EINDHOVEN UNIVERSITY OF TECHNOLOGY

Dynamic networks modeling

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Doctoral School Lyon, France, 11-12 April 2024

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European Research Council

ERC SYSDYNET Team: data-driven modeling in dynamic networks

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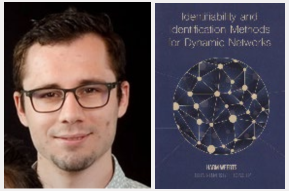
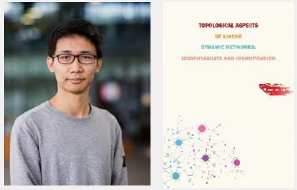


Harm Weerts



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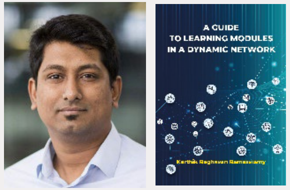

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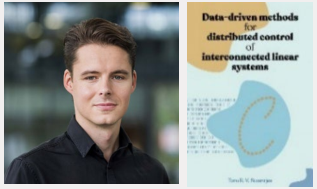

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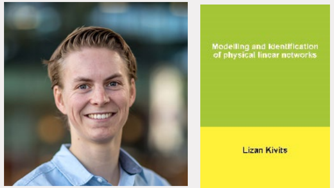

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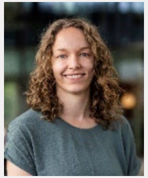

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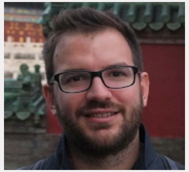

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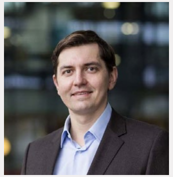

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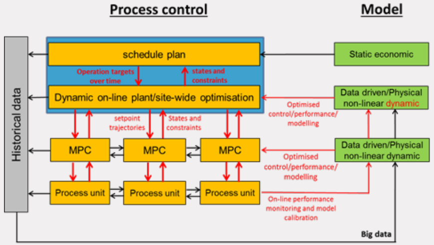
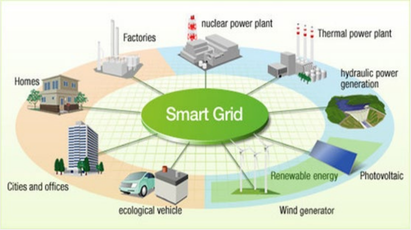
Dynamic networks

Introduction – dynamic networks

Decentralized process control

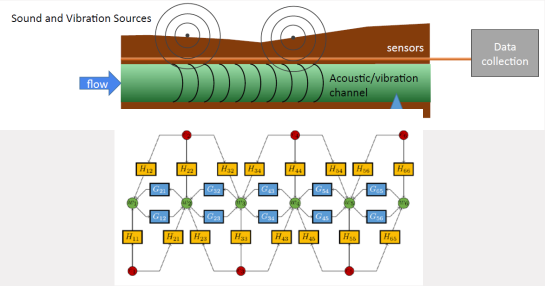


Smart power grid



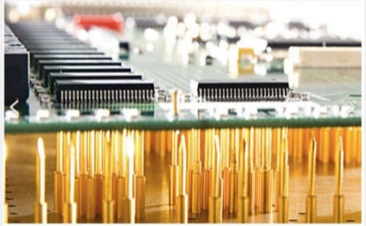
Betterworldsolutions.eu

Gas pipelines



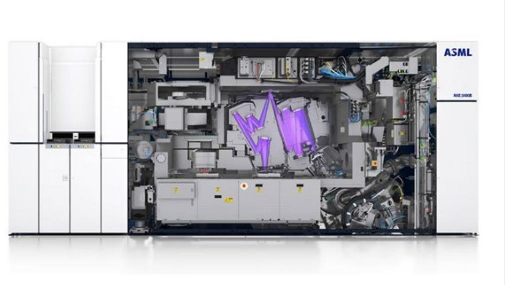
Arne Dankers, Hifi Engineering

PCB testing

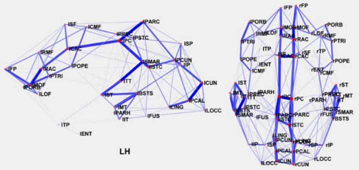


T&M Solutions, Romex BV

Lithography machines

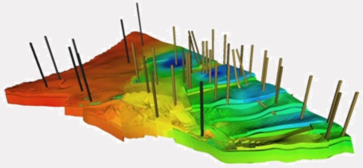


Brain network



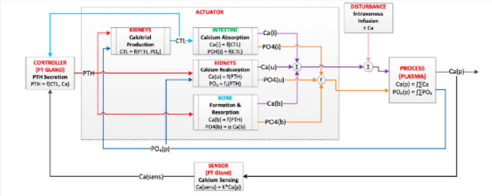
P. Hagmann et al. (2008)

Hydrocarbon reservoirs



Mansoori (2014)

Physiological models



Christie, Achenie and Ogunnaike (2014)

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Introduction

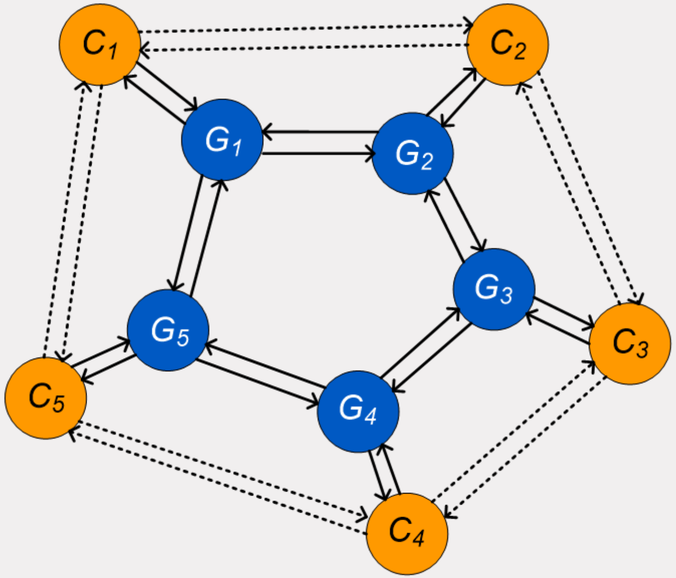
Overall trend:

- Systems become more and more interconnected and large scale
- The scope of system's control and optimization becomes wider
  From components/units to systems-of-systems
- Modeling, monitoring, control and optimization actions become distributed
- Data is playing an increasing role in monitoring, decision making, control of (highly autonomous) smart systems (machine learning, AI)

- → **Learning models/actions from data** (including physical insights when available)

Introduction

Distributed / multi-agent control:



With both physical and communication links between systems G_i and controllers C_i

How to address data-driven modelling problems in such a setting?

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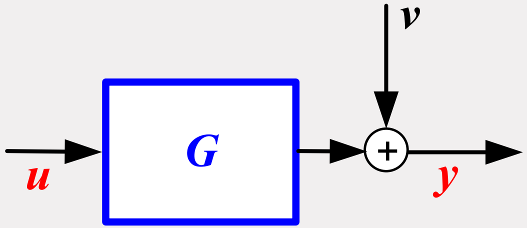
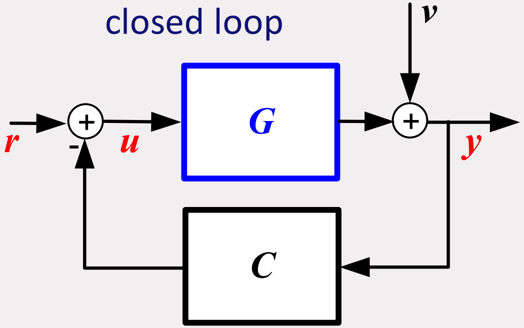
Introduction

