path* connecting vertices i_0 and i_n is a sequence of edges of the form (i_{k-1}, i_k), $k = 1, \cdots, n$, and every

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X. Cheng is with the Control Systems Group, Department of Electrical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands. He is currently a research associate in the Department of Engineering at the University of Cambridge, CB2 1PZ, United Kingdom. xc336@cam.ac.uk

S. Shi and P. M. J. Van den Hof are with the Control Systems Group, Department of Electrical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands. {s.shi, p.m.j.vandenhof}@tue.nl

vertex appears at most once on the path. If there is a directed path from vertex i to j, we say j is *reachable* from i. Two directed paths are *vertex-disjoint* if they do not share any common vertex, including the start and the end vertices. In a simple directed graph $\mathcal{G}$, we denote $b_{\mathcal{U}\to\mathcal{Y}}$ as the maximum number of mutually vertex-disjoint paths from $\mathcal{U} \subseteq \mathcal{V}$ to $\mathcal{Y} \subseteq \mathcal{V}$. Furthermore, a set of m vertex-disjoint paths from $\mathcal{U}$ to $\mathcal{Y}$ is *constrained* if it is unique, and we denote $\hat{b}_{\mathcal{U}\to\mathcal{Y}}$ as the maximum number of constrained vertex-disjoint paths from $\mathcal{U}$ to $\mathcal{Y}$.

In this paper, we concentrate on a class of simple graphs, which are *acyclic*, meaning that there does not exist a pair of vertices i, j which can reach each other via directed paths.

C. Dynamic Network Model

Consider a dynamic network whose topology is captured by a simple directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V} = \{1, 2, \cdots, L\}$ and edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Following the basic setup in [1], each vertex describes an internal variable $w_j(t) \in \mathbb{R}$, and a compact form of the overall network dynamics is

$$\begin{aligned} w(t) &= G(q)w(t) + Rr(t) + v_{\mathrm{e}}(t), \\ y(t) &= Cw(t) + v_{\mathrm{m}}(t), \end{aligned} \tag{1}$$

where q^{-1} is the delay operator, and all the internal signals are stacked in the vector $w(t) := [w_1(t)\ w_2(t)\ \cdots\ w_L(t)]^\top$. $G(q)$ is hollow transfer matrix, in which the (i, j)-th entry, denoted by $G_{ij}(q) \in \mathbb{R}(q)$, indicates the transfer operator from vertex j to vertex i.

Let $\mathcal{R} \subseteq \mathcal{V}$ and $\mathcal{C} \subseteq \mathcal{V}$ be the vertices that are excited and measured, respectively, and $K = |\mathcal{R}|$ and $N = |\mathcal{C}|$. The signals $r(t) \in \mathbb{R}^K$ and $y(t) \in \mathbb{R}^N$ are the external excitation and measurement signals with $R \in \mathbb{R}^{L\times K}$, $C \in \mathbb{R}^{N\times L}$ binary matrices indicating which vertices are excited and measured, respectively, where each row and column of R and C contain only one nonzero entry. The excitation and measurement noises are represented by $v_{\mathrm{e}}(t) \in \mathbb{R}^L$, $v_{\mathrm{m}}(t) \in \mathbb{R}^L$, respectively.

Assumption 1: We consider a dynamic network (1) with the following properties.

1) The network (1) is *well-posed* and stable, i.e., $(I - G(q))^{-1}$ is proper and stable.
2) All the entries of $G(q)$ are proper and stable transfer operators.

The above assumptions are standard in the identification of dynamic networks, see e.g., [1], [5], [9]–[11], which ensure the properness and stability of the mapping from r to w.

Based on the model setting in (1), a problem of interest concerns if all the dynamics of a network, i.e., parameterized transfer functions in $G(q)$, can be consistently identified from the external excitation signals r and the measured output y. Let $M = (G, R, C)$ be a network model of (1) and

$$\mathcal{M} := \{M(q, \theta) = (G(q, \theta), R, C), \theta \in \Theta\} \tag{2}$$

be the network model set with parameterized models $M(q, \theta)$, in which all the nonzero transfer functions in $G(q, \theta)$ are parameterized independently. The network model set $\mathcal{M}$ contains the modeling assumption on the dynamic network including the topology, and locations of actuators and sensors. Then identifiability of the network model set is defined as follows.

