Course: 5SMB0 - System Identification 2020
Exercise/computer session 2: PE identification (Lectures 4-5)

Problem 1: persistence of excitation

Let us consider the following true system $\mathcal{S}$ where $e(t)$ is the realization of a white noise sequence of variance $\sigma_e^2 = 1$:

$$y(t) = \overbrace{\frac{b_1^0 q^{-1} + b_2^0 q^{-2}}{1 + f_1^0 q^{-1} + f_2^0 q^{-2}}}^{G_0(q)} u(t) + \overbrace{\frac{1 + c_1^0 q^{-1}}{1 + d_1^0 q^{-1}}}^{H_0(q)} e(t),$$

We want to identify a model of this true system in a full-order order model structure $\mathcal{M}$.

$$\mathcal{M} = \left\{ G(q,\theta) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + f_1 q^{-1} + f_2 q^{-2}} \ ; \ H(q,\theta) = \frac{1 + c_1 q^{-1}}{1 + d_1 q^{-1}} \ \mid \ \theta = \begin{pmatrix} b_1 \\ b_2 \\ c_1 \\ d_1 \\ f_1 \\ f_2 \end{pmatrix} \right\}$$

1. Is the following **periodic** input signal $u(t)$ sufficient to obtain a consistent estimate of $\mathcal{S}$ in the proposed model structure $\mathcal{M}$?

$$u(t) = \begin{cases} 1 & t = 1 \ and \ 2 \\ -1 & t = 3 \ and \ 4 \end{cases}$$

$$and \ u(t+4) = u(t)$$

2. Let us now consider an input signal made up of the sum of k sinusoids at different pulsations ω_i $(i = 1...k)$ with $\omega_i < \pi \ \forall i$:

$$u(t) = \sum_{i=1}^{k} sin(\omega_i t)$$

   What is the minimum value of k in order to be able to identify appropriately a model of $\mathcal{S}$ in the considered model structure $\mathcal{M}$ using data generated by applying $u(t) = \sum_{i=1}^{k} sin(\omega_i t)$ to $\mathcal{S}$?

Problem 2: noise estimate

Let us consider the true system $\mathcal{S}$:

$$y(t) = G_0(q)u(t) + \overbrace{H_0(q)e(t)}^{v(t)},$$

and a model structure $\mathcal{M} = \{G(q,\theta),\ H(q,\theta)\}$ such that $\mathcal{S} \in \mathcal{M}$.

Using data $u(t)$ and $y(t)$ ($t = 1...N$) collected on the true system $\mathcal{S}$, we have identified $G(q,\hat{\theta}_N)$ and $H(q,\hat{\theta}_N)$.

Using the collected data and/or the identified models, give different methods to determine if the noise $v(t)$ that corrupts the output measurements $y(t)$ is substantial or not (in terms of signal-to-noise ratio at the output).

Problem 3: delay estimate

Let us consider the true system $\mathcal{S}$:

$$y(t) = G_0(q)u(t) + \overbrace{H_0(q)e(t)}^{v(t)},$$

and let us suppose that we have collected input-output data from this unknown true system with a **white** input signal. Show that it is then very easy to determine the delay of the unknown $G_0(q)$ by inspecting $R_{yu}(\tau)$.

Problem 4: computer exercise

Let us consider the following "unknown" true system $\mathcal{S}$ which is in the OE form (i.e. $H_0 = 1$):

$$y(t) = \overbrace{\frac{q^{-3}(0.10276 + 0.18123q^{-1})}{1 - 1.99185q^{-1} + 2.20265q^{-2} - 1.84083q^{-3} + 0.89413q^{-4}}}^{G_0} u(t) + e(t) \qquad (1)$$

The transfer function G_0 corresponds to the transfer function of a flexible transmission system that has been developed by the LAG (Grenoble, France). The objective is to identify a model of the true system $\mathcal{S} = \{G_0, H_0\}$ using PE identification.

Questions.

1. In order to identify a model for the true system, we will need to choose a sufficiently exciting input signal $u(t)$. What is the minimal order of excitation of $u(t)$ in order to perform the identification of the true system (1) with a model structure that satisfies $\mathcal{S} \in \mathcal{M}$?

2. Which model structure with polynomial degrees $(n_a, n_b, n_c, n_d, n_f, n_k)$ needs to be chosen in order to guarantee that $\mathcal{S} \in \mathcal{M}$?

3. Suppose $u(t)$ fulfils the condition of item 1. Define θ^* as the parameter minimizing the power $\bar{E}\epsilon^2(t, \theta)$ of the prediction error when the model structure $\mathcal{M} = \{G(q, \theta),\ H(q, \theta)\ \}$ is chosen as $OE(n_b = 2, n_f = 4, n_k = 3)$. Can we then conclude that $G(q, \theta^*) = G_0(q)$ and $H(q, \theta^*) = H_0(q)$. Why?

4. We have applied a signal $u(t)$ having the property of a white noise of variance 0.1 to the true system (1), and have collected 5000 input-output data. These data are in the file *dataG0oe.mat*. Choose a model structure $OE(n_b = 2, n_f = 4, n_k = 3)$. Using the input-output data and this full-order model structure, identify a model of the true system. Evaluate the residual tests that are available in the Matlab Toolbox *ident*. Interpret the results of this test. Do these residual tests confirm the conclusions of item 3?

5. Consider now an ARX model structure with the following lengths for the polynomials A and B, and the following delay:

$$n_a = 4, \quad n_b = 2, \quad n_k = 3.$$

   Suppose $u(t)$ fulfils the condition of item 1 and define θ^* as the parameter minimizing the power $\bar{E}\epsilon^2(t, \theta)$ of the prediction error when the model structure is this ARX model structure i.e. $ARX(n_a = 4, n_b = 2, n_k = 3)$. Is it true that $G(q, \theta^*) = G_0(q)$ and $H(q, \theta^*) = H_0(q)$? Why?

6. Using the same input-output data as in item 4, identify a new model in the ARX model structure of item 5. Perform the residual tests on the estimated model. Do these residual tests confirm the conclusions of item 5.

7. Compare the bode diagrams of $G_0(q)$ and the two identified models $G(q, \hat{\theta}_N)$. Does this comparison confirm your previous conclusions?