The classical (multivariable) identification problems[1] :

open loop



closed loop



Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

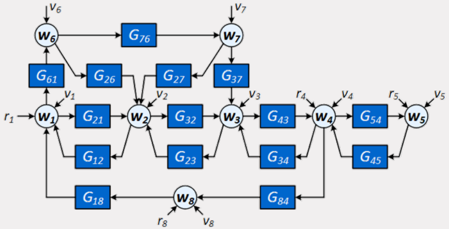
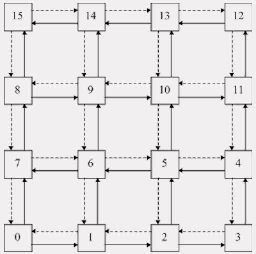
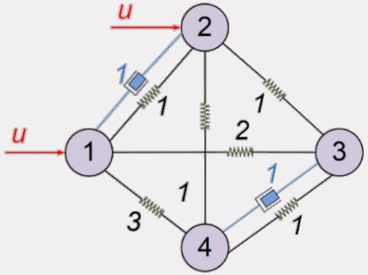
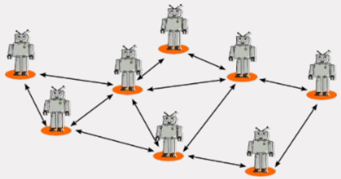
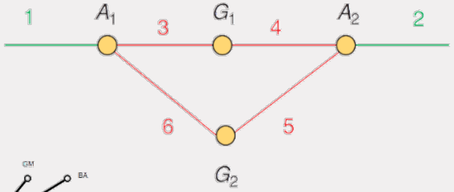
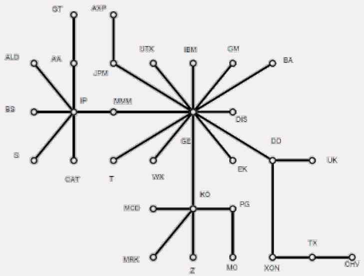
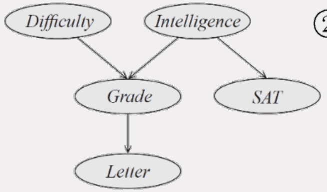
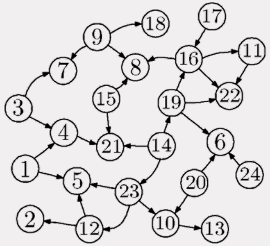
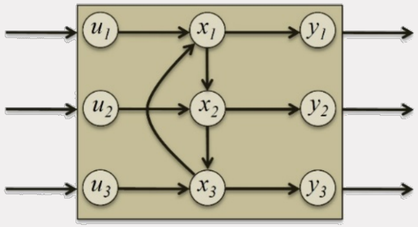
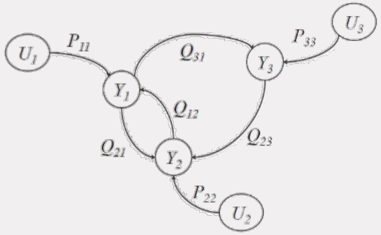
We have to move from a simple and fixed configuration to deal with ***structure*** in the problem.

[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

Dynamic networks for data-driven modeling

Network models

- dynamic elements with cause-effect
- handling feedback loops (cycles)
- centered around measured signals
- allow disturbances and probing signals

D. Materassi and M.V. Salapaka (2012)

R.N. Mantegna (1999)

www.momo.cs.okayama-u.ac.jp

J.C. Willems (2007)

D. Koller and N. Friedman (2009)

E.A. Carara and F.G. Moraes (2008)

P.E. Paré et al (2013)

P.M.J. Van den Hof et al (2013)

X.Cheng (2019)

E. Yeung et al (2010)

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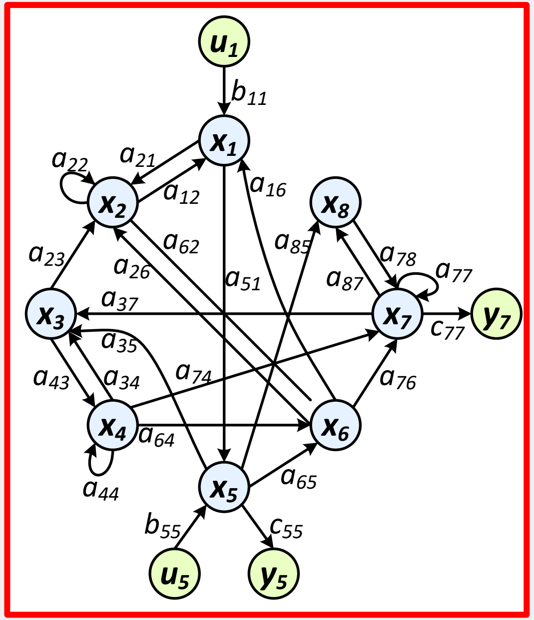
Network models

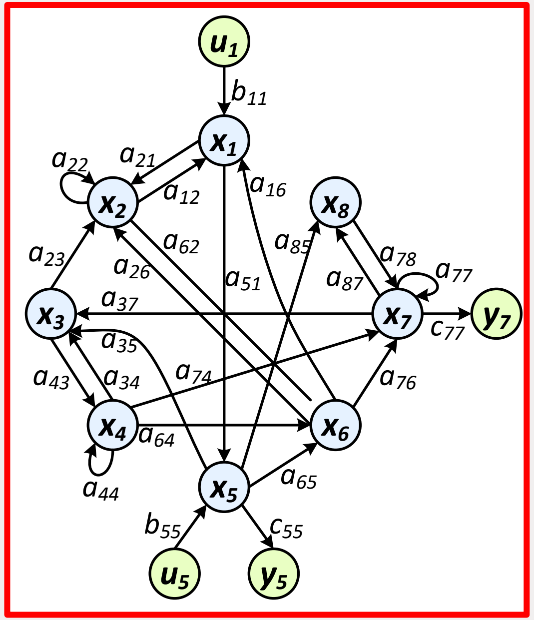


State space representation

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

- States as **nodes** in a (directed graph)
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in **links**
- Effect of transitions are summed in the nodes
- Self loops are allowed
- Actuation (u) and sensing (y) reflected by separate links

Network models



State space representation

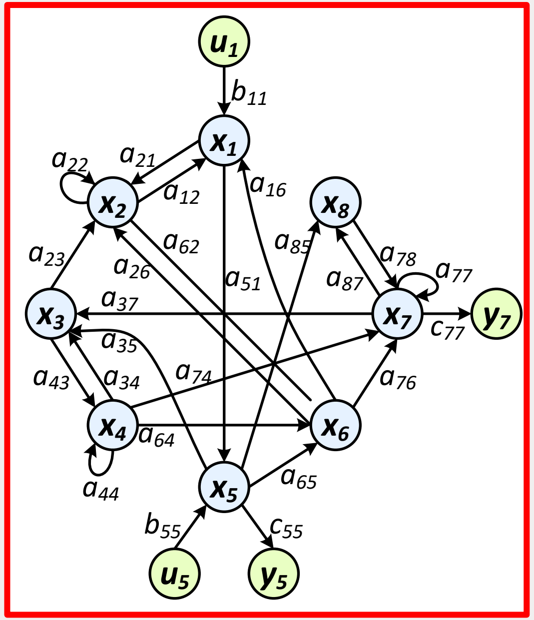
$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

- Ultimate break-down of structure in the system
- to smallest possible level of detail

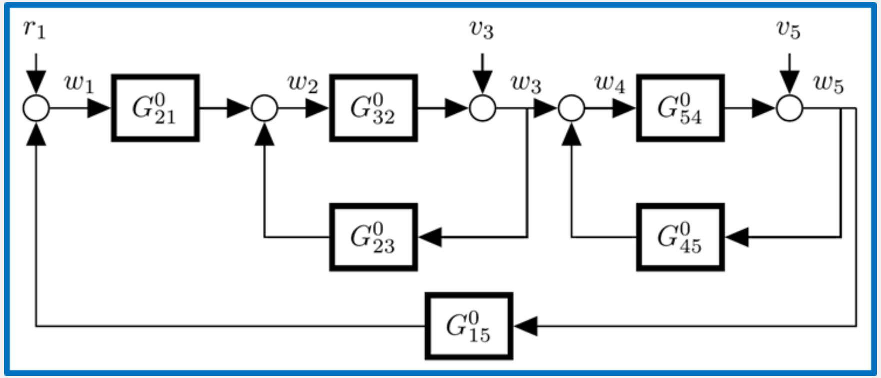
For data-driven modeling problems:

- Stronger role for measurable inputs and outputs
- i/o dynamics can be lumped in dynamic **modules**

Network models



State space representation [1]



Module representation [2]