Definition 1: Denote the transfer matrix

$$T(q, \theta) := (I - G(q, \theta))^{-1}. \tag{3}$$

The network model set $\mathcal{M}$ in (2) is *network identifiable* from the submatrix $[T]_{\mathcal{C},\mathcal{R}}$ at $M_0 := M(\theta_0)$ with $\theta_0 \in \Theta$ if the implication

$$CT(q, \theta_1)R = CT(q, \theta_0)R \Rightarrow M(q, \theta_1) = M(q, \theta_0), \tag{4}$$

holds for all parameter $\theta_1 \in \Theta$. Furthermore, the network model set $\mathcal{M}$ is identifiable if (4) holds for all $\theta_0 \in \Theta$.

Generic identifiability of the network model set $\mathcal{M}$ is defined when the implication (4) holds for almost all $\theta_0 \in \Theta$[1], see more details in [2], [5]. Identifiability of a dynamic network reflects the ability to distinguish between models in the set $\mathcal{M}$ from measurement data, or more preciously, from the transfer matrix $[T]_{\mathcal{C},\mathcal{R}}$ as described in Definition 1. In this sense, network identifiability essentially depends on the presence and location of external excitation signals r and the selection of measured vertex signals y.

III. Main Results

In this section, we present a sufficient condition for generic identifiability of an acyclic network model set. We resort to a graph covering approach as in [4] based on directed trees. A *directed tree*, denoted by $\mathcal{T}$, is special acyclic graph containing a unique *root* vertex, from which there is exactly one directed path to every other vertex in $\mathcal{T}$. The sinks in $\mathcal{T}$ are also called *leaves*, and the vertices that are neither the root nor leaves of $\mathcal{T}$ are *internal vertices*. Analogously, an anti-tree $\breve{\mathcal{T}}$ is defined as such all the vertices in $\breve{\mathcal{T}}$ has exactly one directed path to a unique *root* vertex, while all the sources in $\breve{\mathcal{T}}$ are called *leaves*.

Then, we introduce the concept of disjoint trees.

Definition 2 (Disjoint trees): In a directed acyclic graph $\mathcal{G}$, two trees $\mathcal{T}_1$ and $\mathcal{T}_2$ are called *disjoint* if

1) $\mathcal{T}_1$ and $\mathcal{T}_2$ do not share edges, and
2) the edges incident from a common vertex in $\mathcal{T}_1$ or $\mathcal{T}_2$ are included in the same tree.

In parallel with disjoint trees, we can define disjoint anti-trees. Two anti-trees $\mathcal{T}_1$ and $\mathcal{T}_2$ are *disjoint* if $\mathcal{T}_1$ and $\mathcal{T}_2$ do not share edges, the edges incident to a common vertex in $\mathcal{T}_1$ or $\mathcal{T}_2$ are included in the same anti-tree.

It should be emphasized that any acyclic graph can be decomposed into a set of disjoint trees or disjoint anti-trees, namely, there always exist a disjoint tree/anti-tree covering all the edges of an acyclic graph, and such covering is not unique. Based on that, we present a new identifiability condition for acyclic networks as follows.

[1]"Almost all" refers to the exclusion of parameters that are in a subset of Θ with Lebesgue measure zero.

Theorem 1 (Tree/Anti-Tree Covering): Consider the network model set $\mathcal{M}$ associating with an acyclic graph $\mathcal{G}$ with $\mathcal{V} = \mathcal{R} \cup \mathcal{C}$. Then, $\mathcal{M}$ is generically identifiable if either of the following two conditions holds:

1) $\mathcal{G}$ can be decomposed of κ disjoint trees, and for each tree, its root is excited and all the leaves are measured.
2) $\mathcal{G}$ can be decomposed of κ disjoint anti-trees, and for each anti-tree, its root is measured, and all the leaves are excited.

Proof: We prove that $\mathcal{M}$ is generically identifiable if condition 1) holds. Let $\mathcal{G}$ be a composition of disjoint trees $\mathcal{T}_1, \mathcal{T}_2, \cdots, \mathcal{T}_\kappa$, where all the roots are excited, all the leaves are measured, and the rest of vertices are either excited or measured. Therefore, all the sources in $\mathcal{G}$ are excited and all the sinks in $\mathcal{G}$ are measured. Furthermore, it implies that a vertex has more than one in-neighbor *only if* it is measured. To show generic identifiability of $\mathcal{M}$, we can extend our results in [8] for single module identifiability to the full network case. Specifically, $\mathcal{M}$ is generically identifiable if the following conditions hold.