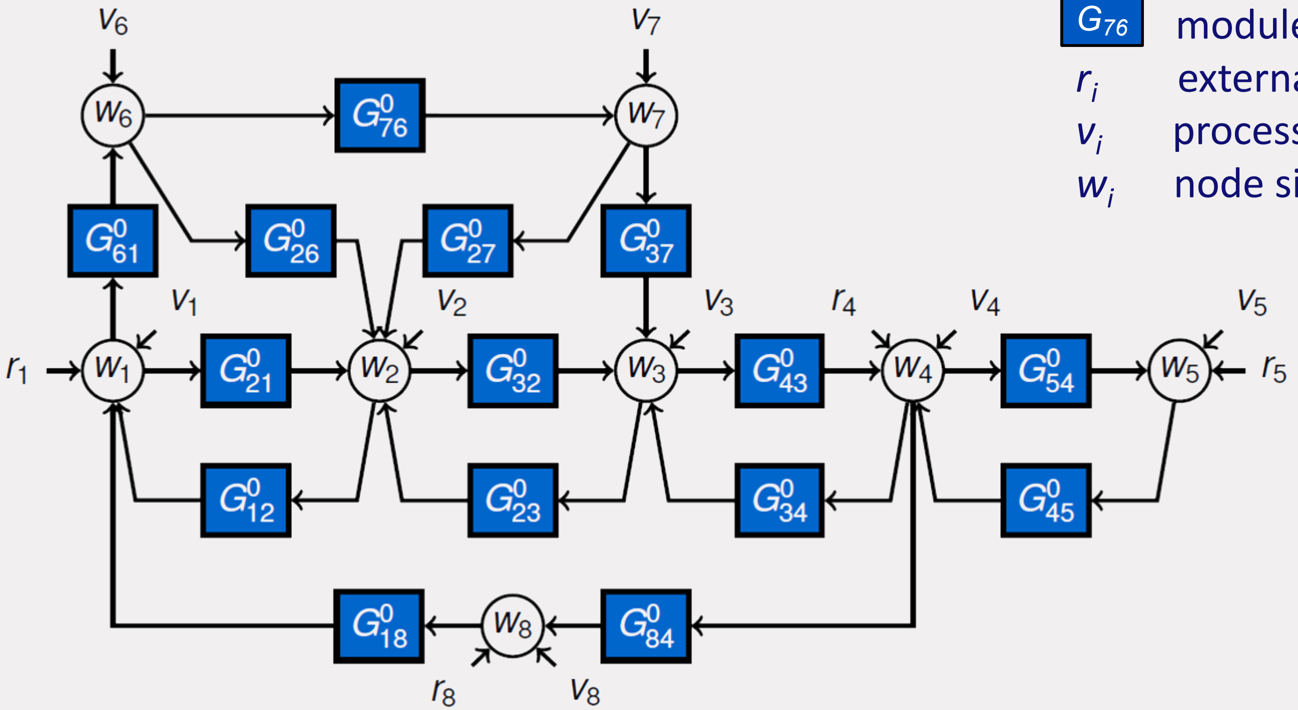
Compare e.g. classical signal flow graphs [3]

[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…

[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,…

[3] S.J. Mason, 1953, 1955.

Dynamic network setup



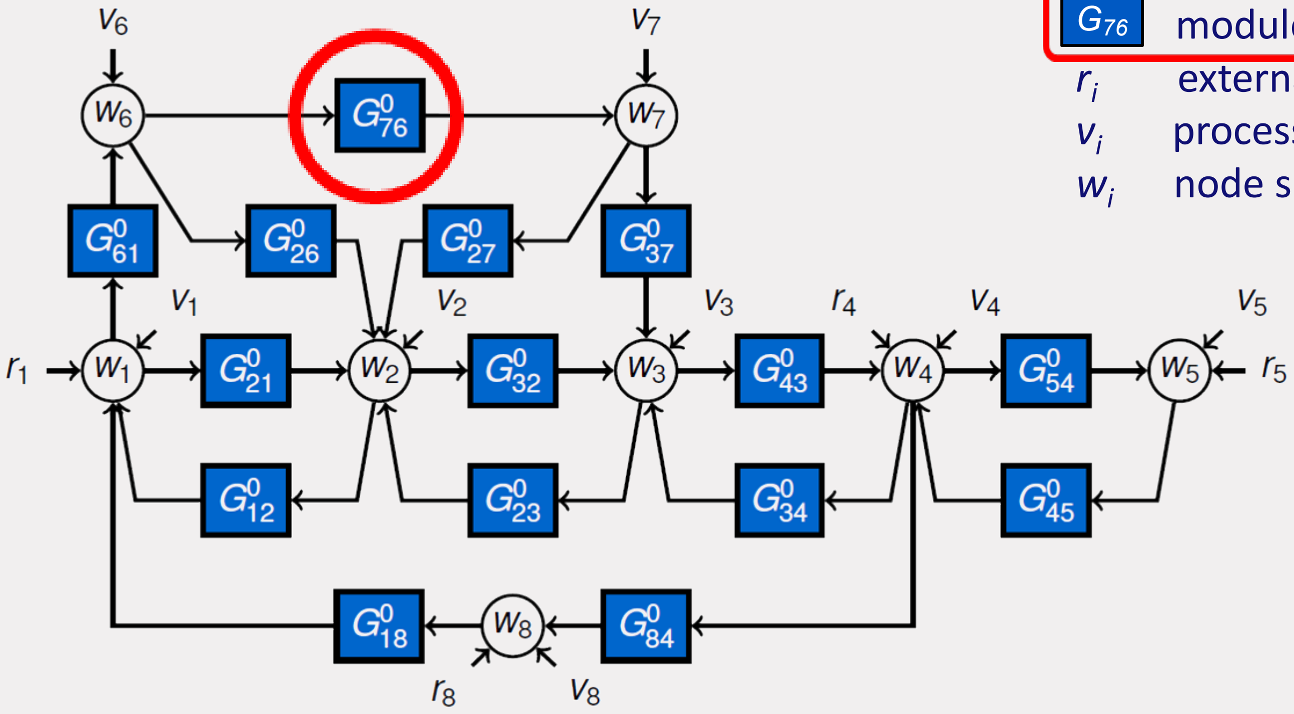
G_{76} module

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



G_{76} module

r_i external excitation

v_i process noise

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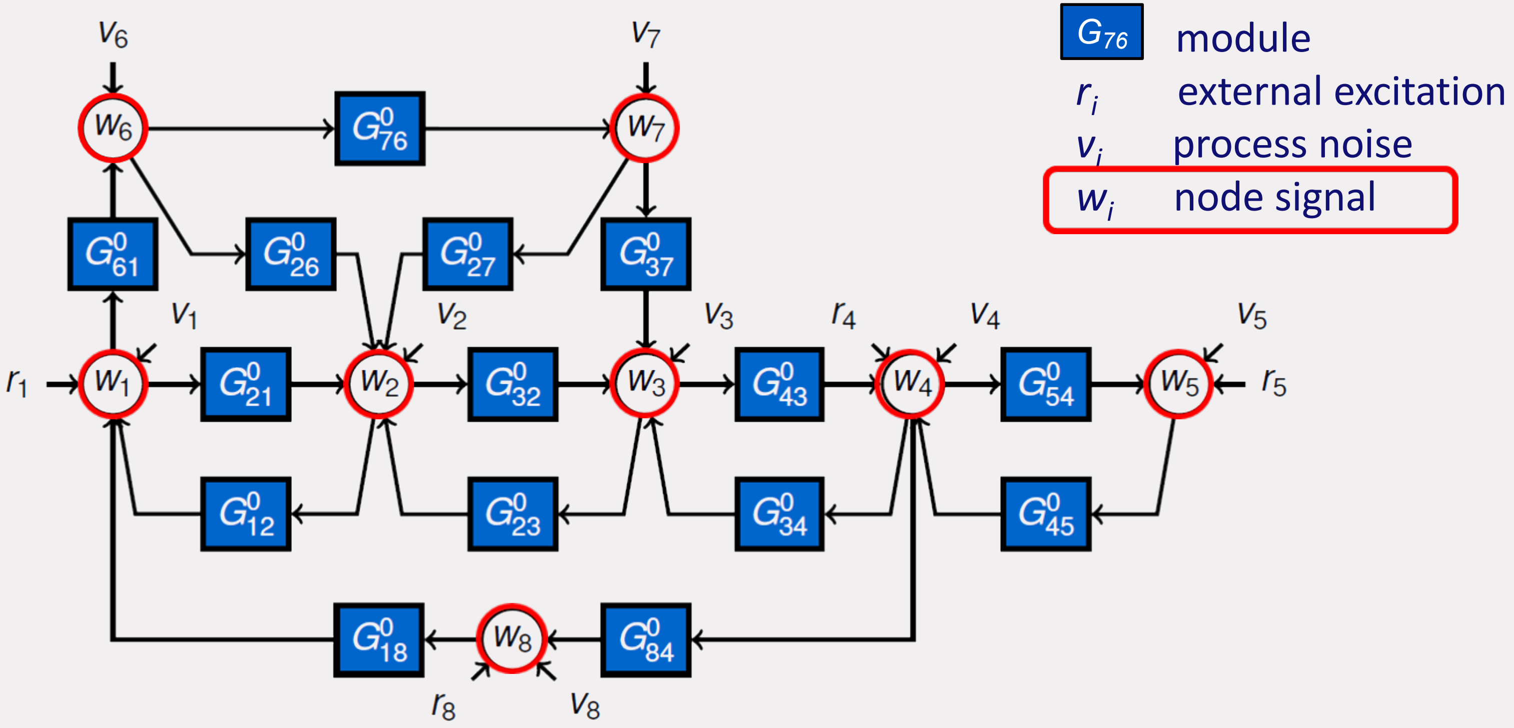
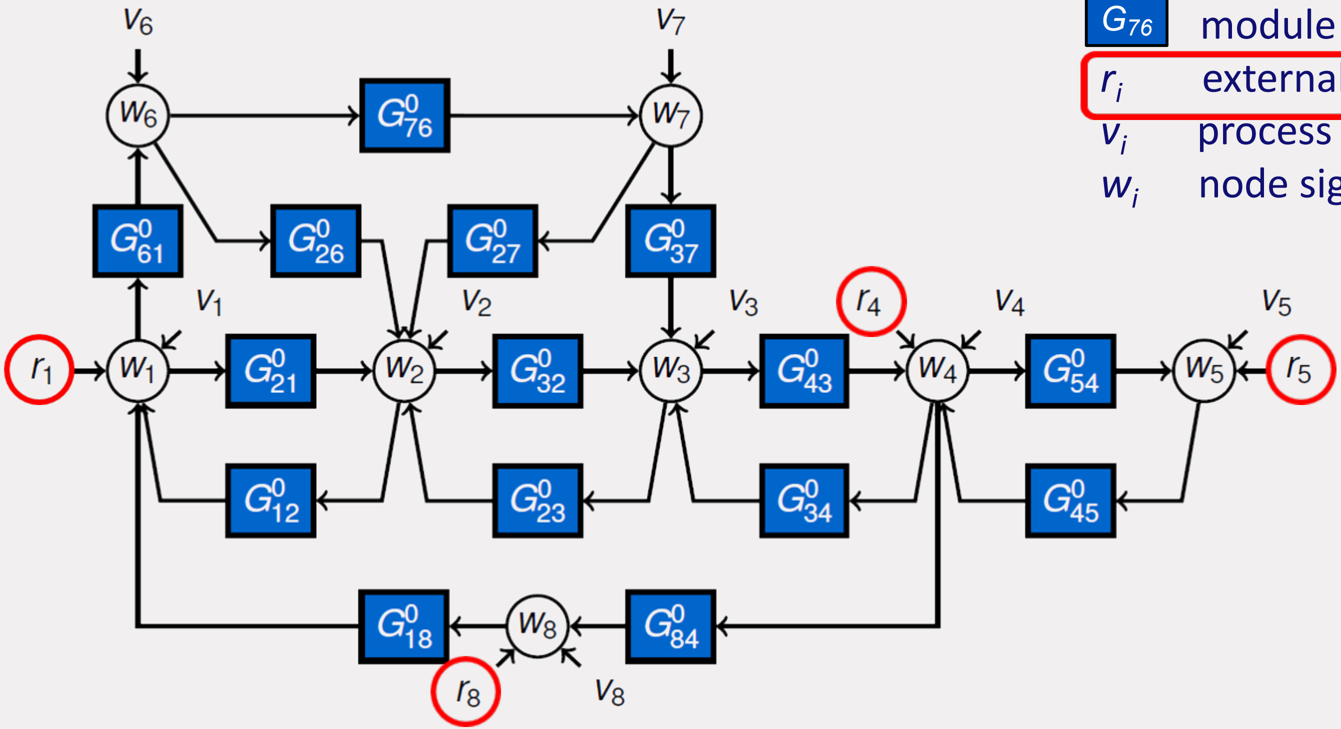
Dynamic network setup

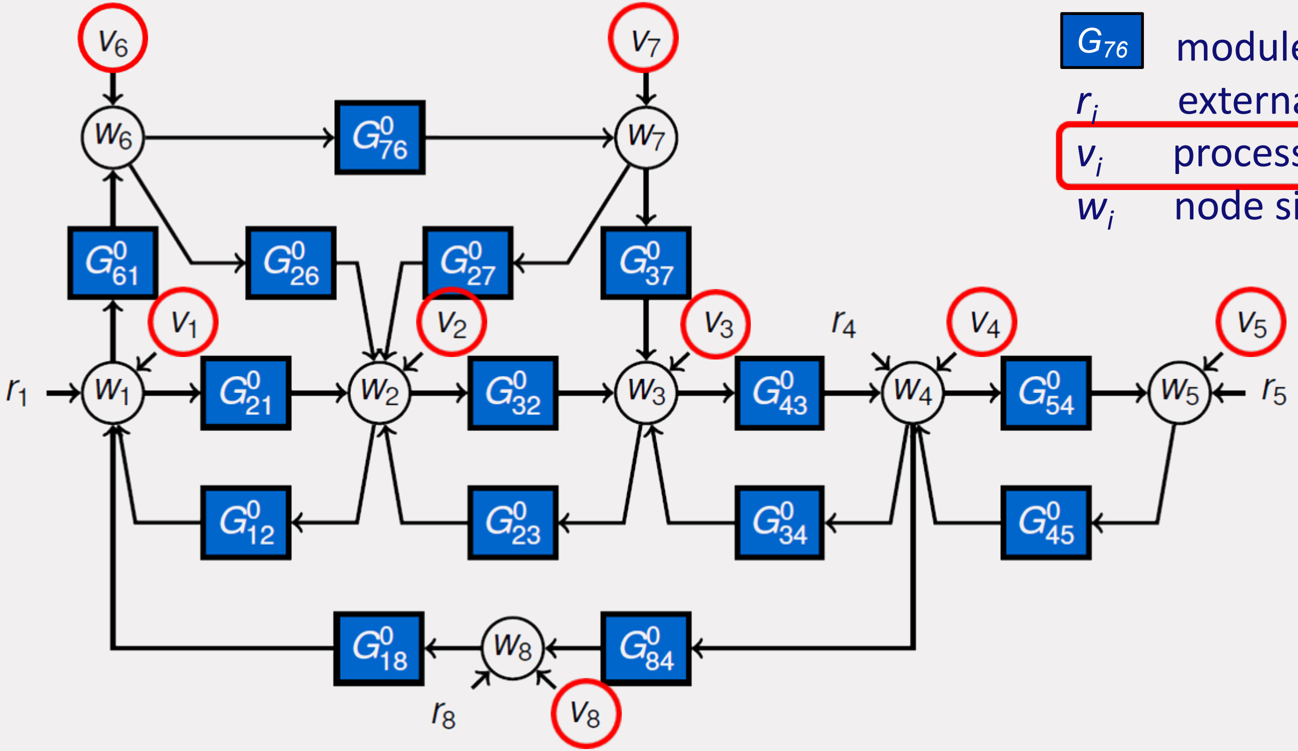


Dynamic network setup



G_{76}	module
r_i	external excitation
v_i	process noise
w_i	node signal

Dynamic network setup



G_{76}	module
r_i	external excitation
v_i	process noise
w_i	node signal

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Dynamic network setup

Basic building block:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + v_j(t) + u_j(t)$$

w_j: node signal
v_j : (unmeasured) disturbance, stationary stochastic process
u_j : external excitation signal
G^0_{jk}: module, rational proper transfer function, $\mathcal{N}_j \subset \{\mathbb{Z} \cap [1, L] \backslash \{j\}\}$

Node signals: $w_1, \cdots w_L$
Interconnection structure / topology of the network is encoded in $\mathcal{N}_j,\ j = 1, \cdots L$

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

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Dynamic network setup

Collecting all equations:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix}$$

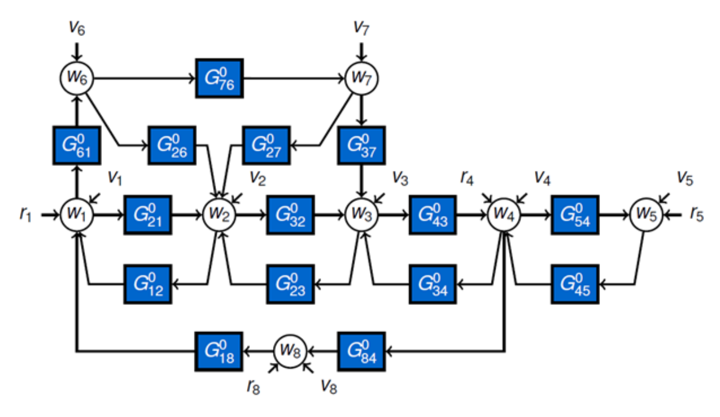
$$w(t) = G^0(q)w(t) + v(t) + \underbrace{R^0(q)r(t)}_{u(t)}; \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r (u) and e are called external signals.