1) Every measured vertex j in the network satisfies
$$b_{\mathcal{R}\to\mathcal{N}_j^-} = |\mathcal{N}_j^-|; \tag{5}$$
2) For each edge (i, j) with j unmeasured, there exist a subset of measured vertices $\mathcal{C}_j$ and excited vertices $\mathcal{R}_j$ including j such that
$$b_{\mathcal{R}_j\to\mathcal{C}_j} = |\mathcal{R}_j|, \tag{6}$$
$$b_{\mathcal{R}_j\setminus j\to\mathcal{C}_j} = b_{(\mathcal{R}_j\cup\mathcal{S}_i)\setminus j\to\mathcal{C}_j}, \tag{7}$$
where $\mathcal{S}_i := \mathcal{N}_i^+$ if $i \in \mathcal{R}$, and $\mathcal{S}_i := \mathcal{N}_i^+ \cup i$ otherwise.

The details of the proof for the above conditions are omitted here due to the limited space.

First, we consider $\mathcal{C}$ as the set of all the measured vertices in the acyclic graph $\mathcal{G}$. It is implied in Definition 2 that each vertex and its out-neighbors are included in the same tree. Thus, for any vertex $j \in \mathcal{C}$, all the edges incident from the vertices in $\mathcal{N}_j^-$ to j should belong to distinct trees. Therefore, there exist at least $|\mathcal{N}_j^-|$ vertex-disjoint paths from the roots of the disjoint trees $\mathcal{T}_1, \mathcal{T}_2, \cdots, \mathcal{T}_\kappa$ to $\mathcal{N}_j^-$. As in condition 1), the roof of each tree is excited, we have $b_{\mathcal{R}_j\to\mathcal{N}_j^-} = |\mathcal{N}_j^-|$, where $\mathcal{R}_j$ is the set of all the roots in the trees. As a result, (5) holds because of $\mathcal{R}_j \subseteq \mathcal{R}$.

Next, all the unmeasured vertices in $\mathcal{G}$ are analyzed. Let j be any an unmeasured vertex, which is excited due to $\mathcal{R}\cup\mathcal{C} = \mathcal{V}$. If condition (1) holds, j has only one in-neighbor in $\mathcal{G}$, which is denoted by i. We then show that G_{ji} is generically identifiable in $\mathcal{M}$. Suppose that both vertices i and j are in a tree $\mathcal{T}_a$, and we can find the first measured descendant of j in $\mathcal{T}_a$, denoted by k, such that (j, k) is an edge in $\mathcal{G}$, or there are no measured vertices on the directed path from j to k. The existence of k is guaranteed because each excited vertex has only one in-neighbor if condition (1) holds. Hereafter, three cases are discussed.

Case I: If the in-neighbor i is excited, and there is a unique directed path from i to k via j, we obtain $G_{ji} = T_{ki}T_{ji}^{-1}$, which immediately gives identifiability of G_{ji} in $\mathcal{M}$.

Case II: If the in-neighbor i is excited, and there are multiple paths from i to k, then we can find a minimum number of measured leaves in $\mathcal{T}_a$, denoted by $\bar{\mathcal{C}}_j$, whose removal will lead to a single path from i to k via j in the entire graph $\mathcal{G}$. Correspondingly, there exits a set of trees $\mathcal{N}_\mathcal{T}$, with $|\bar{\mathcal{C}}_j| = |\mathcal{N}_\mathcal{T}|$, such that each tree $\mathcal{T}_b \in \mathcal{N}_\mathcal{T}$ shares with $\mathcal{T}_a$ a distinct vertex in $\bar{\mathcal{C}}_j$ and the measured vertex k.

Let $\bar{\mathcal{R}}_j$ collect all the roots of the trees in $\mathcal{N}_\mathcal{T}$, where $|\bar{\mathcal{R}}_j| = |\bar{\mathcal{C}}_j|$, and the vertices in $\bar{\mathcal{R}}_j$ are excited by condition (1). It is clear that there is a one-to-one correspondence between $\bar{\mathcal{R}}_j$ and $\bar{\mathcal{C}}_j$, that is, the root of $\mathcal{T}_b \in \mathcal{N}_\mathcal{T}$ has a unique path in $\mathcal{T}_b$ to k via an only measured vertex in $\bar{\mathcal{C}}_j$.