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

Dynamic network setup

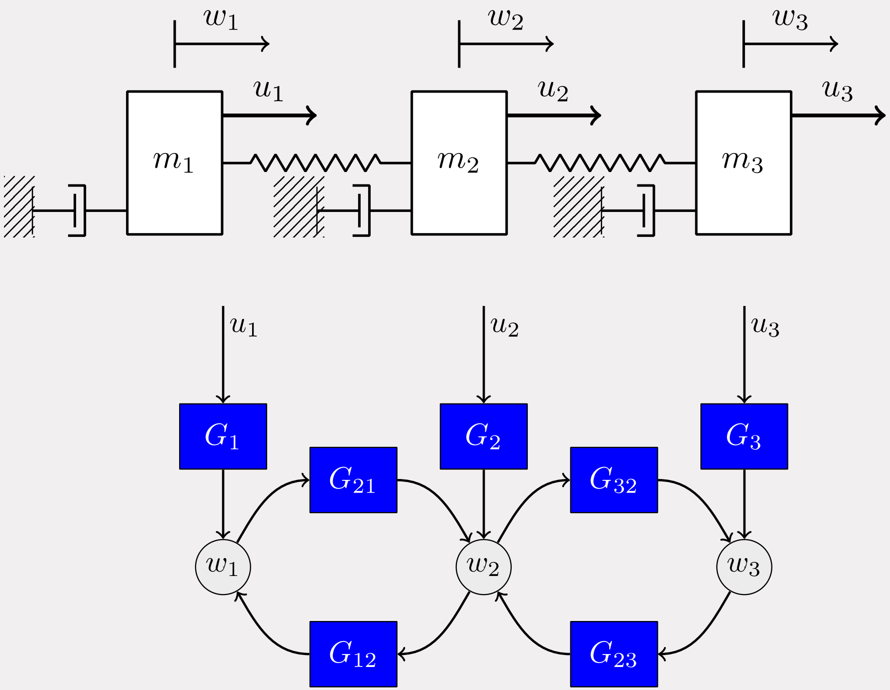
$$w = G^0 w + H^0 e + R^0 r$$



Assumptions:

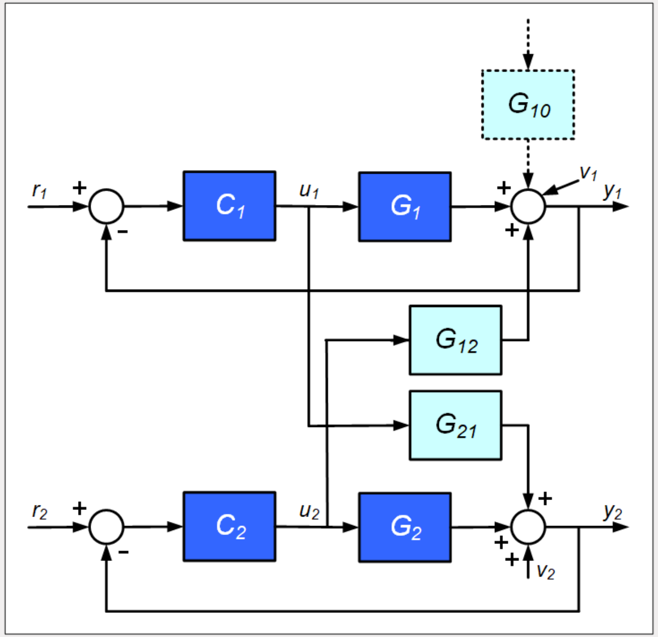
- Total of L nodes, no self-loops
- Network is well-posed and stable, i.e. $(I - G^0)^{-1}$ exists and is stable
- Modules are dynamic, LTI, proper, may be unstable
- Disturbances can be correlated: H^0 not necessarily diagonal

Example: Networks of (damped) oscillators



- Power systems, vehicle platoons, thermal building dynamics, …
- Spatially distributed
- Bilaterally coupled

Example: decentralized MPC

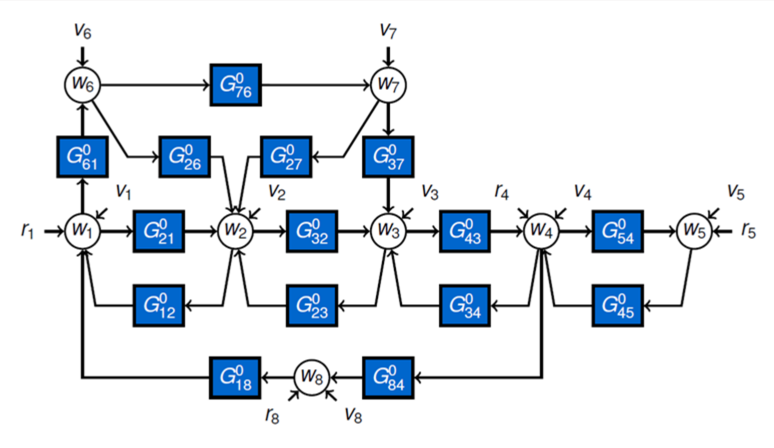


Decentralized MPC

2 interacting MPC loops

R.D. Gudi and J.B. Rawlings, AIChE Journal, 52(6), 2198-2210, 2008.

Data-driven modeling

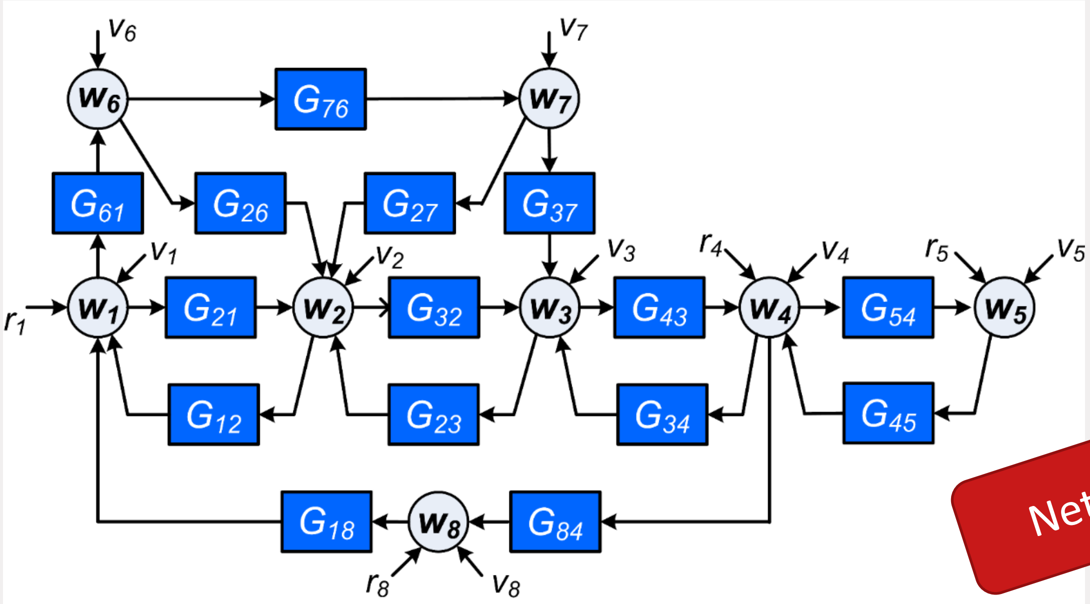


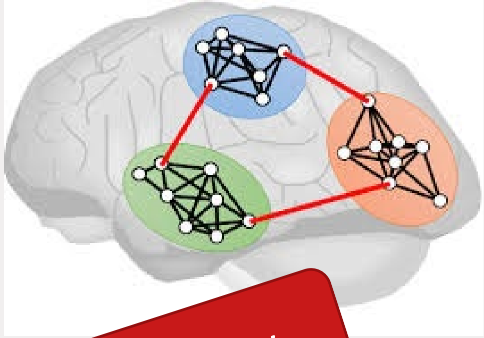
Many data-driven modeling questions can be formulated

Measured time series:

$$\{w_i(t)\}_{i=1,\cdots L}; \;\; \{r_j(t)\}_{j=1,\cdots K}$$

Model learning problems





Network identifiability

Under which conditions can we estimate the topology and/or dynamics of the full network?

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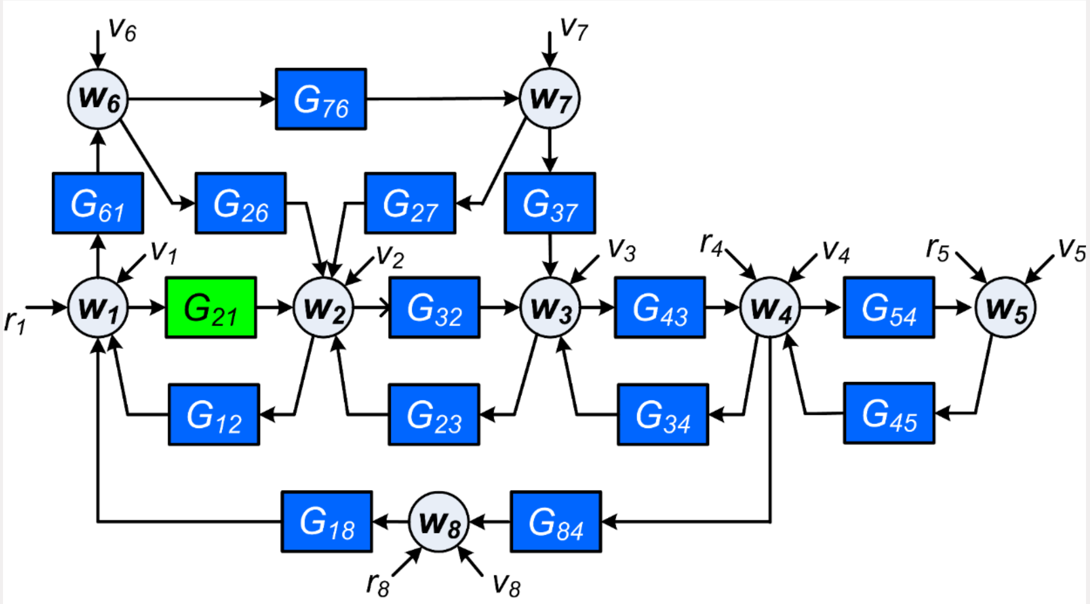
Model learning problems



How/when can we learn a local module from data
(with known/unkown network topology) ? Which signals to measure?

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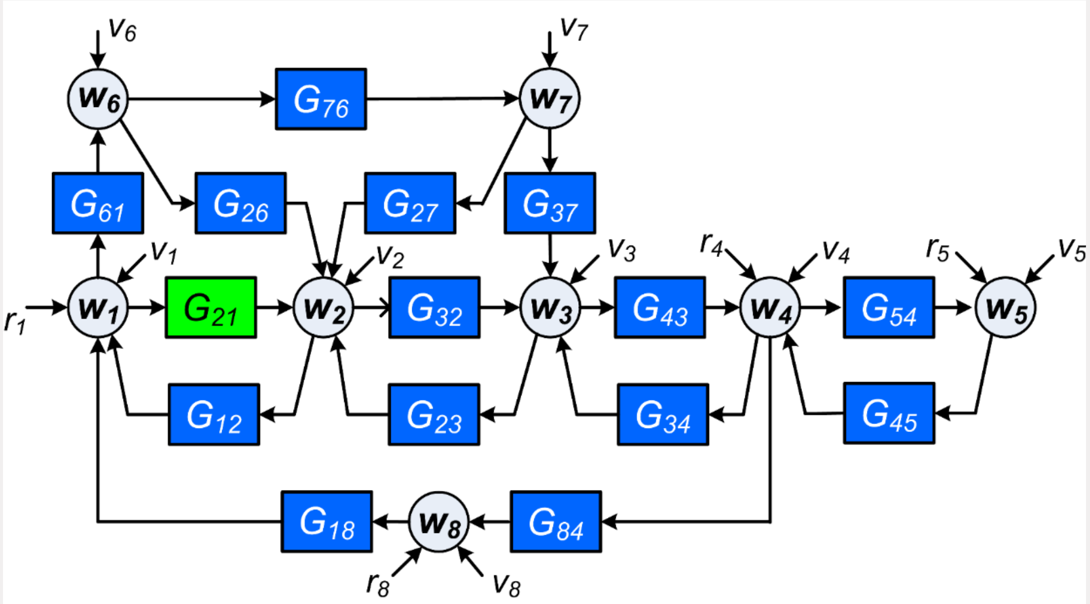
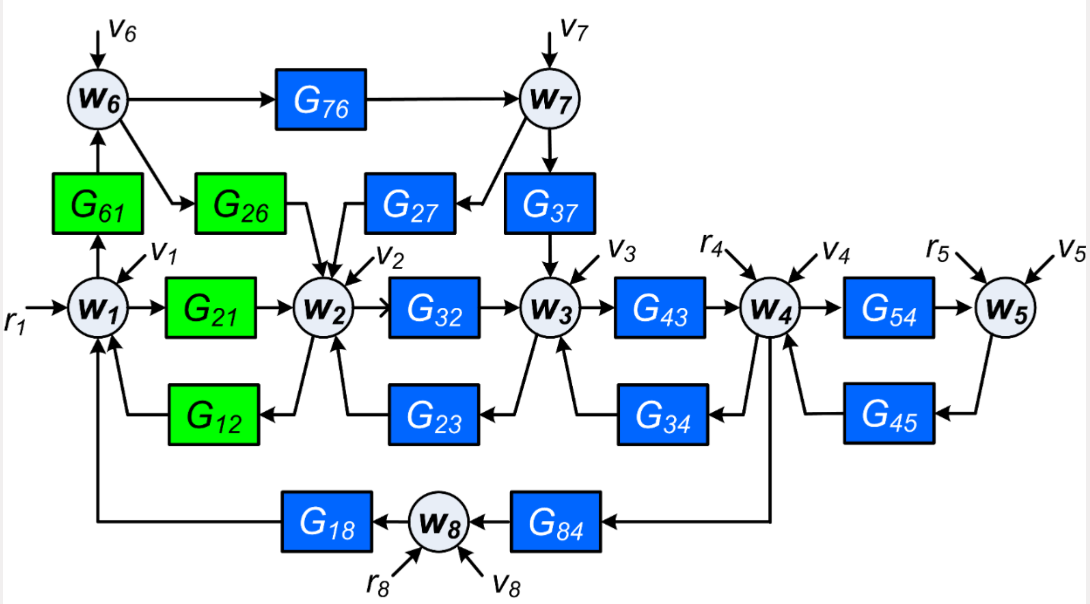
Model learning problems



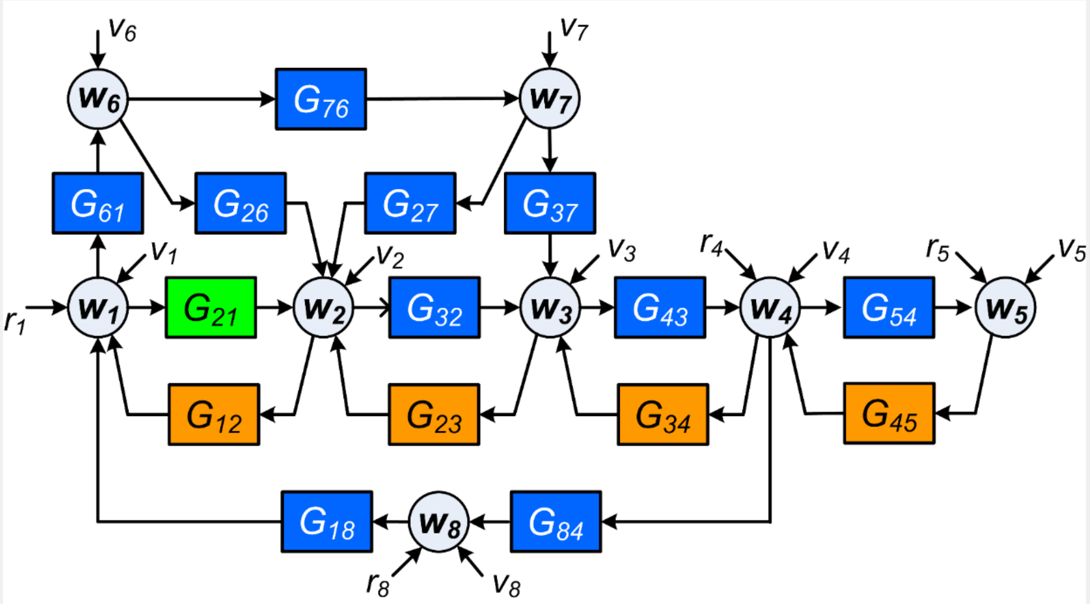
Where to optimally locate sensors and actuators,
and how to design experiments?