When vertex i is not reachable from $\bar{\mathcal{R}}_j$, the set $\bar{\mathcal{C}}_j$ is minimum disconnecting sets from $\bar{\mathcal{R}}_j$ to $\bar{\mathcal{C}}_j \cup k$ and from $\mathcal{N}_i^+ \setminus j$ to $\bar{\mathcal{C}}_j \cup k$, simultaneously. Consequently, we have
$$b_{\mathcal{R}_j\to\mathcal{C}_j} = b_{\bar{\mathcal{R}}_j\to\bar{\mathcal{C}}_j} + b_{j\to k} = |\bar{\mathcal{R}}_j| + 1 = |\mathcal{R}_j|,$$
$$b_{\mathcal{R}_j\setminus j\to\mathcal{C}_j} = |\bar{\mathcal{R}}_j| = |\bar{\mathcal{C}}_j| = b_{(\mathcal{R}_j\cup\mathcal{N}_i^+)\setminus j},$$
where $\mathcal{R}_j = \bar{\mathcal{R}}_j \cup j$, and $\mathcal{C}_j = \bar{\mathcal{C}}_j \cup k$.

When vertex i is reachable from $\bar{\mathcal{R}}_j$, then there exists a measured ascendant vertex ℓ in $\mathcal{T}_a$ such that (ℓ, i) is an edge in $\mathcal{G}$, or every vertex along the path from ℓ to i is excited. Thereby, ℓ disconnects all the paths from $\bar{\mathcal{R}}_j \cup r_a$ to i, where r_a is the root of $\mathcal{T}_a$. Note that $\bar{\mathcal{C}}_j \cap \ell = \emptyset$ as ℓ is not a leaf. Denote $\mathcal{C}_j := \bar{\mathcal{C}}_j \cup \ell \cup k$, and $\mathcal{R}_j := \bar{\mathcal{R}}_j \cup r_a \cup j$, which satisfies
$$b_{\mathcal{R}_j\to\mathcal{C}_j} = b_{\bar{\mathcal{R}}_j\to\bar{\mathcal{C}}_j} + b_{r_a\to\ell} + b_{j\to k} = |\mathcal{R}_j|. \tag{8}$$
Moreover, it is not hard to verify that $\bar{\mathcal{C}}_j \cup \ell$ is a minimum disconnecting set from $\mathcal{R}_j \setminus j$ to $\mathcal{C}_j$ as well as a disconnecting set from $(\mathcal{R}_j \cup \mathcal{N}_i^+) \setminus j$ to $\mathcal{C}_j$. Therefore, (7) is also satisfied.

Case III: If the in-neighbor i is measured but unexcited, then we can still find a minimum number of measured leaves $\bar{\mathcal{C}}_j$ in the tree $\mathcal{T}_a$, and removing the vertices in $\bar{\mathcal{C}}_j$ will lead to a single path from i to k via j in $\mathcal{G}$. Moreover, a set of trees $\mathcal{N}_\mathcal{T}$ exists as in Case II, whose root set $\bar{\mathcal{R}}_j$ has a one-to-one relation with $\bar{\mathcal{C}}_j$, thus $|\bar{\mathcal{C}}_j| = |\mathcal{N}_\mathcal{T}| = |\bar{\mathcal{R}}_j|$.

Let r_a be the root of $\mathcal{T}_a$. Now i disconnects all the paths from $\bar{\mathcal{R}}_j \cup r_a$ to j, as i is the only in-neighbor of j. Consider $\mathcal{C}_j := \bar{\mathcal{C}}_j \cup i \cup k$, and $\mathcal{R}_j := \bar{\mathcal{R}}_j \cup r_a \cup j$. Therefore, we have
$$b_{\mathcal{R}_j\to\mathcal{C}_j} = b_{\bar{\mathcal{R}}_j\to\bar{\mathcal{C}}_j} + b_{r_a\to i} + b_{j\to k} = |\mathcal{R}_j|. \tag{9}$$
Furthermore, similar to the analysis in Case II, we can verify that $\bar{\mathcal{C}}_j \cup i$ is a minimum disconnecting set from $\mathcal{R}_j \setminus j$ to $\mathcal{C}_j$ as well as a disconnecting set from $(\mathcal{R}_j \cup \mathcal{N}_i^+ \cup i) \setminus j$ to $\mathcal{C}_j$, leading to (7).

According to the above analysis, for every edge (i, j) in $\mathcal{G}$ with j an unmeasured vertex, the path-based conditions in (6) and (7) are fulfilled. Therefore, the network model set $\mathcal{M}$ is generically identifiable.