Model learning problems



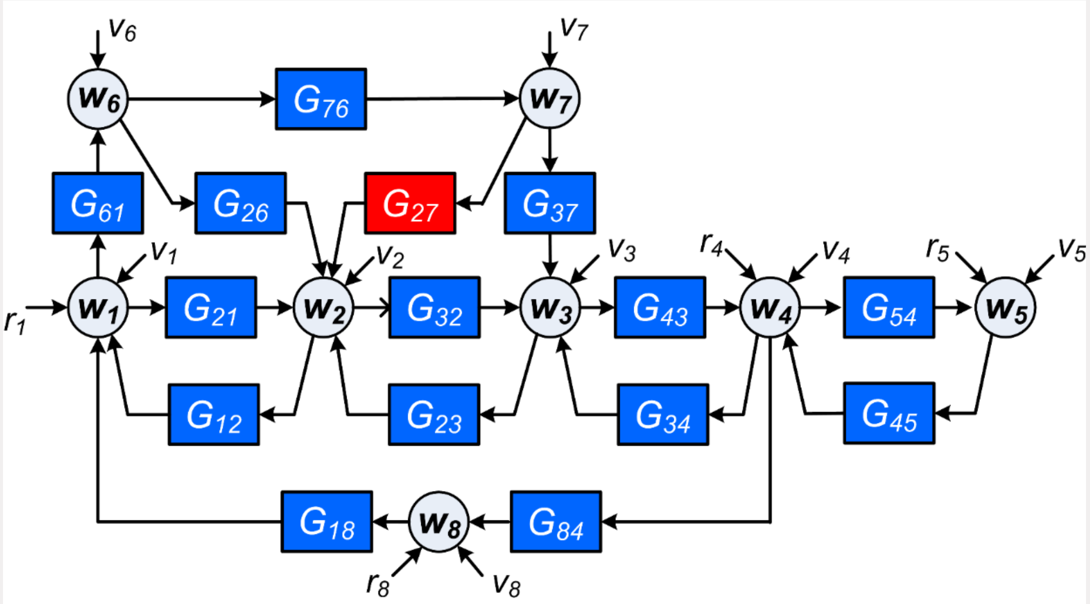
Same questions for a subnetwork

Model learning problems



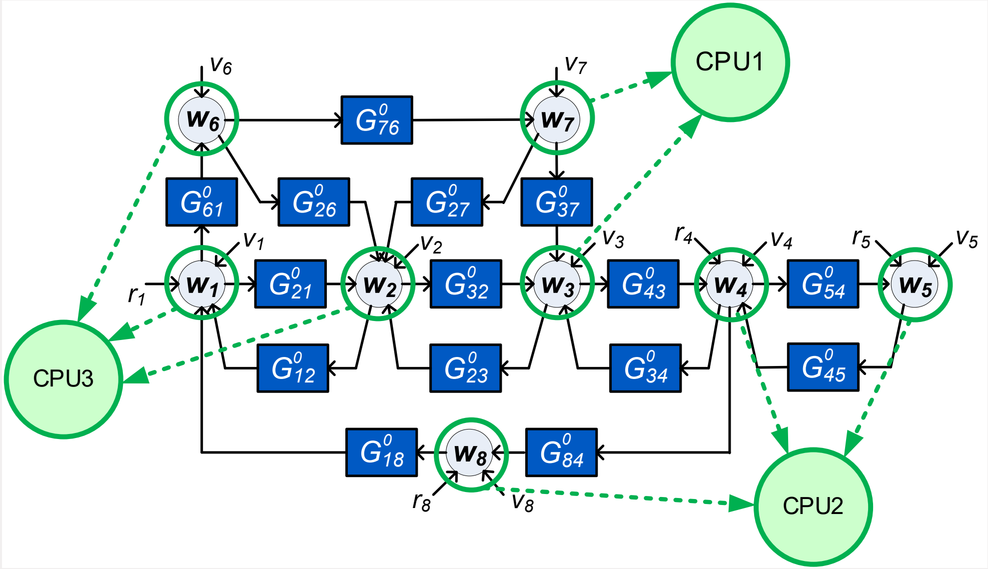
How can we benefit from a priori known modules?

Model learning problems



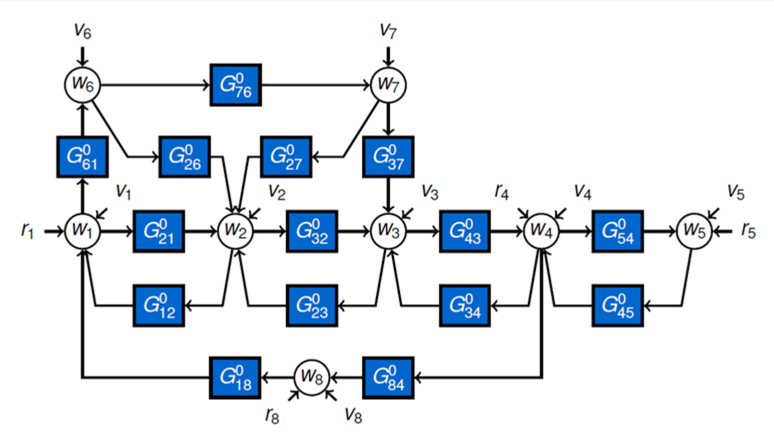
Fault detection and diagnosis; detect/handle nonlinear elements

Model learning problems



Can we distribute the computations?

Dynamic network setup



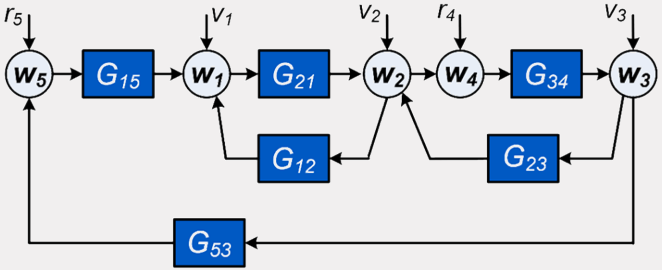
Measured time series:

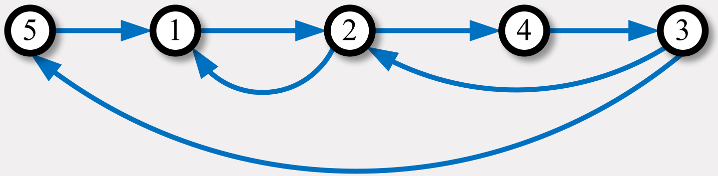
$$\{w_i(t)\}_{i=1,\cdots L}; \ \{r_j(t)\}_{j=1,\cdots K}$$

Many data-driven modeling questions can be formulated

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- **Scalable algorithms**

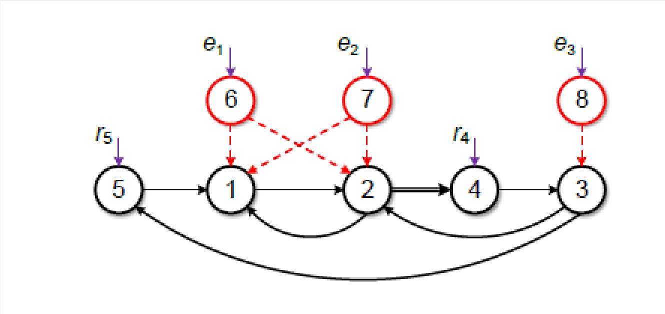
Dynamic network setup - graph





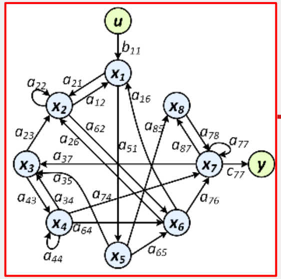
Nodes are vertices; modules/links are edges

Extended graph:
including the external signals
and disturbance correlations

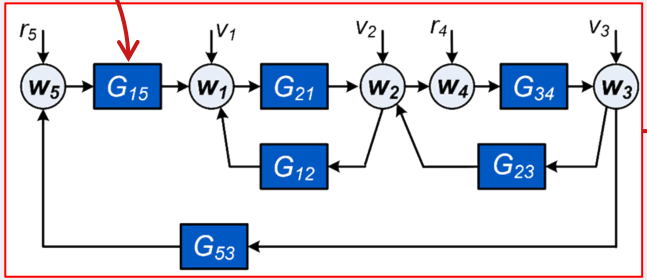


TU/e

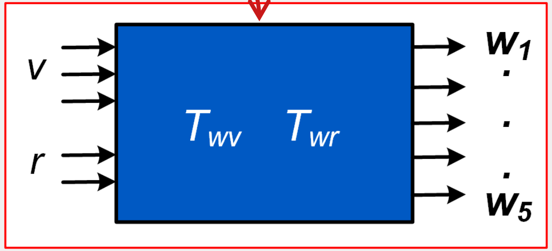
Dynamic network models - zooming



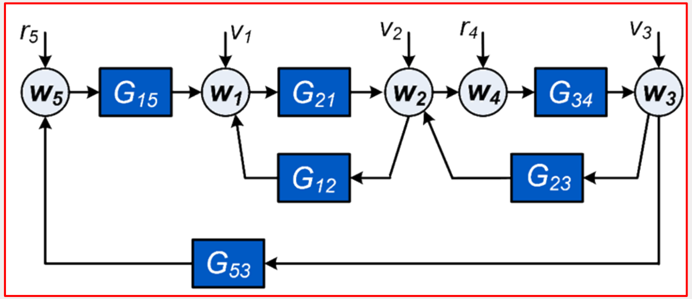
Increasing level of detail

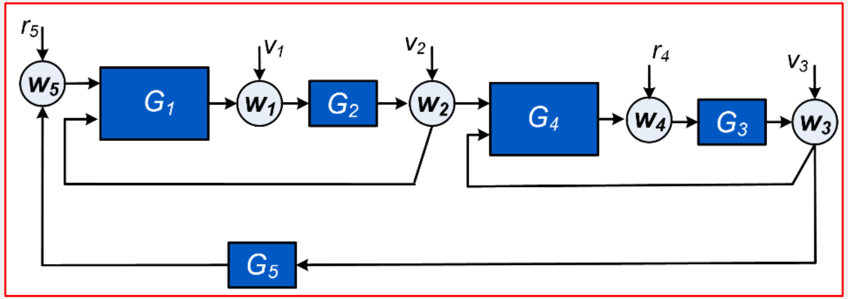


Decreasing structural information



Dynamic network models – From SISO to MISO



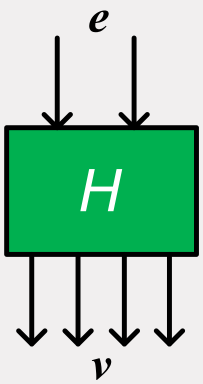


Summation of ``outputs'' →
Linear dynamics

MISO mappings (possibly nonlinear),
with additive disturbances

Disturbance modeling – reduced rank noise

How many white noises do we need to describe an L-th dimensional disturbance process v?



$\dim(e) < \dim(v)$

If $\dim(e) < \dim(v)$: reduced rank process

Appealing when dealing with large dimensions;
finding (few) causes of your observations (cf. dynamic factor analysis[1],[2])

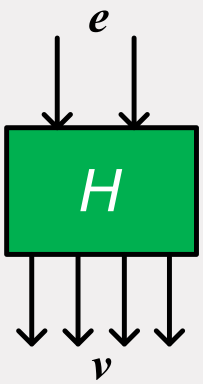
Consequences:

- $H(z)$ is nonsquare
- Spectal density $\Phi_v(z) := H(z)\Lambda H^T(z^{-1})$ is singular over $\mathbb{R}(z)$

[1] Deistler et al., EJC, 2010.
[2] Zorzi and Chiuso, Automatica 2017.

Disturbance modeling – reduced rank noise

There are two unique spectral factorizations now:



$\dim(e) < \dim(v)$

1. $\Phi_v(z) = H(z)\Lambda H^T(z^{-1})$
    - $H \in \mathbb{R}^{L\times L}(z)$, monic;
    - $\Lambda \in \mathbb{R}^{L\times L}$, singular;
2. $\Phi_v(z) := \tilde{H}(z)\tilde{\Lambda}\tilde{H}^T(z^{-1})$
    - $\tilde{H} \in \mathbb{R}^{L\times p}(z)$
    - $\tilde{\Lambda} \in \mathbb{R}^{p\times p}$, regular;
    - monicity scaling either through $\tilde{\Lambda} = I_p$ or through a $p \times p$ submatrix of $\tilde{H}$;

Reduced rank noise has consequences for identification algorithms[1,2]

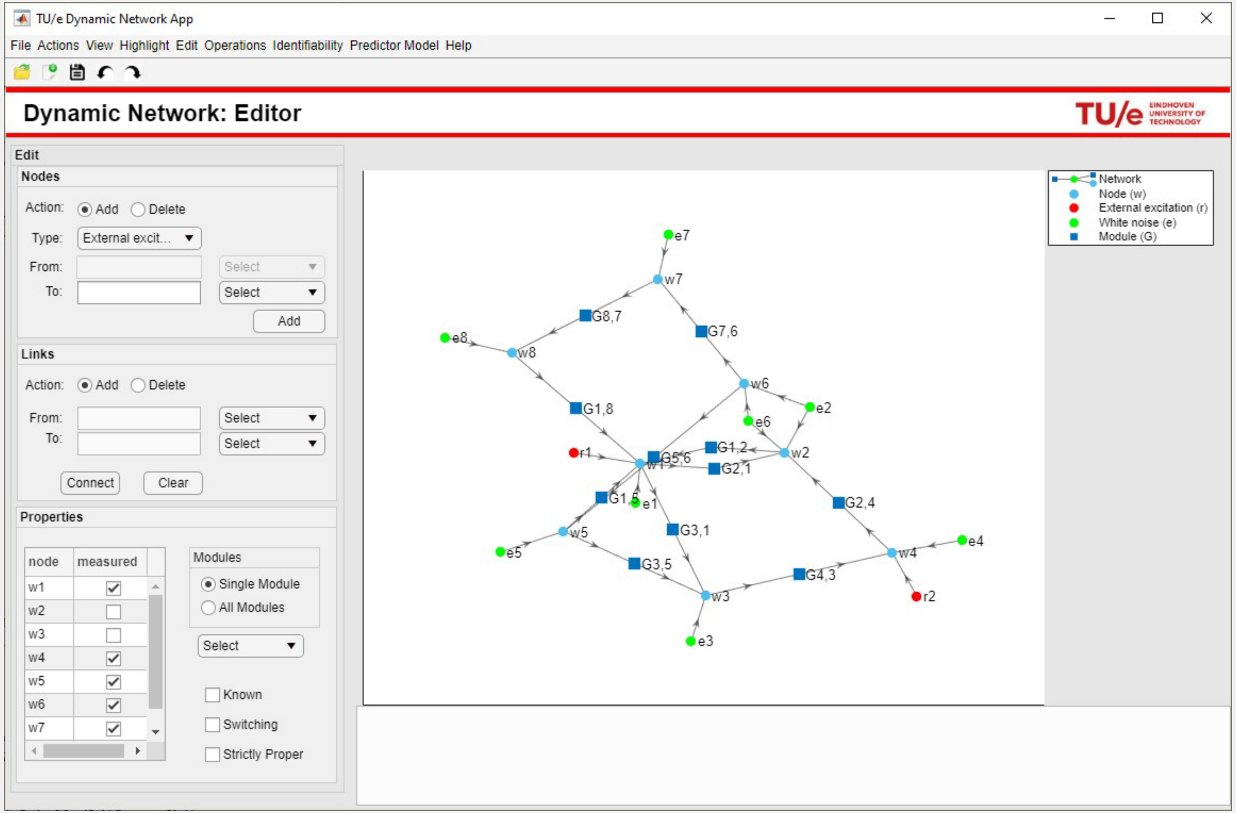
[1] Weerts et al., Automatica, December 2018.
[2] Cao, Picci & Lindquist, Automatica, May 2023.

Summary network modeling

- Several different approaches to network modelling. For now: **module framework**
- In view of data-driven modelling we extend the typical transfer function approach to include structure (topology)
- This raises an abundance of data-driven modeling challenges
- that include structural issues like: selection of measured and excited nodes (sensors and actuators)
- An alternative modelling framework (**diffusively coupled networks**) will be discussed later
- Download and install the MATLAB toolbox SYSDYNET for running examples:

www.sysdynet.net

Algorithms implemented in SYSDYNET App and Toolbox



Structural analysis and operations on dynamic networks

- Edit and manipulate
- Assign properties to nodes and modules
- Immersion of nodes, PPL test
- Generic identifiability analysis and synthesis
- Predictor model construction for single module ID

to be complemented with

- estimation algorithms for single module and full network ID;
- topology estimation

Beta-version to be downloaded from www.sysdynet.net