Then the proof of the condition (1) has been completed, and the proof for the second condition can be proved following a similar reasoning and thus is omitted here. ■

Given an acyclic network without any excitations and measurements, the synthesis problem in this paper is to allocate actuators and sensors to achieve generic identifiability of the

Algorithm 1 Allocation of Actuators and Sensors in Acyclic Networks

1: Decompose the acyclic network into a set of disjoint minimal trees.
2: Merge two trees whose union is also a tree.
3: Repeat Step 2 until there is no pair of trees that can be merged, so that we a set of disjoint trees $\mathcal{T}_1, \mathcal{T}_2, \cdots, \mathcal{T}_\kappa$ is found to cover all the edges of the acyclic graph $\mathcal{G}$.
4: **if** a vertex in $\mathcal{G}$ is a root of a tree **then**
5: Put this vertex in $\mathcal{R}$;
6: **else if** a vertex in $\mathcal{G}$ is a leaf of a tree **then**
7: Put this vertex in $\mathcal{C}$;
8: **else**
9: Put this vertex in either $\mathcal{C}$ or $\mathcal{R}$.
10: **end if**
11: **return** $\mathcal{R}$ and $\mathcal{C}$ as actuator and sensor vertices

network. To tackle this problem, we devise an algorithmic method to allocate actuators and sensors on the basis of Theorem 1, see Algorithm 1.

The procedure in the algorithm is now explained. In Step 1, a minimal tree covering is found, where a minimal tree consists a vertex with all its outgoing edges and neighbors. It follows a similar reasoning as in [4] that any directed acyclic graph can be decomposed into a set of disjoint minimal trees. Then, we can iteratively merge two disjoint minimal trees if their union is still a tree, see Steps 2 and 3. We can adopt the heuristic algorithm in [4], which gives a particular order for the merging. The tree covering in this paper can be regarded as a special case of the pseudotree covering dealt in [4]. The merging stops when there is no pair of trees that can be merged into a new tree. Then, we obtain a disjoint tree covering with only a few number of trees, which are used to allocate sensors and actuators. For each tree, allocate an actuator at its root, and measure all the leaves. The other vertices in the tree can be either excited or measured.

Example 1: We use a ten-vertex acyclic network in Fig. 1 to illustrate the procedure of Algorithm 1. First, a disjoint tree covering of the graph is found with a minimum cardinality. The trees are distinguished by different colors in Fig. 1. Then, we select their roots and leaves to be excited and measured, respectively. Thus, we have $\mathcal{R} = \{1, 2, 4, 7\}$ and $\mathcal{C} = \{2, 4, 5, 6, 7, 8, 9\}$. The remaining vertices 3 and 10 can be either excited or measured. In this example, vertices 3 and vertex 10 are assigned to $\mathcal{C}$ and $\mathcal{R}$, respectively. Now, the network is generically identifiable due to Theorem 1.

Algorithm 1 utilizes the first condition in Theorem 1 to find the two potential sets $\mathcal{R}$ and $\mathcal{C}$ for allocating actuators and sensors, respectively. Alternatively, we can also design a different scheme following the second condition in Theorem 1, where an anti-tree covering of $\mathcal{G}$ is considered. Particularly, we first seek for a minimum set of disjoint anti-trees that cover all the edges of $\mathcal{G}$ and allocate sensors and actuators at the roots and leaves of the anti-trees, respectively.

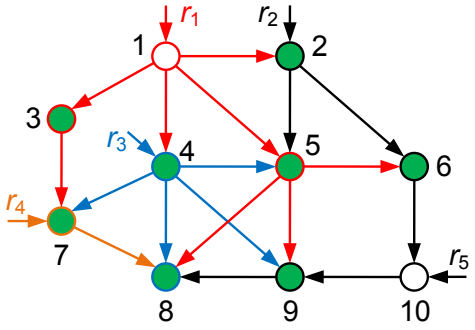

Fig. 1: A tree covering of a ten-vertex acyclic network, where the disjoint trees are indicated by different colors.

IV. Conclusion

In this abstract, we have analyzed identifiability of a dynamic network where only partial excitation and measurement signals are available. The major contribution of this abstract is to present a sufficient condition for identifiability of acyclic networks. This condition is established using the concept of disjoint tree/anti-tree covering and is instrumental to develop an algorithmic procedure, aiming to select potential locations for actuators and sensors such that an acyclic network is generically identifiable.

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